

Online Supplemental Material A

With Bayesian estimation, inference can be made about the parameters of focal interest using the posterior distribution given the observed data. For the models proposed in the manuscript, we work with the augmented posterior density given the observed data \mathbf{Y} :

$$P(\boldsymbol{\Theta} | \mathbf{Y}) \propto P(\mathbf{Y}, \boldsymbol{\Theta}), \quad (\text{A.1})$$

where $\boldsymbol{\Theta}$ is a vector that contains the model parameters including all the fixed effects, variance-covariance of the random effects, as well as the between-level latent random variables (hence unobserved).

In the case of Model A and Model B, we have $\boldsymbol{\Theta} = (\boldsymbol{\theta}^T, \text{vec}(\boldsymbol{\eta}^{(B)})^T)^T$, where the model parameters vector $\boldsymbol{\theta}$ contains both fixed effects and variance-covariance of the random effects: $\boldsymbol{\theta} = (\boldsymbol{\gamma}^{(B)T}, \text{vech}(\boldsymbol{\Phi}_v)^T)^T$. The augmented posterior distribution can be obtained as:

$$P(\boldsymbol{\theta}, \boldsymbol{\eta}^{(B)} | \mathbf{Y}) \propto P(\mathbf{Y}, \boldsymbol{\eta}^{(B)} | \boldsymbol{\theta}) P(\boldsymbol{\theta}). \quad (\text{A.2})$$

The first term on the right side of Eq. (A.2) is the complete data likelihood of the observed and latent data, which can be computed with Eq. (A.3) assuming conditional independence:

$$P(\mathbf{Y}, \boldsymbol{\eta}^{(B)} | \boldsymbol{\theta}) = \prod_{j=1}^J \prod_{i=1}^{n_j} P(y_{ij} | \boldsymbol{\eta}_j^{(B)}, \boldsymbol{\theta}) P(\boldsymbol{\eta}_j^{(B)} | \boldsymbol{\theta}). \quad (\text{A.3})$$

If we assume normality, the first term on the right side of Eq. (A.3) can be computed as

$$P(y_{ij} | \boldsymbol{\eta}_j^{(B)}, \boldsymbol{\theta}) = \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp\left(-\frac{(y_{ij} - \gamma_{0j})^2}{2\sigma_j^2}\right). \quad (\text{A.4})$$

The second term on the right side of Eq. (A.3) is the conditional likelihood of latent variables given model parameters. Based on Eq. (11) in the main text,

$$\begin{aligned}
P(\boldsymbol{\eta}^{(B)} | \boldsymbol{\theta}) &= \prod_{j=1}^J f(\boldsymbol{\eta}_j^{(B)} | \boldsymbol{\theta}) \\
&= \prod_{j=1}^J \frac{1}{2\pi^{m/2} |\boldsymbol{\Phi}_v|^{1/2}} \exp \left\{ -\frac{1}{2} (\boldsymbol{\eta}_j^{(B)} - \boldsymbol{\gamma}^{(B)})^T \boldsymbol{\Phi}_v^{-1} (\boldsymbol{\eta}_j^{(B)} - \boldsymbol{\gamma}^{(B)}) \right\}, \tag{A.5}
\end{aligned}$$

with m indicating the number of level-2 random effects. The second term on the right side of Eq. (A.2) is the prior distribution of model parameters. The joint density of the model parameters assuming independence between the fixed effects $\boldsymbol{\gamma}^{(B)}$ and random effects $\boldsymbol{\Phi}_v$ can be broken down as

$$\begin{aligned}
P(\boldsymbol{\theta}) &= P(\boldsymbol{\gamma}^{(B)}, \boldsymbol{\Phi}_v) \\
&= P(\boldsymbol{\gamma}^{(B)})P(\boldsymbol{\Phi}_v). \tag{A.6}
\end{aligned}$$

