Human Inference in Changing Environments With Temporal Structure

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Supplemental Material

A. Supplemental Tables: Statistical Tests

$\tilde{\tau}$	0	1	2	3	4	5	6	7	8	9	10
W	.307	.448	.265	.335	.180	.072*	.016**	.073*	.426	.160	.008***
MWU	.269	.391	.281	.212	.207	.066*	.034**	.022**	.491	.568	.006***
S	.307	.448	.268	.337	.178	.070*	.015**	.074*	.426	.158	.008***
N(HI)	454	521	613	508	461	436	411	360	318	316	240
N(HD)	603	616	707	632	637	521	499	421	387	300	231
Excluding repetitions											
W	.353	.421	.329	.352	.142	.042**	.008***	.151	.391	.202	.034**
MWU	.357	.431	.484	.170	.073*	.025**	.007***	.058*	.343	.590	.050**
S	.352	.421	.332	.353	.140	.041**	.007***	.153	.391	.201	.035**
N(HI)	412	435	495	402	353	333	307	288	237	231	174
N(HD)	553	529	582	497	493	405	379	318	291	226	186

Table A1. ***p-values for the one-sided statistical tests of equality of the means of the learning rates in the HI and HD conditions, for each run-length (Fig. 2C).*** W: Welch's test. MWU: Mann-Whitney's U test. S: Student's test. The N(HI) and N(HD) lines report the number of observations. The second half of the table reports the same quantities when excluding all occurrences of repetitions (Suppl. Fig. B2B).

Condition 1	Condition 2	Avg_1	Avg_2	W	MWU	S	N_1	N_2
HI, $\tilde{\tau} \in [5,6]$	HI, $\tilde{\tau} \in [9,10]$	0.255	0.208	.0099***	.0476**	.0119**	847	556
HI, $\tilde{\tau} \in [5,6]$	HD, $\tilde{\tau} \in [5,6]$	0.255	0.209	.0055***	.0089***	.0052***	847	1020
HI, $\tilde{\tau} \in [9,10]$	HD, $\tilde{\tau} \in [9,10]$	0.208	0.264	.0103**	.0602*	.0100***	556	531
HD, $\tilde{\tau} \in [5,6]$	HD, $\tilde{\tau} \in [9,10]$	0.209	0.264	.0072***	.0173**	.0050***	1020	531
Excluding repetitions								
HI, $\tilde{\tau} \in [5,6]$	HI, $\tilde{\tau} \in [9,10]$	0.338	0.285	.0203**	.0780*	.0233**	640	405
HI, $\tilde{\tau} \in [5,6]$	HD, $\tilde{\tau} \in [5,6]$	0.338	0.272	.0017***	.0009***	.0016***	640	784
HI, $\tilde{\tau} \in [9,10]$	HD, $\tilde{\tau} \in [9,10]$	0.285	0.340	.0342**	.1721	.0345**	405	412
HD, $\tilde{\tau} \in [5,6]$	HD, $\tilde{\tau} \in [9,10]$	0.272	0.340	.0066***	.0095***	.0047***	784	412

Table A2. ***Learning rates averages under various HI/HD and short/long run-length conditions, and p-values for one-sided statistical tests of equality of the means (Fig. 2B).*** The first two columns indicate the HI/HD and short/long run-length conditions. Learning rates at trials verifying these two conditions have their averages reported in the Avg_1 and Avg_2 columns. Columns W, MWU, and S provide the p-values for the tests of equality of the means between the two conditions. W: Welch's test. MWU: Mann-Whitney's U test. S: Student's test. N_1 and N_2 report the number of observations for each condition. The second half of the table reports the same quantities when excluding all occurrences of repetitions (Suppl. Fig. B2A).

