Pooling Methods for Likelihood Ratio Tests in Multiply Imputed Data Sets

Supplemental Online Material

Contents

A) Simulation with Unequal FMIs and Robust Version of D_4

In this additional simulation study, we evaluated the performance of the pooling methods for LRTs in conditions with equal versus unequal FMIs. In this context, we also include the robust version of D_4 proposed by Chan & Meng (2022; see also Chan and Meng, 2019). This method uses a different (more robust) estimator of the ARIV but requires the stronger assumption that the relative increase in variance is equal for *all* parameters in the full model, whereas the other pooling methods assume equal FMIs only for the parameters being tested.

Pooled LRTs using "stacked" data (robust, $D_4^{\diamond}$)

The $D_4^{\diamond}$ statistic is calculated in a manner similar to D_4. As in the main text, the log-likelihood of the stacked data is denoted by $\ell_S(\theta_S) = m^{-1} \log f(Y_S|\theta_S)$. Then, $D_4^{\diamond}$ is calculated as

$$D_4^{\diamond} = \frac{\hat{d}_S}{k(1 + r_4^{\diamond})} , \tag{1}$$

where $\hat{d}_S = -2\left[\ell_S(\hat{\theta}_{0,S}) - \ell_S(\hat{\theta}_S)\right]$ is the LRT statistic from the stacked data set, and $r_4^{\diamond}$ is an alternative (more robust) estimate of the ARIV:

$$r_4^{\diamond} = \frac{m+1}{h(m-1)}(\bar{\delta} - \hat{\delta}_S) , \tag{2}$$

where h is the number of parameters in the full model, with $\bar{\delta} = 2 \cdot m^{-1} \sum_{l=1}^{m} \ell_l(\hat{\theta}^{(l)})$, and $\hat{\delta}_S = 2 \cdot \ell_S(\hat{\theta}_S)$. $D_4^{\diamond}$ can be compared with an F distribution that has k numerator and $\nu_4^{\diamond}$ denominator degrees of freedom:

$$\nu_4^{\diamond} = k(m-1)\left[1 + (r_4^{\diamond})^{-1}\right]^2 . \tag{3}$$

Chan and Meng (2022) argued that $D_4^{\diamond}$ provides a more robust method for pooling the LRT because its estimator of the ARIV ($r_4^{\diamond}$) is consistent under both the null and alternative hypotheses (for a detailed discussion, see Chan & Meng, 2022). An important caveat with respect to the robust estimator $r_4^{\diamond}$ is that it requires the stronger assumption that the FMI be approximately equal for all parameters in the full model (i.e., not just the elements in θ that are restricted in θ_0).

Simulation Procedure

The procedures for generating data were identical to those used in Study 1 in conditions with MAR data. However, in addition to the conditions used in Study 1, we also simulated conditions with unequal percentages of missing data in the first and second half of the predictor variables. The simulated conditions for the sample size and the number of predictors were the same as in Study 1, but we simulated only conditions with either no effect or a medium effect size (R^2 = 0, .13) and MAR data. In addition, we varied the percentage of missing data by included conditions either with 20% in both halves or with 10% in the first half and 30% in the second half. In the latter case, the overall percentage of cases with missing data was similar to the former case, but the FMIs of the parameters being tested were unqual, which violated the assumptions underlying all the pooling methods for LRTs. In addition, the assumptions underlying $D_4^{\Diamond}$ were always violated, even in the condition with a constant 20% missing data, because the intercept tended to have a lower FMI than the other regression coefficients. Finally, we used `mice` (van Buuren & Groothuis-Oudshoorn, 2011) to impute the missing data as described in the main text, and we used D_2, D_3, D_4, and $D_4^{\Diamond}$ to pool the LRTs. In addition, we included CD, LD, and FIML for comparison.

Results

The results for the Type 1 error rates ($R^2 = 0$) and the statistical power to detect a medium sized effect ($R^2 = .13$) are summarized in Table A1 and Figure A1. For all pooling methods, the results in the condition with an equal versus unequal percentage of missing data were virtually identical. The same was true for LD and FIML.

For LD, FIML, D_2, D_3, and D_4, the remaining results were consistent with those from Study 1 in the main manuscript. By contrast, $D_4^{\Diamond}$ often provided Type 1 error rates above the nominal value, especially in conditions with fewer predictors ($k = 2$). However, consistent with these differences in Type 1 error rates, $D_4^{\Diamond}$ tended to have a higher statistical power than D_3 and D_4. In addition, in conditions with smaller samples ($n \leq 100$) and few predictors ($k = 2$), the power of

Table A1
Study 1: Type 1 Error Rates in % ($R^2 = 0$) in Conditions with MAR Data

							MI			
MD	k	n	FMI	CD	LD	FIML	D_2	D_3	D_4	$D_4^{\diamond}$
20%	2	50	.309	5.9	6.6	6.5	7.0	4.7	4.9	**9.6**
		100	.342	5.4	5.9	6.3	6.6	5.8	5.8	**9.7**
		200	.347	4.9	5.6	5.1	5.5	5.0	5.1	**8.5**
		500	.374	6.8	4.9	5.5	6.3	5.7	5.7	**9.9**
	4	50	.274	7.1	7.5	**8.4**	**10.3**	4.4	4.9	**8.9**
		100	.336	6.5	6.8	6.2	**7.6**	5.0	5.2	**8.2**
		200	.352	5.7	6.7	5.5	6.0	4.6	4.8	**8.7**
		500	.377	4.4	5.8	4.9	5.7	4.9	5.0	7.4
	6	50	.234	**8.1**	**9.8**	**8.9**	**12.1**	2.8	3.7	6.1
		100	.332	6.5	7.2	7.1	**9.4**	4.8	5.1	**7.8**
		200	.344	5.3	5.3	5.1	6.9	4.4	4.6	6.8
		500	.352	5.5	3.8	4.1	5.1	3.6	3.6	6.3
10%/30%	2	50	.321	7.0	7.0	7.1	7.3	5.2	5.3	**9.6**
		100	.371	5.0	6.7	5.5	5.7	5.1	5.2	**9.2**
		200	.363	5.3	4.2	5.0	5.0	4.7	4.8	**8.5**
		500	.369	5.4	4.9	6.2	6.2	4.8	4.8	**9.3**
	4	50	.307	6.6	**8.3**	**7.6**	**9.2**	3.5	3.9	**7.7**
		100	.349	6.2	6.2	6.8	7.1	5.1	5.3	**8.9**
		200	.368	5.6	5.3	5.4	5.7	4.9	4.9	**8.2**
		500	.397	4.8	4.6	4.9	5.7	5.3	5.3	**8.1**
	6	50	.265	**7.8**	**10.0**	**9.2**	**11.5**	2.9	3.7	6.4
		100	.333	6.4	**8.1**	7.0	**8.6**	5.2	5.8	**8.6**
		200	.358	5.8	5.7	5.6	6.1	5.0	5.3	**8.1**
		500	.378	5.6	4.9	5.9	6.0	5.2	5.3	**8.4**

Note. Bold numbers refer to Type 1 error rates outside the [2.5%; 7.5%] interval. k = number of predictors; n = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4, $D_4^{\diamond}$ = pooling methods.

$D_4^{\diamond}$ was also higher than that of FIML and D_2. Overall, these results suggest that the pooling methods were fairly robust against the violation of the assumption that the parameter that are being tested have similar FMIs. However, the results also showed that the violation of this assumption can still have negative effects, for example, for $D_4^{\diamond}$, which is based on stronger assumptions, and the same may be true for stronger violations of this assumption in D_2, D_3, and D_4.

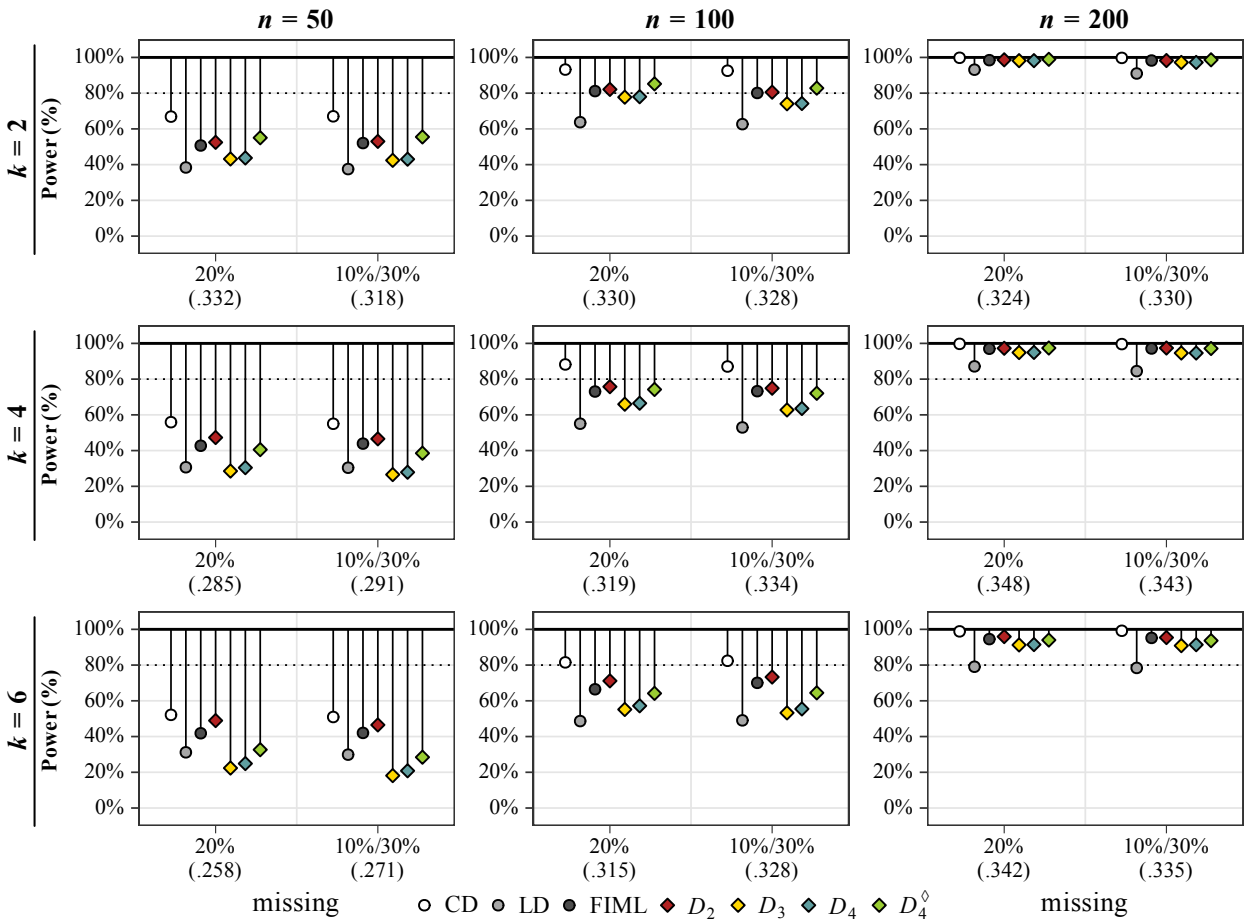

Figure A1

Power (in %) in conditions with medium effect size ($R^2 = .13$) in the supplemental simulation to Study 1. Numbers in parentheses denote the empirical (average) fraction of missing information per condition (FMI). n = sample size; k = number of predictors; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; D_2, D_3, D_4, $D_4^{\Diamond}$ = pooling methods.

