

Online supplemental material: A map of ecologically rational heuristics for uncertain strategic worlds

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**Environments where  $n \leq 10, m \leq 50\%$**

These results are based on the core simulation reported in the main text. However, the summary performance is calculated from a smaller set of environments than the whole set examined in the main text. Specifically, Table 1 reports average policy performance for  $n \leq 10$  and  $m \leq 50\%$ .

Table 1

*Summary of Decision Policies' Performance for Environments where  $n \leq 10$  and  $m \leq 50\%$*

$\rho =$	Indifference				Wald				Comp.–Robustness			
	0	0.5	-0.5	All $\rho$	0	0.5	-0.5	All $\rho$	0	0.5	-0.5	All $\rho$
Random	1.5	10.2	-6.0	1.9	-0.5	-0.2	-19.0	-6.6	3	7	3	4
NE	26.2	<b>50.7</b>	6.9	27.9	10.9	15.1	-4.9	7.0	53	72	37	54
MaxMax	21.7	42.6	9.3	24.5	15.3	17.3	-7.6	8.3	32	40	44	39
MaxMin	24.2	33.3	18.5	25.3	21.3	23.3	<b>7.5</b>	<b>17.4</b>	43	25	67	45
SocMax	26.0	<b>51.9</b>	5.8	27.9	12.0	16.8	-8.8	6.7	47	74	32	51
Eq	2.6	10.6	-4.4	2.9	-0.2	-0.4	-15.0	-5.2	15	11	15	14
L1	<b>34.1</b>	<b>50.9</b>	<b>22.5</b>	<b>35.8</b>	<b>29.4</b>	<b>32.3</b>	-13.5	16.1	79	71	79	76
D1	<b>35.1</b>	<b>51.5</b>	<b>23.4</b>	<b>36.7</b>	<b>28.1</b>	<b>30.9</b>	-8.9	<b>16.7</b>	<b>92</b>	<b>78</b>	<b>91</b>	<b>87</b>
L2	31.9	48.2	18.8	33.0	13.0	13.7	-27.2	-0.2	73	61	71	68
L3	29.8	48.7	15.5	31.3	13.5	20.5	1.6	11.9	65	61	60	62

*Note.* The set of top-performing policies are in bold.

**Inferred games where at least one PSNE exists**

These results are based on the core simulation reported in the main text. However, the summary performance in Table 2 is calculated only over the inferred games for which at least one pure-strategy Nash equilibrium exists. Consequently, the NE policy always proposes a specific course of action in these games and never resorts to the backup random choice mechanism triggered when a game does not have a PSNE.

Table 2

*Summary of Decision Policies' Performance for Inferred Games with at Least One PSNE*

$\rho=$	Indifference				Wald				Comp.-Robustness			
	0	0.5	-0.5	All $\rho$	0	0.5	-0.5	All $\rho$	0	0.5	-0.5	All $\rho$
Random	1.3	7.5	-5.2	1.2	-0.5	-0.2	-14.1	-4.9	5	8	5	6
NE	<b>26.7</b>	<b>45.1</b>	<b>17.4</b>	<b>29.7</b>	8.5	10.0	6.3	8.3	74	<b>77</b>	75	<b>75</b>
MaxMax	15.6	33.1	4.2	17.6	9.6	11.0	-6.7	4.6	32	43	31	35
MaxMin	17.2	24.1	12.9	18.1	15.0	16.4	<b>7.2</b>	12.9	35	26	51	37
SocMax	23.6	<b>45.9</b>	7.7	25.7	7.5	10.8	-4.1	4.7	59	<b>79</b>	39	59
Eq	1.9	7.6	-4.0	1.8	-0.2	-0.2	-11.9	-4.1	13	10	13	12
L1	<b>27.4</b>	41.4	<b>17.4</b>	<b>28.7</b>	<b>23.1</b>	<b>25.3</b>	-4.6	<b>14.6</b>	73	69	74	72
D1	<b>27.8</b>	41.6	<b>17.8</b>	<b>29.1</b>	<b>22.6</b>	<b>24.7</b>	-3.4	<b>14.6</b>	<b>79</b>	69	<b>82</b>	<b>77</b>
L2	25.6	39.4	15.2	26.7	7.9	8.4	-12.4	1.3	66	61	62	63
L3	25.1	38.6	16.7	26.8	10.6	14.7	4.7	10.0	65	58	70	64

*Note.* The set of top-performing policies are in bold.

***S'*: Alternative assumption for payoff uncertainty**

This section presents the results of an additional simulation with an alternative assumption regarding the prevalence of missing payoffs. An asymmetry is introduced in payoff uncertainty, namely that each player knows their own payoffs perfectly, but is ignorant about a percentage  $m$  of their opponent's payoffs. Table 3 summarizes performance over  $n$  and  $m$ , and Figures 1–9 present detailed maps of policy performance.

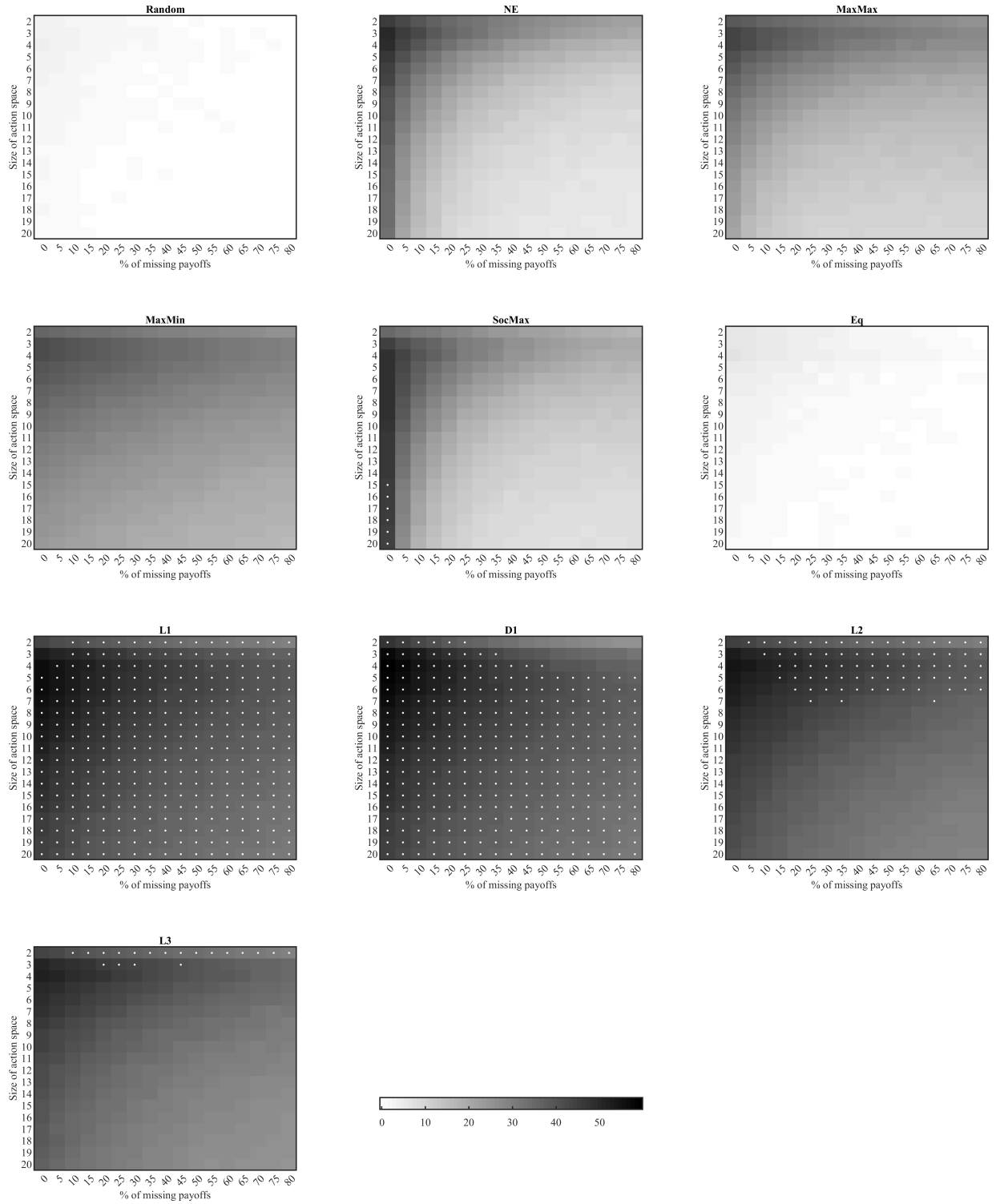
Table 3

*Summary of Decision Policies' Performance in S'*

$\rho=$	Indifference				Wald				Comp.–Robustness			
	0	0.5	-0.5	All $\rho$	0	0.5	-0.5	All $\rho$	0	0.5	-0.5	All $\rho$
Random	0.6	8.7	-6.2	1.0	-0.6	-0.3	-20.0	-7.0	5	7	4	5
NE	16.8	34.7	1.2	17.6	10.4	14.1	-11.4	4.4	30	34	27	30
MaxMax	20.0	35.7	8.8	21.5	16.8	17.6	-7.3	9.0	42	45	46	44
MaxMin	26.5	34.4	20.1	27.0	24.7	25.4	<b>9.2</b>	<b>19.8</b>	55	44	67	55
SocMax	19.0	36.5	4.8	20.1	12.5	15.3	-9.4	6.1	38	44	36	39
Eq	1.2	8.9	-5.3	1.6	-0.4	-0.3	-17.6	-6.1	13	11	14	13
L1	<b>40.9</b>	<b>56.7</b>	<b>26.9</b>	<b>41.5</b>	<b>38.3</b>	<b>39.2</b>	-22.0	18.5	<b>86</b>	<b>86</b>	<b>83</b>	<b>85</b>
D1	<b>40.7</b>	<b>56.3</b>	<b>26.9</b>	<b>41.3</b>	<b>37.4</b>	<b>38.3</b>	-20.0	18.6	<b>83</b>	79	<b>84</b>	<b>82</b>
L2	38.4	<b>55.5</b>	22.0	38.6	12.9	13.3	-44.5	-6.1	78	79	71	76
L3	34.5	51.0	20.8	35.4	13.2	19.1	1.2	11.2	70	72	68	70

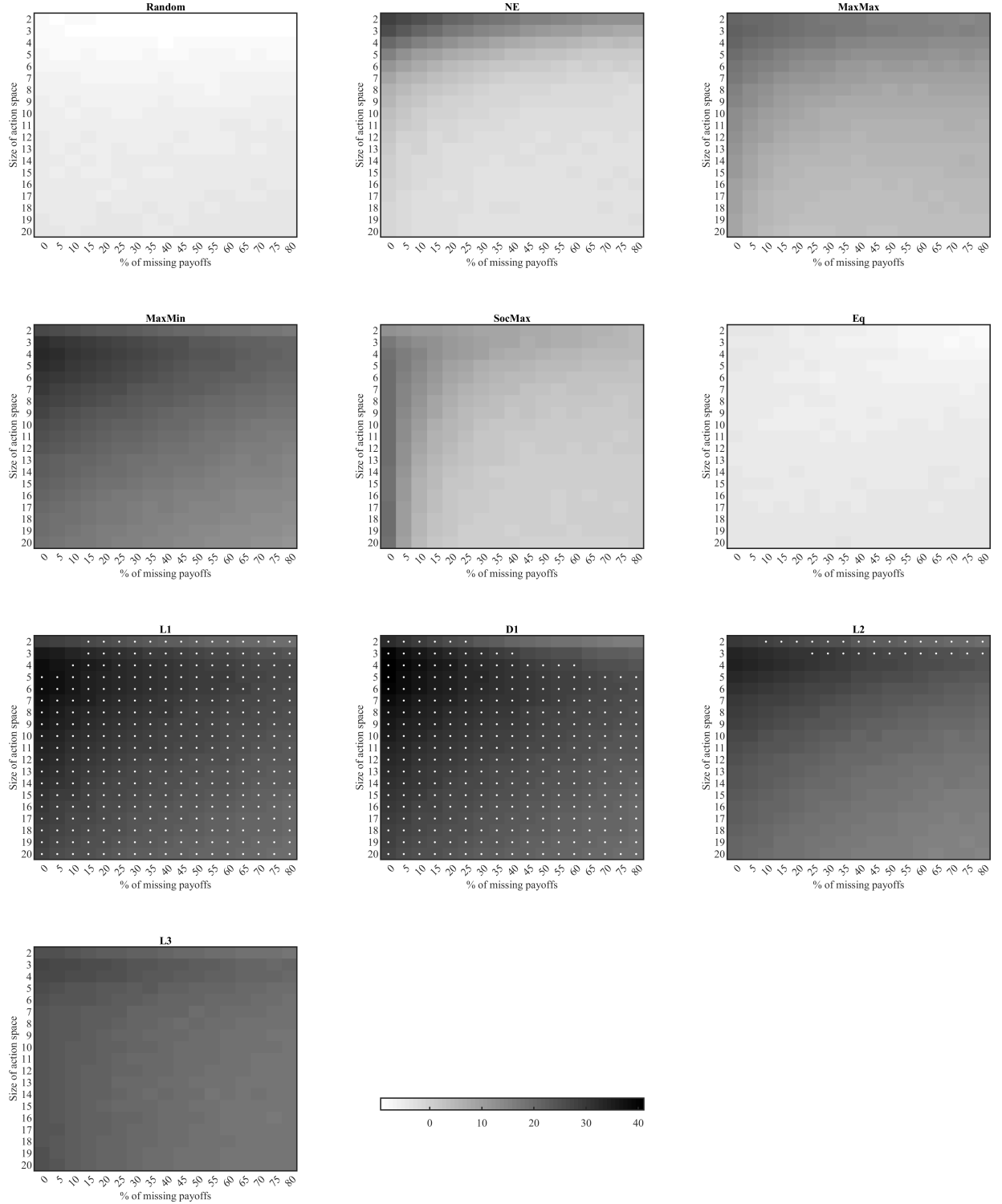
*Note.* The set of top-performing policies are in bold.

