Supplemental Material

Supplemental Material A: The Random Experiment in LST-R Theory

LST and LST-R theory differ in their formulation of the random experiment. For the sake of simplicity, we considered a design with two measurement occasions. The random experiment defined in LST-R theory implies a temporal ordering of the elements in Ω. The set of possible outcomes Ω is defined as,

$$\Omega = \Omega_{U_0} \times \Omega_{E_1} \times \Omega_{S_1} \times \Omega_{O_1} \times \Omega_{E_2} \times \Omega_{S_2} \times \Omega_{O_2}, \quad (1)$$

where Ω_{U_t} is the set of persons at the time of measurement t, and Ω_{E_t} is the set of experiences between measurements $t = t - 1$ and t. Ω_{S_t} is the set of situations at measurement t, and Ω_{O_t} is the set of observations at measurement t. The person variable at measurement t=1 is defined by the mapping $U_1 : \Omega \rightarrow \Omega_{U_0} \times \Omega_{E_1}$, meaning that U_1 assigns the Cartesian product of Ω_{U_0} and Ω_{E_1} to each outcome Ω. In other words, a person at measurement time $t = 1$ results from the same person at measurement time $t = 0$ and the experiences the person has had in the meantime (Eid, Holtmann, Santangelo, & Ebner-Priemer, 2017; Steyer, Mayer, Geiser, & Cole, 2015).

For measurement occasion t=2, the mapping is $U_2 : \Omega \rightarrow \Omega_{U_0} \times \Omega_{E_1} \times \Omega_{S_1} \times \Omega_{O_1} \times \Omega_{E_2}$, meaning that U_2 assigns the Cartesian product of Ω_{U_0}, Ω_{E_1}, Ω_{S_1}, Ω_{O_1}, and Ω_{E_2} to each outcome Ω. In other words, a person at measurement time $t = 2$ results from the same person at measurement time $t = 0$, the experiences the person has had between $t = 0$ and $t = 1$, as well as the experiences the person has had between $t = 1$ and $t = 2$, the situation the person was in at measurement time $t = 1$, and lastly, the observations of the person at measurement time $t = 1$. An element ω of Ω implies that a person-at-time-1 (u_1) is a part of the-person-at-time-2 (u_2). Compared to u_1, u_2 also encompasses the situation s_1, the observations o_1, and the experiences e_1 (Eid et al., 2017; Steyer et al., 2015). Lastly, the situation variable is defined for measurement t as the mapping $S_t : \Omega \rightarrow \Omega_{S_t}$, meaning that

S_t assigns Ω_{S_t} to each outcome Ω. Based on these mappings, the variables of LST-R theory can be defined (Eid et al., 2017; Steyer et al., 2015).

Supplemental Material B: Variables and Model Parameters

Table 1

Overview of the variables and the model parameters

Parameters	Names
LST-R Theory	
Y_{it}	manifest observed variable of measurement i and time t
ϵ_{it}	residual variable (measurement error)
ξ_{it}	latent trait variable
τ_{it}	latent state variable
ζ_{it}	state residual variable
TSO Model	
Y_{it}	manifest observed variable of measurement i and time t
δ_{it}	residual variable (measurement error)
T	latent trait variable
S_t	latent state variable
O_t	occasion-specific variable
ϵ_t	occasion-specific residual variable
λ^*_{t1}	trait effects or trait loadings
β_t	autoregressive effects
LST-AR Model	
Y_{it}	manifest observed variable of measurement i and time t
ϵ_{it}	residual variable (measurement error)
ξ	latent trait variable
τ_t	latent state variable
ζ_t	state residual variable
λ'_{t1}	trait effects or trait loadings
β_t	autoregressive effects

Supplemental Material C: Reliability, Consistency, and Specificity Based on Different Models

We illustrate the different ways to compute reliability $Rel(Y_{it})$, consistency $Con(Y_{it})$, pseudo-consistency $Con^*(Y_{it})$, specificity $Spe(Y_{it})$ and pseudo-specificity $Spe^*(Y_{it})$ coefficients using Y_{12} as an example. $Rel(Y_{it})$, $Con(Y_{it})$, and $Spe(Y_{it})$ are in line with LST-R theory, whereas the pseudo-coefficients $Con^*(Y_{it})$ and $Spe^*(Y_{it})$ are usually computed in the original TSO and LST-AR models.

