Supplemental Materials for "Meta-Analysis of Correlation Coefficients: A Cautionary Tale on Treating Measurement Error"

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1 Bias and coverage rates

Simulation study 1: Examining the influence of true score distribution with respect to bias and coverage probabilities of $\bar{\rho}$ with 12 continuous items on scales of X and Y.

		Meta-analysis using raw correlations						Meta-analysis using z transformed scores					
		$\bar{\rho} = 0$				$\bar{\rho} = .5$		$\bar{\rho} = 0$				$\bar{\rho} = .5$	
		T-Norm		T-Skewed		T-Norm		T-Skewed		T-Norm		T-Skewed	
K	$\bar{N}$	Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP
									Homogeneous case						
	30	.001	.957	.000	.953	-.012	.973	-.011	**.919**	.001	.965	.001	.959	-.004	.976	-.005	.927
	50	.000	.954	-.001	.957	-.004	.966	-.008	**.911**	.000	.959	-.001	.958	-.001	.969	-.004	**.918**
5	100	.002	.961	.004	.964	-.003	.956	-.005	.928	.002	.964	.004	.966	.000	.957	-.003	.929
	200	.000	.961	.001	.963	-.001	.961	-.005	**.909**	.000	.961	.001	.963	.000	.962	-.004	**.910**
	30	.002	.973	-.001	.969	-.006	.955	-.011	.941	.002	.975	.001	.970	.005	.955	.002	.952
	50	.001	.959	.002	.965	-.002	.955	-.007	.927	.001	.957	.003	.964	.003	.955	.002	.927
20	100	.000	.958	.000	.959	-.001	.963	-.004	.936	.000	.959	.000	.959	.001	.965	.000	.941
	200	.000	.955	.000	.957	-.002	.955	-.003	.944	.000	.956	.000	.958	.000	.956	.000	.948
	30	.000	.950	.001	.954	-.006	.949	-.012	.925	.000	.953	.003	.953	.006	.939	.002	.942
	50	.000	.959	.001	.960	-.004	.943	-.008	.937	.000	.960	.002	.959	.004	.940	.000	.954
50	100	-.001	.945	-.001	.972	-.002	.954	-.003	.943	-.001	.945	.000	.973	.001	.959	.001	.948
	200	-.001	.953	.000	.958	-.001	.954	-.002	.937	-.001	.954	.000	.958	.001	.959	.000	.946
									Heterogeneous case						
	30	.004	**.909**	.002	**.919**	-.007	**.921**	-.015	**.912**	.004	**.915**	.002	.926	.001	.932	-.008	**.921**
	50	.001	**.921**	.008	**.917**	-.004	.926	-.008	**.904**	.000	.927	.008	**.920**	.000	**.923**	-.005	**.912**
5	100	.002	**.907**	.003	**.887**	.000	**.884**	-.003	**.878**	.002	**.910**	.003	**.887**	.002	**.893**	-.002	**.882**
	200	-.001	**.886**	.003	**.892**	-.001	**.880**	-.001	**.892**	-.001	**.886**	.003	**.892**	-.001	**.880**	-.002	**.888**
	30	-.003	.930	-.001	.934	-.007	.939	-.015	.927	-.003	.931	.000	.935	.006	.932	-.004	.945
	50	-.002	**.923**	.003	.940	-.004	.928	-.008	.932	-.002	**.922**	.004	.939	.003	.926	.000	.943
20	100	.002	.949	-.001	.941	-.003	.939	-.007	.934	.002	.948	.000	.942	.000	.933	-.004	.940
	200	.001	.942	.001	.936	-.002	.931	-.003	.932	.001	.941	.001	.934	-.001	.928	-.001	.936
	30	.000	.941	.000	.950	-.007	.943	-.014	.930	.000	.943	.001	.950	.007	.932	-.002	.951
	50	-.001	.938	.000	.938	-.003	.955	-.007	.940	-.001	.937	.001	.935	.004	.945	.000	.945
50	100	-.001	.948	.000	.940	-.002	.944	-.006	.940	-.001	.950	.001	.941	.002	.941	-.002	.954
	200	-.001	.948	.000	.938	-.001	.944	-.003	.935	-.001	.948	.000	.938	.001	.940	-.002	.946

Note. T-Norm: Underlying true scores for X and Y are normal; T-Skewed: underlying true scores for X and Y are skewed; CP: Coverage probability. Coverage probability not in the range of [.925, .975] is highlighted in bold.

Simulation study 2: Meta-analysis with correction of measurement errors using the r-based approach: bias and coverage probabilities of $\bar{\rho}$ when the tau-equivalence assumption is met with 12 continuous items on scales of X and Y.