B) Computer Code for the Example Analysis

Given below is the computer code needed to run the example analysis presented in the main text. For a text-only version of the code that can be copied and pasted into a text editor, we refer readers to the OSF project page (https://osf.io/u9s4k).

```
# # # # # # # # # # # # # # # # # # # # #
#                                       #
#    Example Analysis of the EIKA Data  #
#        (Measurement Invariance)       #
#                                       #
# # # # # # # # # # # # # # # # # # # # #

# ***
# Setup
#

# load packages
library(mice)
library(lavaan)
library(mitml)
library(semTools)

# read data
load(file = "eika_data.rda")
# alternatively: eika <- read.csv("eika_data.csv")

# ***
# Multiple imputation (MI)
#

# run MI
eika.mice <- mice(data = eika, maxit = 50, m = 100, seed = 2944)

# convert to list
eika.imp <- mids2mitml.list(eika.mice)

# ***
# Specification of analysis models
#

# configural invariance model (form only)
configural <- "

  # measurement models
  E =~ e1_r + e2 + e3_r + e4 + e5_r + e6 + e7_r + e8
  PE =~ pe1_r + pe2 + pe3_r + pe4 + pe5_r + pe6 + pe7_r + pe8

  # factor means and variances
  E + PE ~ 0*1
  E ~~ 1*E
```

```
  PE ~~ 1*PE

  # residual covariances between raters
  e1_r ~~ pe1_r
  e2 ~~ pe2
  e3_r ~~ pe3_r
  e4 ~~ pe4
  e5_r ~~ pe5_r
  e6 ~~ pe6
  e7_r ~~ pe7_r
  e8 ~~ pe8

  # residual covariances between reverse-coded items
  e1_r ~~ e3_r + e5_r + e7_r
  e3_r ~~ e5_r + e7_r
  e5_r ~~ e7_r
  pe1_r ~~ pe3_r + pe5_r + pe7_r
  pe3_r ~~ pe5_r + pe7_r
  pe5_r ~~ pe7_r

"

# metric invariance model (loadings)
metric <- "

  # measurement models
  E =~ l1*e1_r + l2*e2 + l3*e3_r + l4*e4 + l5*e5_r + l6*e6 + l7*e7_r + l8*e8
  PE =~ l1*pe1_r + l2*pe2 + l3*pe3_r + l4*pe4 + l5*pe5_r + l6*pe6 + l7*pe7_r + l8*pe8

  # factor means and variances
  E + PE ~ 0*1
  E ~~ 1*E
  PE ~~ NA*PE

  # residual covariances between raters
  e1_r ~~ pe1_r
  e2 ~~ pe2
  e3_r ~~ pe3_r
  e4 ~~ pe4
  e5_r ~~ pe5_r
  e6 ~~ pe6
  e7_r ~~ pe7_r
  e8 ~~ pe8

  # residual covariances between reverse-coded items
  e1_r ~~ e3_r + e5_r + e7_r
  e3_r ~~ e5_r + e7_r
  e5_r ~~ e7_r
  pe1_r ~~ pe3_r + pe5_r + pe7_r
  pe3_r ~~ pe5_r + pe7_r
  pe5_r ~~ pe7_r

"
```

```
# scalar invariance model (loadings + intercepts)
scalar <- "

  # measurement models
  E =~ l1*e1_r + l2*e2 + l3*e3_r + l4*e4 + l5*e5_r + l6*e6 + l7*e7_r + l8*e8
  PE =~ l1*pe1_r + l2*pe2 + l3*pe3_r + l4*pe4 + l5*pe5_r + l6*pe6 + l7*pe7_r + l8*pe8

  # factor means and variances
  E ~ 0*1
  PE ~ NA*1
  E ~~ 1*E
  PE ~~ NA*PE

  # item intercepts
  e1_r + pe1_r ~ t1*1
  e2 + pe2 ~ t2*1
  e3_r + pe3_r ~ t3*1
  e4 + pe4 ~ t4*1
  e5_r + pe5_r ~ t5*1
  e6 + pe6 ~ t6*1
  e7_r + pe7_r ~ t7*1
  e8 + pe8 ~ t8*1

  # residual covariances between raters
  e1_r ~~ pe1_r
  e2 ~~ pe2
  e3_r ~~ pe3_r
  e4 ~~ pe4
  e5_r ~~ pe5_r
  e6 ~~ pe6
  e7_r ~~ pe7_r
  e8 ~~ pe8

  # residual covariances between reverse-coded items
  e1_r ~~ e3_r + e5_r + e7_r
  e3_r ~~ e5_r + e7_r
  e5_r ~~ e7_r
  pe1_r ~~ pe3_r + pe5_r + pe7_r
  pe3_r ~~ pe5_r + pe7_r
  pe5_r ~~ pe7_r

"

# scalar invariance model (loadings + intercepts)
partial.scalar <- "

  # measurement models
  E =~ l1*e1_r + l2*e2 + l3*e3_r + l4*e4 + l5*e5_r + l6*e6 + l7*e7_r + l8*e8
  PE =~ l1*pe1_r + l2*pe2 + l3*pe3_r + l4*pe4 + l5*pe5_r + l6*pe6 + l7*pe7_r + l8*pe8

  # factor means and variances
  E ~ 0*1
```

```
  PE ~ NA*1
  E ~~ 1*E
  PE ~~ NA*PE

  # item intercepts
  e1_r + pe1_r ~ t1*1
  e2 + pe2 ~ t2*1
  e3_r + pe3_r ~ NA*1
  e4 + pe4 ~ t4*1
  e5_r + pe5_r ~ t5*1
  e6 + pe6 ~ t6*1
  e7_r + pe7_r ~ t7*1
  e8 + pe8 ~ t8*1

  # residual covariances between raters
  e1_r ~~ pe1_r
  e2 ~~ pe2
  e3_r ~~ pe3_r
  e4 ~~ pe4
  e5_r ~~ pe5_r
  e6 ~~ pe6
  e7_r ~~ pe7_r
  e8 ~~ pe8

  # residual covariances between reverse-coded items
  e1_r ~~ e3_r + e5_r + e7_r
  e3_r ~~ e5_r + e7_r
  e5_r ~~ e7_r
  pe1_r ~~ pe3_r + pe5_r + pe7_r
  pe3_r ~~ pe5_r + pe7_r
  pe5_r ~~ pe7_r

"

# ***
# LRTs in MI (mitml)
#

# fit configural, metric, and scalar invariance models
mi.configural <- with(
  eika.imp,
  cfa(model = configural, estimator = "ML", std.lv = TRUE),
  include.data = TRUE
)

mi.metric <- with(
  eika.imp,
  cfa(model = metric, estimator = "ML", std.lv = TRUE),
  include.data = TRUE
)

mi.scalar <- with(
  eika.imp,
```

```
  cfa(model = scalar, estimator = "ML", std.lv = TRUE),
  include.data = TRUE
)

# pool parameter estimates
testEstimates(mi.configural)
testEstimates(mi.metric)
testEstimates(mi.scalar)

# compare models
anova(mi.configural, mi.metric, mi.scalar, method = "D2")
anova(mi.configural, mi.metric, mi.scalar, method = "D3")
anova(mi.configural, mi.metric, mi.scalar, method = "D4")

# fit partial (scalar) invariance model
mi.partial.scalar <- with(
  eika.imp,
  cfa(model = partial.scalar, estimator = "ML", std.lv = TRUE),
  include.data = TRUE
)

# compare models for metric and partial scalar invariance
anova(mi.metric, mi.partial.scalar, method = "D2")
anova(mi.metric, mi.partial.scalar, method = "D3")
anova(mi.metric, mi.partial.scalar, method = "D4")

# ***
# LRTs in MI (semTools)
#

# fit configural, metric, and scalar invariance models
mi.semTools.configural <- cfa.mi(
  model = configural, data = eika.imp, estimator = "ML", std.lv = TRUE
)

mi.semTools.metric <- cfa.mi(
  model = metric, data = eika.imp, estimator = "ML", std.lv = TRUE
)

mi.semTools.scalar <- cfa.mi(
  model = scalar, data = eika.imp, estimator = "ML", std.lv = TRUE
)

# compare models
lavTestLRT.mi(mi.semTools.metric, h1 = mi.semTools.configural, test = "D2")
lavTestLRT.mi(mi.semTools.scalar, h1 = mi.semTools.metric, test = "D2")
lavTestLRT.mi(mi.semTools.metric, h1 = mi.semTools.configural, test = "D3")
lavTestLRT.mi(mi.semTools.scalar, h1 = mi.semTools.metric, test = "D3")

# fit partial (scalar) invariance model
mi.semTools.partial.scalar <- cfa.mi(
  model = partial.scalar, data = eika.imp, estimator = "ML", std.lv = TRUE
)
```

```
# compare models for metric and partial scalar invariance
lavTestLRT.mi(mi.semTools.metric, h1 = mi.semTools.partial.scalar, test = "D2")
lavTestLRT.mi(mi.semTools.metric, h1 = mi.semTools.partial.scalar, test = "D3")

# ***
# LRTs in FIML (lavaan)
#

# syntax snippet for auxiliary variables
auxiliary <- "

  # covariances between items and auxiliary variables
  sex + read + math + iq ~~ e1_r + e2 + e3_r + e4 + e5_r + e6 + e7_r + e8 +
                            pe1_r + pe2 + pe3_r + pe4 + pe5_r + pe6 + pe7_r + pe8

  # covariances between auxiliary variables
  sex ~~ read + math + iq
  read ~~ math + iq
  math ~~ iq

"

# fit configural, metric, and scalar invariance models
fiml.configural <- cfa(
  eika, model = paste(configural, auxiliary), estimator = "ML", missing = "FIML",
  std.lv = TRUE, fixed.x = FALSE
)

fiml.metric <- cfa(
  eika, model = paste(metric, auxiliary), estimator = "ML", missing = "FIML",
  std.lv = TRUE, fixed.x = FALSE
)

fiml.scalar <- cfa(
  eika, model = paste(scalar, auxiliary), estimator = "ML", missing = "FIML",
  std.lv = TRUE, fixed.x = FALSE
)

# print parameter estimates
parameterEstimates(fiml.configural, fmi = TRUE, remove.nonfree = TRUE)
parameterEstimates(fiml.metric, fmi = TRUE, remove.nonfree = TRUE)
parameterEstimates(fiml.scalar, fmi = TRUE, remove.nonfree = TRUE)

# compare models
anova(fiml.configural, fiml.metric, fiml.scalar)

# fit partial (scalar) invariance model
fiml.partial.scalar <- cfa(
  eika, model = paste(partial.scalar, auxiliary), estimator = "ML", missing = "FIML",
  std.lv = TRUE, fixed.x = FALSE
)
```

```
# compare models for metric and partial scalar invariance
anova(fiml.metric, fiml.partial.scalar)
```