LST-R Models (TSO and LST-AR)

$$Rel(Y_{12}) = Var(\tau_2)/Var(Y_{12})$$
$$Con(Y_{12}) = Var(\xi_2)/Var(Y_{12})$$
$$Spe(Y_{12}) = Var(\zeta_2)/Var(Y_{12})$$

TSO Model

$$\begin{aligned}
Y_{12} &= \nu_2 + S_2 + \delta_{12} \\
&= \nu_2 + \lambda^*_{20} + \lambda^*_{21} T + O_2 + \delta_{12} \\
&= \nu_2 + \lambda^*_{20} + \lambda^*_{21} T + \beta_1 O_1 + \epsilon_2 + \delta_{12}
\end{aligned}$$

$$Rel(Y_{12}) = Var(S_2)/Var(Y_{12})$$
$$Con^*(Y_{12}) = Var(\lambda^*_{21}\,T)/Var(Y_{12})$$
$$= [Var(S_2) - Var(O_2)]/Var(Y_{12})$$
$$Con(Y_{12}) = [Var(S_2) - Var(\epsilon_2)]/Var(Y_{12})$$
$$Spe^*(Y_{12}) = Var(O_2)/Var(Y_{12})$$
$$Spe(Y_{12}) = Var(\epsilon_2)/Var(Y_{12})$$

LST-AR Model

$$Y_{12} = \nu_2 + \tau_2 + \epsilon_{12},$$
$$= \nu_2 + \lambda^{'}_{20} + \lambda^{'}_{21}\,\xi + \beta_1\,\tau_1 + \zeta_2 + \epsilon_{12}$$
$$= \nu_2 + \lambda^{'}_{20} + \lambda^{'}_{21}\,\xi + \beta_1\,\xi + \beta_1\,\zeta_1 + \zeta_2 + \epsilon_{12}$$

$$Rel(Y_{12}) = Var(\tau_2)/Var(Y_{12})$$
$$Con^{'}(Y_{12}) = Var(\lambda^{'}_{21}\,\xi + \beta_1\,\xi)/Var(Y_{12})$$
$$Con(Y_{12}) = [Var(\tau_2) - Var(\zeta_2)]/Var(Y_{12})$$
$$Spe^{'}(Y_{12}) = Var(\beta_1\,\zeta_1 + \zeta_2)/Var(Y_{12})$$
$$Spe(Y_{12}) = Var(\zeta_2)/Var(Y_{12})$$

Conclusions

- $Rel(Y_{12})$ is the same in all models
- $Con^*(Y_{12})$ in the TSO model is equal to $Con^{'}(Y_{12})$ in the LST-AR model. It is the proportion of variance explained by the trait ξ_1 (the common trait in these models).

This does not correspond to the LST-R definition of $Con(Y_{12})$ which is the proportion of variance explained by ξ_2.

- As shown in the equations, $Con(Y_{12})$ could also be computed based on the original TSO and LST-AR models with time-varying trait loadings, but this is usually not done.
- $Spe^{*}(Y_{12})$ in the TSO model is equal to $Spe^{'}(Y_{12})$ in the LST-AR model. This does not correspond to the LST-R definition of $Spe(Y_{12})$.
- As shown in the equations, $Spe(Y_{12})$ could also be computed based on the original TSO and LST-AR models with time-varying trait loadings, but this is usually not done.