K	$\bar{N}$	$\bar{\rho}=0$				$\bar{\rho}=.1$				$\bar{\rho}=.3$				$\bar{\rho}=.5$			
		T-Norm		T-Skewed		T-Norm		T-Skewed		T-Norm		T-Skewed		T-Norm		T-Skewed	
		Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP
									Homogeneous case								
5	30	-.003	.952	-.006	.953	-.005	.955	.003	.958	-.007	.954	.014	.932	-.002	.953	.001	**.923**
	50	.000	.948	-.001	.958	.001	.963	.002	.969	.001	.962	.014	.938	-.001	.972	.006	**.906**
	100	.000	.962	-.001	.960	.001	.967	.004	.949	-.001	.963	.008	.944	-.001	.968	.002	.927
	200	.001	.964	.000	.971	-.001	.957	.000	.960	-.001	.966	.002	.941	-.001	.969	.001	.930
20	30	.002	.953	-.001	.949	-.003	.952	.008	.949	-.001	.954	.012	.936	.001	.963	.009	.926
	50	-.001	.951	.001	.973	.000	.957	.004	.960	-.002	.958	.010	.929	-.001	.969	.007	.936
	100	-.001	.963	-.001	.960	.001	.967	.004	.957	.000	.954	.007	.937	.000	.965	.004	**.923**
	200	.000	.962	.000	.961	.000	.962	.002	.962	-.001	.962	.002	.949	.000	.965	.001	.942
50	30	.000	.954	.001	.964	.000	.953	.009	.942	-.001	.961	.016	**.922**	-.001	.966	.009	**.923**
	50	-.001	.965	.000	.959	-.001	.957	.004	.948	-.001	.957	.011	.928	-.001	.957	.007	.937
	100	.000	.959	.000	.961	.000	.960	.003	.943	-.001	.944	.005	.937	.000	.965	.003	.946
	200	.000	.945	.000	.960	.000	.968	.002	.952	.000	.957	.003	.954	.000	.961	.002	.932
									Heterogeneous case								
5	30	.000	**.916**	.006	**.911**	-.001	**.906**	.002	**.888**	-.007	**.907**	.014	.925	-.006	**.913**	.001	**.899**
	50	.004	**.918**	-.001	**.903**	-.004	**.918**	.002	**.906**	.004	**.901**	.003	.940	-.002	**.911**	-.002	**.898**
	100	.001	**.882**	.001	**.874**	-.001	**.891**	.004	**.884**	.001	**.896**	.004	**.904**	.001	**.897**	-.001	**.882**
	200	.001	**.868**	.003	**.894**	.001	**.878**	.001	**.883**	.001	**.888**	.003	**.894**	.000	**.869**	.005	**.878**
20	30	.004	.941	.000	.926	.004	**.923**	.010	.945	.001	.929	.010	**.923**	-.003	.944	.006	.928
	50	.001	.943	.001	.948	-.002	.932	.005	.946	-.002	.936	.011	**.916**	-.002	.928	.001	**.921**
	100	.001	.944	.002	**.921**	-.003	.926	.002	.932	-.001	.931	.006	.949	-.001	.929	.002	.935
	200	.001	.936	.000	.936	.000	.938	.002	.927	-.001	.947	.002	.931	.000	.933	.001	.938
50	30	.002	.956	.002	.947	-.003	.931	.007	.937	.000	.957	.013	.930	-.002	.946	.006	.937
	50	.003	.937	.000	.927	-.001	.947	.006	.931	-.001	.947	.007	.929	-.003	.948	.004	.952
	100	.001	.940	.001	.945	.001	.936	.001	.934	.000	.946	.005	.946	-.001	.943	.000	.958
	200	-.001	.951	.000	.943	.001	.926	.000	.935	.001	.938	.002	.948	-.001	.935	.000	.945

Note. T-Norm: Underlying true scores for X and Y are normal; T-Skewed: underlying true scores for X and Y are skewed; CP: Coverage probability. Coverage probability not in the range of [.925, .975] is highlighted in bold.

Simulation study 2: Meta-analysis with correction of measurement errors using the z-based approach: bias and coverage probabilities of $\bar{\rho}$ when the tau-equivalence assumption is met with 12 continuous items on scales of X and Y.

K	$\bar{N}$	$\bar{\rho}=0$				$\bar{\rho}=.1$				$\bar{\rho}=.3$				$\bar{\rho}=.5$			
		T-Norm		T-Skewed		T-Norm		T-Skewed		T-Norm		T-Skewed		T-Norm		T-Skewed	
		Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP
									Homogeneous case								
5	30	-.003	.944	-.005	.926	-.003	.946	.006	.934	-.003	.946	.015	.925	.003	.932	-.011	**.912**
	50	.000	.939	-.001	.942	.002	.945	.003	.956	.003	.945	.016	.933	.002	.950	.005	**.893**
	100	.000	.948	-.001	.942	.002	.942	.004	.929	.000	.938	.009	**.923**	.001	.941	.002	**.918**
	200	.001	.951	.000	.954	-.001	.943	.001	.939	-.001	.947	.002	**.923**	.001	.941	.002	**.921**
20	30	.002	.947	.001	.942	-.001	.942	.015	.942	.007	.936	.019	.928	.010	.925	.011	.948
	50	-.001	.940	.002	.955	.002	.944	.007	.951	.002	.937	.014	.930	.006	.952	.010	.943
	100	-.001	.946	.000	.947	.002	.947	.005	.943	.002	.939	.009	.934	.003	.943	.006	.926
	200	.000	.951	.000	.948	.001	.951	.002	.945	.000	.944	.004	.941	.002	.943	.002	.943
50	30	.000	.941	.004	.954	.002	.947	.016	.929	.008	.933	.024	**.911**	.009	.928	.011	.951
	50	-.001	.956	.001	.947	.001	.947	.007	.939	.004	.939	.017	**.915**	.006	.933	.011	.952
	100	.000	.953	.000	.953	.001	.947	.004	.931	.001	.931	.008	.927	.003	.939	.005	.940
	200	.000	.932	.000	.950	.001	.952	.003	.942	.001	.944	.004	.946	.002	.937	.003	.932
									Heterogeneous case								
5	30	.001	**.903**	.008	**.906**	.000	**.900**	.003	**.889**	-.004	**.900**	.013	**.923**	-.003	**.901**	-.005	**.886**
	50	.004	**.900**	.000	**.901**	-.003	**.912**	.002	**.893**	.005	**.897**	.003	**.924**	.000	**.897**	-.004	**.903**
	100	.001	**.882**	.001	**.873**	-.001	**.891**	.004	**.885**	.002	**.889**	.004	**.896**	.001	**.890**	-.002	**.889**
	200	.001	**.868**	.003	**.892**	.000	**.874**	.001	**.883**	.001	**.890**	.002	**.891**	-.001	**.863**	.003	**.883**
20	30	.004	.937	.003	.934	.006	**.921**	.013	.949	.008	.932	.015	.931	.005	.930	.003	.952
	50	.001	.941	.003	.946	-.001	.927	.008	.952	.002	.939	.015	.930	.004	**.922**	.003	.946
	100	.002	.945	.002	**.924**	-.003	.925	.003	.934	.001	.935	.008	.956	.002	.937	.003	.944
	200	.001	.934	.000	.938	.000	.939	.002	.930	.000	.945	.002	.936	.001	.937	.002	.941
50	30	.002	.958	.005	.952	.000	.931	.012	.944	.007	.954	.019	.944	.008	.939	.006	.964
	50	.003	.938	.001	.930	.001	.947	.009	.936	.003	.940	.012	.935	.004	.943	.006	.963
	100	.001	.934	.002	.948	.002	.936	.002	.939	.003	.943	.007	.947	.002	.940	.002	.957
	200	-.001	.951	.000	.947	.001	.925	.000	.936	.002	.936	.003	.954	.001	.934	.001	.950