Putting Eqs. (A.3)-(A.6) together, the posterior distribution in Eq. (A.2) can be expressed as

$$\begin{aligned}
P(\boldsymbol{\theta}, \boldsymbol{\eta}^{(B)} | \mathbf{Y}) &\propto P(\mathbf{Y} | \boldsymbol{\eta}^{(B)}, \boldsymbol{\theta}) P(\boldsymbol{\eta}^{(B)} | \boldsymbol{\theta}) P(\boldsymbol{\theta}) \\
&= \left[\prod_{j=1}^J \prod_{i=1}^{n_j} \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp \left(-\frac{(y_{ij} - \gamma_{0j})^2}{2\sigma_j^2} \right) \right] \times \\
&\quad \left[\prod_{j=1}^J \frac{1}{2\pi^{m/2} |\boldsymbol{\Phi}_v|^{1/2}} \exp \left\{ -\frac{1}{2} (\boldsymbol{\eta}_j^{(B)} - \boldsymbol{\gamma}^{(B)})^T \boldsymbol{\Phi}_v^{-1} (\boldsymbol{\eta}_j^{(B)} - \boldsymbol{\gamma}^{(B)}) \right\} \right] \times P(\boldsymbol{\gamma}^{(B)})P(\boldsymbol{\Phi}_v) \tag{A.7}
\end{aligned}$$

In the case of Model C and Model D, we have $\boldsymbol{\Theta} = (\boldsymbol{\theta}^T, \text{vec}(\boldsymbol{\xi}^{(B)})^T, \boldsymbol{\varepsilon}_B^T)^T$, where the model parameters vector $\boldsymbol{\theta} = (\boldsymbol{\Gamma}^T, \boldsymbol{\gamma}^{(B)T}, \boldsymbol{\lambda}_B^T, \boldsymbol{\lambda}_W^T, \text{vech}(\boldsymbol{\Phi}_v)^T, \text{vech}(\boldsymbol{\Theta}_B)^T, \text{vech}(\boldsymbol{\Theta}_W)^T)^T$. The augmented posterior distribution is obtained as Eq. (A.8) using Bayes rule:

$$P(\boldsymbol{\theta}, \boldsymbol{\xi}^{(B)}, \boldsymbol{\varepsilon}_B | \mathbf{Y}) \propto P(\mathbf{Y}, \boldsymbol{\xi}^{(B)}, \boldsymbol{\varepsilon}_B | \boldsymbol{\theta}) P(\boldsymbol{\theta}). \tag{A.8}$$

The first term on the right side of Eq. (A.8) is the complete data likelihood of the observed variables and between-level latent variables, which can be computed as follows assuming observations are independent after conditioning on clusters and that they are independent across clusters:

$$P(\mathbf{Y}, \boldsymbol{\xi}^{(B)}, \boldsymbol{\varepsilon}_B | \boldsymbol{\theta}) = \prod_{j=1}^J \prod_{i=1}^{n_j} P(\mathbf{Y}_{ij} | \boldsymbol{\xi}_j^{(B)}, \boldsymbol{\varepsilon}_{B,j}, \boldsymbol{\theta}) P(\boldsymbol{\xi}^{(B)} | \boldsymbol{\theta}) P(\boldsymbol{\varepsilon}_B | \boldsymbol{\theta}). \quad (\text{A.9})$$

Assuming multivariate normality, the first term in Eq. (A.9) can be written as

$$P(\mathbf{Y}_{ij} | \boldsymbol{\xi}_j^{(B)}, \boldsymbol{\varepsilon}_j^{(B)}, \boldsymbol{\theta}) = \frac{1}{2\pi^{K/2} |\boldsymbol{\lambda}_w \sigma_j^2 \boldsymbol{\lambda}_w^T + \boldsymbol{\Theta}_w|^{1/2}} \exp \left\{ -\frac{1}{2} (\mathbf{Y}_{ij} - (\boldsymbol{\gamma}_{00} + \boldsymbol{\lambda}_B \boldsymbol{\xi}_{B,j} + \boldsymbol{\varepsilon}_{B,j}))^T (\boldsymbol{\lambda}_w \sigma_j^2 \boldsymbol{\lambda}_w^T + \boldsymbol{\Theta}_w)^{-1} (\mathbf{Y}_{ij} - (\boldsymbol{\gamma}_{00} + \boldsymbol{\lambda}_B \boldsymbol{\xi}_{B,j} + \boldsymbol{\varepsilon}_{B,j})) \right\} \quad (\text{A.10})$$