## Indifference Criterion



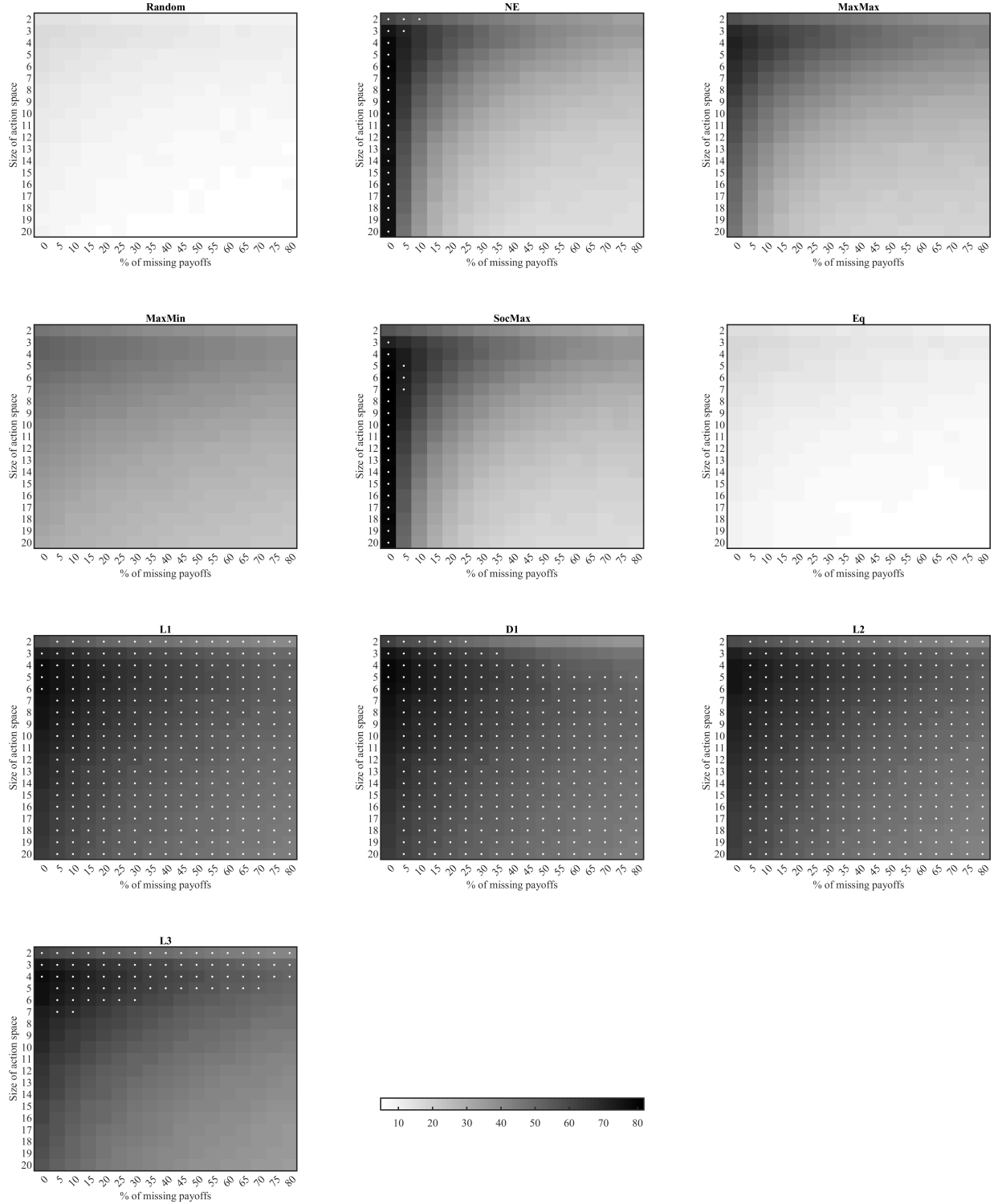
*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

*Figure 1.* Decision policy performance in  $S'$  according to the Indifference criterion ( $\bar{\pi}$ ) for neutral environments ( $\rho = 0$ ).



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

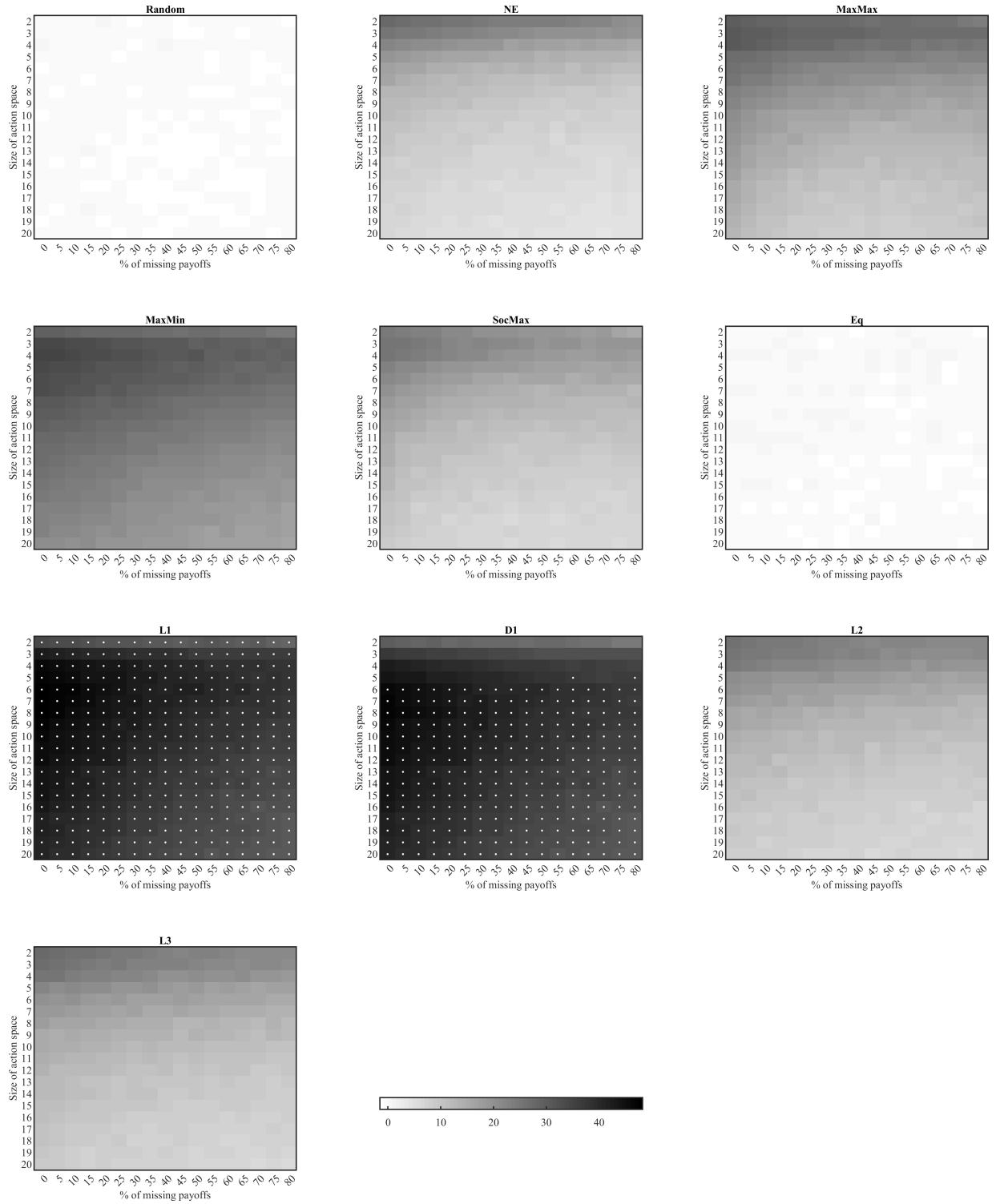
*Figure 2.* Decision policy performance in  $S'$  according to the Indifference criterion ( $\bar{\pi}$ ) for discordant environments ( $\rho = -0.5$ ).



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

*Figure 3.* Decision policy performance in  $S'$  according to the Indifference criterion ( $\bar{\pi}$ ) for harmonious environments ( $\rho = 0.5$ ).

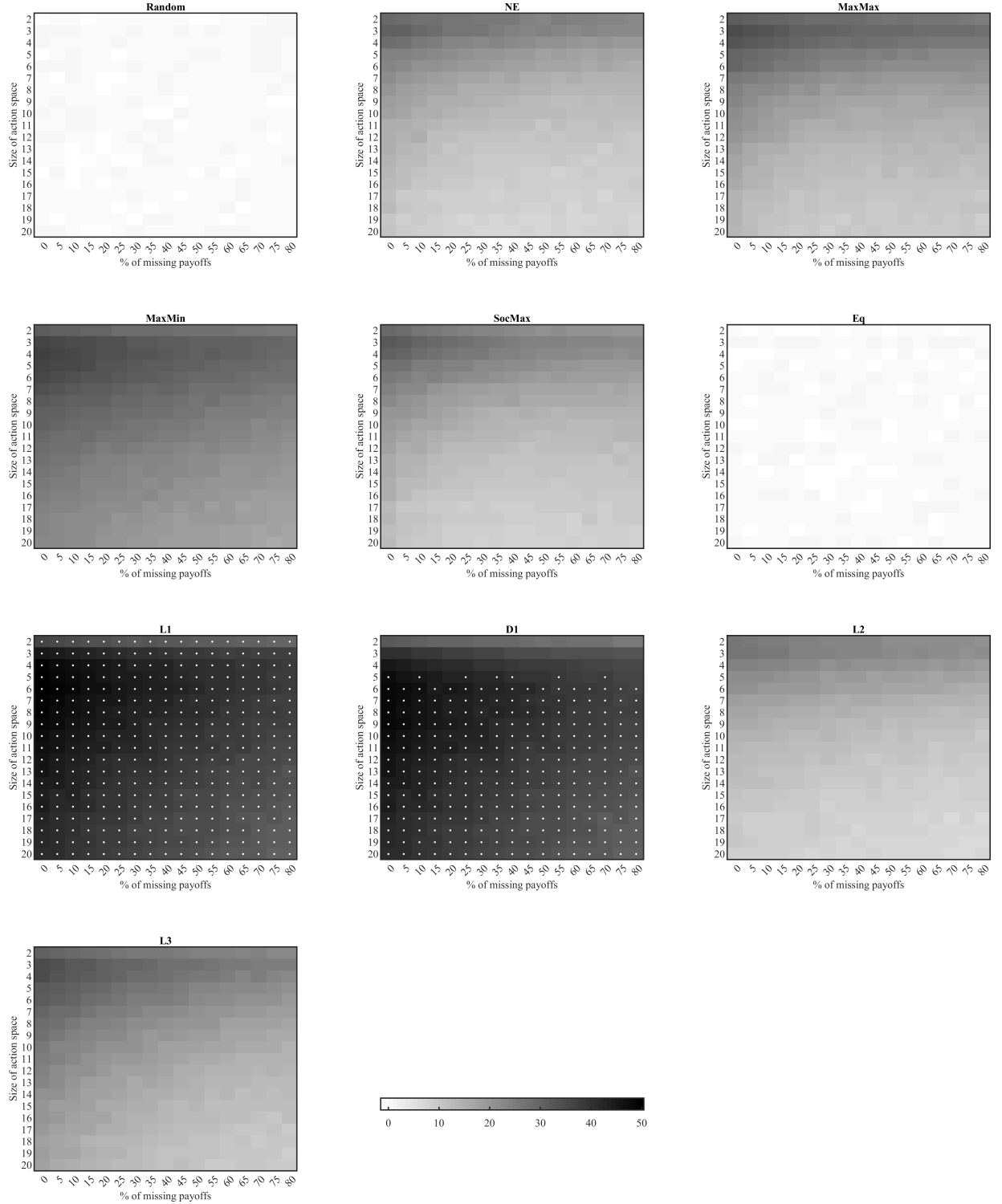
## Wald Criterion



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

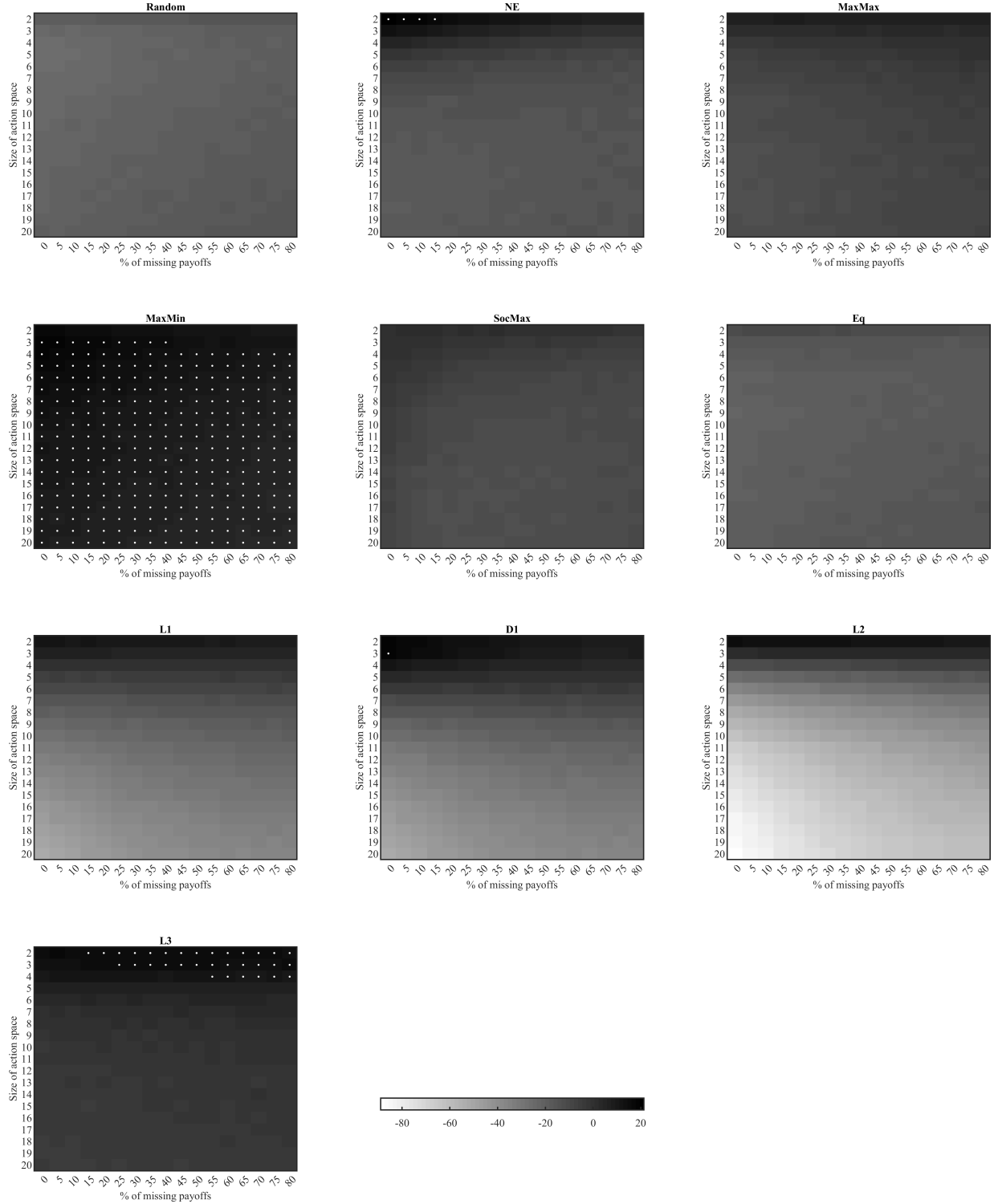
*Figure 4.* Decision policy performance in  $S'$  according to the Wald criterion ( $\pi_w$ ) for neutral environments ( $\rho = 0$ ).