Note. T-Norm: Underlying true scores for X and Y are normal; T-Skewed: underlying true scores for X and Y are skewed; CP: Coverage probability. Coverage probability not in the range of [.925, .975] is highlighted in bold.

Simulation study 2: Meta-analysis with correction of measurement error using the z-based approach: bias and coverage probabilities of $\bar{\rho}$ when the tau-equivalence assumption is NOT met with 4 continuous items on scales of X and Y.

K	$\bar{N}$	$\bar{\rho}=0$				$\bar{\rho}=.1$				$\bar{\rho}=.3$				$\bar{\rho}=.5$			
		T-Norm		T-Skewed		T-Norm		T-Skewed		T-Norm		T-Skewed		T-Norm		T-Skewed	
		Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP
									Homogeneous case								
5	30	.002	**.921**	.003	**.895**	.008	**.905**	.010	**.912**	.013	**.901**	.013	**.905**	.028	**.882**	-.011	**.902**
	50	-.006	**.912**	-.001	**.913**	.009	**.908**	.011	**.905**	.032	**.895**	.030	**.911**	.042	**.861**	.029	**.887**
	100	.001	**.906**	-.005	**.910**	.015	**.895**	.010	**.894**	.031	**.898**	.039	**.845**	.051	**.837**	.049	**.849**
	200	.001	**.906**	.000	**.904**	.011	**.889**	.015	**.904**	.028	**.825**	.032	**.859**	.050	**.717**	.048	**.835**
20	30	.001	.939	-.001	.936	.009	.936	.013	.933	.014	.934	.031	**.917**	.030	**.896**	.014	.939
	50	.001	.931	.002	.948	.012	**.923**	.017	.933	.035	**.896**	.040	**.895**	.049	**.821**	.031	**.913**
	100	.001	**.924**	.001	.926	.010	**.921**	.015	.929	.033	**.833**	.039	**.865**	.053	**.627**	.052	**.798**
	200	.000	**.921**	.000	.940	.010	**.917**	.011	**.919**	.030	**.744**	.036	**.734**	.050	**.464**	.051	**.601**
50	30	.000	.951	-.002	.946	.012	.935	.011	.946	.032	**.885**	.021	**.923**	.028	**.895**	-.009	.946
	50	.001	.955	-.001	.947	.014	.932	.017	.943	.037	**.807**	.038	**.844**	.049	**.695**	.033	**.870**
	100	-.001	**.918**	.002	.944	.012	**.914**	.015	**.894**	.033	**.734**	.040	**.660**	.052	**.321**	.053	**.585**
	200	.000	.957	.001	.953	.010	**.900**	.012	**.879**	.031	**.508**	.035	**.531**	.050	**.045**	.050	**.263**
									Heterogeneous case								
5	30	.000	**.889**	.006	**.896**	.002	**.895**	-.011	**.888**	.000	**.895**	-.028	**.905**	.019	**.894**	-.034	**.902**
	50	-.002	**.893**	.011	**.908**	.011	**.906**	.018	**.888**	.027	**.884**	.022	**.895**	.039	**.859**	.020	**.880**
	100	.003	**.886**	-.009	**.889**	.009	**.869**	.007	**.876**	.035	**.866**	.032	**.894**	.040	**.857**	.036	**.867**
	200	.001	**.901**	.004	**.875**	.013	**.884**	.007	**.878**	.030	**.878**	.028	**.865**	.047	**.834**	.046	**.851**
20	30	-.001	.946	.001	.936	.004	.935	.007	.952	.019	**.920**	.015	.933	.019	.937	-.015	.955
	50	.002	.945	-.002	.942	.012	.935	.017	.941	.025	.925	.025	.935	.044	**.878**	.012	.942
	100	.003	.942	.000	.944	.010	.926	.012	.954	.031	**.885**	.034	**.901**	.046	**.827**	.044	**.886**
	200	.000	.929	-.004	.928	.011	.937	.012	**.905**	.028	**.890**	.033	**.882**	.048	**.739**	.046	**.811**
50	30	-.003	.936	-.002	.948	.009	.947	.002	.946	.018	.931	.007	.937	.017	**.924**	-.018	.934
	50	-.002	.942	-.001	.954	.009	.934	.015	.932	.033	**.897**	.032	**.897**	.043	**.806**	.023	.925
	100	.000	.936	.001	.957	.009	.936	.012	.940	.032	**.845**	.035	**.846**	.050	**.642**	.043	**.797**
	200	.001	.942	-.001	.934	.010	**.915**	.012	**.922**	.029	**.809**	.034	**.816**	.048	**.513**	.047	**.600**