$\tilde{\tau}$	0	1	2	3	4	5	6	7	8	9	10
p	.0266**	.0331**	.0123**	.0550*	.0048***	.0000***	.0387**	.213	.445	.0331**	.0000***
N(HI)	1255	1425	1611	1440	1347	1268	1111	975	883	795	722
N(HD)	1591	1674	1943	1804	1650	1493	1333	1168	983	792	584
Excluding repetitions											
p	.0243**	.0293**	.0556*	.4847	.0094***	.0000***	.4853	.272	.953	.0560*	.0004***
N(HI)	1123	1147	1291	1127	1003	963	818	748	643	583	533
N(HD)	1455	1405	1559	1390	1255	1115	978	890	723	614	458

Table A3. ***p-values for Levene's statistical test of equality of the variances of the corrections in the HI and HD conditions, for each run-length (Fig. 4C).*** The N(HI) and N(HD) lines report the number of observations. The second part of the table reports the same quantities when excluding all occurrences of repetitions (Suppl. Fig. B2).

$\tilde{\tau}$	0	1	2	3	4	5	6	7	8	9	10	11
p	0.561	0.114	0.843	0.245	0.420	0.333	0.636	1.000	0.848	0.194	0.091*	0.005***
N(HI)	2635	2211	1984	1786	1620	1471	1351	1215	1098	978	875	791
N(HD)	2647	2339	2135	1943	1804	1661	1534	1404	1240	1111	946	781

Table A4. ***p-values for Fisher's exact test of independence of the repetition propensities in the HI and HD conditions, at each run-length (Fig. 3B).*** The N(HI) and N(HD) lines report the number of observations.

B. Supplemental Figures

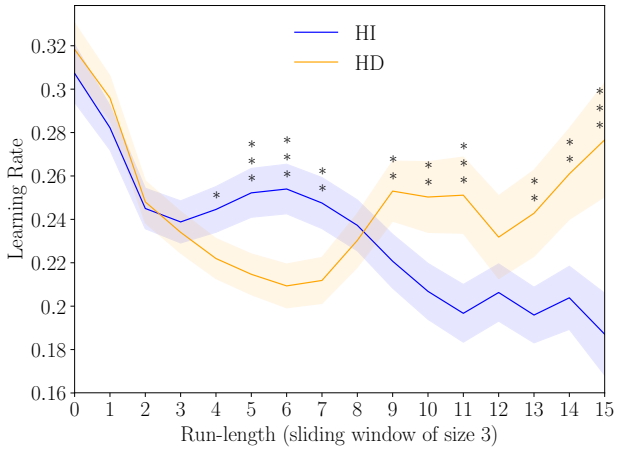

Figure B1. **Learning rates at run-lengths greater than 10.** In order to curb the fluctuations due to the decreasing amount of data at high run-lengths, we use a sliding window of size 3 over the run-lengths (i.e., we pool together the learning rates corresponding to three consecutive run-lengths). The number reported on the x-axis is the center of this sliding window. In the HD condition, the learning rate, after run-length 10, remains large and appears to increase. In the HI condition, the learning rate keeps decreasing after run-length 10. Although presumably the learning rate in this condition should eventually plateau and remain constant, it does not seem that our subjects reach this stage over the run-lengths for which we have sufficient data to allow for analysis (up to 15).

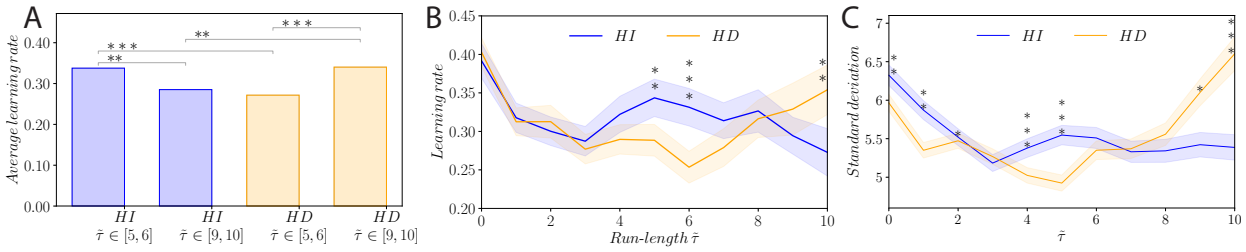

Figure B2. **Human learning rates and standard deviations of responses, excluding all occurrences of repetitions.** (**A,B**) as in Figs. 2B,C; (**C**) as in Fig. 4C.