*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

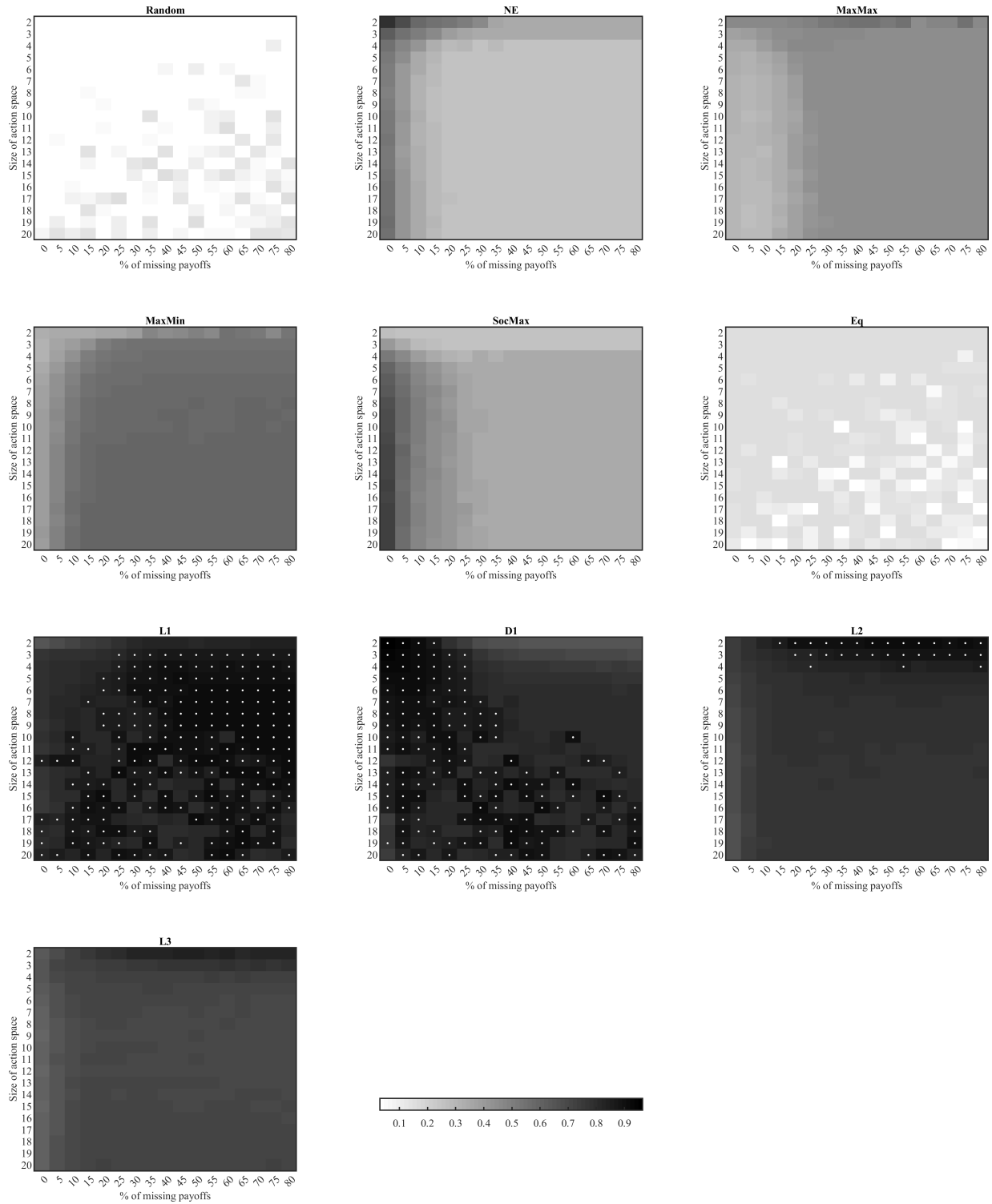
*Figure 5.* Decision policy performance in  $S'$  according to the Wald criterion ( $\pi_w$ ) for harmonious environments ( $\rho = 0.5$ ).



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

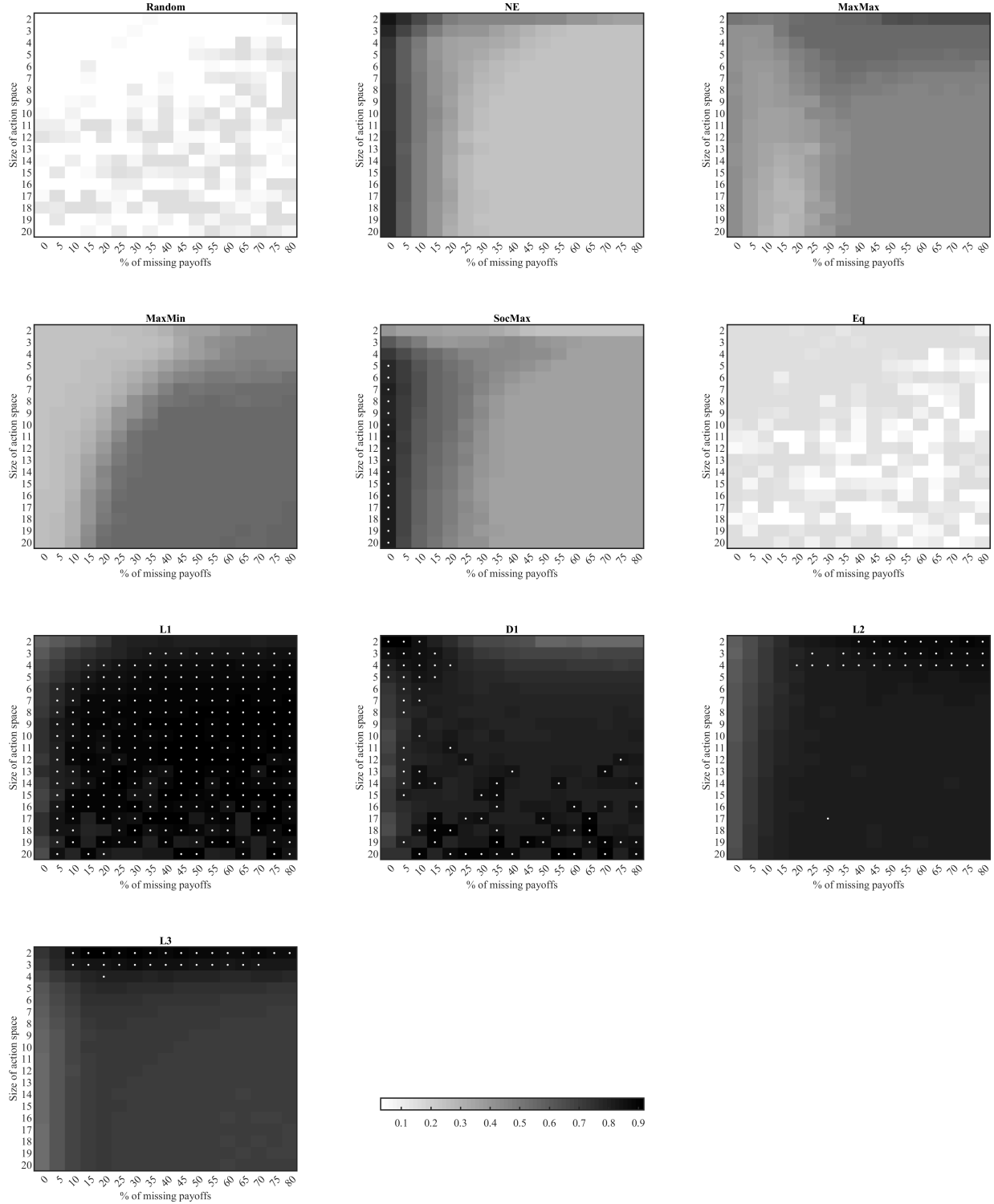
*Figure 6.* Decision policy performance in  $S'$  according to the Wald criterion ( $\pi_w$ ) for discordant environments ( $\rho = -0.5$ ).

## Composition–Robustness Criterion



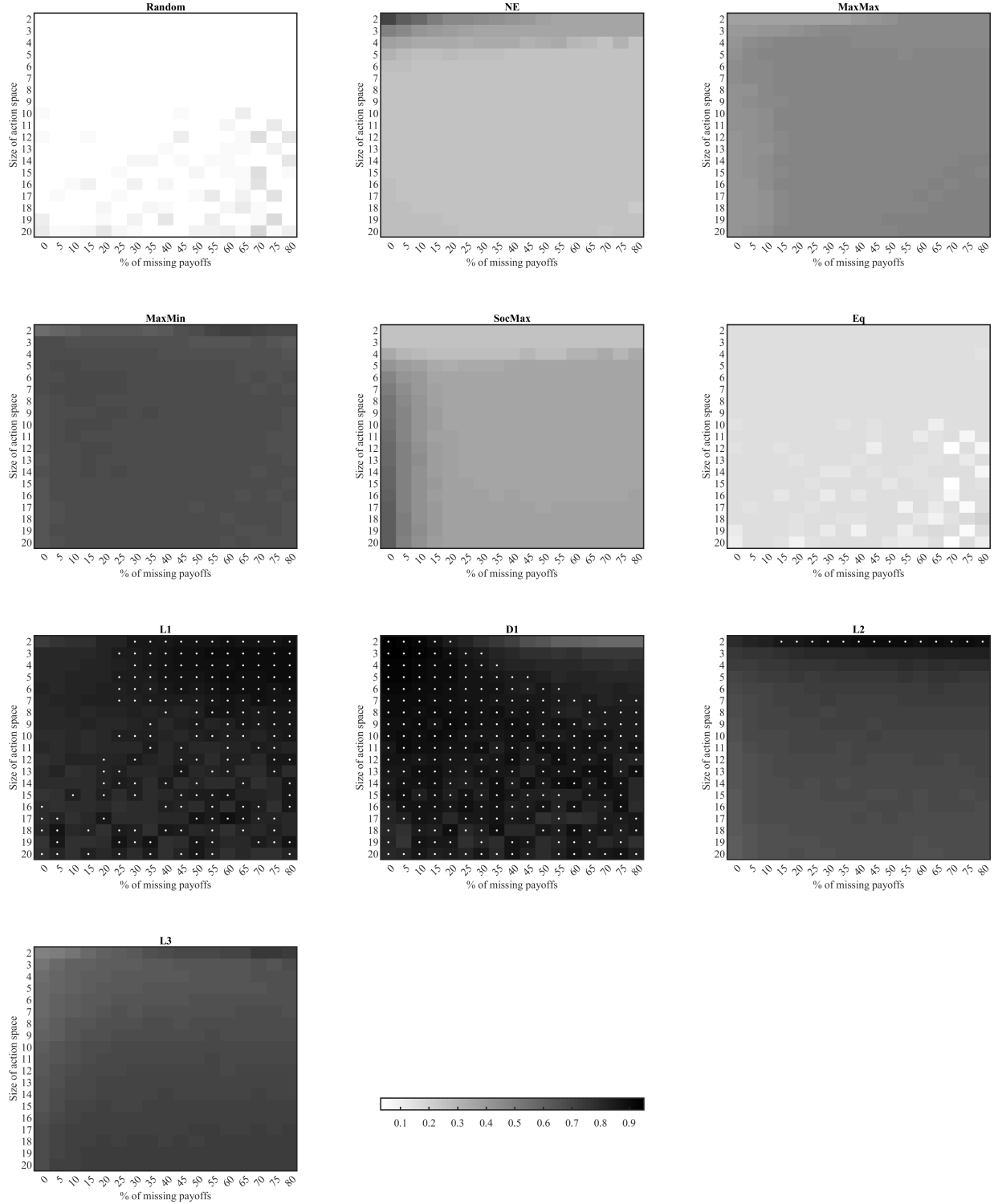
*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

*Figure 7.* Decision policy performance in  $S'$  according to the Composition–Robustness criterion for neutral environments ( $\rho = 0$ ).



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

*Figure 8.* Decision policy performance in  $S'$  according to the Composition-Robustness criterion for harmonious environments ( $\rho = 0.5$ ).



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

*Figure 9.* Decision policy performance in  $S'$  according to the Composition-Robustness criterion for discordant environments ( $\rho = -0.5$ ).

**$S''$ : Alternative assumption for the expected quality of actions**

In this additional simulation, each action varies in expected quality (or average performance across an opponent's actions). A game is constructed by drawing payoffs for each action from a distribution with an action-specific mean; in the core simulation, the mean is the same (zero) for all actions. The mean associated with each action is now drawn from a normal distribution with mean zero and variance equal to 100. No other changes were made with respect to the generation of games in the environments of the core simulation.