C) Additional Tables

This supplement contains additional tables for all simulation results from Studies 1, 2, and 3.

List of Tables

Table C1-1

Study 1: Type 1 Error Rates in % ($R^2 = 0$) in Conditions with MCAR Data

							MI	
MD	k	n	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	2	50	.121	6.1	6.8	6.1	5.9	5.5	5.5
		100	.127	5.5	5.6	5.3	5.3	5.1	5.1
		200	.111	5.8	5.3	5.4	5.2	5.3	5.3
		500	.089	5.0	5.0	5.0	4.9	4.7	4.7
	4	50	.108	6.6	6.6	6.9	6.6	4.9	5.0
		100	.106	5.5	5.0	5.2	5.3	4.5	4.6
		200	.112	5.8	6.1	5.6	5.3	5.6	5.6
		500	.095	5.8	4.6	5.0	5.0	5.0	5.0
	6	50	.106	**9.3**	**9.3**	**8.9**	**8.6**	6.0	6.1
		100	.114	6.5	6.9	6.7	6.4	5.6	5.6
		200	.101	5.1	5.5	4.8	5.1	4.7	4.7
		500	.106	6.7	6.6	6.9	6.8	6.6	6.6
20%	2	50	.232	6.5	7.2	6.8	6.3	5.0	5.0
		100	.234	5.1	4.9	5.3	5.1	4.6	4.6
		200	.247	5.0	5.5	5.9	5.9	5.7	5.8
		500	.198	5.0	5.1	4.1	4.2	4.1	4.1
	4	50	.237	6.9	**8.7**	**8.4**	**7.7**	4.6	4.7
		100	.203	5.7	6.3	6.5	6.2	4.9	5.0
		200	.230	4.9	5.7	5.2	5.5	4.7	4.7
		500	.199	5.9	5.3	4.8	5.2	4.5	4.5
	6	50	.242	**7.9**	**10.0**	**9.4**	**8.4**	3.4	3.9
		100	.221	7.4	**7.7**	**8.5**	**8.4**	5.9	6.1
		200	.218	5.1	5.5	5.7	5.5	4.7	4.8
		500	.221	5.9	6.0	5.4	5.7	5.5	5.5
30%	2	50	.360	5.3	6.8	6.7	6.5	4.1	4.3
		100	.324	5.8	6.7	6.8	6.4	4.9	5.1
		200	.326	6.1	5.4	5.3	5.3	5.3	5.3
		500	.350	5.4	5.9	5.3	6.0	5.6	5.6
	4	50	.345	7.0	**9.1**	**9.2**	**8.1**	**2.3**	2.8
		100	.345	6.0	5.7	7.0	6.9	4.6	4.7
		200	.313	5.7	6.3	5.8	5.9	4.6	4.6
		500	.333	4.4	5.5	5.2	5.5	4.9	4.9
	6	50	.395	**7.7**	**13.8**	**10.7**	**9.4**	**0.8**	**1.5**
		100	.348	6.0	6.5	7.0	7.0	3.4	3.6
		200	.342	6.6	6.9	5.8	5.8	3.5	3.7
		500	.326	5.7	4.9	5.4	5.4	5.1	5.1

Note. Bold numbers refer to Type 1 error rates outside the $[2.5\%; 7.5\%]$ interval. k = number of predictors; n = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C1-2
Study 1: Power in % (R^2 = 0.02) in Conditions with MCAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	2	50	.116	14.7	12.9	13.9	13.9	12.6	12.7
		100	.105	23.5	19.4	21.2	21.1	19.9	19.9
		200	.113	40.6	33.6	38.3	38.1	37.0	37.0
		500	.080	81.3	69.1	76.6	77.2	75.6	75.6
	4	50	.115	12.9	13.0	12.9	12.9	10.4	10.4
		100	.105	18.8	16.0	18.1	18.4	15.3	15.3
		200	.126	31.0	25.7	30.1	30.2	28.1	28.1
		500	.112	72.3	63.3	71.1	71.5	68.8	68.8
	6	50	.109	12.8	12.5	12.5	12.1	8.2	8.2
		100	.111	16.3	15.4	16.1	16.2	14.2	14.3
		200	.107	28.3	23.7	27.6	28.2	25.1	25.1
		500	.128	67.0	54.6	63.2	64.4	58.8	58.8
20%	2	50	.235	13.7	12.3	12.8	12.7	9.6	9.6
		100	.234	22.9	15.8	21.2	21.8	17.8	17.9
		200	.247	41.2	29.1	36.9	37.9	33.7	33.7
		500	.217	82.1	62.3	74.0	74.6	71.3	71.3
	4	50	.215	12.4	11.6	12.8	12.1	6.5	6.7
		100	.233	18.5	14.8	18.1	18.4	13.4	13.6
		200	.225	33.8	22.5	29.6	30.9	25.6	25.7
		500	.220	72.9	51.5	67.2	68.2	63.8	63.8
	6	50	.243	12.9	13.9	13.1	12.9	5.5	5.9
		100	.224	16.6	13.9	16.8	17.0	10.7	11.1
		200	.220	28.1	19.5	23.9	25.5	20.0	20.1
		500	.220	65.5	45.2	61.6	63.2	55.5	55.5
30%	2	50	.349	15.1	12.4	13.7	13.7	7.9	8.0
		100	.324	22.6	13.5	18.2	18.9	14.8	14.9
		200	.309	39.6	21.0	29.3	29.8	25.2	25.3
		500	.307	82.5	47.3	70.4	70.9	65.3	65.3
	4	50	.352	11.9	12.5	12.4	11.9	4.0	4.5
		100	.322	18.9	11.7	15.6	16.1	10.1	10.2
		200	.327	33.8	19.0	27.1	28.2	20.1	20.2
		500	.328	74.2	39.8	62.1	63.3	53.8	54.1
	6	50	.388	13.3	14.8	15.3	13.6	2.1	3.0
		100	.341	16.9	12.6	16.0	16.0	7.1	7.4
		200	.335	28.5	16.9	25.1	26.7	16.5	16.7
		500	.347	62.0	33.2	53.1	55.6	42.5	42.5

Note. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C1-3
Study 1: Power in % (R^2 = 0.13) in Conditions with MCAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	2	50	.127	66.7	57.3	63.8	64.0	60.9	60.9
		100	.107	92.8	86.6	90.5	90.8	90.0	90.0
		200	.086	99.9	99.3	99.7	99.7	99.7	99.7
		500	.097	100.0	100.0	100.0	100.0	100.0	100.0
	4	50	.125	56.0	47.5	52.9	52.9	46.4	46.5
		100	.122	87.0	78.1	83.8	83.8	81.4	81.5
		200	.112	99.7	98.2	99.4	99.4	99.3	99.3
		500	.102	100.0	100.0	100.0	100.0	100.0	100.0
	6	50	.122	50.8	44.9	50.4	50.4	40.9	41.1
		100	.117	83.7	73.7	81.5	82.4	77.5	77.7
		200	.120	98.9	96.7	98.7	98.8	98.3	98.3
		500	.104	100.0	100.0	100.0	100.0	100.0	100.0
20%	2	50	.216	65.3	46.1	57.5	57.9	49.6	49.9
		100	.236	93.3	76.0	87.0	87.8	84.9	84.9
		200	.203	99.9	97.6	99.3	99.4	99.1	99.2
		500	.224	100.0	100.0	100.0	100.0	100.0	100.0
	4	50	.246	56.3	41.3	52.1	52.3	37.7	38.4
		100	.220	87.2	67.2	82.2	83.0	75.9	75.9
		200	.221	99.7	94.0	98.9	98.9	98.5	98.5
		500	.222	100.0	99.9	100.0	100.0	100.0	100.0
	6	50	.258	50.5	39.0	48.2	48.4	27.0	28.8
		100	.218	83.6	60.9	77.8	79.0	67.0	67.6
		200	.217	99.3	90.3	98.3	98.5	96.8	96.9
		500	.201	100.0	99.9	100.0	100.0	100.0	100.0
30%	2	50	.348	66.4	38.9	54.2	54.4	40.7	41.3
		100	.323	92.7	66.0	84.8	85.2	79.7	79.8
		200	.339	99.9	92.6	99.1	99.0	98.9	98.9
		500	.361	100.0	100.0	100.0	100.0	100.0	100.0
	4	50	.363	55.6	33.3	45.7	46.6	23.2	24.8
		100	.345	86.7	55.8	78.1	79.0	66.4	66.8
		200	.350	99.5	86.2	97.9	98.2	96.2	96.3
		500	.332	100.0	99.9	100.0	100.0	100.0	100.0
	6	50	.387	50.9	34.9	44.6	44.3	11.8	15.1
		100	.347	82.6	51.8	74.2	75.4	55.1	56.6
		200	.327	99.3	83.4	97.6	98.0	94.1	94.3
		500	.340	100.0	99.9	100.0	100.0	100.0	100.0