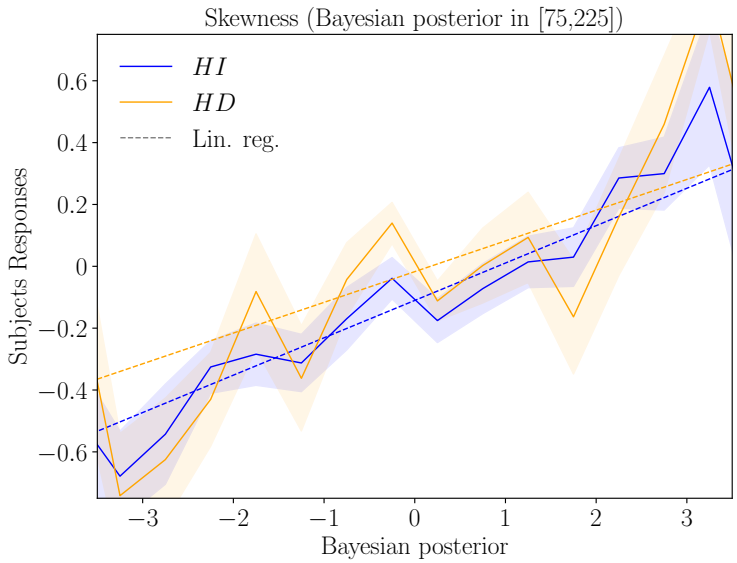

Figure B3. **Skewness in subjects' responses away from the bounds of the response range.** Same analysis of the skewness as in Fig. 6B, but restricted to the trials in which the support of the Bayesian posterior is entirely contained in the middle interval of width half that of the state space (i.e., [75,225], to be compared to the state space, [0,300]). In both conditions, the correlation between the skewness of the Bayesian posterior and the empirical skewness of the subjects' responses is positive and significant (HI: Pearson's r = 0.23, p = 2e-12; HD: r = 0.14, p = 4e-4).

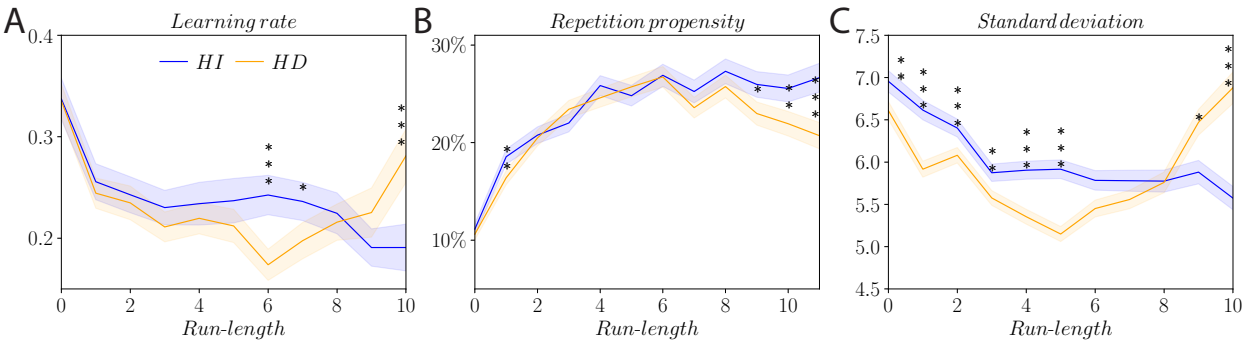

Figure B4. **Human learning rates, repetition propensities, and standard deviations of responses, from the complete contingent of subjects including the four subjects excluded from the analysis presented in the main text (see Methods).** (**A**) as in Fig. 2C; (**B**) as in Fig. 3B; (**C**) as in Fig. 4C.