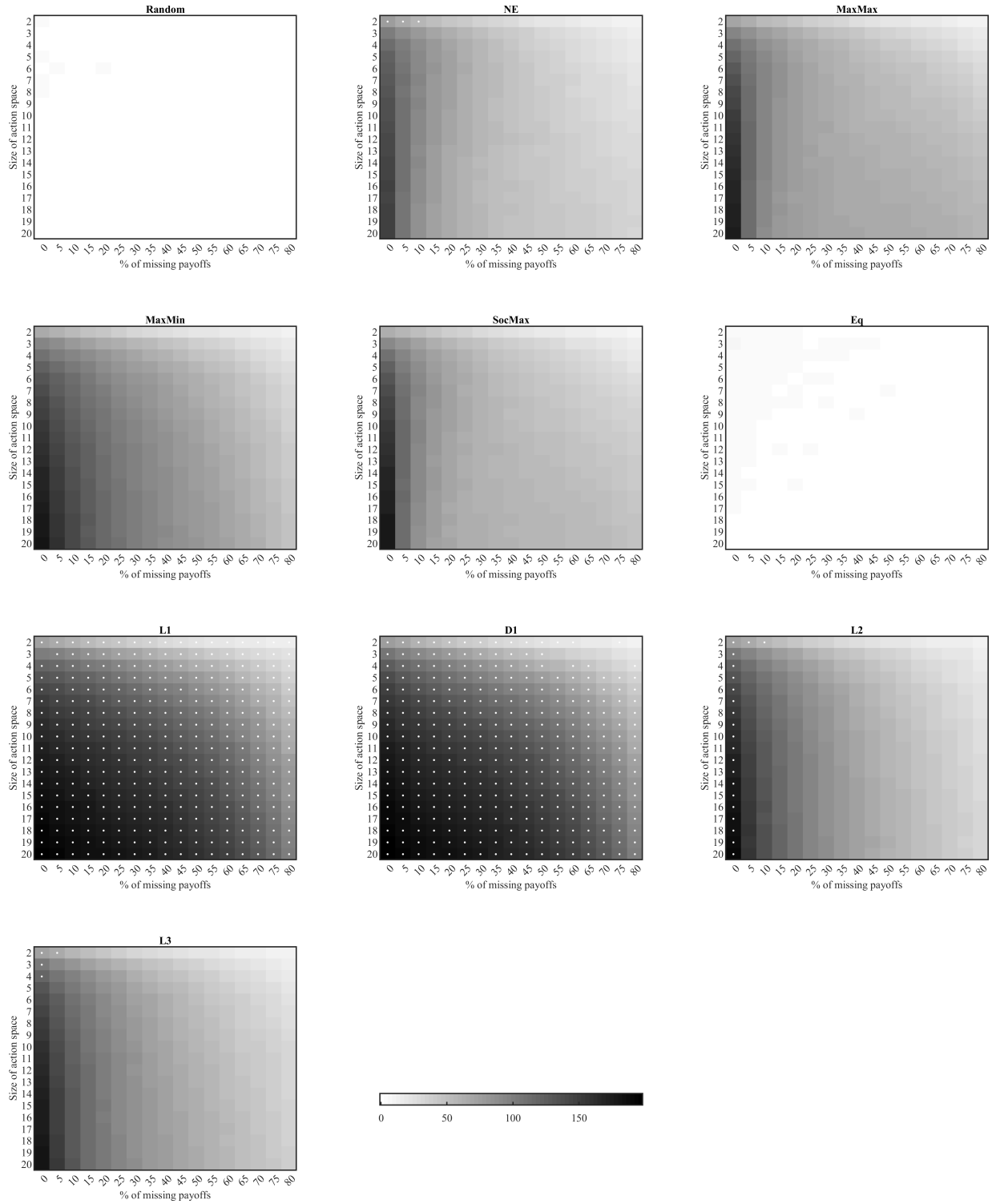
Table 4

*Summary of Decision Policies' Performance in  $S''$*

$\rho =$	Indifference				Wald				Comp.–Robustness			
	0	0.5	-0.5	All $\rho$	0	0.5	-0.5	All $\rho$	0	0.5	-0.5	All $\rho$
Random	0.7	3.9	-2.3	0.8	-0.5	-0.2	-6.4	-2.4	5	8	5	6
NE	55.7	74.8	34.5	55.0	42.5	50.7	28.8	40.7	30	39	28	32
MaxMax	67.1	76.0	61.1	68.1	62.8	64.0	50.5	59.1	53	47	55	52
MaxMin	81.5	86.6	77.4	81.8	78.9	79.9	67.0	75.3	69	59	72	67
SocMax	61.3	77.0	48.3	62.2	49.0	52.5	40.5	47.3	38	46	37	40
Eq	1.4	4.1	-1.2	1.4	-0.3	-0.3	-4.3	-1.6	13	10	13	12
L1	<b>120.8</b>	<b>129.4</b>	<b>113.6</b>	<b>121.3</b>	<b>117.1</b>	<b>118.4</b>	<b>86.7</b>	<b>107.4</b>	<b>95</b>	<b>95</b>	<b>95</b>	<b>95</b>
D1	<b>120.2</b>	<b>128.5</b>	<b>113.1</b>	<b>120.6</b>	<b>114.5</b>	<b>115.8</b>	<b>89.6</b>	<b>106.6</b>	87	86	87	87
L2	73.0	78.8	68.3	73.4	56.5	54.4	35.5	48.8	54	44	61	53
L3	72.9	89.5	58.8	73.7	58.3	69.7	48.0	58.7	55	66	48	56

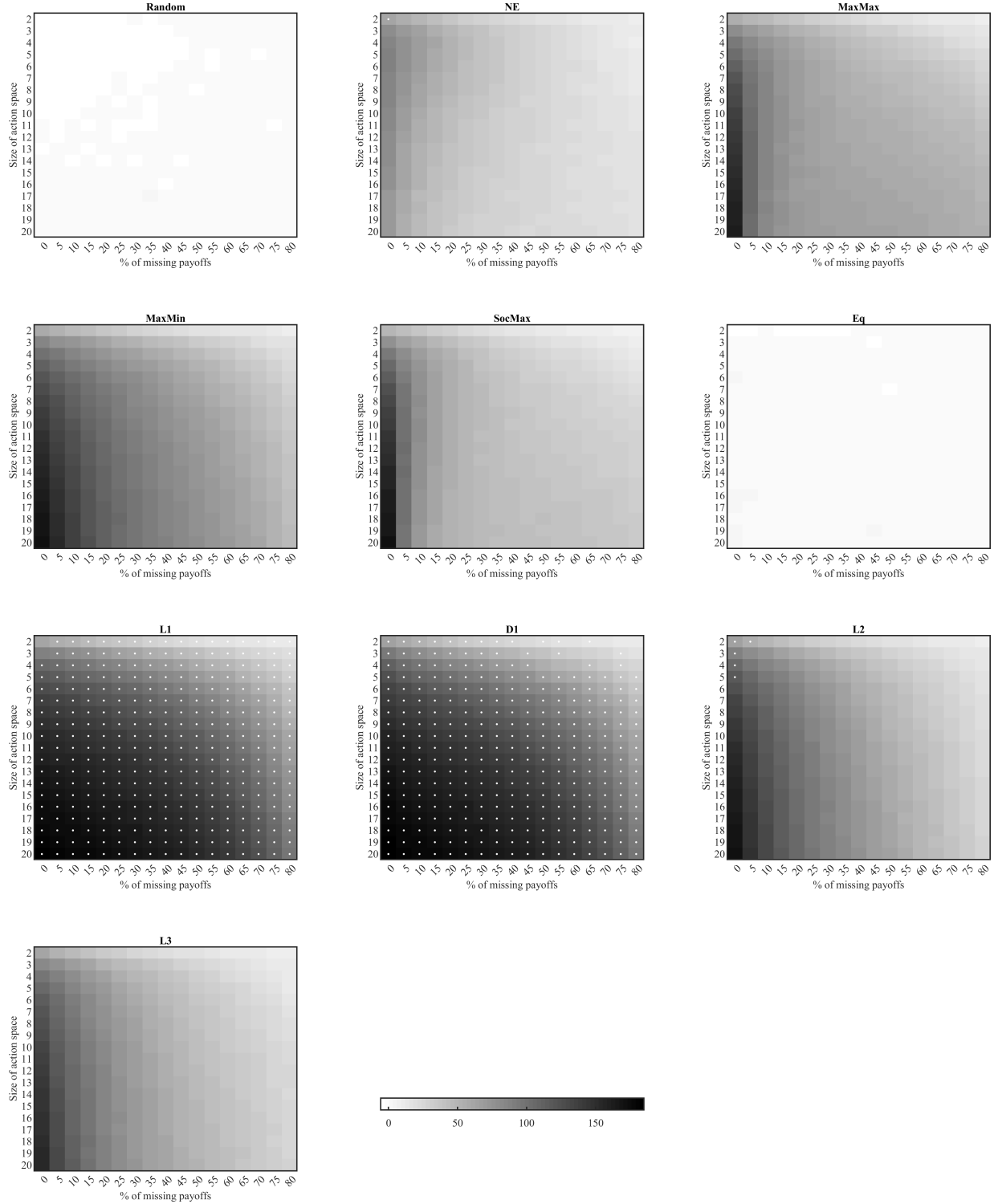
*Note.* The set of top-performing policies are in bold.

## Indifference Criterion



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

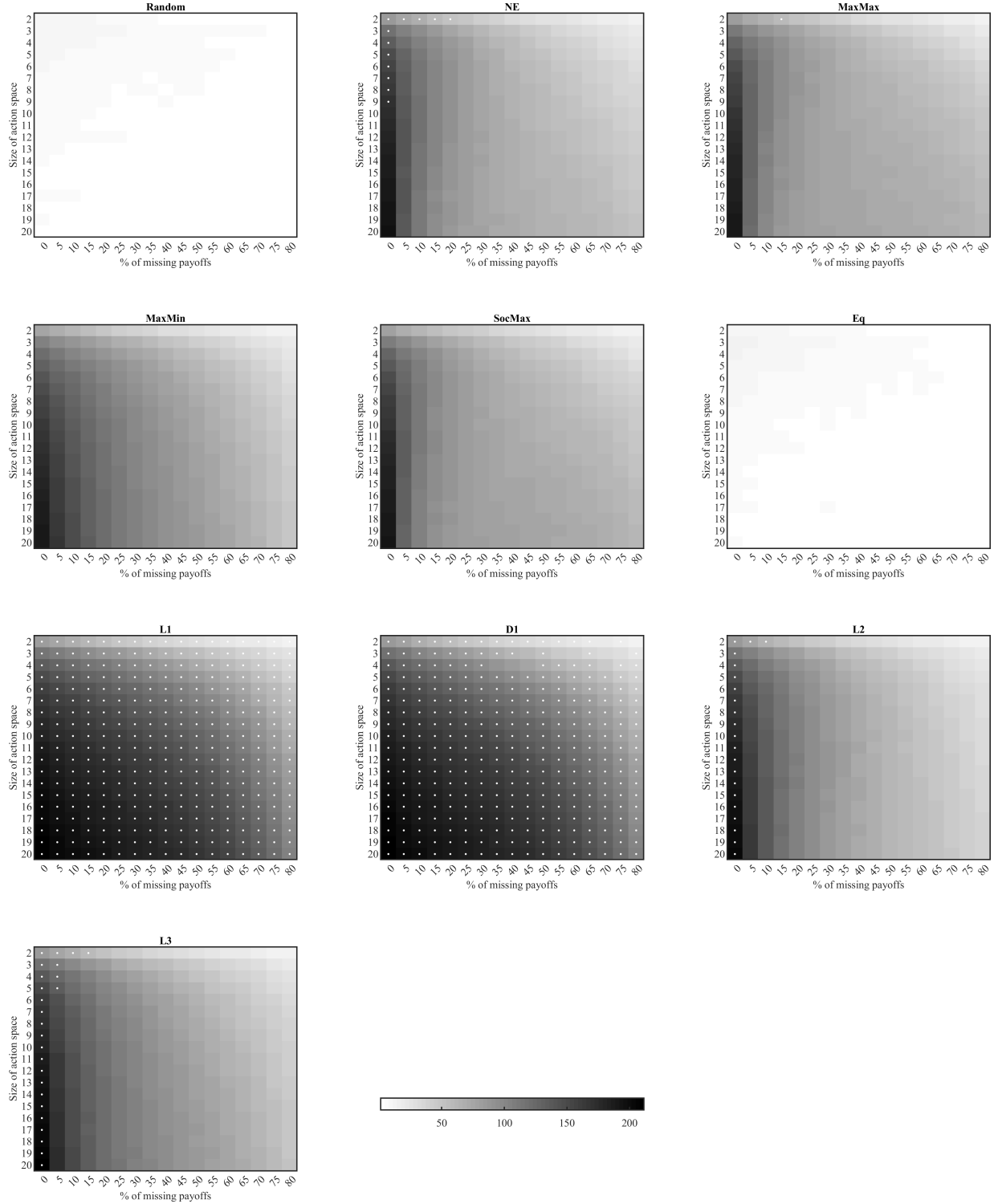
*Figure 10.* Decision policy performance in  $S''$  according to the Indifference criterion ( $\bar{\pi}$ ) for neutral environments ( $\rho = 0$ ).



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

*Figure 11.* Decision policy performance in  $S''$  according to the Indifference criterion ( $\bar{\pi}$ ) for discordant environments ( $\rho = -0.5$ ).