Note. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C1-4

Study 1: Power in % ($R^2 = 0.26$) in Conditions with MCAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	2	50	.103	96.1	90.6	94.1	94.1	93.1	93.1
		100	.113	99.9	99.7	99.8	99.8	99.6	99.6
		200	.085	100.0	100.0	100.0	100.0	100.0	100.0
		500	.089	100.0	100.0	100.0	100.0	100.0	100.0
	4	50	.123	91.5	84.5	89.4	89.4	85.8	85.9
		100	.113	99.8	99.0	99.7	99.7	99.7	99.7
		200	.103	100.0	100.0	100.0	100.0	100.0	100.0
		500	.120	100.0	100.0	100.0	100.0	100.0	100.0
	6	50	.109	88.8	81.0	87.2	87.2	80.5	80.8
		100	.124	99.6	98.0	99.2	99.4	98.9	98.9
		200	.105	100.0	100.0	100.0	100.0	100.0	100.0
		500	.116	100.0	100.0	100.0	100.0	100.0	100.0
20%	2	50	.209	95.7	82.0	91.8	91.6	87.5	87.7
		100	.216	100.0	98.2	99.7	99.8	99.7	99.7
		200	.181	100.0	100.0	100.0	100.0	100.0	100.0
		500	.194	100.0	100.0	100.0	100.0	100.0	100.0
	4	50	.255	90.7	73.8	86.4	86.7	76.0	76.9
		100	.228	99.7	96.2	99.4	99.4	98.5	98.5
		200	.214	100.0	100.0	100.0	100.0	100.0	100.0
		500	.218	100.0	100.0	100.0	100.0	100.0	100.0
	6	50	.254	87.6	70.6	82.8	83.5	63.2	65.4
		100	.236	99.8	94.0	99.1	99.2	98.0	98.0
		200	.227	100.0	99.9	100.0	100.0	100.0	100.0
		500	.219	100.0	100.0	100.0	100.0	100.0	100.0
30%	2	50	.369	95.0	71.5	88.2	88.3	79.7	80.1
		100	.318	100.0	94.2	99.4	99.4	98.7	98.7
		200	.298	100.0	99.9	100.0	100.0	100.0	100.0
		500	.309	100.0	100.0	100.0	100.0	100.0	100.0
	4	50	.372	90.6	64.0	82.1	82.1	59.7	62.0
		100	.333	99.9	91.0	98.6	98.8	96.8	96.9
		200	.360	100.0	99.7	100.0	100.0	100.0	100.0
		500	.338	100.0	100.0	100.0	100.0	100.0	100.0
	6	50	.411	87.4	60.2	79.3	78.8	40.1	45.9
		100	.354	99.6	85.9	97.9	98.2	93.8	94.0
		200	.314	100.0	99.5	100.0	100.0	100.0	100.0
		500	.333	100.0	100.0	100.0	100.0	100.0	100.0

Note. k = number of predictors; n = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C1-5

Study 1: Type 1 Error Rates in % ($R^2 = 0$) in Conditions with MAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	2	50	.194	6.3	5.5	5.7	6.2	5.3	5.4
		100	.194	6.7	5.9	6.3	6.8	5.9	5.9
		200	.248	5.4	4.9	5.3	5.3	5.5	5.5
		500	.200	5.9	6.3	5.6	5.6	5.3	5.3
	4	50	.176	6.9	6.8	6.6	**7.5**	5.3	5.5
		100	.230	6.1	6.7	6.7	6.9	6.4	6.5
		200	.213	5.3	5.2	4.9	4.9	4.7	4.7
		500	.221	5.3	5.8	4.5	4.6	4.5	4.5
	6	50	.154	**7.7**	**9.1**	**7.9**	**9.1**	5.6	5.9
		100	.200	6.2	7.5	7.2	**8.1**	6.0	6.3
		200	.217	6.7	**7.6**	6.9	7.4	7.0	7.1
		500	.230	5.2	4.9	4.6	5.0	4.3	4.3
20%	2	50	.307	6.6	6.5	6.8	7.3	5.4	5.6
		100	.346	6.0	6.5	6.3	6.6	5.9	5.9
		200	.358	6.1	5.8	6.0	6.3	6.0	6.0
		500	.370	4.5	4.8	4.6	4.9	4.6	4.6
	4	50	.260	7.5	7.1	6.7	**8.2**	3.7	4.1
		100	.335	7.1	6.7	6.8	**7.9**	5.3	5.5
		200	.356	5.9	5.6	5.8	6.9	5.3	5.3
		500	.364	5.0	5.0	4.4	5.3	4.0	4.0
	6	50	.250	**7.8**	**9.7**	**8.8**	**12.4**	3.5	4.1
		100	.293	7.2	7.2	7.3	**9.6**	5.1	5.5
		200	.354	5.7	6.0	6.0	7.1	4.9	5.1
		500	.385	4.9	5.1	4.3	5.3	4.3	4.4
30%	2	50	.418	5.5	5.9	5.6	6.2	2.9	3.1
		100	.472	4.7	5.8	5.5	6.3	4.5	4.7
		200	.476	4.9	5.3	5.0	5.3	4.1	4.2
		500	.512	5.5	4.3	5.5	5.5	5.3	5.3
	4	50	.381	6.6	**9.2**	**7.9**	**11.1**	2.5	3.2
		100	.440	5.4	6.6	6.5	**8.9**	3.8	4.3
		200	.453	6.3	5.9	5.6	6.7	4.9	5.3
		500	.483	4.8	4.8	4.9	5.6	4.6	4.7
	6	50	.339	**7.8**	**11.0**	**10.3**	**16.5**	**1.2**	**1.9**
		100	.413	6.3	7.5	6.9	**10.7**	3.6	4.2
		200	.465	5.7	6.0	6.2	**9.6**	4.6	5.0
		500	.481	5.2	5.8	5.3	**7.9**	5.1	5.3

Note. Bold numbers refer to Type 1 error rates outside the [2.5%; 7.5%] interval. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C1-6
Study 1: Power in % (R^2 = 0.02) in Conditions with MAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	2	50	.162	14.6	10.2	12.2	12.3	10.6	10.7
		100	.205	23.4	18.4	20.9	21.5	19.5	19.6
		200	.230	38.3	28.7	33.3	33.7	31.5	31.5
		500	.251	84.2	63.9	75.4	76.1	73.1	73.1
	4	50	.176	13.0	11.8	12.7	13.5	10.0	10.3
		100	.193	18.9	14.1	15.8	17.0	13.2	13.4
		200	.196	31.0	20.8	25.1	26.8	23.3	23.4
		500	.206	72.3	50.8	62.2	64.3	58.2	58.2
	6	50	.158	12.1	11.6	11.8	13.7	8.1	8.5
		100	.193	16.9	13.7	15.2	16.7	12.2	12.6
		200	.217	27.1	18.5	23.2	25.3	19.7	19.9
		500	.219	65.5	44.2	55.8	58.1	52.4	52.4
20%	2	50	.301	14.3	10.5	11.4	12.4	8.5	8.6
		100	.342	23.4	14.4	17.9	18.9	15.6	15.7
		200	.333	43.2	22.3	30.1	31.6	28.3	28.4
		500	.433	82.6	51.2	66.4	66.8	63.6	63.6
	4	50	.284	12.5	11.2	12.0	14.4	7.2	8.0
		100	.336	18.4	11.6	15.1	17.1	12.2	12.6
		200	.344	32.8	17.9	25.0	27.5	21.1	21.7
		500	.373	75.1	39.7	56.2	59.8	49.0	49.3
	6	50	.261	12.7	13.0	14.0	17.6	4.9	6.4
		100	.311	17.2	12.4	14.6	18.4	10.7	11.2
		200	.343	27.7	15.1	19.5	22.8	15.1	15.3
		500	.354	66.1	33.0	49.2	54.1	43.1	43.5
30%	2	50	.409	14.5	9.8	11.3	13.2	7.4	7.8
		100	.429	23.5	11.3	15.7	16.8	11.9	12.2
		200	.483	42.7	18.5	27.8	29.5	24.2	24.4
		500	.503	83.6	39.0	58.2	59.9	54.4	54.5
	4	50	.367	13.5	11.1	11.6	16.3	4.1	4.9
		100	.441	17.8	10.5	14.0	17.6	8.6	9.1
		200	.467	32.5	14.1	22.0	25.4	16.7	17.3
		500	.477	71.5	28.1	47.1	51.3	39.1	39.4
	6	50	.334	12.6	14.0	13.1	20.4	1.4	2.2
		100	.419	16.7	11.1	14.0	19.8	6.5	7.5
		200	.449	26.5	13.1	17.0	22.9	12.1	12.7
		500	.483	65.8	28.1	44.6	49.7	35.4	36.5