Note. T-Norm: Underlying true scores for X and Y are normal; T-Skewed: underlying true scores for X and Y are skewed; CP: Coverage probability. Coverage probability not in the range of [.925, .975] is highlighted in bold.

Simulation study 2: Meta-analysis with correction of measurement errors using the z-based approach: bias and coverage probabilities of $\bar{\rho}$ when the tau-equivalence assumption is NOT met with 12 continuous items on scales of X and Y.

K	$\bar{N}$	$\bar{\rho}=0$				$\bar{\rho}=.1$				$\bar{\rho}=.3$				$\bar{\rho}=.5$			
		T-Norm		T-Skewed		T-Norm		T-Skewed		T-Norm		T-Skewed		T-Norm		T-Skewed	
		Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP	Bias	CP
									Homogeneous case								
5	30	.000	.943	-.004	.949	.003	.945	.011	.943	.012	.929	.023	**.920**	.020	.928	.009	**.883**
	50	-.003	.936	.002	.941	.002	.947	.013	.943	.007	.944	.022	**.918**	.013	.941	.012	**.903**
	100	-.001	.938	.003	.941	.003	.945	.008	**.924**	.007	.952	.016	**.901**	.013	**.915**	.016	**.893**
	200	-.001	.940	-.001	.943	.003	.947	.003	.937	.007	.945	.011	**.915**	.010	.934	.017	**.873**
20	30	.002	.946	.002	.936	.004	.946	.020	.948	.014	.925	.026	.926	.022	**.915**	.018	.936
	50	.001	.943	.003	.949	.003	.945	.012	.930	.011	.930	.022	**.924**	.017	**.890**	.020	**.916**
	100	.002	.936	-.001	.951	.005	.931	.008	.943	.010	**.907**	.016	**.905**	.015	**.879**	.018	**.898**
	200	.000	.949	.000	.927	.004	.940	.005	.937	.008	**.915**	.013	**.908**	.013	**.854**	.015	**.883**
50	30	-.001	.952	.004	.949	.005	.950	.016	.938	.015	**.924**	.033	**.880**	.024	**.851**	.019	**.921**
	50	-.001	.937	.001	.956	.005	.937	.011	.935	.011	**.918**	.022	**.881**	.017	**.845**	.020	**.902**
	100	.000	.948	.000	.944	.003	.941	.007	.934	.009	**.897**	.015	**.866**	.015	**.799**	.019	**.844**
	200	.000	.944	.000	.955	.003	.935	.004	.936	.008	**.887**	.012	**.865**	.013	**.655**	.015	**.828**
									Heterogeneous case								
5	30	.004	**.879**	-.004	**.916**	.001	**.915**	.002	**.909**	.009	**.912**	.018	**.904**	.011	**.916**	.004	**.917**
	50	-.002	**.907**	.005	**.877**	.000	**.903**	.009	**.893**	.005	**.884**	.015	**.902**	.013	**.883**	.006	**.903**
	100	-.001	**.871**	-.001	**.910**	.007	**.865**	.003	**.880**	.011	**.873**	.011	**.897**	.012	**.874**	.015	**.877**
	200	.004	**.871**	.000	**.890**	-.001	**.881**	.006	**.865**	.004	**.889**	.008	**.886**	.011	**.857**	.006	**.893**
20	30	.001	.940	.004	.932	.005	.926	.011	.941	.015	.930	.024	.949	.018	.926	.005	.957
	50	.001	.936	.003	.936	.006	.934	.008	.928	.011	.939	.018	.934	.015	**.917**	.015	.940
	100	-.001	.937	-.001	.927	.000	.929	.007	.941	.008	**.916**	.013	.933	.013	**.907**	.014	**.923**
	200	.000	.933	.000	.927	.001	.943	.005	.932	.009	**.924**	.011	**.916**	.013	**.905**	.017	**.901**
50	30	-.002	.950	.004	.939	.003	.944	.014	.934	.013	.935	.025	.942	.019	**.888**	.010	.949
	50	-.002	.941	.002	.945	.006	.933	.009	.934	.008	.931	.020	.925	.016	**.890**	.018	.939
	100	-.001	.935	-.002	.942	.002	.940	.003	.947	.009	.932	.013	.934	.014	**.892**	.015	.931
	200	.000	.949	-.001	.954	.002	.941	.004	.954	.007	**.920**	.009	.935	.012	**.876**	.014	**.919**

Note. T-Norm: Underlying true scores for X and Y are normal; T-Skewed: underlying true scores for X and Y are skewed; CP: Coverage probability. Coverage probability not in the range of [.925, .975] is highlighted in bold.