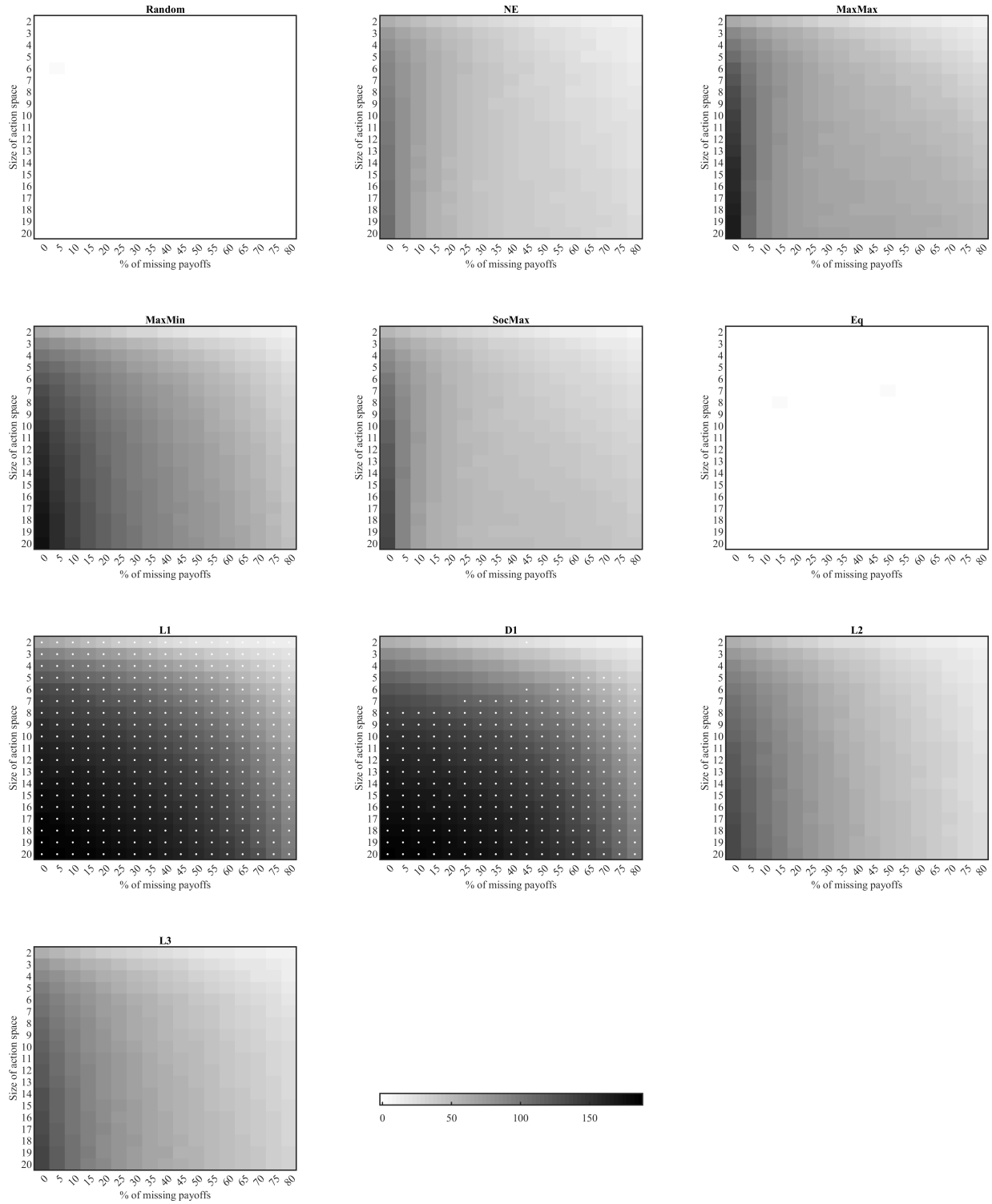




*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

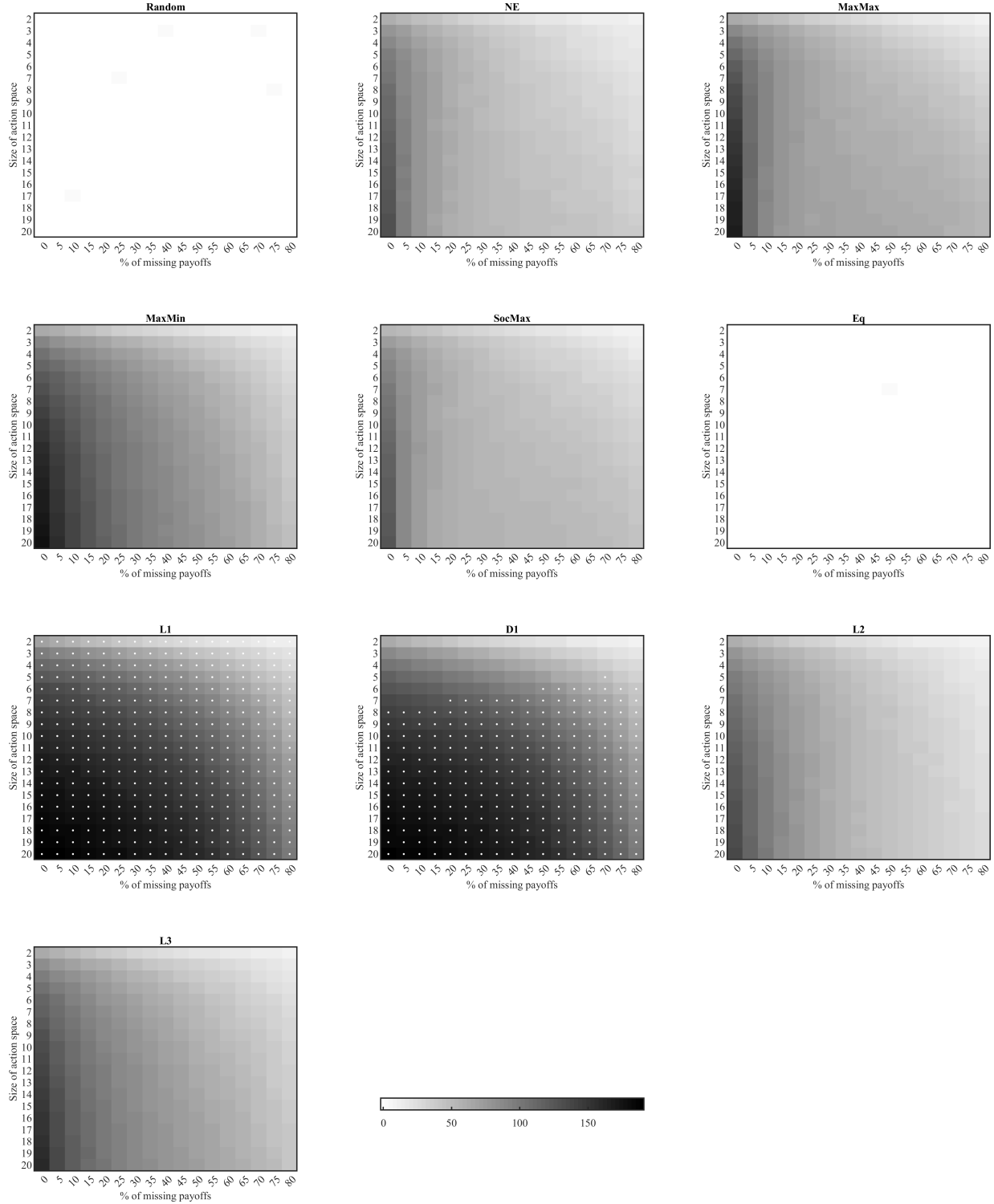
*Figure 12.* Decision policy performance in  $S''$  according to the Indifference criterion ( $\bar{\pi}$ ) for harmonious environments ( $\rho = 0.5$ ).

## Wald Criterion



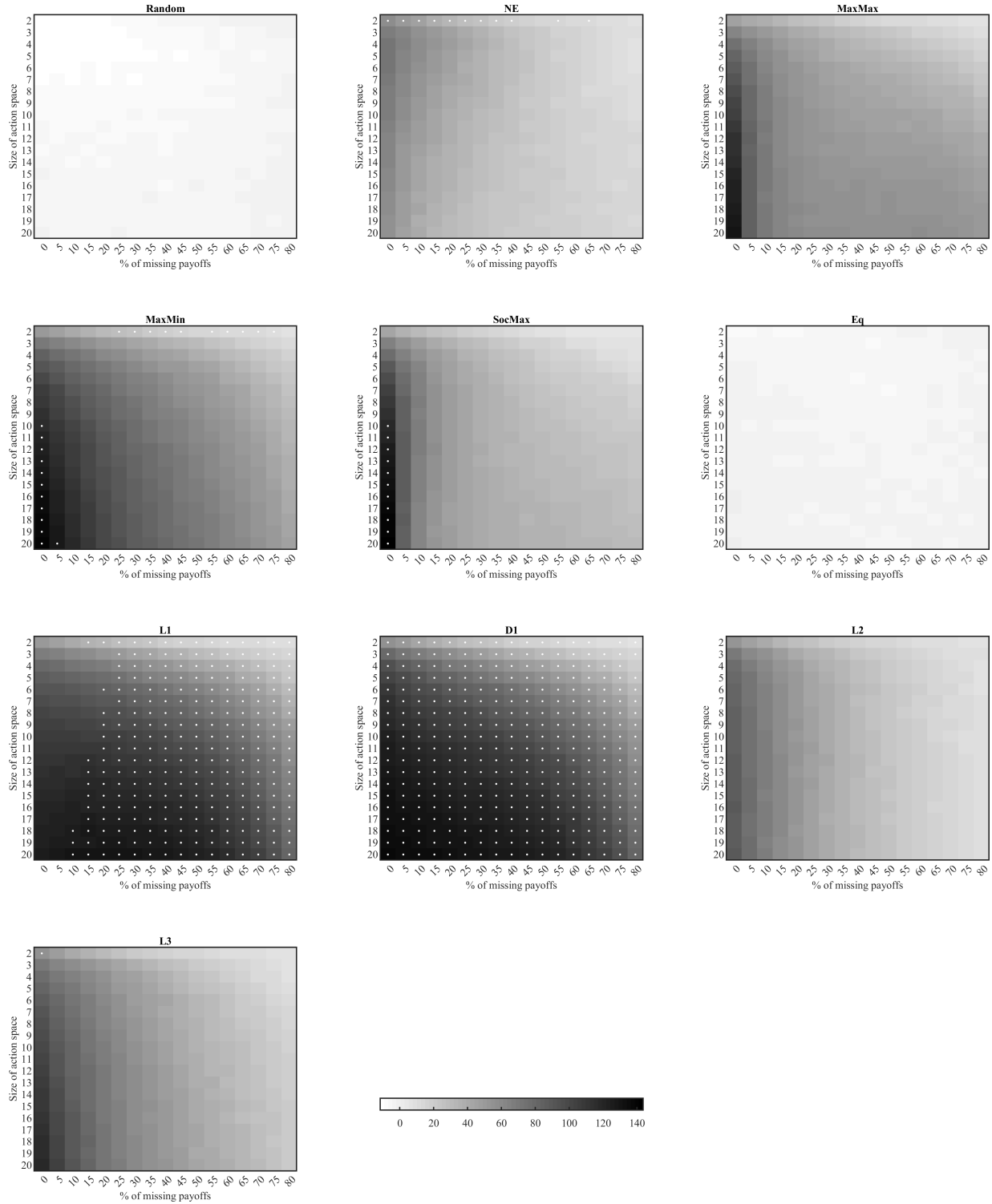
*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

*Figure 13.* Decision policy performance in  $S''$  according to the Wald criterion ( $\pi_w$ ) for neutral environments ( $\rho = 0$ ).



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

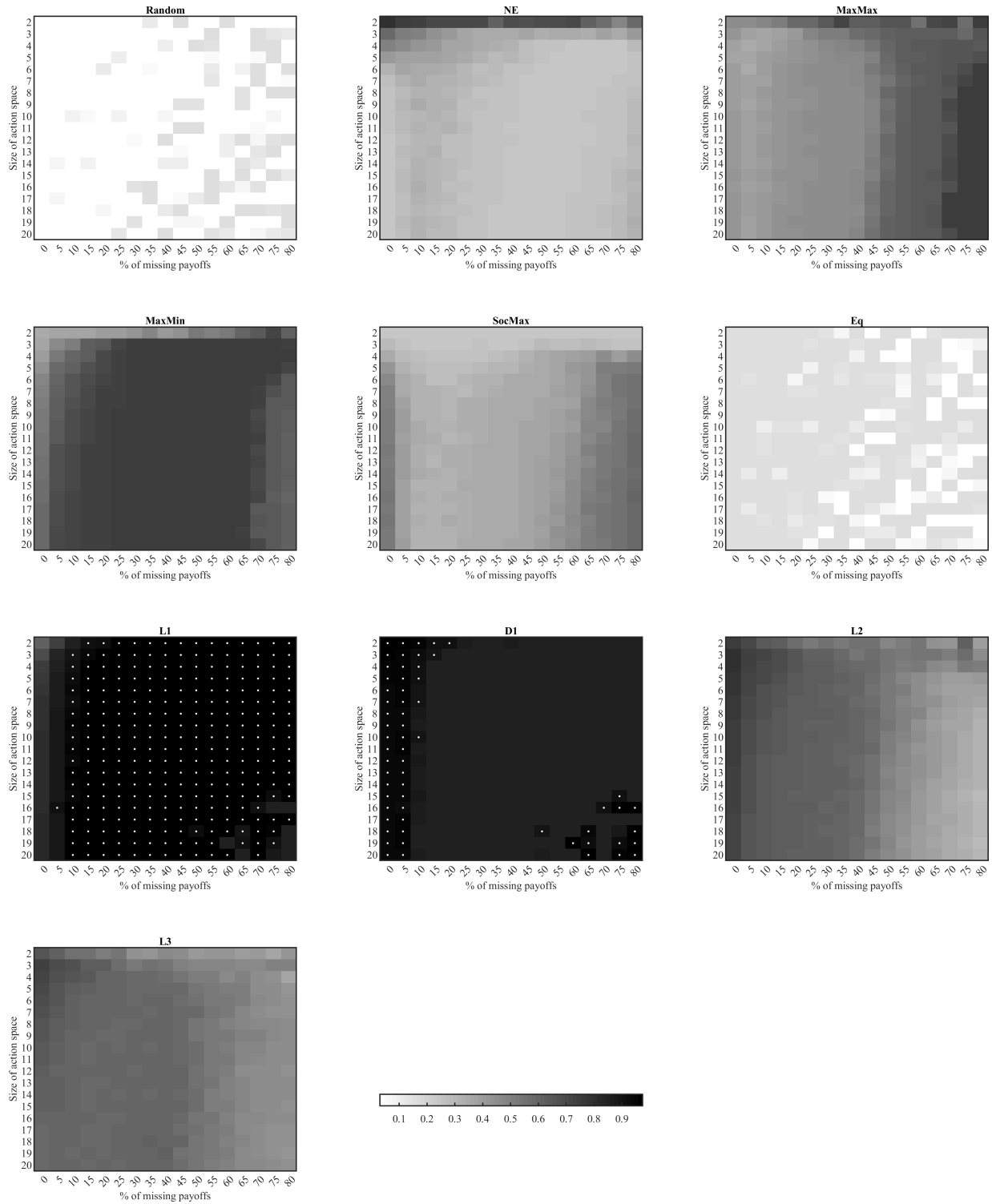
*Figure 14.* Decision policy performance in  $S''$  according to the Wald criterion ( $\pi_w$ ) for harmonious environments ( $\rho = 0.5$ ).