Note. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C1-7
Study 1: Power in % (R^2 = 0.13) in Conditions with MAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	2	50	.160	68.9	50.5	59.1	59.6	55.0	55.2
		100	.180	93.2	79.2	87.5	87.8	85.7	85.7
		200	.179	99.8	98.0	99.3	99.4	99.3	99.3
		500	.207	100.0	100.0	100.0	100.0	100.0	100.0
	4	50	.175	56.2	41.0	49.8	51.5	42.7	43.3
		100	.188	87.6	69.9	80.3	81.4	76.8	76.8
		200	.219	99.4	94.6	98.2	98.5	97.7	97.7
		500	.221	100.0	100.0	100.0	100.0	100.0	100.0
	6	50	.154	49.9	36.7	44.3	47.2	33.6	34.8
		100	.183	83.2	61.5	74.2	76.6	68.8	69.5
		200	.218	99.0	91.2	97.7	98.0	96.9	96.9
		500	.205	100.0	100.0	100.0	100.0	100.0	100.0
20%	2	50	.316	66.6	37.6	49.7	51.3	42.2	42.9
		100	.338	93.0	65.1	79.8	80.7	76.7	77.0
		200	.312	99.9	91.0	98.0	98.1	97.3	97.3
		500	.317	100.0	100.0	100.0	100.0	100.0	100.0
	4	50	.280	57.4	32.6	43.4	47.3	29.1	30.6
		100	.305	87.5	55.3	72.5	75.6	64.6	65.5
		200	.342	99.3	84.7	96.0	97.1	94.1	94.4
		500	.345	100.0	99.8	100.0	100.0	100.0	100.0
	6	50	.254	51.7	31.1	42.4	48.7	20.5	23.1
		100	.313	81.5	47.9	65.0	69.8	54.0	55.6
		200	.346	99.1	81.3	94.9	95.7	92.0	92.4
		500	.371	100.0	99.8	100.0	100.0	100.0	100.0
30%	2	50	.408	67.1	30.4	43.4	46.1	32.0	33.1
		100	.448	93.2	53.0	72.2	74.2	65.7	66.5
		200	.454	99.9	82.1	95.0	95.1	92.9	93.1
		500	.391	100.0	99.8	100.0	100.0	100.0	100.0
	4	50	.378	56.9	25.1	36.9	44.2	17.5	19.8
		100	.437	88.8	43.9	65.6	71.8	51.6	54.1
		200	.456	99.5	70.4	91.5	92.7	87.2	87.5
		500	.456	100.0	99.3	100.0	100.0	100.0	100.0
	6	50	.342	50.1	27.8	36.3	46.6	9.8	12.9
		100	.407	81.8	37.7	59.9	69.0	39.7	43.0
		200	.435	99.0	68.5	90.3	93.3	82.4	83.5
		500	.464	100.0	98.3	99.9	100.0	99.8	99.8

Note. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C1-8
Study 1: Power in % (R^2 = 0.26) in Conditions with MAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	2	50	.172	95.2	83.2	90.4	90.5	88.6	88.8
		100	.185	100.0	98.8	99.9	99.8	99.8	99.8
		200	.165	100.0	100.0	100.0	100.0	100.0	100.0
		500	.182	100.0	100.0	100.0	100.0	100.0	100.0
	4	50	.176	90.4	75.3	84.1	85.3	79.4	79.8
		100	.197	99.7	96.6	98.9	98.9	98.3	98.3
		200	.182	100.0	100.0	100.0	100.0	100.0	100.0
		500	.191	100.0	100.0	100.0	100.0	100.0	100.0
	6	50	.147	87.4	68.8	80.6	82.8	71.6	72.9
		100	.174	99.5	95.0	98.5	98.7	97.9	98.0
		200	.200	100.0	99.9	100.0	100.0	100.0	100.0
		500	.192	100.0	100.0	100.0	100.0	100.0	100.0
20%	2	50	.293	95.6	70.0	83.9	84.5	78.5	79.0
		100	.322	100.0	95.2	98.9	99.2	98.5	98.6
		200	.279	100.0	99.9	100.0	100.0	100.0	100.0
		500	.317	100.0	100.0	100.0	100.0	100.0	100.0
	4	50	.285	91.7	62.4	78.2	81.7	65.5	67.0
		100	.307	99.9	88.6	97.5	98.1	96.2	96.4
		200	.314	100.0	99.7	100.0	100.0	100.0	100.0
		500	.333	100.0	100.0	100.0	100.0	100.0	100.0
	6	50	.276	85.8	54.9	73.0	78.8	49.5	53.1
		100	.305	99.7	86.2	96.5	97.5	93.5	94.0
		200	.328	100.0	99.2	99.9	100.0	99.9	99.9
		500	.345	100.0	100.0	100.0	100.0	100.0	100.0
30%	2	50	.396	95.2	58.0	77.2	80.0	66.9	68.1
		100	.404	100.0	87.4	97.4	97.7	96.2	96.2
		200	.405	100.0	99.5	100.0	100.0	100.0	100.0
		500	.405	100.0	100.0	100.0	100.0	100.0	100.0
	4	50	.394	91.1	49.6	69.9	75.6	46.2	50.2
		100	.424	99.9	78.8	94.2	95.7	90.1	90.9
		200	.426	100.0	98.4	99.9	99.9	99.8	99.9
		500	.445	100.0	100.0	100.0	100.0	100.0	100.0
	6	50	.352	87.3	47.4	66.4	75.7	28.7	35.3
		100	.402	99.9	74.5	92.2	95.5	82.8	85.8
		200	.427	100.0	96.5	99.9	99.9	99.7	99.7
		500	.432	100.0	100.0	100.0	100.0	100.0	100.0

Note. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C2-1
Study 2: Type 1 Error Rates in % ($R^2 = 0$) in Conditions with MCAR Data

							MI		
MD	k	n	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	2	100	.125	5.8	5.7	5.6	5.6	5.2	5.3
		200	.116	5.3	5.5	5.8	5.8	5.9	5.9
		500	.115	5.0	4.9	5.4	5.7	5.5	5.5
		1000	.113	5.3	5.2	4.8	5.3	5.1	5.1
	4	100	.130	6.0	6.2	6.2	6.0	5.6	5.7
		200	.129	5.2	6.3	5.9	5.5	5.3	5.3
		500	.111	5.3	4.5	4.9	4.8	4.8	4.8
		1000	.118	4.0	4.6	5.1	4.9	4.9	4.9
	6	100	.151	6.2	6.2	6.3	6.0	5.4	5.4
		200	.140	6.0	5.7	6.5	6.4	5.9	5.9
		500	.121	4.1	5.2	5.3	5.8	5.6	5.6
		1000	.115	4.3	5.6	4.8	5.5	5.5	5.5
20%	2	100	.246	5.4	5.5	5.2	5.5	4.9	5.0
		200	.205	6.2	5.2	5.6	5.8	5.3	5.3
		500	.213	5.9	5.7	5.3	5.5	5.1	5.1
		1000	.197	4.3	5.6	3.6	4.7	4.8	4.8
	4	100	.284	5.8	6.5	5.8	5.9	4.8	4.8
		200	.254	5.3	6.1	5.8	6.1	5.1	5.1
		500	.216	5.7	5.9	5.4	6.1	5.5	5.5
		1000	.229	4.8	5.3	4.4	5.3	5.3	5.3
	6	100	.302	6.2	7.0	6.4	6.7	4.2	4.5
		200	.250	5.9	6.5	6.3	6.2	5.3	5.5
		500	.236	5.5	5.1	4.5	4.3	4.5	4.5
		1000	.230	4.4	6.1	4.6	5.1	5.0	5.0
30%	2	100	.370	5.8	5.8	5.2	5.4	4.2	4.3
		200	.349	5.2	5.5	4.6	5.1	4.7	4.7
		500	.325	5.1	5.4	4.5	5.0	4.7	4.7
		1000	.343	4.1	4.1	4.3	4.3	4.0	4.0
	4	100	.411	6.1	6.8	6.9	7.1	4.8	5.1
		200	.380	6.2	6.7	6.7	7.4	5.8	5.9
		500	.331	5.1	4.8	5.1	5.1	4.5	4.5
		1000	.355	5.5	5.1	5.3	5.9	5.5	5.5
	6	100	.445	6.5	**8.2**	6.9	7.2	3.3	3.6
		200	.382	5.9	6.0	5.5	6.5	4.2	4.2
		500	.353	4.9	5.9	4.9	5.7	4.5	4.5
		1000	.328	5.3	5.0	4.2	4.6	4.5	4.5

Note. Bold numbers refer to Type 1 error rates outside the [2.5%; 7.5%] interval. k = number of predictors; n = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C2-2
Study 2: Power in % (R^2 = 0.02) in Conditions with MCAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	2	100	.132	19.8	17.3	19.3	19.4	18.2	18.2
		200	.111	35.7	29.4	32.6	33.1	32.0	32.0
		500	.128	73.9	63.6	68.8	71.0	70.3	70.3
		1000	.137	96.6	93.2	94.5	96.0	95.7	95.7
	4	100	.135	17.0	14.5	15.8	16.2	14.0	14.1
		200	.115	26.8	22.1	25.1	26.5	24.1	24.2
		500	.117	62.5	51.1	58.5	60.5	58.2	58.2
		1000	.093	93.9	86.0	89.0	92.0	90.7	90.7
	6	100	.133	15.9	14.0	15.2	15.1	13.0	13.0
		200	.126	22.4	19.2	20.6	21.2	18.2	18.2
		500	.116	56.5	44.7	51.1	54.8	51.3	51.3
		1000	.125	87.1	78.5	83.1	84.7	83.6	83.6
20%	2	100	.251	19.5	13.8	17.1	17.7	15.9	15.9
		200	.239	35.6	24.8	30.5	32.3	29.3	29.4
		500	.210	73.1	53.1	63.6	66.2	63.5	63.5
		1000	.219	95.8	82.4	90.2	92.7	90.9	90.9
	4	100	.276	16.4	12.5	14.5	15.2	11.7	11.7
		200	.232	26.4	17.9	22.6	24.1	19.5	19.7
		500	.234	61.6	41.8	53.6	57.7	51.7	51.7
		1000	.232	91.7	74.1	81.9	88.6	83.5	83.5
	6	100	.291	14.8	11.2	14.1	14.0	9.5	9.6
		200	.242	22.9	15.2	20.7	22.1	16.4	16.5
		500	.237	52.1	33.9	45.5	50.1	41.9	41.9
		1000	.237	89.0	68.9	79.8	86.3	80.9	80.9
30%	2	100	.395	18.7	12.9	15.7	16.6	12.4	12.7
		200	.338	32.9	18.9	24.9	26.8	23.0	23.1
		500	.338	74.3	42.8	56.4	60.5	55.2	55.2
		1000	.328	96.5	71.5	85.5	90.4	87.1	87.1
	4	100	.436	15.7	12.3	14.8	15.5	10.0	10.4
		200	.379	27.6	16.8	21.4	23.8	17.9	18.0
		500	.345	59.5	32.4	45.8	51.5	42.6	42.6
		1000	.355	92.4	63.0	78.9	86.1	81.0	81.0
	6	100	.444	14.0	11.3	13.6	13.7	6.1	6.7
		200	.377	22.9	13.1	18.1	21.3	12.4	12.7
		500	.347	55.1	27.7	41.2	46.7	35.5	35.5
		1000	.323	87.2	52.8	70.6	81.3	71.6	71.6