with K denoting the number of observed indicators for the measurement model. The second and third terms on the right side of Eq. (A.9) constitute the conditional likelihood of between-level latent variables given model parameters. Based on Eq. (18) in the main text, the second term can be expressed as

$$P(\boldsymbol{\xi}^{(B)} | \boldsymbol{\theta}) = \prod_{j=1}^J f(\boldsymbol{\xi}_j^{(B)} | \boldsymbol{\gamma}^{(B)}, \boldsymbol{\Phi}_\nu) = \prod_{j=1}^J \frac{1}{2\pi^{m/2} |\boldsymbol{\Phi}_\nu|^{1/2}} \exp \left\{ -\frac{1}{2} (\boldsymbol{\xi}_j^{(B)} - \boldsymbol{\gamma}^{(B)})^T \boldsymbol{\Phi}_\nu^{-1} (\boldsymbol{\xi}_j^{(B)} - \boldsymbol{\gamma}^{(B)}) \right\}. \quad (\text{A.11})$$

Also, because the between-level residuals are assumed to following multivariate normal distribution, the third term can thus be expressed as

$$P(\boldsymbol{\varepsilon}_B | \boldsymbol{\theta}) = \prod_{j=1}^J f(\boldsymbol{\varepsilon}_{B,j} | \boldsymbol{\Gamma}, \boldsymbol{\Theta}_B) = \prod_{j=1}^J \frac{1}{2\pi^{K/2} |\boldsymbol{\Theta}_B|^{1/2}} \exp \left\{ -\frac{1}{2} (\boldsymbol{\varepsilon}_{B,j} - \boldsymbol{\Gamma})^T \boldsymbol{\Theta}_B^{-1} (\boldsymbol{\varepsilon}_{B,j} - \boldsymbol{\Gamma}) \right\}. \quad (\text{A.12})$$

The second term on the right side of Eq. (A.8) is the prior distribution of model parameters. The joint density of the model parameters can be further written in the following form assuming independence between the fixed effects and random effects:

$$P(\boldsymbol{\theta}) = P(\boldsymbol{\Gamma}, \boldsymbol{\gamma}^{(B)}, \boldsymbol{\lambda}_B, \boldsymbol{\lambda}_W, \boldsymbol{\Phi}_\nu, \boldsymbol{\Theta}_B, \boldsymbol{\Theta}_W) = P(\boldsymbol{\Gamma}) P(\boldsymbol{\gamma}^{(B)}) P(\boldsymbol{\lambda}_B, \boldsymbol{\lambda}_W) P(\boldsymbol{\Phi}_\nu) P(\boldsymbol{\Theta}_B) P(\boldsymbol{\Theta}_W). \quad (\text{A.13})$$

Combining Eqs. (A.8)-(A.13), the augmented posterior distribution for Model C and Model D is:

$$\begin{aligned}
P(\boldsymbol{\theta}, \boldsymbol{\xi}^{(B)}, \boldsymbol{\varepsilon}_B \mid \mathbf{Y}) &\propto P(\mathbf{Y} \mid \boldsymbol{\xi}^{(B)}, \boldsymbol{\varepsilon}_B, \boldsymbol{\theta}) P(\boldsymbol{\xi}^{(B)} \mid \boldsymbol{\theta}) P(\boldsymbol{\varepsilon}_B \mid \boldsymbol{\theta}) P(\boldsymbol{\theta}) \\
&= \left[\prod_{j=1}^J \prod_{i=1}^{n_j} \frac{1}{2\pi^{K/2} |\boldsymbol{\lambda}_w \sigma_j^2 \boldsymbol{\lambda}_w^T + \boldsymbol{\Theta}_w|^{1/2}} \exp \left\{ -\frac{1}{2} (\mathbf{Y}_{ij} - (\boldsymbol{\gamma}_{00} + \boldsymbol{\lambda}_B \boldsymbol{\xi}_{B,j} + \boldsymbol{\varepsilon}_{B,j}))^T (\boldsymbol{\lambda}_w \sigma_j^2 \boldsymbol{\lambda}_w^T + \boldsymbol{\Theta}_w)^{-1} (\mathbf{Y}_{ij} - (\boldsymbol{\gamma}_{00} + \boldsymbol{\lambda}_B \boldsymbol{\xi}_{B,j} + \boldsymbol{\varepsilon}_{B,j})) \right\} \right] \times \\
&\quad \left[\prod_{j=1}^J \frac{1}{2\pi^{m/2} |\boldsymbol{\Phi}_v|^{1/2}} \exp \left\{ -\frac{1}{2} (\boldsymbol{\xi}_j^{(B)} - \boldsymbol{\gamma}^{(B)})^T \boldsymbol{\Phi}_v^{-1} (\boldsymbol{\xi}_j^{(B)} - \boldsymbol{\gamma}^{(B)}) \right\} \right] \times \\
&\quad \left[\prod_{j=1}^J \frac{1}{2\pi^{K/2} |\boldsymbol{\Theta}_B|^{1/2}} \exp \left\{ -\frac{1}{2} (\boldsymbol{\varepsilon}_{B,j} - \boldsymbol{\Gamma})^T \boldsymbol{\Theta}_B^{-1} (\boldsymbol{\varepsilon}_{B,j} - \boldsymbol{\Gamma}) \right\} \right] \times P(\boldsymbol{\Gamma}) P(\boldsymbol{\gamma}^{(B)}) P(\boldsymbol{\lambda}_B, \boldsymbol{\lambda}_w) P(\boldsymbol{\Phi}_v) P(\boldsymbol{\Theta}_B) P(\boldsymbol{\Theta}_w)
\end{aligned}
\tag{A.14}$$

Online Supplemental Material B**Mplus Code for Example 1—Unconditional Model**

```

TITLE: Illustrative Example 1:
unconditional random variability model for
daily curiosity, using log variance approach

DATA: FILE = ex1-dat.csv;

VARIABLE:
NAMES = IDNUM DayOfStudy CurAve_d Happ_d Dep_d Anx_d
CEIMean AgeYears GenderF LSMean FSMean CESDMean id;
USEVARIABLES = CurAve_d;
CLUSTER = IDNUM;           !observations nested within individuals

ANALYSIS:
TYPE = TWOLEVEL RANDOM;
ESTIMATOR = BAYES;         !Bayesian estimation
CHAINS = 3;                !three chains
BITERATIONS = 50000 (2000); !max and min number of iterations each chain
PROCESSORS = 2;

MODEL:
%WITHIN%                   !within-level model (daily)
LOGV | CurAve_d;           !log residual variance modeled as random
coefficients

%BETWEEN%                  !between-level model (person)
[CurAve_d] (gamma00);      !between-level intercept of daily curiosity
[LOGV] (gamma10);          !between-level intercept of log residual variance
CurAve_d (phi_0);         !variance of between-level daily curiosity
LOGV (phi_1);              !variance of between-level log residual variance
CurAve_d WITH LOGV (phi_10);
!covariance between individual curiosity and log residual variance

MODEL CONSTRAINT:
NEW (mean_v var_v);        !monitor the mean and variance of within-level
residual variance
mean_v = exp(gamma10 + (1/2)*phi_1); !mean of the within variance
var_v = exp(2*gamma10 + phi_1)*(exp(phi_1)-1); !variance of the variance

OUTPUT:                    !request iteration history and PSR report
TECH8 CINTERVAL(HPD);      !request highest posterior density credibility
interval;