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

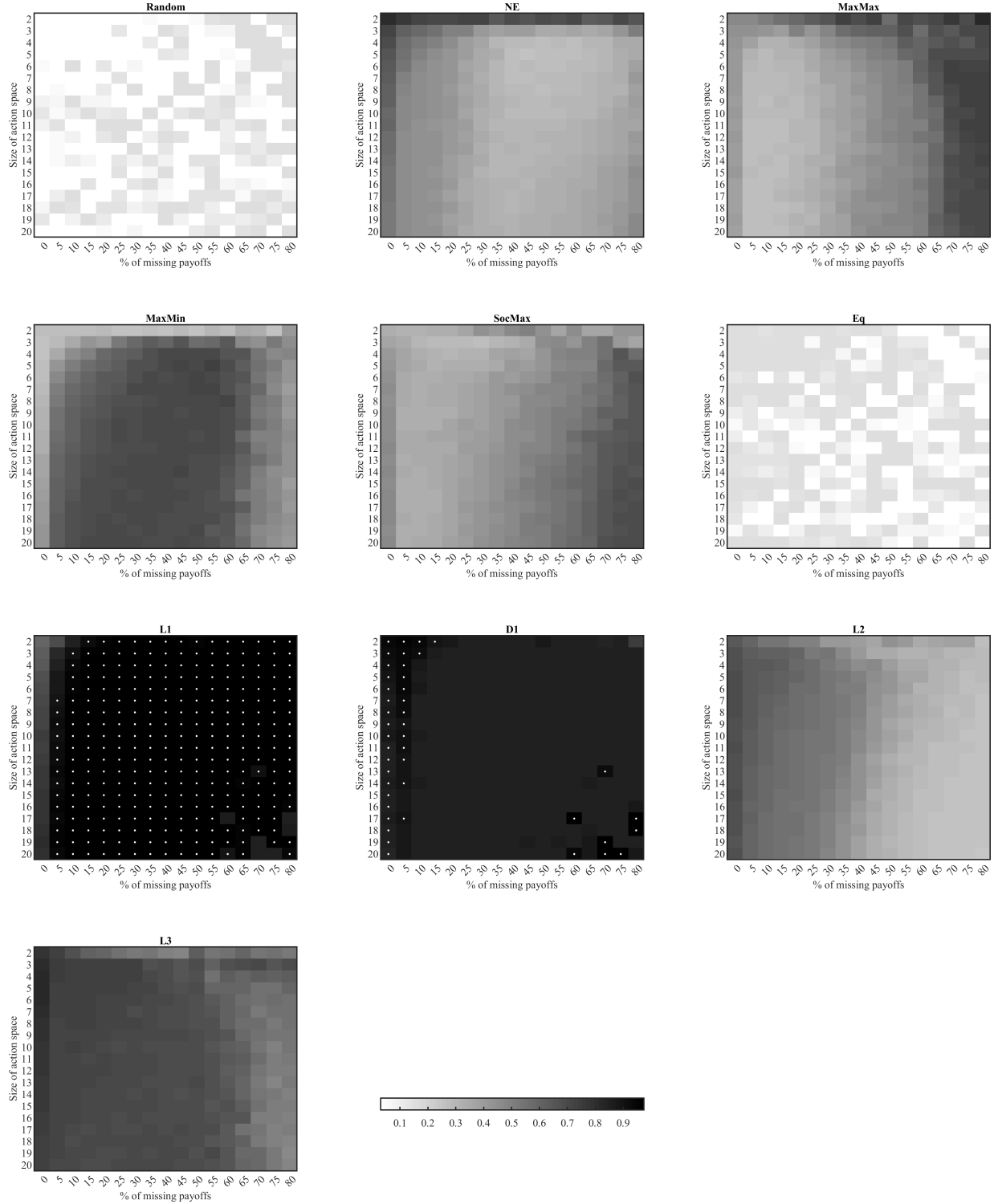
*Figure 15.* Decision policy performance in  $S''$  according to the Wald criterion ( $\pi_w$ ) for discordant environments ( $\rho = -0.5$ ).

## Composition–Robustness Criterion



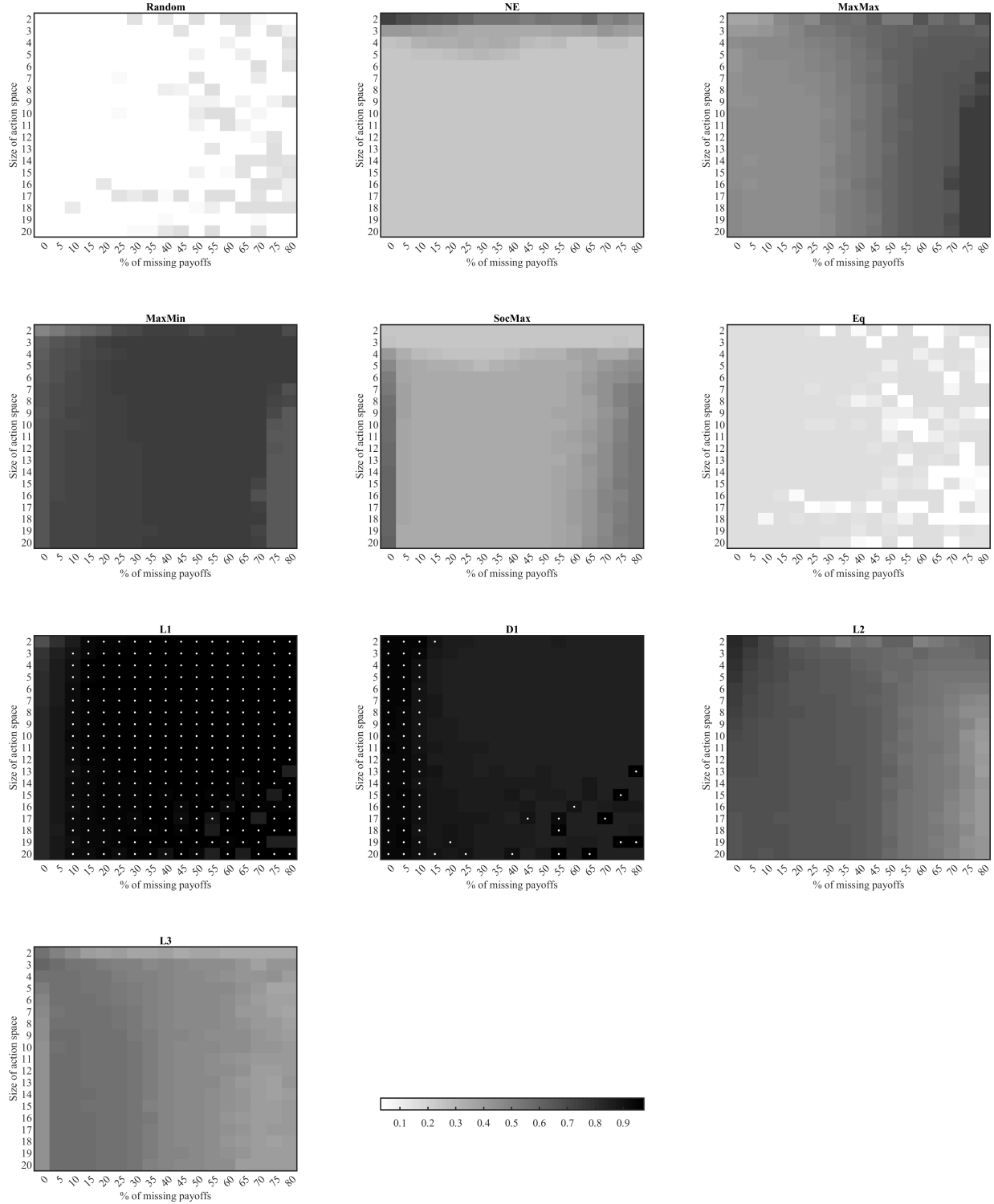
*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

*Figure 16.* Decision policy performance in  $S''$  according to the Composition–Robustness criterion for neutral environments ( $\rho = 0$ ).



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

*Figure 17.* Decision policy performance in  $S''$  according to the Composition-Robustness criterion for harmonious environments ( $\rho = 0.5$ ).



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

*Figure 18.* Decision policy performance in  $S''$  according to the Composition-Robustness criterion for discordant environments ( $\rho = -0.5$ ).

**$S'''$ : Alternative mechanism for missing payoff inference**

This section presents the results of an additional simulation with an alternative (simpler) mechanism for missing payoff inference, where only  $\hat{\mu}$  is imputed in each game, that is, players do not use information about the variance or correlation of payoffs. Missing values are replaced by  $\hat{\mu}$  plus a very small random error  $\epsilon \sim N(0, 10^{-6})$  for each payoff to avoid complications with ties. Table 5 summarizes performance over  $n$  and  $m$ , and Figures 19–27 present detailed maps of policy performance.

Table 5

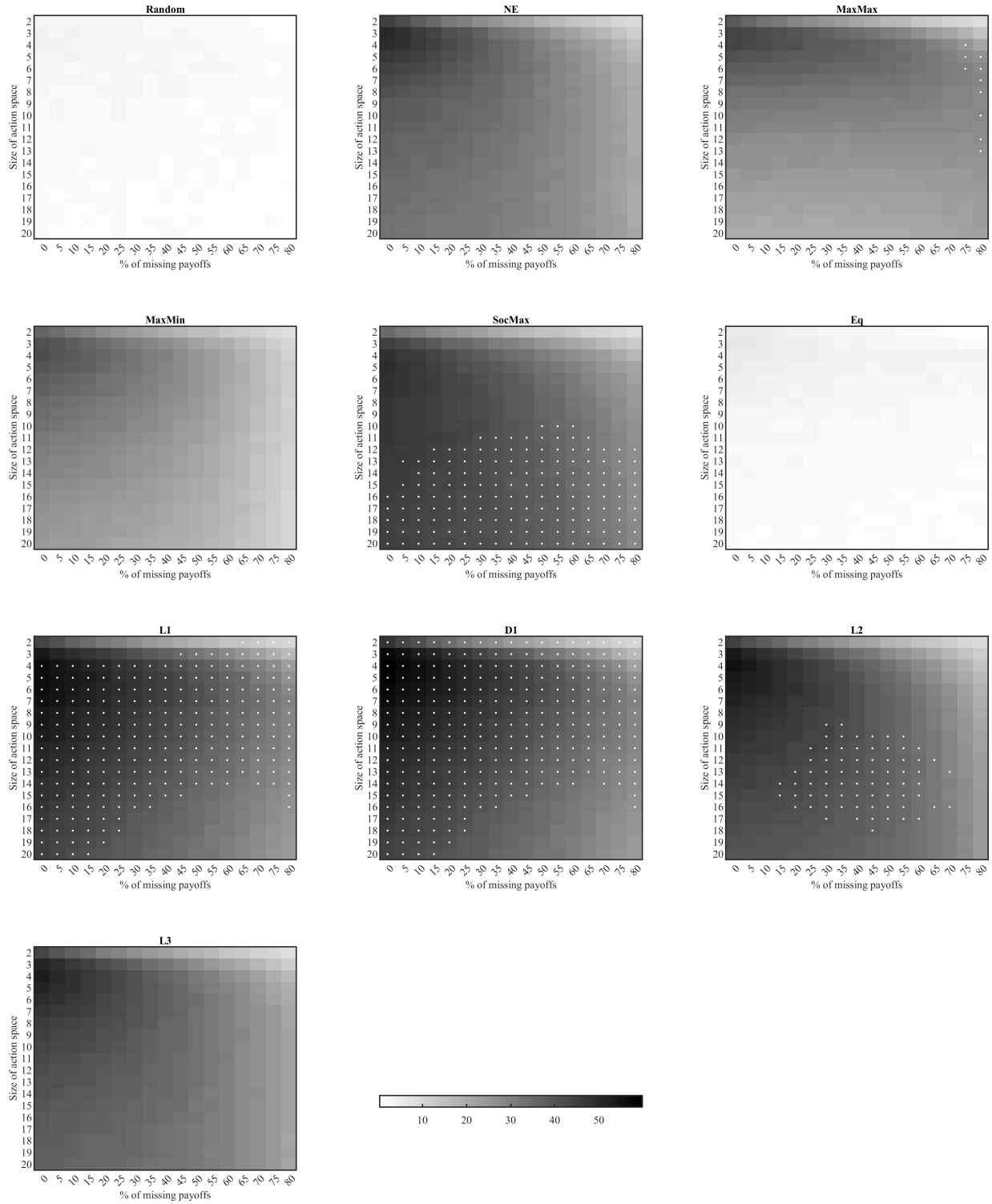
*Summary of Decision Policies' Performance in  $S'''$*

$\rho =$	Indifference				Wald				Comp.–Robustness			
	0	0.5	-0.5	All $\rho$	0	0.5	-0.5	All $\rho$	0	0.5	-0.5	All $\rho$
Random	1.6	8.7	-4.8	1.8	-0.6	-0.3	-17.3	-6.1	5	6	5	5
NE	30.6	56.9	7.4	31.6	9.4	12.5	-5.6	5.4	53	67	35	52
MaxMax	27.6	45.4	16.6	29.9	18.1	18.8	-4.2	10.9	45	46	59	50
MaxMin	23.3	30.8	16.5	23.5	20.3	20.9	<b>5.0</b>	<b>15.4</b>	32	25	56	38
SocMax	36.5	<b>63.7</b>	8.3	36.2	12.9	16.6	-22.0	2.5	65	<b>84</b>	36	62
Eq	2.2	9.1	-4.1	2.4	-0.4	-0.3	-15.3	-5.3	13	12	15	13
L1	<b>38.6</b>	54.4	<b>24.8</b>	<b>39.3</b>	<b>32.5</b>	<b>33.4</b>	-24.4	13.8	77	71	78	75
D1	<b>39.0</b>	54.7	<b>25.2</b>	<b>39.6</b>	<b>31.8</b>	<b>32.6</b>	-22.2	14.1	<b>84</b>	73	<b>86</b>	81
L2	36.5	53.1	21.0	36.9	11.0	11.5	-41.9	-6.5	67	62	66	65
L3	33.4	49.5	19.8	34.2	11.5	16.5	1.1	9.7	59	53	64	59

*Note.* The set of top-performing policies are in bold.