Note. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C2-3
Study 2: Power in % (R^2 = 0.13) in Conditions with MCAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	2	100	.128	84.7	76.1	80.9	81.2	79.7	79.7
		200	.114	99.2	97.2	98.7	98.8	98.6	98.6
		500	.127	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.116	100.0	100.0	100.0	100.0	100.0	100.0
	4	100	.143	74.5	64.0	71.3	71.8	68.6	68.7
		200	.122	97.7	93.8	96.9	97.5	96.1	96.1
		500	.116	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.107	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.147	69.4	59.9	66.7	68.2	62.4	62.5
		200	.128	96.2	90.3	94.3	95.0	93.8	93.8
		500	.119	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.123	100.0	100.0	100.0	100.0	100.0	100.0
20%	2	100	.254	85.2	63.8	76.7	78.0	73.4	73.5
		200	.241	98.7	92.0	96.4	96.8	96.5	96.5
		500	.225	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.230	100.0	100.0	100.0	100.0	100.0	100.0
	4	100	.280	74.6	54.2	67.2	69.3	61.0	61.2
		200	.248	98.0	86.4	95.0	96.0	93.4	93.5
		500	.224	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.217	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.302	69.0	49.7	62.5	64.9	50.9	51.4
		200	.253	95.8	80.3	91.7	93.1	88.3	88.3
		500	.232	100.0	99.8	100.0	100.0	100.0	100.0
		1000	.202	100.0	100.0	100.0	100.0	100.0	100.0
30%	2	100	.381	85.7	53.2	71.4	73.3	65.9	66.1
		200	.341	98.9	83.1	95.1	96.3	94.5	94.5
		500	.331	100.0	99.9	100.0	100.0	100.0	100.0
		1000	.331	100.0	100.0	100.0	100.0	100.0	100.0
	4	100	.429	77.1	44.9	64.5	66.7	52.7	53.3
		200	.375	98.1	74.7	92.2	93.9	88.5	88.6
		500	.347	100.0	99.5	99.9	100.0	100.0	100.0
		1000	.330	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.450	69.2	37.4	58.2	61.7	39.4	40.6
		200	.383	95.3	67.8	88.7	90.7	82.6	82.7
		500	.348	100.0	98.7	100.0	100.0	100.0	100.0
		1000	.333	100.0	100.0	100.0	100.0	100.0	100.0

Note. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C2-4
Study 2: Power in % (R^2 = 0.26) in Conditions with MCAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	2	100	.125	99.4	97.8	98.9	99.0	98.7	98.7
		200	.115	100.0	100.0	100.0	100.0	100.0	100.0
		500	.098	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.125	100.0	100.0	100.0	100.0	100.0	100.0
	4	100	.141	98.2	94.0	97.2	97.4	96.5	96.5
		200	.117	100.0	100.0	100.0	100.0	100.0	100.0
		500	.110	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.103	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.148	97.1	91.8	95.4	96.0	94.0	94.0
		200	.126	100.0	99.9	100.0	100.0	100.0	100.0
		500	.119	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.125	100.0	100.0	100.0	100.0	100.0	100.0
20%	2	100	.260	99.4	93.5	97.7	97.9	97.3	97.3
		200	.244	100.0	100.0	100.0	100.0	100.0	100.0
		500	.224	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.204	100.0	100.0	100.0	100.0	100.0	100.0
	4	100	.285	97.7	87.9	95.5	96.0	93.5	93.5
		200	.243	100.0	99.7	100.0	100.0	100.0	100.0
		500	.227	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.234	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.298	96.4	82.1	92.7	93.8	88.2	88.6
		200	.247	100.0	99.1	100.0	100.0	100.0	100.0
		500	.241	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.233	100.0	100.0	100.0	100.0	100.0	100.0
30%	2	100	.392	99.3	84.9	96.2	96.5	94.3	94.3
		200	.348	100.0	99.3	100.0	100.0	100.0	100.0
		500	.341	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.335	100.0	100.0	100.0	100.0	100.0	100.0
	4	100	.389	97.9	75.3	92.9	94.2	87.1	87.5
		200	.358	100.0	97.3	99.9	100.0	99.8	99.9
		500	.343	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.350	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.447	96.5	70.5	90.8	91.9	80.1	81.2
		200	.386	100.0	96.1	99.8	99.9	99.7	99.7
		500	.334	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.331	100.0	100.0	100.0	100.0	100.0	100.0

Note. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C2-5

Study 2: Type 1 Error Rates in % ($R^2 = 0$) in Conditions with MAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	2	100	.138	5.7	5.6	5.5	5.2	5.0	5.0
		200	.153	5.5	5.5	5.1	5.5	5.5	5.5
		500	.135	4.3	4.9	4.9	4.9	4.9	4.9
		1000	.163	5.5	5.4	7.0	6.5	7.1	7.1
	4	100	.147	6.2	6.4	5.5	5.7	5.0	5.0
		200	.149	4.8	5.0	4.8	4.3	4.3	4.3
		500	.120	5.3	4.2	4.1	4.3	4.2	4.2
		1000	.138	4.6	4.4	4.5	4.6	4.6	4.6
	6	100	.154	6.5	6.9	6.3	6.2	5.3	5.3
		200	.144	5.9	5.5	5.9	5.9	5.3	5.3
		500	.142	6.5	6.4	6.9	6.9	6.8	6.8
		1000	.140	3.7	3.7	4.1	4.1	4.1	4.1
20%	2	100	.335	5.5	5.3	5.4	5.8	4.8	4.9
		200	.323	4.6	4.9	5.7	5.4	4.7	4.8
		500	.293	5.9	5.3	5.1	5.1	4.9	4.9
		1000	.288	4.7	4.5	4.8	4.4	4.7	4.7
	4	100	.334	6.2	5.7	5.6	5.6	3.9	4.2
		200	.329	5.3	5.8	5.8	5.6	5.1	5.2
		500	.302	6.0	5.2	4.9	5.8	4.9	5.0
		1000	.309	4.2	5.3	4.4	5.1	4.3	4.4
	6	100	.361	6.0	7.3	6.2	7.2	4.1	4.4
		200	.332	5.1	5.9	4.8	5.6	4.1	4.2
		500	.311	5.1	5.9	5.3	5.7	5.2	5.2
		1000	.278	4.7	5.1	4.4	4.4	4.1	4.1
30%	2	100	.519	5.9	5.9	5.5	6.1	4.4	4.9
		200	.485	5.8	5.1	5.5	6.0	5.4	5.5
		500	.491	4.8	6.2	5.7	5.7	5.1	5.2
		1000	.485	5.8	3.9	6.2	6.5	6.4	6.4
	4	100	.544	6.4	**7.8**	6.7	**8.0**	3.9	4.2
		200	.502	5.2	6.0	5.4	6.4	4.3	4.5
		500	.465	5.2	5.3	5.5	6.1	5.3	5.4
		1000	.474	6.2	5.5	4.0	5.5	5.0	5.0
	6	100	.553	6.3	**7.5**	6.7	**8.6**	2.8	3.6
		200	.497	5.9	6.3	5.5	6.7	4.2	4.7
		500	.490	5.2	5.5	5.5	6.6	5.3	5.4
		1000	.469	4.5	6.1	4.7	6.2	5.2	5.3

Note. Bold numbers refer to Type 1 error rates outside the [2.5%; 7.5%] interval. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C2-6
Study 2: Power in % (R^2 = 0.02) in Conditions with MAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	2	100	.156	19.6	16.1	17.5	18.4	17.0	17.1
		200	.141	36.9	28.0	31.2	33.1	31.6	31.6
		500	.133	72.9	59.7	64.1	66.8	65.0	65.0
		1000	.132	96.0	88.9	92.5	94.5	93.8	93.8
	4	100	.176	14.7	12.9	14.4	14.6	12.5	12.5
		200	.156	24.8	20.1	24.2	23.8	21.5	21.6
		500	.128	60.8	48.4	56.7	58.1	54.6	54.6
		1000	.134	92.5	81.9	89.0	90.4	88.9	88.9
	6	100	.167	13.8	13.2	13.5	13.5	10.8	11.0
		200	.144	24.0	19.0	23.5	23.5	20.3	20.3
		500	.136	56.1	43.9	52.7	53.7	49.3	49.3
		1000	.142	87.2	76.4	84.6	85.5	83.1	83.1
20%	2	100	.334	19.6	12.0	15.2	16.9	14.0	14.1
		200	.339	34.0	20.4	26.7	29.3	25.3	25.4
		500	.316	71.7	43.8	62.0	63.3	59.1	59.2
		1000	.269	97.1	74.0	89.4	91.8	89.3	89.4
	4	100	.363	15.7	11.5	14.5	15.0	10.6	10.9
		200	.320	27.7	16.3	23.8	24.1	18.7	19.0
		500	.295	63.7	33.4	52.5	52.9	45.9	46.0
		1000	.301	90.5	60.1	82.8	85.8	80.1	80.1
	6	100	.381	14.1	11.8	13.9	14.8	8.8	9.2
		200	.325	22.7	14.8	21.6	22.1	14.6	14.8
		500	.303	57.0	28.1	48.1	50.5	39.5	39.7
		1000	.291	89.7	55.3	80.1	82.8	75.1	75.4
30%	2	100	.506	20.2	9.2	13.8	15.5	10.9	11.1
		200	.492	34.2	11.9	21.6	24.2	18.1	18.5
		500	.475	73.3	24.5	51.7	54.7	46.5	46.6
		1000	.479	96.2	48.0	82.8	83.4	77.9	78.0
	4	100	.550	15.6	10.1	13.3	15.1	7.0	8.0
		200	.499	26.6	11.8	19.9	21.1	13.6	14.2
		500	.471	62.3	19.1	44.3	45.6	33.3	33.7
		1000	.476	92.9	36.7	76.9	79.6	67.9	68.1
	6	100	.560	13.9	10.9	12.5	14.9	4.5	6.1
		200	.520	22.4	10.2	18.0	20.3	10.5	11.5
		500	.493	55.8	18.2	37.4	42.7	27.4	27.8
		1000	.490	87.1	31.8	67.3	73.4	58.2	58.4