```

Online Supplemental Material B (Continued)**Mplus Code for Example 1—Conditional Model**

```

TITLE: Illustrative Example 1:
conditional random variability model for
daily curiosity, using log variance approach,
level-2 predictor: trait-level curiosity;
level-2 outcome: flourishing;

DATA: FILE = ex1-dat.csv;

VARIABLE:
NAMES = IDNUM DayOfStudy CurAve_d Happ_d Dep_d Anx_d
CEIMean AgeYears GenderF LSMean FSMean CESDMean id;
USEVARIABLES = CurAve_d FSMean CEIMean;
CLUSTER = IDNUM;                !observations nested within individuals
BETWEEN = FSMean CEIMean;       !flourishing and trait-level curiosity are both
between-level variables;

ANALYSIS:
TYPE = TWOLEVEL RANDOM;
ESTIMATOR = BAYES;               !Bayesian estimation
CHAINS = 3;                     !three chains
BITERATIONS = 50000 (2000);     !max and min number of iterations each chain
PROCESSORS = 2;
ALGORITHM=GIBBS (RW);

MODEL:
%WITHIN%                         !within-level model (daily)
LOGV | CurAve_d;                 !log residual variance modeled as random
coefficients

%BETWEEN%                       !between-level model (person)
[CurAve_d] (gamma00);           !between-level intercept of daily curiosity
[LOGV] (gamma10);               !between-level intercept of log residual
variance
[CEIMean] (gamma20);            !between-level intercept of trait-level
curiosity

CurAve_d (phi_0);               !variance of between-level daily curiosity
LOGV (phi_1);                   !variance of between-level log residual
variance
CEIMean (phi_2)                 !variance of between-level curiosity
CurAve_d WITH LOGV (phi_10)    !covariance between individual average
curiosity and log residual variance
CEIMean (phi_12);              !covariance between individual average
curiosity and trait-level curiosity;
LOGV WITH CEIMean;              !covariance between log residual variance and
trait-level curiosity;

FSMean ON CurAve_d LOGV CEIMean; !predict flourishing with individual
average curiosity, log residual variance, and trait-level curiosity;

MODEL CONSTRAINT:

```

```
NEW (mean_v var_v);           !monitor the mean and variance of within-level
residual variance
mean_v = exp(gamma10 + (1/2)*phi_1);    !mean of the within variance
var_v = exp(2*gamma10 + phi_1)*(exp(phi_1)-1);  !variance of the variance

OUTPUT:      !request iteration history and PSR report
TECH8 CINTERVAL(HPD);      !request highest posterior density credibility
interval;
```

Online Supplemental Material B (Continued)**Mplus Code for Example 2**

```

TITLE: Illustrative Example 2:
unconditional random variability model for
PISA, using phantom variable approach

DATA: FILE = teach_Mplus.csv;

VARIABLE:
NAMES = SCHOOLID STUDENTID GENDER YEAR teach1-teach5 AGE MEAN;
USEVAR = SCHOOLID teach2-teach4;
CLUSTER = SCHOOLID;          !students nested within schools

ANALYSIS:
TYPE = TWOLEVEL RANDOM;
ESTIMATOR = BAYES;           !Bayesian estimation
CHAINS = 3;                  !three chains
BITERATIONS = 50000 (4000); !max and min number of iterations each chain
PROCESSORS = 2;

MODEL:
%WITHIN%                     !within-level model
FW BY teach2 (a)             !define within-level latent construct
    teach3 (b)
    teach4 (c);

BETA | PHAN BY FW;           !random scaling factor defined as the loading on a
phantom variable
PHAN@1;                      !phantom variable has a variance of 1
FW@0;                        !the within-level construct has zero residual
variance

%BETWEEN%                   !between-level model (school-level)
FB BY teach2 (a)             !define between-level latent construct
    teach3 (b)               !constrain cross-level loadings to be equal
    teach4 (c);

FB;                          !between-level latent construct variance

!between-level intercepts for each item
[teach2];
[teach3];
[teach4];

[BETA] (gamma10);           !between-level intercept of scaling factor;
BETA (phi_1);               !variance of between-level scaling factor;
BETA WITH FB;               !covariance between the scaling factor and level-2
latent construct

!between-level residual variance for each item
teach2;
teach3;
teach4;

```



```
MODEL CONSTRAINT:
NEW (mean_v var_v);           !monitor the mean and variance of within-level
residual variance
mean_v = gamma10^2 + phi_1; !mean of the within variance
var_v = gamma10^4 + 6*phi_1*gamma10^2 + 3*phi_1^2 - (gamma10^2 + phi_1)^2;
!variance of the variance

OUTPUT:                       !request iteration history and PSR report
TECH8 CINTERVAL(HPD);         !request highest posterior density credibility
interval;
```