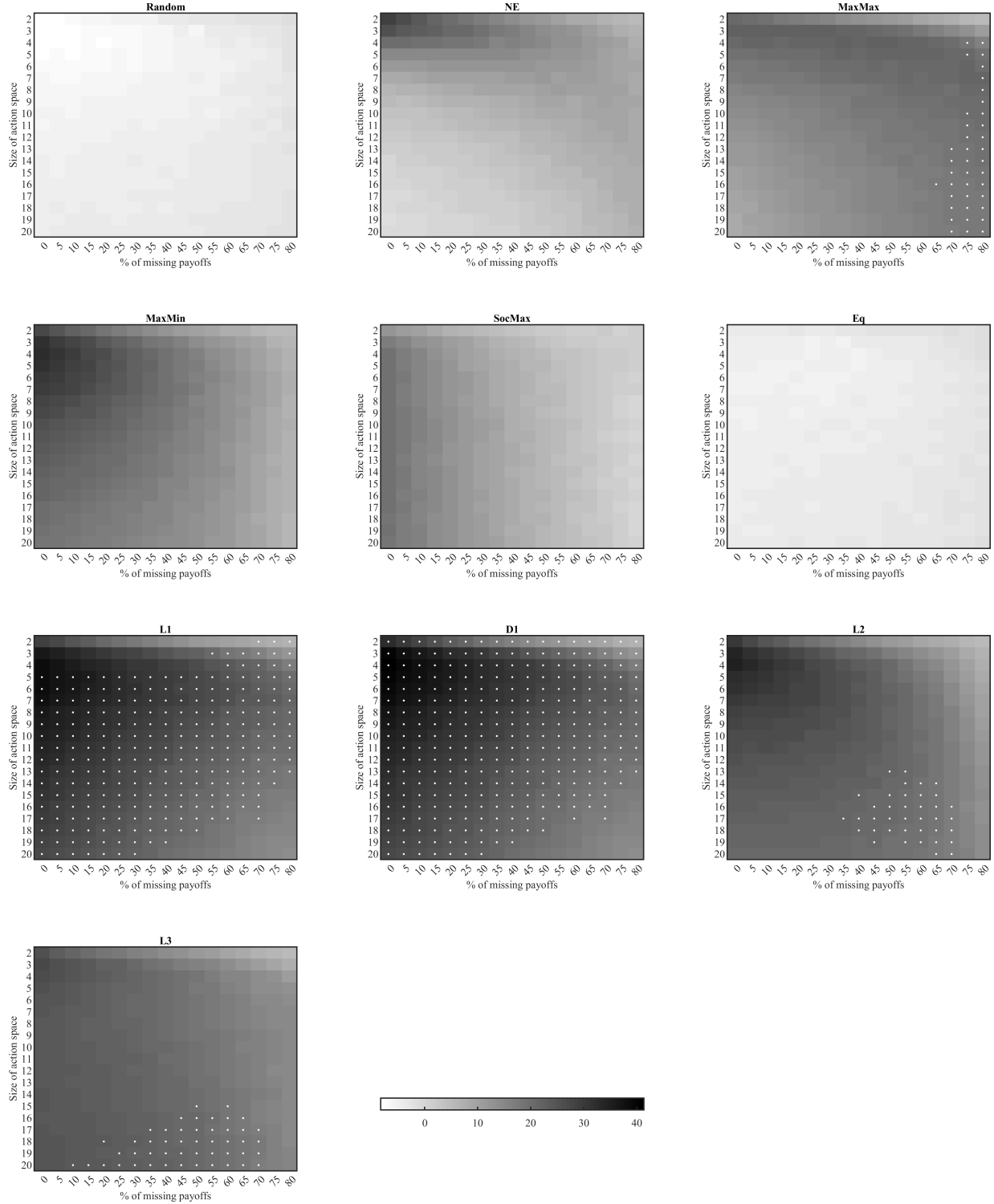


## Indifference Criterion



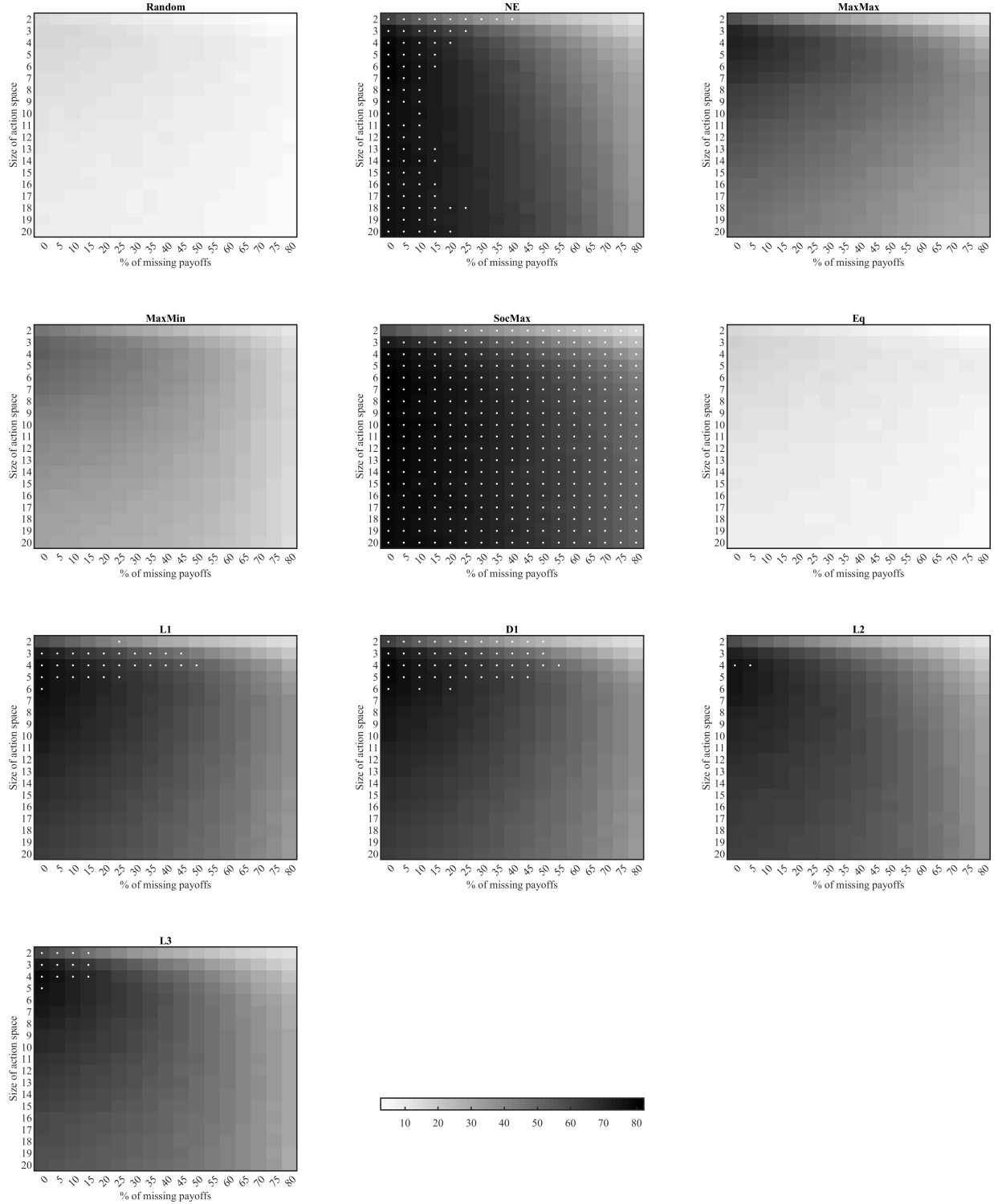
*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

*Figure 19.* Decision policy performance in  $S'''$  according to the Indifference criterion ( $\bar{\pi}$ ) for neutral environments ( $\rho = 0$ ).



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

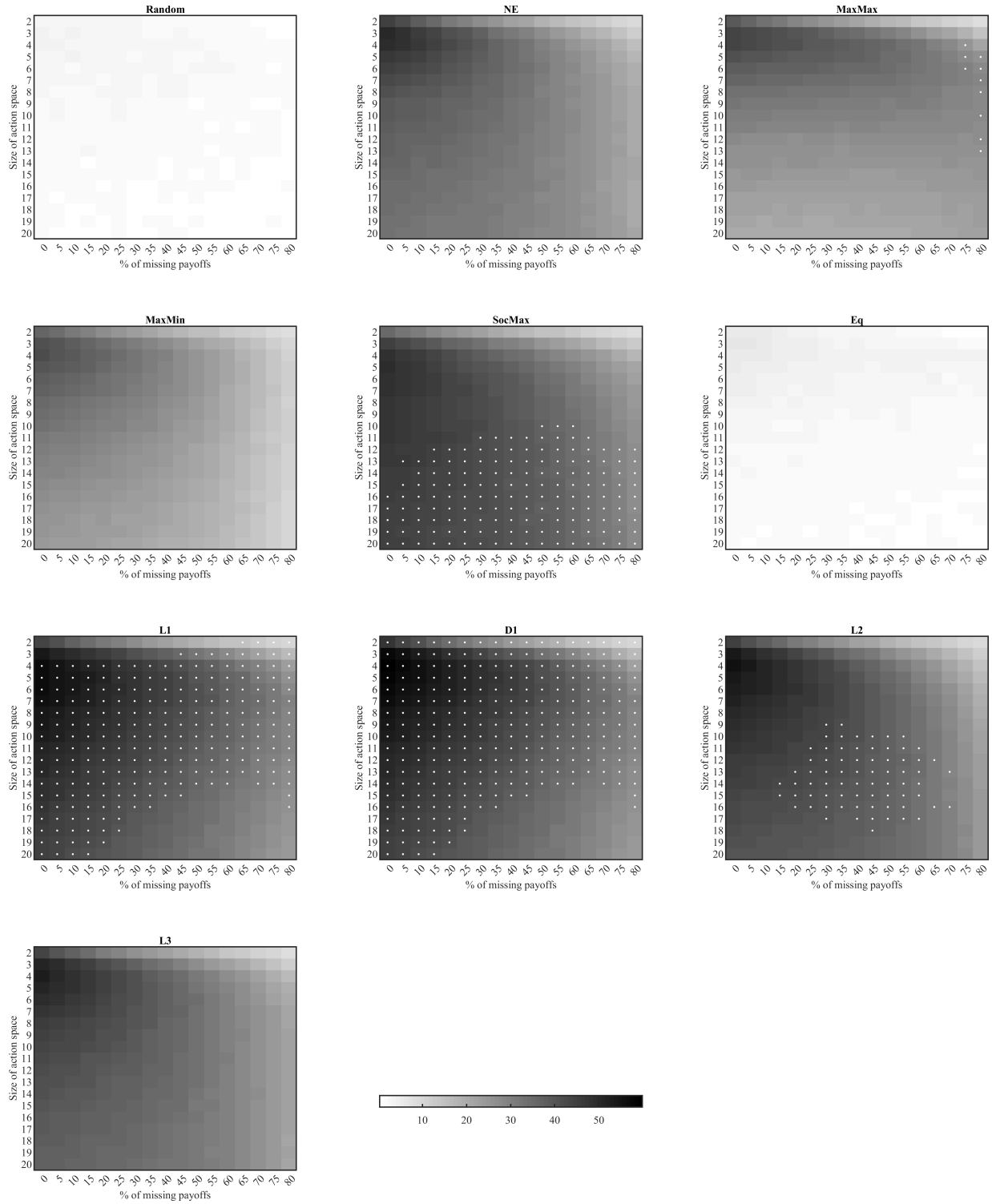
*Figure 20.* Decision policy performance in  $S'''$  according to the Indifference criterion ( $\bar{\pi}$ ) for discordant environments ( $\rho = -0.5$ ).



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

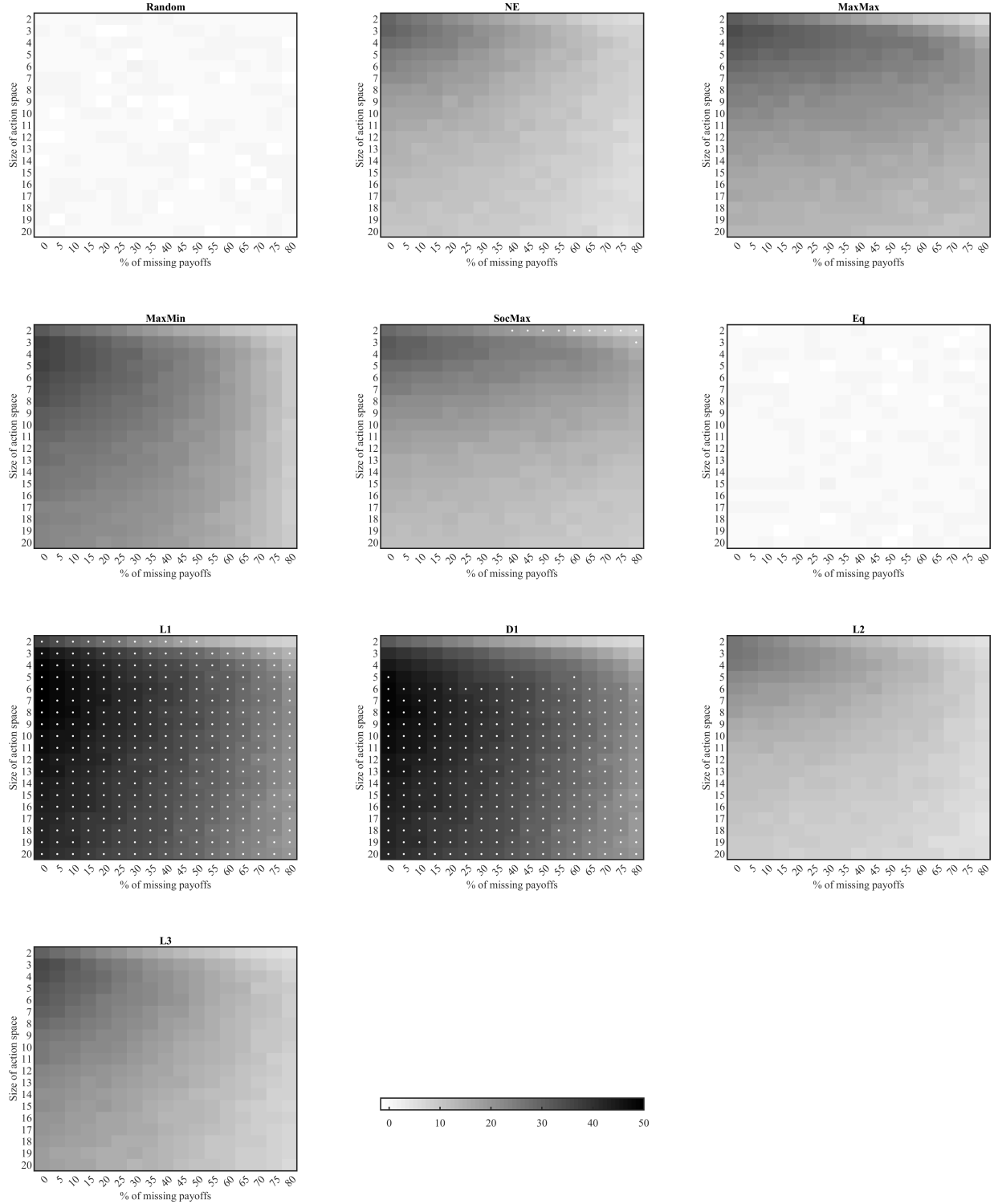
*Figure 21.* Decision policy performance in  $S'''$  according to the Indifference criterion ( $\bar{\pi}$ ) for harmonious environments ( $\rho = 0.5$ ).