Supplemental Material D: Correlation Matrices

Table 2

Model-implied correlation matrix of the TSO model and the LST-AR model, with freely estimated trait loadings, as depicted in Figure 1a and Figure 1b, respectively

Latent Variables	Latent Variables					
	τ_1	τ_2	τ_3	τ_4	τ_5	ξ
τ_1	1.00					
τ_2	.72	1.00				
τ_3	.68	.79	1.00			
τ_4	.61	.76	.80	1.00		
τ_5	.59	.74	.78	.82	1.00	
ξ	.73	.91	.97	.83	.81	1.00

Note. The R code to calculate the prediction matrix is provided in OSF (Model 3 and Model 4).

Table 3

Model-implied correlation matrix of the TSO model and the LST-AR Model, with freely estimated trait loadings, as depicted in Figure 1c and Figure 1d, respectively

Latent Variables	Latent Variables														
	τ_1	τ_2	τ_3	τ_4	τ_5	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5
τ_1	1.00														
τ_2	.72	1.00													
τ_3	.68	.79	1.00												
τ_4	.61	.76	.80	1.00											
τ_5	.59	.74	.78	.82	1.00										
ξ_1	.73	.91	.97	.83	.81	1.00									
ξ_2	.79	.91	.97	.83	.80	1.00	1.00								
ξ_3	.68	.79	1.00	.80	.78	.97	.97	1.00							
ξ_4	.73	.92	.97	.83	.81	1.00	1.00	.97	1.00						
ξ_5	.70	.87	.92	.96	.85	.95	.95	.92	.95	1.00					
ζ_1	.68	.08	-.04	.00	.00	.00	.09	-.04	.00	.00	1.00				
ζ_2	.00	.41	-.22	.02	.01	.00	.00	-.22	.02	.01	.00	1.00			
ζ_3	.00	.00	.03	.00	.00	.00	.00	.00	.00	.00	.00	.00	1.00		
ζ_4	.00	.00	.00	.56	.27	.00	.00	.00	.00	.32	.00	.00	.00	1.00	
ζ_5	.00	.00	.00	.00	.52	.00	.00	.00	.00	.00	.00	.00	.00	.00	1.00

Note. The R code to calculate the prediction matrix is provided in OSF (Model 5 and Model 6).

Supplemental Material E: Results of the Bivariate Models

Table 4

Model fit and χ^2 difference test for bivariate LST-AR models and bivariate TSO models

Models	Model Fit						χ^2 Difference Test		
	χ^2	*df*	*p*	RMSEA [90% CI]	CFI	TLI	χ^2_d	df_d	p_d
Model 1	1821.34	241	.00	.04 [.04, .05]	.97	.96	-	-	-
Model 2	1770.51	241	.00	.04 [.04, .04]	.97	.96	-	-	-
Model 3	1747.49	233	.00	.04 [.04, .04]	.97	.96	73.85	8	<.001
Model 4	1747.49	233	.00	.04 [.04, .04]	.97	.96	23.02	8	<.01

Note. Model 1 = bivariate TSO model as introduced by Cole, Martin, and Steiger (2005), with constrained trait loadings over time (represented in Figure 1a). Model 2 = bivariate LST-AR model as introduced by Steyer and Schmitt (1994), with constrained trait loadings over time (represented in Figure 1b). Model 3 = in terms of LST-R theory reformulated bivariate TSO model as introduced by (Eid et al., 2017, represented in Figure 1c). Model 4 = in terms of LST-R theory reformulated bivariate LST-AR model as introduced in this article (represented in Figure 1d). *df* = degrees of freedom. RMSEA = root mean square error of approximation. CI = confidence interval. CFI = comparative fit index. TLI = Tucker-Lewis index. χ^2_d = difference in χ^2 values with respect to the less restrictive model. df_d = *df* for the χ^2 difference test. p_d = *p*-value of the χ^2 difference test. The R code to calculate the models is provided in OSF.