Online Supplemental Material B (Continued)**Mplus Code for Example 3**

```

TITLE: Illustrative Example 3:
conditional random variability model for
MIDUS sleep, using log variance approach

DATA: FILE = sleep_Mplus_male.csv;

VARIABLE:
NAMES = MRID RA4BCRP RA4BIL6 RA4BFGN RA1PRSEX
RA1PRAGE RA4XTCS_55 RA4XTCS_249 RA4XTCS_19 RA4PBMI
time OL SNT EFF WSO WAK;
USEVAR = MRID RA4BCRP RA4BIL6 RA4BFGN OL WSO EFF;
BETWEEN = RA4BCRP RA4BIL6 RA4BFGN;
CLUSTER = MRID; !repeated measures nested within individuals

DEFINE: !divide by SD to rescale the observed indicators

OL = OL/42.70933;
EFF = EFF/13.99498;
WSO = WSO/38.50612;

RA4BCRP = RA4BCRP/4.059313;
RA4BIL6 = RA4BIL6/2.730577;
RA4BFGN = RA4BFGN/72.965199;

ANALYSIS:
TYPE = TWOLEVEL RANDOM;
ESTIMATOR = BAYES; !Bayesian estimation
CHAINS = 2; !two chains
BITERATIONS = 50000 (13000); !max and min number of iterations each chain
PROCESSORS = 2;
ALGORITHM = GIBBS(RW);
BCONVERGENCE = 0.05;

MODEL:
%WITHIN% !within-level model

FW BY OL* !within-level sleep quality
EFF@1
WSO;

LOGV | FW; !log variance as random coefficient

%BETWEEN% !between-level model (person)

FB BY OL* !person-level sleep quality
EFF@1
WSO;

FB; !variance of sleep quality

```

```

INF BY RA4BCRP RA4BIL6 RA4BFGN;      !Inflammation latent factor

LOGV;

LOGV WITH FB;                          !covariance between log variance and person level
sleep quality

INF ON FB LOGV;                        !predict inflammation with average sleep quality
and consistency

      OL;                              !residual variances
      EFF;
      WSO;

      RA4BCRP;
      RA4BIL6;
      RA4BFGN;

      [LOGV];                          !average log variance
      [INF@0];

      [OL];                            !between-level intercepts for each item
      [EFF];
      [WSO];

      [RA4BCRP];
      [RA4BIL6];
      [RA4BFGN];

OUTPUT:
TECH1 TECH8 CINTERVAL(HPD); !request highest posterior density credibility
interval;

```

Online Supplemental Material B (Continued)

The BUGS Code for Example 4

```

model
{
  for (i in 1:nobs){

    # growth factors are assumed to follow multivariate normal distribution

    eta[i, 1:3] ~ dmnorm(eta_means[1:3], sigma2_inv_eta) #the precision
    sigma2_inv_eta is to be defined

    # level-1 latent factor disturbances (multivariate normal)

    for(t in 1:8){
      u[i, t] ~ dnorm(0, sigma2_inv_u[i]) #the person-specific precision depends
      on the log variance factor
    }

    # level-1 latent factors

    for(t in 1:8){
      pa[i, t] <- eta[i,1] + (t-1)*eta[i, 2] + u[i, t]
    }

    # observed indicators expected values

    for(t in 1:8){

      m7[i, t] <- a7 + g7*pa[i, t]      # intercept a and loadings g to be defined
      m8[i, t] <- a8 + g8*pa[i, t]
      m9[i, t] <- a9 + g9*pa[i, t]
      m10[i, t] <- a10 + g10*pa[i, t]
      m11[i, t] <- a11 + g11*pa[i, t]
      m12[i, t] <- a12 + g12*pa[i, t]
      m21[i, t] <- a21 + g21*pa[i, t]
      m22[i, t] <- a22 + g22*pa[i, t]
      m23[i, t] <- a23 + g23*pa[i, t]
      m24[i, t] <- a24 + g24*pa[i, t]
      m25[i, t] <- a25 + g25*pa[i, t]
      m26[i, t] <- a26 + g26*pa[i, t]
      m27[i, t] <- a27 + g27*pa[i, t]
    }

    # observed indicators (multivariate normal)

    y7[i, 1:8] ~ dmnorm(m7[i, 1:8], sigma2_inv7)
    y8[i, 1:8] ~ dmnorm(m8[i, 1:8], sigma2_inv8)
    y9[i, 1:8] ~ dmnorm(m9[i, 1:8], sigma2_inv9)
    y10[i, 1:8] ~ dmnorm(m10[i, 1:8], sigma2_inv10)
    y11[i, 1:8] ~ dmnorm(m11[i, 1:8], sigma2_inv11)
    y12[i, 1:8] ~ dmnorm(m12[i, 1:8], sigma2_inv12)
    y21[i, 1:8] ~ dmnorm(m21[i, 1:8], sigma2_inv21)
  }
}