2 Type I error rates of homogeneity test

Simulation study 1: Type I error rate when $J = 4$.

K	$\bar{N}$	$\bar{\rho} = 0$		$\bar{\rho} = .1$		$\bar{\rho} = 3$		$\bar{\rho} = .5$	
		T-Norm	T-Skewed	T-Norm	T-Skewed	T-Norm	T-Skewed	T-Norm	T-Skewed
					Meta-analysis using raw correlations				
5	30	.058	.045	.050	.063	.045	**.096**	.053	**.213**
	50	.044	.043	.048	.062	.053	**.100**	.055	**.197**
	100	.055	.049	.053	.069	.041	**.097**	.043	**.218**
	200	.047	.048	.060	.062	.041	**.114**	.049	**.256**
20	30	.041	.045	.041	.047	.032	**.126**	.050	**.388**
	50	.044	.048	.042	.066	.045	**.162**	.062	**.422**
	100	.050	.052	.047	.070	.055	**.176**	.041	**.488**
	200	.056	.057	.050	**.076**	.043	**.197**	.060	**.545**
50	30	.043	.046	.049	.052	.044	**.230**	.057	**.614**
	50	.046	.069	.049	.057	.047	**.256**	.064	**.728**
	100	.042	.059	.048	.064	.065	**.324**	.050	**.793**
	200	.046	.066	.050	**.078**	.051	**.350**	.059	**.847**
					Meta-analysis using z transformed scores				
5	30	.059	.057	.056	.075	.052	**.101**	.058	**.231**
	50	.048	.051	.047	.067	.051	**.107**	.052	**.206**
	100	.057	.053	.052	.070	.046	**.107**	.043	**.226**
	200	.047	.048	.060	.066	.042	**.115**	.050	**.262**
20	30	.053	.061	.057	.066	.040	**.158**	.048	**.439**
	50	.050	.059	.051	**.082**	.051	**.189**	.054	**.462**
	100	.055	.058	.048	**.084**	.056	**.192**	.041	**.503**
	200	.056	.061	.050	**.080**	.048	**.199**	.060	**.560**
50	30	.056	.071	.056	**.085**	.045	**.279**	.042	**.659**
	50	.050	**.084**	.055	**.082**	.047	**.289**	.065	**.766**
	100	.045	.063	.052	**.078**	.070	**.343**	.042	**.815**
	200	.047	.067	.051	**.084**	.048	**.359**	.060	**.859**

Note. T-Norm: Underlying true scores for X and Y are normal; T-Skewed: underlying true scores for X and Y are skewed. Type I error rate not in the range of [.025, .075] is highlighted in bold.

Simulation study 1: Type I error rate when $J = 12$.

K	$\bar{N}$	$\bar{\rho} = 0$		$\bar{\rho} = .1$		$\bar{\rho} = .3$		$\bar{\rho} = .5$	
		T-Norm	T-Skewed	T-Norm	T-Skewed	T-Norm	T-Skewed	T-Norm	T-Skewed
					Meta-analysis using raw correlations				
5	30	.044	**.078**	.058	**.077**	.041	**.099**	.060	**.227**
	50	.054	.063	.051	.064	.058	**.127**	.050	**.276**
	100	.042	.060	.044	.060	.049	**.132**	.047	**.285**
	200	.052	.060	.048	.062	.052	**.169**	.054	**.318**
20	30	.048	.035	.039	.057	.041	**.202**	.050	**.540**
	50	.029	.061	.054	**.084**	.049	**.251**	.056	**.590**
	100	.040	.057	.053	**.081**	.045	**.284**	.048	**.696**
	200	.063	.067	.045	**.081**	.049	**.300**	.050	**.725**
50	30	.041	.064	.042	.071	.051	**.318**	**.080**	**.834**
	50	.056	.061	.051	**.078**	.041	**.378**	.053	**.907**
	100	.051	.066	.051	**.087**	.060	**.442**	.059	**.945**
	200	.049	.052	.047	**.080**	.059	**.471**	.055	**.961**
5	30	.048	**.092**	.066	**.087**	.042	**.119**	.063	**.264**
	50	.054	.069	.058	.065	.059	**.141**	.053	**.284**
	100	.043	.063	.046	.066	.054	**.132**	.036	**.299**
	200	.053	.061	.051	.064	.054	**.166**	.053	**.325**
20	30	.061	.064	.051	**.088**	.048	**.273**	.057	**.584**
	50	.038	**.080**	.059	**.104**	.053	**.308**	.048	**.630**
	100	.040	.059	.059	**.095**	.045	**.316**	.044	**.713**
	200	.064	.070	.044	**.083**	.051	**.310**	.048	**.739**
50	30	.050	**.098**	.055	**.137**	.055	**.447**	.054	**.880**
	50	.063	**.088**	.063	**.118**	.043	**.451**	.050	**.927**
	100	.054	.069	.056	**.107**	.062	**.488**	.049	**.948**
	200	.049	.055	.047	**.096**	.058	**.493**	.053	**.962**

Note. T-Norm: Underlying true scores for X and Y are normal; T-Skewed: underlying true scores for X and Y are skewed. Type I error rate not in the range of [.025, .075] is highlighted in bold.

Simulation study 2: Type I error rate when $J = 4$ and tau-equivalence is met.