## Wald Criterion



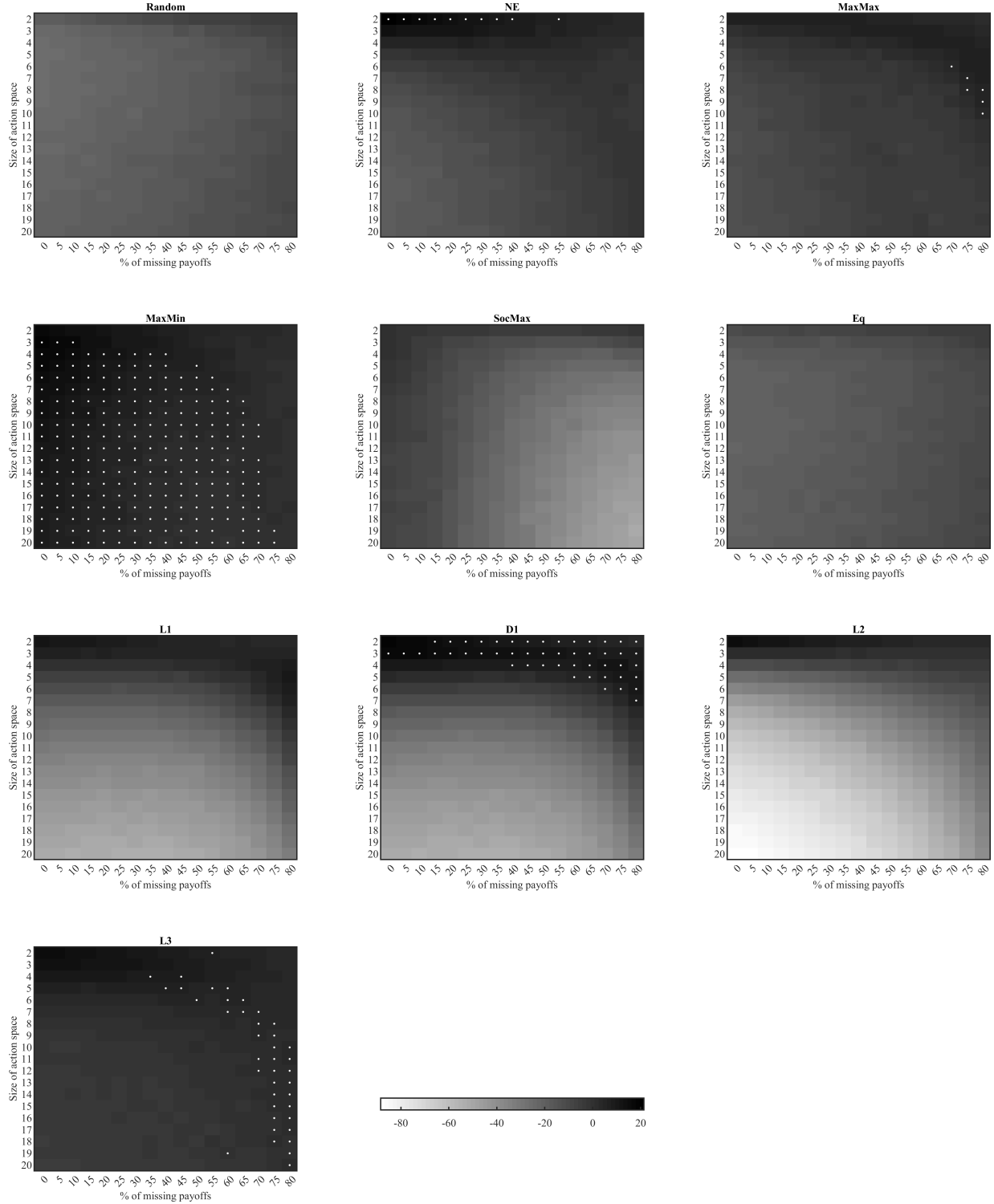
*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

*Figure 22.* Decision policy performance in  $S'''$  according to the Wald criterion ( $\pi_w$ ) for neutral environments ( $\rho = 0$ ).



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

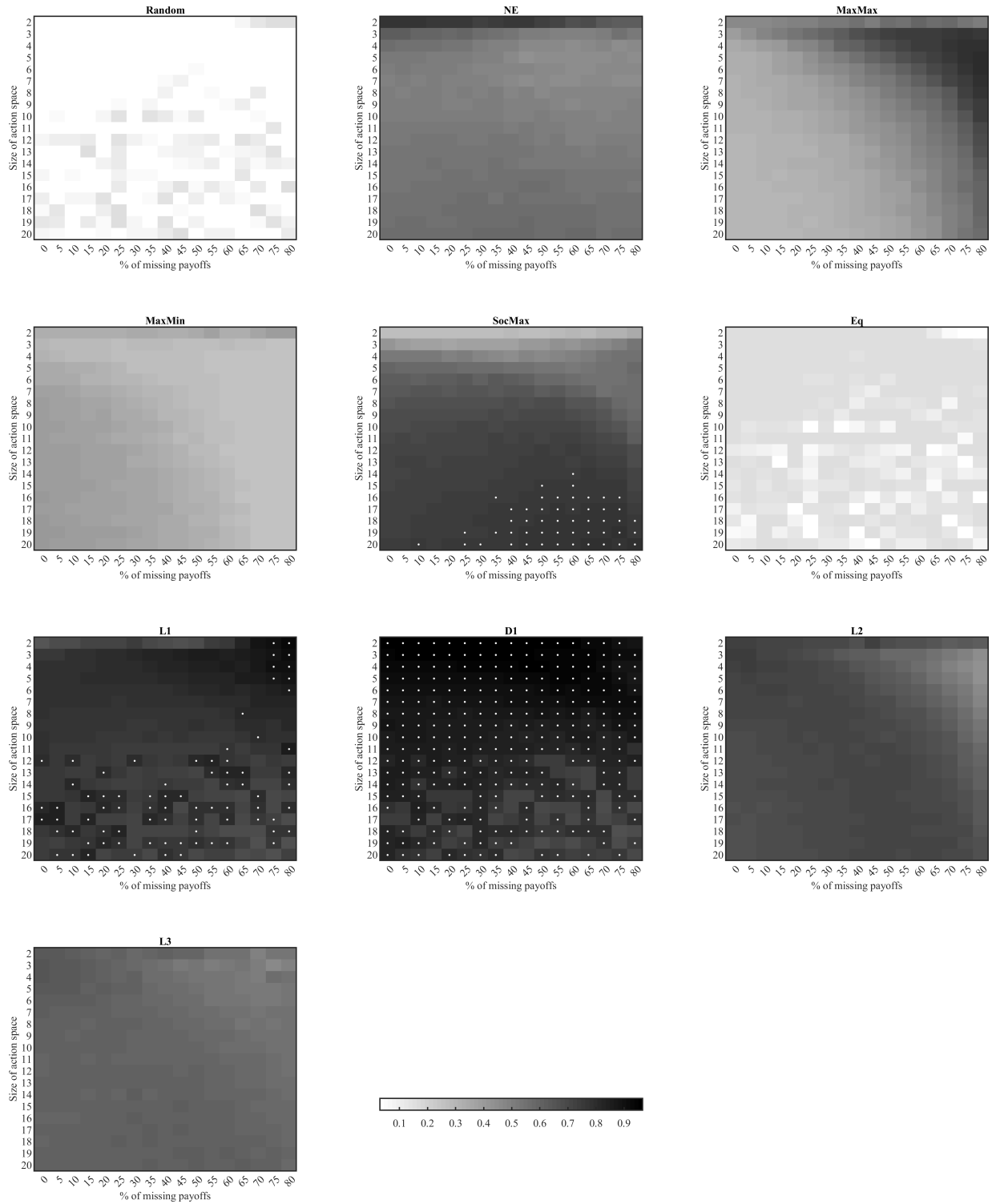
*Figure 23.* Decision policy performance in  $S'''$  according to the Wald criterion ( $\pi_w$ ) for harmonious environments ( $\rho = 0.5$ ).



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

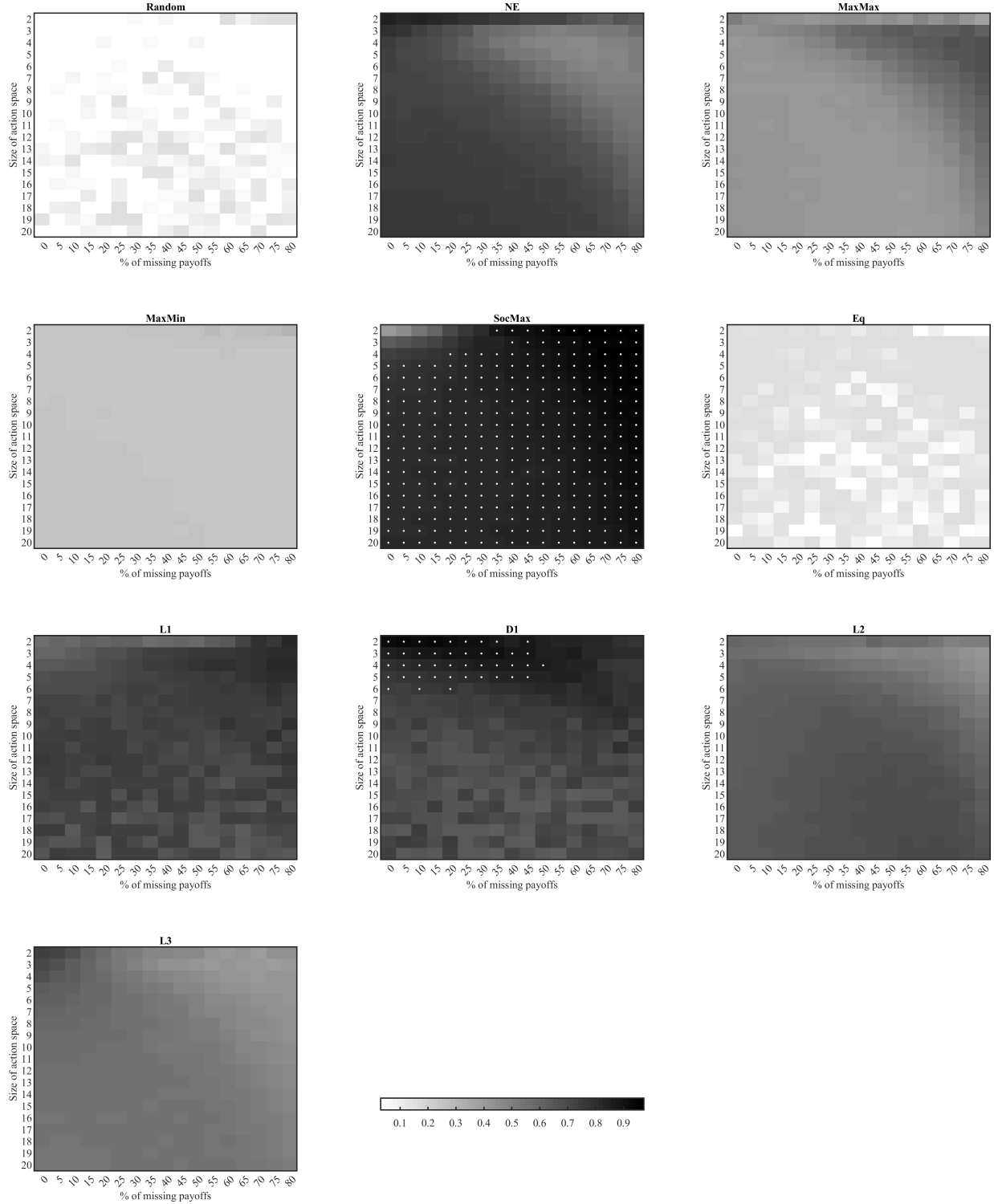
*Figure 24.* Decision policy performance in  $S'''$  according to the Wald criterion ( $\pi_w$ ) for discordant environments ( $\rho = -0.5$ ).

## Composition–Robustness Criterion



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

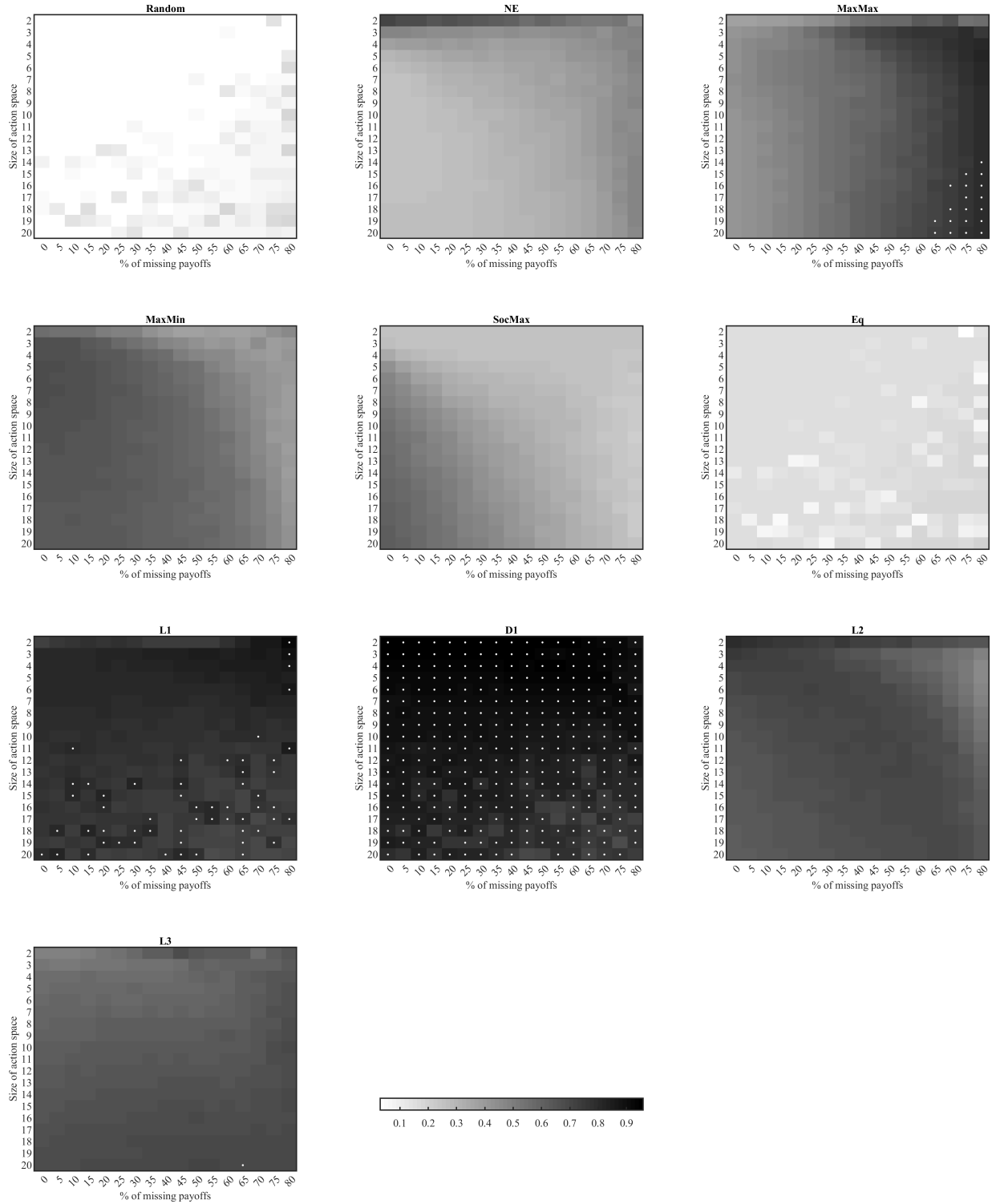
*Figure 25.* Decision policy performance in  $S'''$  according to the Composition–Robustness criterion for neutral environments ( $\rho = 0$ ).



*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

*Figure 26.* Decision policy performance in  $S'''$  according to the Composition–Robustness criterion for harmonious environments ( $\rho = 0.5$ ).





*Note.* The set of top-performing policies for each environment is marked by an overlaid white dot.

*Figure 27.* Decision policy performance in  $S'''$  according to the Composition–Robustness criterion for discordant environments ( $\rho = -0.5$ ).