```

```

y22[i, 1:8] ~ dmnorm(m22[i, 1:8], sigma2_inv22)
y23[i, 1:8] ~ dmnorm(m23[i, 1:8], sigma2_inv23)
y24[i, 1:8] ~ dmnorm(m24[i, 1:8], sigma2_inv24)
y25[i, 1:8] ~ dmnorm(m25[i, 1:8], sigma2_inv25)
y26[i, 1:8] ~ dmnorm(m26[i, 1:8], sigma2_inv26)
y27[i, 1:8] ~ dmnorm(m27[i, 1:8], sigma2_inv27)
}

# prior for between-level latent random effects variances

sigma2_inv_eta ~ dwish(Imat, 4)
sigma2_eta <- inverse(sigma2_inv_eta)    # variances

# intra-individual variances

for(i in 1:nobs){
  sigma2_u[i] <- exp(eta[i, 3])
  sigma_u[i] <- pow(sigma2_u[i],1/2)      #standard deviation
  sigma2_inv_u[i] <- pow(sigma2_u[i], -1)  # precision
}

# priors for residual variances & covariances

sigma2_inv7 ~ dwish(Id, 9)
sigma2_inv8 ~ dwish(Id, 9)
sigma2_inv9 ~ dwish(Id, 9)
sigma2_inv10 ~ dwish(Id, 9)
sigma2_inv11 ~ dwish(Id, 9)
sigma2_inv12 ~ dwish(Id, 9)
sigma2_inv21 ~ dwish(Id, 9)
sigma2_inv22 ~ dwish(Id, 9)
sigma2_inv23 ~ dwish(Id, 9)
sigma2_inv24 ~ dwish(Id, 9)
sigma2_inv25 ~ dwish(Id, 9)
sigma2_inv26 ~ dwish(Id, 9)
sigma2_inv27 ~ dwish(Id, 9)

sigma2_7 <- inverse(sigma2_inv7)
sigma2_8 <- inverse(sigma2_inv8)
sigma2_9 <- inverse(sigma2_inv9)
sigma2_10 <- inverse(sigma2_inv10)
sigma2_11 <- inverse(sigma2_inv11)
sigma2_12 <- inverse(sigma2_inv12)
sigma2_21 <- inverse(sigma2_inv21)
sigma2_22 <- inverse(sigma2_inv22)
sigma2_23 <- inverse(sigma2_inv23)
sigma2_24 <- inverse(sigma2_inv24)
sigma2_25 <- inverse(sigma2_inv25)
sigma2_26 <- inverse(sigma2_inv26)
sigma2_27 <- inverse(sigma2_inv27)

# priors for latent factor means
eta_means[1] ~ dnorm(0, 1e-07)
eta_means[2] ~ dnorm(0, 1e-07)
eta_means[3] ~ dnorm(0, 1e-07)