Note. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C2-7
Study 2: Power in % (R^2 = 0.13) in Conditions with MAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	2	100	.147	85.0	74.1	79.7	81.3	79.5	79.6
		200	.131	99.5	96.9	98.4	98.9	98.5	98.5
		500	.136	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.110	100.0	100.0	100.0	100.0	100.0	100.0
	4	100	.155	76.6	63.4	71.8	72.7	68.4	68.4
		200	.128	97.8	92.2	96.4	96.5	95.5	95.5
		500	.137	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.113	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.175	67.9	55.6	64.4	65.2	58.6	58.8
		200	.149	95.2	88.9	94.2	94.2	92.7	92.7
		500	.136	100.0	99.9	100.0	100.0	100.0	100.0
		1000	.132	100.0	100.0	100.0	100.0	100.0	100.0
20%	2	100	.309	85.2	54.5	73.0	75.3	69.8	70.0
		200	.302	98.9	83.3	95.3	96.5	95.2	95.2
		500	.279	100.0	99.8	100.0	100.0	100.0	100.0
		1000	.309	100.0	100.0	100.0	100.0	100.0	100.0
	4	100	.358	75.6	45.0	67.7	68.5	56.3	57.3
		200	.317	97.9	75.9	94.6	95.1	92.0	92.1
		500	.310	100.0	99.3	100.0	100.0	100.0	100.0
		1000	.299	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.356	70.1	38.9	60.9	63.1	46.1	47.7
		200	.325	95.2	69.6	90.2	90.5	84.2	84.6
		500	.299	100.0	99.0	100.0	100.0	99.9	99.9
		1000	.287	100.0	100.0	100.0	100.0	100.0	100.0
30%	2	100	.484	85.2	35.1	61.0	65.7	54.6	55.4
		200	.483	99.5	59.9	90.5	92.2	88.5	88.6
		500	.455	100.0	92.9	99.9	100.0	100.0	100.0
		1000	.447	100.0	100.0	100.0	100.0	100.0	100.0
	4	100	.520	75.6	27.7	55.8	58.2	37.7	40.2
		200	.479	97.9	46.4	88.0	90.0	78.2	79.0
		500	.462	100.0	87.5	100.0	100.0	99.9	99.9
		1000	.445	100.0	99.7	100.0	100.0	100.0	100.0
	6	100	.565	69.3	27.4	53.5	57.4	27.5	32.2
		200	.501	95.5	43.0	83.9	87.1	69.0	70.5
		500	.452	100.0	83.6	100.0	99.9	99.7	99.7
		1000	.456	100.0	99.4	100.0	100.0	100.0	100.0

Note. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C2-8
Study 2: Power in % (R^2 = 0.26) in Conditions with MAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	2	100	.163	99.5	96.9	98.6	98.9	98.5	98.5
		200	.134	100.0	100.0	100.0	100.0	100.0	100.0
		500	.131	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.108	100.0	100.0	100.0	100.0	100.0	100.0
	4	100	.164	98.1	93.4	97.0	97.2	96.2	96.2
		200	.144	100.0	99.9	100.0	100.0	100.0	100.0
		500	.142	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.117	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.158	97.0	89.6	95.2	95.7	93.2	93.3
		200	.158	100.0	99.7	100.0	100.0	100.0	100.0
		500	.127	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.124	100.0	100.0	100.0	100.0	100.0	100.0
20%	2	100	.284	99.4	85.6	96.3	96.8	95.7	95.8
		200	.293	100.0	99.1	100.0	100.0	100.0	100.0
		500	.273	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.261	100.0	100.0	100.0	100.0	100.0	100.0
	4	100	.335	97.6	75.7	94.2	94.6	89.4	89.7
		200	.320	100.0	97.2	100.0	100.0	100.0	100.0
		500	.293	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.270	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.359	96.4	71.8	92.1	92.8	82.8	84.0
		200	.310	100.0	95.1	99.9	99.9	99.8	99.8
		500	.303	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.284	100.0	100.0	100.0	100.0	100.0	100.0
30%	2	100	.473	99.2	59.7	90.9	93.1	87.3	87.8
		200	.452	100.0	88.4	100.0	100.0	99.9	99.9
		500	.457	100.0	99.9	100.0	100.0	100.0	100.0
		1000	.415	100.0	100.0	100.0	100.0	100.0	100.0
	4	100	.526	98.3	50.3	89.9	91.1	77.3	79.1
		200	.471	100.0	80.8	99.6	99.6	98.5	98.6
		500	.444	100.0	99.5	100.0	100.0	100.0	100.0
		1000	.442	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.551	95.9	45.4	86.3	88.2	64.1	68.7
		200	.470	100.0	73.9	99.8	99.7	98.0	98.2
		500	.461	100.0	99.1	100.0	100.0	100.0	100.0
		1000	.449	100.0	100.0	100.0	100.0	100.0	100.0

Note. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C3-1

Study 3: Type 1 Error Rates in % (ρ = 0) in Conditions with MCAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	4	100	.147	5.4	5.7	6.2	5.6	5.2	5.0
		200	.126	4.7	4.9	4.7	4.5	4.3	4.2
		500	.118	5.6	4.5	5.3	5.5	5.4	5.4
		1000	.131	5.8	5.6	6.3	6.8	6.6	6.5
	6	100	.147	6.2	7.1	7.1	6.6	5.9	5.7
		200	.138	5.9	6.1	5.9	5.5	5.3	5.3
		500	.124	4.8	5.0	4.9	4.8	4.4	4.4
		1000	.123	6.2	5.4	5.4	5.0	5.0	5.0
	8	100	.134	7.0	7.1	6.8	5.6	4.7	4.6
		200	.125	5.8	5.9	5.7	4.8	4.6	4.5
		500	.127	5.1	5.0	4.4	4.3	4.2	4.2
		1000	.117	5.7	5.8	5.3	5.8	5.3	5.3
20%	4	100	.307	6.5	7.3	7.1	6.9	5.0	4.8
		200	.268	5.9	6.1	5.7	5.6	5.1	5.1
		500	.244	5.3	4.7	4.7	5.3	4.5	4.5
		1000	.240	6.2	5.8	5.7	5.3	4.8	4.8
	6	100	.276	5.9	7.0	6.3	5.1	3.4	3.1
		200	.248	5.3	5.7	5.1	5.1	3.9	3.8
		500	.232	6.3	5.7	6.2	5.7	5.3	5.3
		1000	.218	4.0	4.1	3.1	3.3	2.7	2.7
	8	100	.281	6.3	**7.9**	7.4	5.3	3.6	3.4
		200	.238	6.5	6.1	5.5	5.3	4.5	4.3
		500	.240	5.7	5.9	6.1	5.7	5.3	5.3
		1000	.220	6.0	6.2	6.7	6.7	6.3	6.2
30%	4	100	.426	6.0	**7.5**	6.4	5.5	3.0	2.8
		200	.403	5.5	5.9	5.1	5.7	3.9	3.7
		500	.374	5.1	5.0	5.3	5.3	4.5	4.5
		1000	.376	6.0	6.0	5.6	6.1	4.8	4.8
	6	100	.423	5.9	**8.1**	7.1	5.9	3.0	2.6
		200	.373	6.5	6.4	5.3	5.2	3.5	3.3
		500	.352	4.6	5.7	4.7	5.1	4.1	4.1
		1000	.356	4.8	4.4	5.2	5.9	5.1	5.1
	8	100	.418	6.5	**8.9**	6.7	4.4	**1.8**	**1.7**
		200	.368	5.9	7.3	6.4	5.5	3.5	3.4
		500	.342	5.7	5.1	5.2	4.9	3.9	3.9
		1000	.356	5.4	6.4	5.2	6.3	5.3	5.3

Note. Bold numbers refer to Type 1 error rates outside the [2.5%; 7.5%] interval. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C3-2
Study 3: Power in % (ρ = 0.10) in Conditions with MCAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	4	100	.148	26.4	21.8	23.3	22.8	21.3	21.0
		200	.130	47.4	39.5	41.9	41.6	41.0	40.8
		500	.123	89.5	81.1	84.1	84.1	84.3	84.1
		1000	.121	99.9	98.7	98.8	99.0	98.8	98.8
	6	100	.140	37.2	30.9	33.1	30.5	28.1	27.6
		200	.124	66.0	53.9	59.8	57.9	57.0	56.8
		500	.129	98.9	94.1	97.0	96.9	96.8	96.8
		1000	.103	100.0	99.9	100.0	100.0	100.0	100.0
	8	100	.129	44.4	36.4	39.7	35.0	32.5	32.1
		200	.117	77.8	68.3	72.2	71.0	69.8	69.7
		500	.117	99.6	98.1	99.2	98.9	99.0	99.0
		1000	.110	100.0	100.0	100.0	100.0	100.0	100.0
20%	4	100	.289	26.7	19.9	21.7	20.2	16.3	15.7
		200	.255	49.5	33.2	38.1	38.0	34.6	34.2
		500	.248	89.1	70.7	78.7	78.8	77.4	77.4
		1000	.243	100.0	96.3	98.1	97.9	97.9	97.8
	6	100	.277	34.5	25.5	27.9	24.0	19.4	18.8
		200	.256	65.0	45.1	52.8	51.2	47.8	47.5
		500	.242	98.9	87.1	93.9	93.7	92.8	92.7
		1000	.227	100.0	99.2	99.9	100.0	100.0	100.0
	8	100	.274	42.5	30.8	34.8	27.3	21.9	21.2
		200	.246	78.5	56.2	64.6	61.6	58.1	57.7
		500	.239	99.9	94.3	97.3	96.9	96.6	96.6
		1000	.229	100.0	100.0	100.0	100.0	100.0	100.0
30%	4	100	.432	27.9	17.6	20.4	18.8	11.4	10.3
		200	.384	47.9	26.2	32.1	31.4	26.2	25.6
		500	.386	90.9	59.0	71.2	70.4	67.3	67.0
		1000	.370	100.0	88.2	94.9	94.5	95.0	94.9
	6	100	.425	34.9	21.8	25.7	21.5	13.2	12.2
		200	.381	65.2	36.9	46.2	43.4	38.2	37.4
		500	.361	97.8	75.0	89.1	88.0	86.3	86.2
		1000	.339	100.0	97.8	99.4	99.6	99.6	99.6
	8	100	.413	43.5	27.1	30.8	22.7	13.1	12.3
		200	.375	76.4	42.4	55.9	51.0	44.2	43.6
		500	.323	99.5	87.5	95.6	94.9	94.1	94.1
		1000	.334	100.0	99.7	100.0	100.0	100.0	100.0