K	$\bar{N}$	$\bar{\rho} = 0$		$\bar{\rho} = .1$		$\bar{\rho} = .3$		$\bar{\rho} = .5$	
		T-Norm	T-Skewed	T-Norm	T-Skewed	T-Norm	T-Skewed	T-Norm	T-Skewed
Meta-analysis using raw correlations									
5	30	.042	.028	.025	.057	.043	.051	**.016**	.070
	50	.053	.053	.045	.051	.043	.055	.036	**.079**
	100	.044	.052	.059	.062	.049	**.094**	.038	**.141**
	200	.051	.045	.042	.052	.039	**.112**	.029	**.170**
20	30	.028	.023	.025	**.024**	.030	.043	**.015**	**.082**
	50	.049	.044	.034	.045	.029	**.111**	**.024**	**.161**
	100	.048	.061	.035	.045	.036	**.132**	**.019**	**.297**
	200	.053	.058	.052	.051	.044	**.169**	.035	**.358**
50	30	.026	**.023**	.026	**.023**	**.020**	.048	**.004**	**.107**
	50	.034	.045	.049	.053	.031	**.118**	**.022**	**.319**
	100	.050	.053	.045	.057	**.023**	**.194**	**.018**	**.481**
	200	.046	.053	.055	.057	.030	**.275**	**.018**	**.598**
Meta-analysis using z transformed scores									
5	30	**.358**	**.450**	**.434**	**.449**	**.434**	**.591**	**.509**	**.678**
	50	**.365**	**.419**	**.351**	**.463**	**.394**	**.562**	**.466**	**.709**
	100	**.316**	**.356**	**.309**	**.396**	**.375**	**.517**	**.425**	**.696**
	200	**.300**	**.314**	**.311**	**.336**	**.319**	**.485**	**.401**	**.656**
20	30	**.831**	**.928**	**.819**	**.926**	**.870**	**.960**	**.918**	**.991**
	50	**.775**	**.895**	**.783**	**.875**	**.816**	**.955**	**.927**	**.991**
	100	**.704**	**.793**	**.738**	**.817**	**.757**	**.923**	**.874**	**.991**
	200	**.674**	**.738**	**.725**	**.751**	**.762**	**.918**	**.875**	**.989**
50	30	**.992**	**.997**	**.981**	**.996**	**.994**	**1.000**	**1.000**	**1.000**
	50	**.974**	**.990**	**.981**	**.992**	**.985**	**.999**	**1.000**	**1.000**
	100	**.958**	**.973**	**.959**	**.987**	**.981**	**.999**	**.999**	**1.000**
	200	**.950**	**.968**	**.961**	**.972**	**.975**	**.997**	**.998**	**1.000**

Note. T-Norm: Underlying true scores for X and Y are normal; T-Skewed: underlying true scores for X and Y are skewed. Type I error rate not in the range of [.025, .075] is highlighted in bold.

Simulation study 2: Type I error rate when $J = 12$ and tau-equivalence is met.

K	$\bar{N}$	$\bar{\rho} = 0$		$\bar{\rho} = .1$		$\bar{\rho} = .3$		$\bar{\rho} = .5$	
		T-Norm	T-Skewed	T-Norm	T-Skewed	T-Norm	T-Skewed	T-Norm	T-Skewed
Meta-analysis using raw correlations									
5	30	.060	.054	.046	.064	.046	**.086**	.034	**.128**
	50	.054	.055	.042	.061	.048	**.106**	.038	**.236**
	100	.058	.048	.048	.065	.035	**.111**	.055	**.229**
	200	.047	.055	.039	.054	.048	**.128**	.042	**.285**
20	30	.048	.048	.058	.045	.041	**.155**	.035	**.436**
	50	.040	.065	.053	.073	.047	**.201**	.036	**.482**
	100	.049	.060	.049	.075	.040	**.251**	.047	**.601**
	200	.056	.050	.046	.067	.044	**.256**	.033	**.628**
50	30	.038	.068	.044	.050	.035	**.220**	.037	**.649**
	50	.041	.058	.036	**.089**	.040	**.336**	.027	**.773**
	100	.049	.063	.050	**.084**	.031	**.424**	.029	**.908**
	200	.044	.062	.050	**.081**	.044	**.446**	.027	**.918**
Meta-analysis using z transformed scores									
5	30	**.133**	**.219**	**.154**	**.212**	**.153**	**.332**	**.174**	**.495**
	50	**.137**	**.145**	**.117**	**.179**	**.128**	**.325**	**.170**	**.507**
	100	**.139**	**.129**	**.126**	**.146**	**.105**	**.273**	**.178**	**.476**
	200	**.102**	**.145**	**.112**	**.142**	**.121**	**.236**	**.155**	**.511**
20	30	**.278**	**.448**	**.299**	**.527**	**.307**	**.717**	**.403**	**.938**
	50	**.250**	**.322**	**.234**	**.407**	**.268**	**.684**	**.314**	**.940**
	100	**.214**	**.256**	**.253**	**.313**	**.232**	**.654**	**.290**	**.939**
	200	**.216**	**.239**	**.202**	**.265**	**.252**	**.612**	**.312**	**.939**
50	30	**.464**	**.688**	**.479**	**.800**	**.544**	**.957**	**.650**	**1.000**
	50	**.413**	**.587**	**.430**	**.593**	**.486**	**.948**	**.606**	**.999**
	100	**.400**	**.430**	**.391**	**.509**	**.424**	**.927**	**.543**	**1.000**
	200	**.374**	**.428**	**.385**	**.514**	**.410**	**.904**	**.512**	**1.000**

Note. T-Norm: Underlying true scores for X and Y are normal; T-Skewed: underlying true scores for X and Y are skewed. Type I error rate not in the range of [.025, .075] is highlighted in bold.

