

# The Similarity-Updating Model of Probability Judgment and Belief Revision

## Supplement: Preliminary Model Testing

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Several different variants of similarity and updating processes were tested in a preliminary explorative analysis only using the data of Study 1. In this preliminary analysis we tested similarity processes separately by fitting them to the first probability judgments in each trial. The best similarity component (the similarity component that is described in the main article) was then used to test several different updating components by estimating the models to both probability judgments in each trial.

**Similarity processes.** In past research various similarity processes have been used to predict probability judgments and we tested the models that could be adapted for our data. We implemented the similarity heuristic by Read and Grushka-Cockayne (2011) as:

$$s_{ij} = 1 - \frac{|x_{im}-x_{jm}|+|x_{in}-x_{jn}|+|x_{il}-x_{jl}|}{3}, \quad (S1)$$

$$p(A|E_1) = \frac{s_1}{s_1+s_2}, \quad (S2)$$

where the  $x_i$  and  $x_j$  are the frequency distributions of the red, blue, and green cards in the deck and the sample, and  $s_1$  is the similarity between sample  $E_1$  and deck  $A$  and  $s_2$  is the similarity between sample  $E_1$  and deck  $B$ .

According to the representativeness heuristic as a prototype similarity model (Nilsson et al., 2005), similarity is computed by

$$sim(T, y) = \prod_{j=1}^J d_j, \text{ where } d_j = \begin{cases} 1 & \text{if } t_j = y_j, \\ s_j & \text{if } t_j \neq y_j \end{cases},$$

(S3)

where  $T$  is the sample,  $y$  is the deck and  $s_j$  is a free parameter between 0 and 1, which reflects the impact of the perceived similarity  $sim(t, y)$  of a mismatch on feature dimension  $j$ . The probability judgment is then computed by

$$p(A|E_1) = \frac{0.5 \times \phi + sim(T|A)}{\phi + sim(T|A) + sim(T|B)} \quad (S4)$$

where  $\phi$  is a free dampening parameter, which pulls predictions towards 0.5.

According to the representativeness heuristic as a relative likelihood model (Nilsson et al., 2005), the probability of sample  $E_1$  to stem from deck A is computed by

$$p(CatA|E_1) = \frac{0.5 \times \phi + l(T|A)}{0.5 \times \phi + l(T|A) + l(T|B)}, \quad (S5)$$

where  $\phi$  is a free dampening parameter, which pulls predictions towards 0.5, and  $l(T|A)$  and  $l(T|B)$  are the relative likelihoods.

**Updating processes.** In the previous literature, belief revision has been modeled in different ways. Updating processes have been modeled with the sigma model (Juslin et al., 2008) and the belief-updating model (Hogarth & Einhorn 1992). We implemented these models assuming that the single probability judgments are computed according to the similarity component of the similarity-updating model (which was the best fitting similarity process, see next section). According to the sigma model a probability judgment is updated by

$$p(A|E_1, E_2) = p(A|E_1) + \eta \times (\delta - p(A|E_1)), \quad (S6)$$

where  $\eta$  is the importance attached to the new piece of evidence and  $\eta$  is computed by

$$\eta = \frac{s_2}{s_1 + s_2}, \quad (S7)$$

where  $s_1$  is the similarity between deck A and evidence  $E_1$  and  $s_2$  is the similarity between deck A and evidence  $E_2$ , and  $\delta$  is the criterion when sample 2 is presented, here  $\delta = p(A|E_2)$ .

Further, we implemented the belief-updating model

$$p(A|E_1, E_2) = p(A|E_1) + \alpha \times (p(A|E_2) - R), \quad (S9)$$

where  $\alpha$  is an adjustment weight, which can take values between 0 and 1, and  $R$  is the reference point against which the impact of the new piece of evidence is measured, here  $p(A|E_1)$ . This belief updating mechanism is mathematically equivalent to an averaging mechanism.

**Results.** To compare the different similarity processes, we fitted them to the first probability judgment in each trial. In a model comparison based on median BIC, the similarity component used in the similarity updating model was clearly best (-144), compared to representativeness as relative likelihood (-84), representativeness as prototype similarity (-12), and the similarity heuristic (4).

For the evaluation of the updating mechanisms we used the best-faring similarity process to obtain single probability judgments and fitted the complete models to both probability judgments in each trial. All three models describe the updating process similarly well according to median BIC (updating process used in the similarity updating -242; belief updating mechanism/simple averaging -240; updating process from the sigma model -235). However, the belief updating model and the sigma model suffer from the general problem that they cannot predict combined probabilities that lie outside the range of single probabilities. Specifically, they cannot predict the confirmation effect which we find in our data and is a key aspect of the similarity-updating model.

## References

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