Note. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C3-3
Study 3: Power in % (ρ = 0.20) in Conditions with MCAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	4	100	.136	81.6	72.0	75.4	74.3	72.7	72.3
		200	.128	99.0	96.2	97.6	97.5	97.2	97.2
		500	.126	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.133	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.131	94.2	88.4	91.1	89.4	88.9	88.7
		200	.131	99.9	99.5	99.7	99.7	99.7	99.7
		500	.101	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.115	100.0	100.0	100.0	100.0	100.0	100.0
	8	100	.123	98.8	96.5	97.5	96.8	96.3	96.2
		200	.117	100.0	99.9	100.0	100.0	100.0	100.0
		500	.115	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.113	100.0	100.0	100.0	100.0	100.0	100.0
20%	4	100	.288	82.8	62.3	70.4	66.9	60.8	59.7
		200	.269	98.6	90.8	95.2	94.5	94.2	94.0
		500	.247	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.230	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.266	94.4	78.7	87.3	83.5	79.8	79.3
		200	.239	100.0	98.6	99.8	99.6	99.4	99.2
		500	.228	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.213	100.0	100.0	100.0	100.0	100.0	100.0
	8	100	.263	98.7	88.3	94.8	90.6	87.2	86.4
		200	.238	100.0	99.9	100.0	100.0	99.9	99.9
		500	.227	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.208	100.0	100.0	100.0	100.0	100.0	100.0
30%	4	100	.404	80.3	50.0	61.0	56.6	45.5	44.2
		200	.370	99.1	80.8	91.6	89.3	87.2	86.7
		500	.355	100.0	99.8	100.0	100.0	100.0	100.0
		1000	.372	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.400	95.1	67.4	81.3	74.2	63.5	62.0
		200	.356	100.0	94.7	99.5	99.2	98.7	98.5
		500	.359	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.335	100.0	100.0	100.0	100.0	100.0	100.0
	8	100	.396	98.5	78.3	90.7	81.5	71.4	70.2
		200	.356	100.0	97.5	99.9	99.7	99.4	99.3
		500	.327	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.321	100.0	100.0	100.0	100.0	100.0	100.0

Note. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C3-4

Study 3: Type 1 Error Rates in % (ρ = 0) in Conditions with MAR Data

							MI	
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	4	100	.153	6.0	6.4	6.3	6.1	5.3	5.3
		200	.129	5.1	5.5	5.6	5.5	5.1	5.1
		500	.132	4.7	5.9	5.4	5.4	5.3	5.3
		1000	.118	5.5	4.4	4.7	4.9	4.6	4.6
	6	100	.135	6.3	6.8	6.8	5.8	4.8	4.7
		200	.132	4.9	5.2	5.1	4.9	4.5	4.5
		500	.121	5.9	6.5	6.1	6.2	6.0	6.0
		1000	.122	4.7	5.6	5.4	4.9	4.8	4.8
	8	100	.129	6.9	7.2	6.9	5.5	5.0	4.9
		200	.126	5.9	5.9	6.0	5.4	5.0	4.9
		500	.114	5.8	5.3	4.9	4.8	4.6	4.6
		1000	.110	4.9	4.1	4.8	4.4	4.3	4.3
20%	4	100	.292	5.6	7.0	6.6	6.4	4.8	4.5
		200	.257	6.2	6.8	6.1	5.9	5.0	4.8
		500	.235	5.5	5.3	5.2	5.0	4.8	4.7
		1000	.256	4.7	6.1	5.9	6.0	5.6	5.5
	6	100	.272	6.9	7.0	6.4	6.1	4.2	3.9
		200	.250	6.0	5.8	5.3	4.9	4.0	3.8
		500	.236	3.9	4.7	4.6	4.5	3.8	3.8
		1000	.233	5.8	6.1	6.4	6.2	5.5	5.5
	8	100	.279	6.8	**8.3**	**7.6**	5.5	3.5	3.0
		200	.241	5.2	6.2	5.5	4.9	3.9	3.9
		500	.231	5.3	4.6	4.9	4.9	4.5	4.3
		1000	.222	5.0	5.7	5.9	6.1	5.6	5.6
30%	4	100	.413	6.6	**7.7**	7.0	6.5	3.5	2.9
		200	.383	5.9	6.8	6.3	6.5	4.9	4.8
		500	.364	5.1	5.9	5.4	5.5	4.4	4.3
		1000	.368	5.3	5.2	5.1	5.4	4.7	4.6
	6	100	.407	6.1	**8.9**	**7.7**	5.8	3.0	**2.4**
		200	.366	5.3	6.7	5.6	6.0	3.5	3.4
		500	.347	5.1	5.4	5.1	5.4	4.3	4.1
		1000	.333	4.5	5.2	4.6	5.5	4.1	4.1
	8	100	.406	6.0	**8.2**	7.3	4.4	**1.6**	**1.3**
		200	.373	5.1	6.2	6.3	5.7	3.5	3.3
		500	.318	5.6	5.3	4.5	4.0	3.1	3.0
		1000	.332	4.7	4.9	4.8	5.2	4.0	4.0

Note. Bold numbers refer to Type 1 error rates outside the [2.5%; 7.5%] interval. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C3-5
Study 3: Power in % (ρ = 0.10) in Conditions with MAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	4	100	.145	27.4	24.4	25.3	24.7	22.7	22.3
		200	.139	46.0	40.2	42.1	41.0	40.4	40.2
		500	.116	88.8	81.1	84.7	84.3	83.9	83.9
		1000	.112	99.6	99.2	99.4	99.4	99.4	99.4
	6	100	.144	34.7	30.2	31.6	29.6	27.0	26.5
		200	.128	64.3	56.6	58.9	57.9	57.2	57.0
		500	.121	98.1	95.0	96.4	95.9	95.9	95.9
		1000	.120	100.0	100.0	100.0	100.0	100.0	100.0
	8	100	.141	44.3	37.5	38.9	35.0	31.8	31.5
		200	.120	76.1	66.5	71.0	68.5	67.3	67.0
		500	.113	99.7	98.9	99.0	99.1	99.1	99.1
		1000	.110	100.0	100.0	100.0	100.0	100.0	100.0
20%	4	100	.278	25.8	20.1	20.9	19.0	15.7	15.1
		200	.263	46.8	33.5	36.9	35.8	32.4	32.0
		500	.259	89.6	74.3	80.1	78.9	78.5	78.3
		1000	.212	100.0	97.3	98.6	98.3	98.3	98.3
	6	100	.278	35.7	26.8	28.7	24.5	19.7	18.7
		200	.237	66.1	49.5	54.5	51.6	49.0	48.6
		500	.228	97.7	89.4	93.0	92.3	92.4	92.3
		1000	.226	100.0	99.8	100.0	100.0	99.9	99.9
	8	100	.269	44.6	32.5	36.2	28.5	21.4	20.3
		200	.235	78.2	59.5	66.2	62.1	58.7	58.2
		500	.216	99.5	96.5	98.1	98.0	97.9	97.9
		1000	.201	100.0	100.0	100.0	100.0	100.0	100.0
30%	4	100	.403	26.5	18.1	18.6	16.5	10.6	9.7
		200	.382	48.9	29.3	33.8	32.2	27.8	26.7
		500	.354	89.6	64.6	71.9	71.2	69.2	68.6
		1000	.361	99.8	93.5	96.2	95.8	95.9	95.9
	6	100	.400	35.8	23.2	26.4	21.1	12.9	11.7
		200	.357	66.3	39.8	46.6	44.1	39.2	38.1
		500	.344	97.9	80.1	89.7	88.8	87.7	87.7
		1000	.337	100.0	98.8	100.0	99.9	99.8	99.8
	8	100	.393	43.7	26.4	31.4	20.7	11.9	10.2
		200	.354	77.4	46.3	56.4	52.0	45.1	43.6
		500	.340	99.7	90.8	97.1	96.3	95.6	95.5
		1000	.326	100.0	99.8	99.9	100.0	99.9	99.9

Note. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

Table C3-6
Study 3: Power in % (ρ = 0.20) in Conditions with MAR Data

							MI		
MD	*k*	*n*	FMI	CD	LD	FIML	D_2	D_3	D_4
10%	4	100	.133	81.1	73.4	75.7	74.2	72.2	71.9
		200	.122	98.8	96.8	97.6	97.4	97.4	97.4
		500	.122	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.110	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.127	94.5	89.5	91.6	90.4	88.9	88.8
		200	.120	99.9	99.9	99.8	99.8	99.8	99.8
		500	.108	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.119	100.0	100.0	100.0	100.0	100.0	100.0
	8	100	.125	98.5	96.5	97.7	96.7	96.0	95.9
		200	.108	100.0	100.0	100.0	100.0	100.0	100.0
		500	.110	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.114	100.0	100.0	100.0	100.0	100.0	100.0
20%	4	100	.261	82.0	64.7	70.1	67.9	63.0	61.7
		200	.239	98.6	92.3	95.4	94.7	94.0	93.8
		500	.232	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.232	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.261	94.6	82.3	87.2	83.3	79.2	78.4
		200	.228	100.0	99.4	99.8	99.7	99.7	99.7
		500	.222	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.201	100.0	100.0	100.0	100.0	100.0	100.0
	8	100	.255	98.8	91.8	95.6	91.9	88.0	87.2
		200	.231	100.0	99.9	99.9	99.9	99.8	99.8
		500	.216	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.207	100.0	100.0	100.0	100.0	100.0	100.0
30%	4	100	.393	81.7	53.1	61.6	57.7	47.7	45.2
		200	.369	98.8	87.2	92.1	91.8	89.2	88.2
		500	.340	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.343	100.0	100.0	100.0	100.0	100.0	100.0
	6	100	.389	94.8	73.0	82.7	75.5	66.0	63.4
		200	.363	99.9	96.6	99.0	98.7	98.2	97.9
		500	.335	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.322	100.0	100.0	100.0	100.0	100.0	100.0
	8	100	.386	98.2	84.2	91.6	83.5	72.7	70.2
		200	.346	100.0	99.2	99.9	99.7	99.5	99.5
		500	.329	100.0	100.0	100.0	100.0	100.0	100.0
		1000	.310	100.0	100.0	100.0	100.0	100.0	100.0

Note. *k* = number of predictors; *n* = sample size; FMI = empirical fraction of missing information per condition; CD = complete data; LD = listwise deletion; FIML = full-information maximum likelihood; MI = multiple imputation; D_2, D_3, D_4 = pooling methods.

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