Simulation study 2: Type I error rate when $J = 4$ and tau-equivalence is NOT met.

K	$\bar{N}$	$\bar{\rho} = 0$		$\bar{\rho} = .1$		$\bar{\rho} = .3$		$\bar{\rho} = .5$	
		T-Norm	T-Skewed	T-Norm	T-Skewed	T-Norm	T-Skewed	T-Norm	T-Skewed
Meta-analysis using raw correlations									
5	30	.035	.046	.044	.031	**.024**	.034	**.017**	.049
	50	.047	.045	.033	.045	.044	.061	**.020**	**.077**
	100	.043	.059	.039	.053	.039	**.085**	.048	**.113**
	200	.055	.042	.053	.056	.039	**.082**	.032	**.140**
20	30	.028	**.024**	.037	**.022**	**.010**	.052	**.004**	.073
	50	.047	.031	.041	.041	.026	**.087**	**.020**	**.137**
	100	.039	.047	.057	.047	.028	**.128**	.029	**.253**
	200	.050	.060	.050	.059	.041	**.166**	.028	**.329**
50	30	.028	**.021**	**.011**	**.006**	**.017**	.031	**.004**	.037
	50	.039	.031	.039	.026	.030	**.114**	**.012**	**.169**
	100	.049	.038	.046	.049	.038	**.209**	**.024**	**.451**
	200	.049	.057	.047	.062	.044	**.247**	**.017**	**.604**
Meta-analysis using z transformed scores									
5	30	**.471**	**.526**	**.467**	**.558**	**.556**	**.637**	**.616**	**.761**
	50	**.465**	**.532**	**.485**	**.550**	**.507**	**.643**	**.645**	**.824**
	100	**.439**	**.477**	**.418**	**.476**	**.507**	**.605**	**.659**	**.807**
	200	**.390**	**.424**	**.419**	**.462**	**.438**	**.571**	**.586**	**.786**
20	30	**.933**	**.962**	**.902**	**.964**	**.960**	**.991**	**.982**	**.997**
	50	**.905**	**.944**	**.918**	**.948**	**.935**	**.983**	**.976**	**.997**
	100	**.866**	**.895**	**.871**	**.919**	**.917**	**.980**	**.987**	**.999**
	200	**.841**	**.880**	**.869**	**.898**	**.895**	**.972**	**.978**	**.997**
50	30	**.999**	**1.000**	**.999**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
	50	**.999**	**.999**	**.997**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
	100	**.996**	**.994**	**.993**	**.998**	**.999**	**1.000**	**1.000**	**1.000**
	200	**.990**	**.995**	**.990**	**.995**	**.999**	**1.000**	**1.000**	**1.000**

Note. T-Norm: Underlying true scores for X and Y are normal; T-Skewed: underlying true scores for X and Y are skewed. Type I error rate not in the range of [.025, .075] is highlighted in bold.

Simulation study 2: Type I error rate when $J = 12$ and tau-equivalence is NOT met.

K	$\bar{N}$	$\bar{\rho} = 0$		$\bar{\rho} = .1$		$\bar{\rho} = 3$		$\bar{\rho} = .5$	
		T-Norm	T-Skewed	T-Norm	T-Skewed	T-Norm	T-Skewed	T-Norm	T-Skewed
Meta-analysis using raw correlations									
5	30	.037	.062	.044	.067	.048	**.100**	.038	**.110**
	50	.046	.049	.054	.064	.058	**.113**	.042	**.193**
	100	.042	.052	.054	.064	.036	**.125**	.043	**.249**
	200	.047	.050	.049	.067	.050	**.131**	.040	**.280**
20	30	.038	.050	.043	.048	.046	**.141**	.030	**.369**
	50	.039	.061	.055	**.077**	.041	**.194**	.049	**.499**
	100	.044	.068	.045	.069	.046	**.224**	**.022**	**.591**
	200	.043	.062	.052	.069	.040	**.272**	.031	**.642**
50	30	.048	.061	.040	.038	.036	**.202**	.031	**.663**
	50	.046	.058	.042	**.079**	.039	**.347**	.028	**.800**
	100	.040	.059	.038	**.078**	.044	**.449**	.026	**.906**
	200	.046	.064	.045	.071	.046	**.438**	.026	**.905**
Meta-analysis using z transformed scores									
5	30	**.150**	**.261**	**.182**	**.233**	**.186**	**.347**	**.220**	**.535**
	50	**.146**	**.180**	**.153**	**.241**	**.168**	**.377**	**.214**	**.544**
	100	**.141**	**.164**	**.132**	**.173**	**.150**	**.303**	**.191**	**.510**
	200	**.111**	**.133**	**.153**	**.168**	**.150**	**.298**	**.180**	**.536**
20	30	**.332**	**.550**	**.347**	**.565**	**.393**	**.755**	**.510**	**.957**
	50	**.299**	**.414**	**.329**	**.433**	**.354**	**.723**	**.458**	**.960**
	100	**.292**	**.298**	**.273**	**.391**	**.334**	**.712**	**.428**	**.952**
	200	**.237**	**.280**	**.269**	**.352**	**.298**	**.688**	**.409**	**.957**
50	30	**.553**	**.794**	**.585**	**.845**	**.644**	**.977**	**.831**	**1.000**
	50	**.519**	**.621**	**.504**	**.705**	**.587**	**.966**	**.766**	**.998**
	100	**.487**	**.541**	**.495**	**.620**	**.520**	**.955**	**.696**	**1.000**
	200	**.457**	**.486**	**.474**	**.579**	**.520**	**.939**	**.682**	**1.000**

Note. T-Norm: Underlying true scores for X and Y are normal; T-Skewed: underlying true scores for X and Y are skewed. Type I error rate not in the range of [.025, .075] is highlighted in bold.