Table 5

Parameter estimates, variance, and correlation coefficients for bivariate LST-AR models and bivariate TSO models

Models	λ^f_{11}	λ^f_{21}	λ^f_{31}	λ^f_{41}	λ^f_{51}	$\lambda^f_{T_1}$	$\lambda^f_{T_2}$	$\lambda^f_{T_3}$	$\lambda^f_{T_4}$	$\lambda^f_{T_5}$
Model 1	1	1	1	1	1	.	.	.	.	.
Model 2	1	1	1	1	1	.	.	.	.	.
Model 3	.	.	.	.	.	1	1.14**	1.22**	1.2**	1.22**
Model 4	.	.	.	.	.	1	.91**	1.00**	.88**	.91**

Models	λ^c_{11}	λ^c_{21}	λ^c_{31}	λ^c_{41}	λ^c_{51}	$\lambda^c_{T_1}$	$\lambda^c_{T_2}$	$\lambda^c_{T_3}$	$\lambda^c_{T_4}$	$\lambda^c_{T_5}$
Model 1	1	1	1	1	1	.	.	.	.	.
Model 2	1	1	1	1	1	.	.	.	.	.
Model 3	.	.	.	.	.	1	1.09**	1.11**	1.1**	1.17**
Model 4	.	.	.	.	.	1	.87**	.88**	.69**	.77**

Models	$\text{Var}(T)^f$	β^f_1	β^f_2	β^f_3	β^f_4	$\lambda^f_{S_1}$	$\lambda^f_{S_2}$	$\lambda^f_{S_3}$	$\lambda^f_{S_4}$	λ^f_1
Model 1	.36**	.20**	.30**	.43**	.40**	.	.	.	.	2.01**
Model 2	.27**	.20**	.21**	.20**	.21**	.	.	.	.	2.01**
Model 3	.29**	.	.	.	.	.22**	.19**	.26**	.21**	2.01**
Model 4	.29**	.	.	.	.	.22**	.19**	.26**	.21**	2.01**

Models	$\text{Var}(T)^c$	β^c_1	β^c_2	β^c_3	β^c_4	$\lambda^c_{S_1}$	$\lambda^c_{S_2}$	$\lambda^c_{S_3}$	$\lambda^c_{S_4}$	λ^c_1
Model 1	.34**	.17*	.30**	.51**	.49**	.	.	.	.	1.61**
Model 2	.28**	.14**	.14**	.14**	.20**	.	.	.	.	1.61**
Model 3	.29**	.	.	.	.	.21**	.21**	.28**	.36**	1.61**
Model 4	.29**	.	.	.	.	.21**	.21**	.38**	.36**	1.61**

Models	$\text{Cor}(T^f, T^c)$
Model 1	.95**
Model 2	.91**
Model 3	.92**
Model 4	.92**

Note. $^*p \leq .05$. $^{**}p \leq .01$. Model 1 = bivariate TSO model as introduced by Cole et al. (2005), with constrained trait loadings over time (represented in Figure 1a). Model 2 = bivariate LST-AR model as introduced by Steyer and Schmitt (1994), with constrained trait loadings over time (represented in Figure 1b). Model 3 = in terms of LST-R theory reformulated bivariate TSO model as introduced by (Eid et al., 2017, represented in Figure 1c). Model 4 = in terms of LST-R theory reformulated bivariate LST-AR model as introduced in this article (represented in Figure 1d). i = measurement indicator. t = time of measurement. λ_{t1} = unstandardized trait loading of Model 1 and Model 2. λ_{T_t} = unstandardized trait loading of Model 3 and Model 4. β_t = unstandardized autoregressive effect of Model 1 and Model 2. λ_{S_t} = unstandardized autoregressive effect of Model 3 and Model 4. λ_1 = intercept of the latent trait (for $t = 1$ in Model 3 and Model 4). $\text{Var}(T)$ = variance of the latent trait (for $t = 1$ in Model 3 and Model 4). $\text{Cor}(T^{FA}, T^{CF})$ = correlation of both latent traits (for $t = 1$ in Model 3 and Model 4). T = trait. f = cancer-related fatigue. c = cognitive functioning. The R code to calculate the models is provided in OSF.

References

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