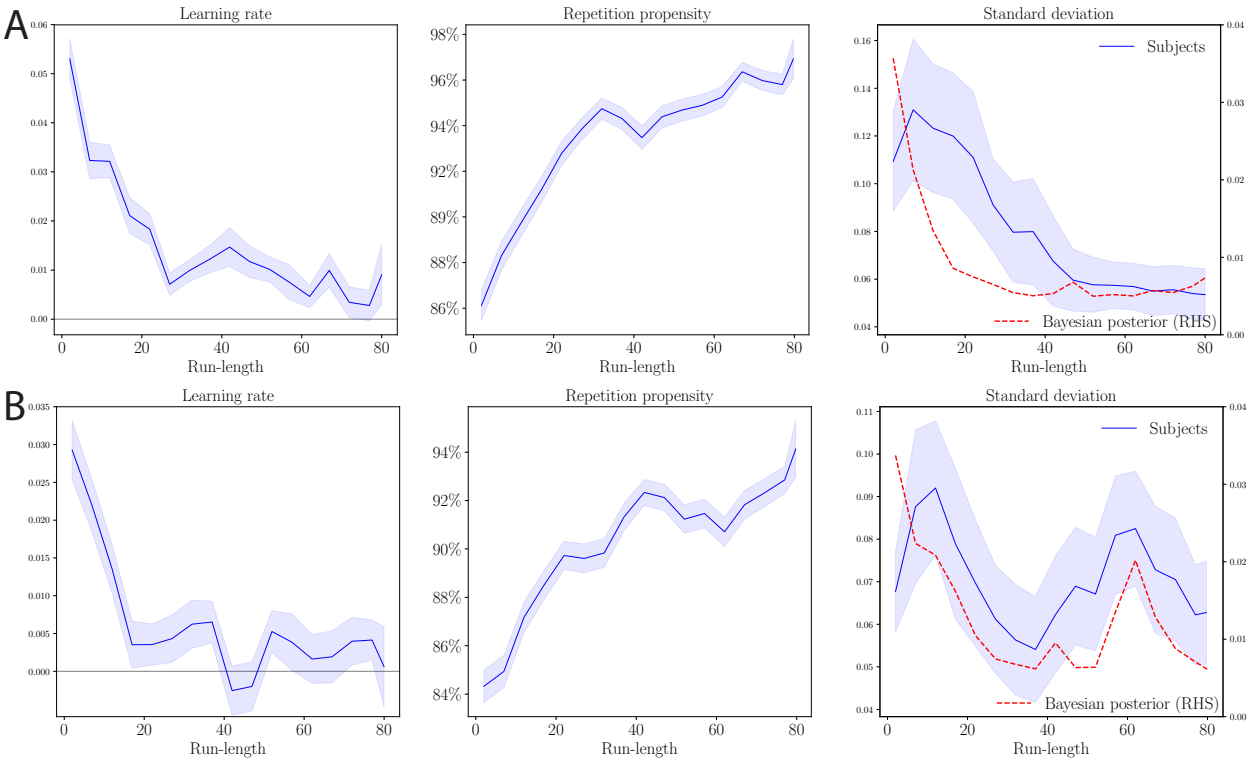

Figure B5. **Behavior of human subjects in the online inference tasks conducted by Gallistel et al., 2014 (A), and Khaw et al., 2017 (B).** *Left panels*: Learning rates as a function of the run-lengths, in trials in which the surprise is greater than .25. *Middle panels*: Repetition propensities. *Right panels*: Standard deviations of responses, averaged across subjects. In these studies, the stimulus is binary and Bernoulli-distributed. The task is to infer the parameter of the Bernoulli distribution, which is subject to change points with a constant probability of 0.5% (to be compared to 10% in the HI condition of our task.) Moreover, some subjects were presented several times with the same sequence of stimuli (in different sessions). It is thus possible to examine the variability of responses within subjects (right panels, across-subject mean of the within-subject standard-deviations). In all panels, in order to mitigate noise, we pool together the responses in windows of five consecutive run-lengths.