Simulation study 3: Type I error rate when $K = 50$, $\bar{N} = 200$, and $J = 12$.

Dist of Items	$\bar{\rho}$	Tau equivalent				Non-tau equivalent			
		r-based approach		z-based approach		r-based approach		z-based approach	
		Number of categories per item is two							
		T-Norm	T-Skew	T-Norm	T-Skew	T-Norm	T-Skew	T-Norm	T-Skew
All Sym	0	.047	.056	**.721**	**.968**	.050	.061	**.758**	**.975**
	.1	.047	.049	**.756**	**.961**	.046	.033	**.774**	**.961**
	.3	.047	.046	**.802**	**.988**	.052	.033	**.854**	**.990**
	.5	.027	.060	**.912**	**1.000**	.049	.043	**.949**	**.999**
All RSke	0	.049	.047	**.830**	**.630**	.048	.053	**.863**	**.712**
	.1	**.090**	**.087**	**.917**	**.766**	**.093**	**.101**	**.915**	**.816**
	.3	**.200**	**.427**	**.974**	**.962**	**.233**	**.377**	**.979**	**.981**
	.5	**.252**	**.788**	**1.000**	**1.000**	**.372**	**.646**	**.998**	**1.000**
LSke+RSke	0	.056	.041	**1.000**	**1.000**	.046	.049	**.771**	**1.000**
	.1	.056	.030	**.999**	**1.000**	**.117**	.043	**.891**	**1.000**
	.3	**.108**	.026	**1.000**	**1.000**	**.324**	.061	**.981**	**1.000**
	.5	**.106**	**.013**	**1.000**	**1.000**	**.522**	.040	**1.000**	**1.000**
		Number of categories per item is five							
All Sym	0	.044	.043	**.423**	**.660**	.047	.049	**.536**	**.738**
	.1	.040	.029	**.435**	**.617**	.046	.034	**.517**	**.736**
	.3	.043	**.120**	**.478**	**.858**	.033	.072	**.582**	**.893**
	.5	.030	**.309**	**.616**	**.988**	.029	**.179**	**.763**	**.991**
All RSke	0	.047	.047	**.476**	**.469**	.051	.049	**.552**	**.587**
	.1	.059	**.086**	**.536**	**.528**	.056	.054	**.587**	**.626**
	.3	.050	**.257**	**.607**	**.860**	**.083**	**.172**	**.754**	**.887**
	.5	.066	**.687**	**.790**	**.991**	**.088**	**.430**	**.882**	**.993**
LSke+RSke	0	.066	.043	**.703**	**.988**	.049	.061	**.553**	**.939**
	.1	.056	.025	**.711**	**.982**	.054	**.020**	**.584**	**.918**
	.3	.059	.029	**.793**	**.996**	.068	.026	**.716**	**.981**
	.5	.041	.043	**.907**	**1.000**	.067	.032	**.870**	**.997**

Note. T-Norm: Underlying true scores for X and Y are normal; T-Skewed: underlying true scores for X and Y are skewed. Dist of Items: Distribution of items; All Sym: All symmetric; All RSke: All right skewed; LSym+RSke: Half of the items are left skewed, and the other are right skewed. T-Norm: Underlying true scores for X and Y are normal; T-Skewed: underlying true scores for X and Y are skewed. Type I error rate not in the range of [.025, .075] is highlighted in bold.

3 Reliability estimates in simulation study 3

Simulation study 3: Reliability estimates averaged across different $\bar{\rho}$ values when $K = 50$, $\bar{N} = 200$, and $J = 12$.

Cat	Dist	tau-equivalence	T-norm		T-skew	
			X	Y	X	Y
2	All Sym	Tau	.806	.806	.701	.716
		NonTau	.794	.794	.702	.719
	All RSke	Tau	.772	.771	.825	.825
		NonTau	.767	.767	.808	.808
	LSke+RSke	Tau	.599	.599	.369	.391
		NonTau	.789	.789	.492	.537
5	All Sym	Tau	.870	.870	.823	.829
		NonTau	.850	.850	.800	.808
	All RSke	Tau	.861	.861	.863	.866
		NonTau	.844	.844	.835	.840
	LSke+RSke	Tau	.816	.816	.676	.694
		NonTau	.846	.846	.717	.738

Note. Cat: number of categories per item. Tau: tau-equivalence is satisfied; NonTau: Tau-equivalence is not satisfied. X: reliability estimate for the scale of X; Y: reliability estimate for the scale of Y. Dist of Items: Distribution of items; All Sym: All symmetric; All RSke: All right skewed; LSym+RSke: Half of the items are left skewed, and the other are right skewed. T-Norm: Underlying true scores for X and Y are normal; T-Skewed: underlying true scores for X and Y are skewed.