```

```
# priors for item intercepts
a7 <- 0
a8 ~ dnorm(0, 1e-07)
a9 ~ dnorm(0, 1e-07)
a10 ~ dnorm(0, 1e-07)
a11 ~ dnorm(0, 1e-07)
a12 ~ dnorm(0, 1e-07)
a21 ~ dnorm(0, 1e-07)
a22 ~ dnorm(0, 1e-07)
a23 ~ dnorm(0, 1e-07)
a24 ~ dnorm(0, 1e-07)
a25 ~ dnorm(0, 1e-07)
a26 ~ dnorm(0, 1e-07)
a27 ~ dnorm(0, 1e-07)

# priors for factor loadings

g7 <- 1
g8 ~ dnorm(0, 1e-07)
g9 ~ dnorm(0, 1e-07)
g10 ~ dnorm(0, 1e-07)
g11 ~ dnorm(0, 1e-07)
g12 ~ dnorm(0, 1e-07)
g21 ~ dnorm(0, 1e-07)
g22 ~ dnorm(0, 1e-07)
g23 ~ dnorm(0, 1e-07)
g24 ~ dnorm(0, 1e-07)
g25 ~ dnorm(0, 1e-07)
g26 ~ dnorm(0, 1e-07)
g27 ~ dnorm(0, 1e-07)

# inference about the variance
# lognormal distribution

# mean
mean.v <- exp(eta_means[3] + sigma2_eta[3,3]/2)

# variance
var.v <- (exp(sigma2_eta[3,3]) - 1) * exp(2 * eta_means[3] +
sigma2_eta[3,3])
}
```

Online Supplemental Material B (Continued)**The Stan Code for Example 4**

```

//stan code for illustrative example 4

// ----- data block -----;

data {
  int<lower=0> nobs;          // number of subjects

  matrix[nobs,8] y7;         // observed indicators
  matrix[nobs,8] y8;
  matrix[nobs,8] y9;
  matrix[nobs,8] y10;
  matrix[nobs,8] y11;
  matrix[nobs,8] y12;
  matrix[nobs,8] y21;
  matrix[nobs,8] y22;
  matrix[nobs,8] y23;
  matrix[nobs,8] y24;
  matrix[nobs,8] y25;
  matrix[nobs,8] y26;
  matrix[nobs,8] y27;
}

// ----- parameter block -----;

parameters {
  vector[3] eta_means;      // latent means

  // indicator intercepts
  real a8;
  real a9;
  real a10;
  real a11;
  real a12;
  real a21;
  real a22;
  real a23;
  real a24;
  real a25;
  real a26;
  real a27;

  // factor loadings
  real g8;
  real g9;
  real g10;
  real g11;
  real g12;
  real g21;
  real g22;
  real g23;
  real g24;
  real g25;

```

```

real g26;
real g27;

// covariance matrix for 2nd order latent variables
cov_matrix[3] sigma2_eta;

// covariance matrix for observed indicator residual variances
cov_matrix[8] sigma2_7;
cov_matrix[8] sigma2_8;
cov_matrix[8] sigma2_9;
cov_matrix[8] sigma2_10;
cov_matrix[8] sigma2_11;
cov_matrix[8] sigma2_12;
cov_matrix[8] sigma2_21;
cov_matrix[8] sigma2_22;
cov_matrix[8] sigma2_23;
cov_matrix[8] sigma2_24;
cov_matrix[8] sigma2_25;
cov_matrix[8] sigma2_26;
cov_matrix[8] sigma2_27;

// latent variables as parameters

matrix[nobs, 3] eta;      // latent growth constructs
matrix[nobs, 8] u;        // latent construct disturbances

}

// ----- transformed parameters block -----;

transformed parameters {

// declare the parameters

real mean_v;      // mean of the intra-individual variance
real var_v;        // variance of the intra-individual variance

matrix[nobs, 8] pa;    // latent construct

// define transformation

mean_v = exp(eta_means[3] + sigma2_eta[3,3]/2);
var_v = (exp(sigma2_eta[3,3]) - 1) * exp(2 * eta_means[3] +
sigma2_eta[3,3]);

for (i in 1:nobs){
  for(t in 1:8){
    pa[i, t] = eta[i,1] + (t-1)*eta[i, 2] + u[i, t];
  }
}
}

// ----- model block -----;

model {

```



```

// priors for latent factor means
eta_means[1] ~ normal(0, 1e+07);
eta_means[2] ~ normal(0, 1e+07);
eta_means[3] ~ normal(0, 1e+07);

// priors for item intercepts
a8 ~ normal(0, 1e+07);
a9 ~ normal(0, 1e+07);
a10 ~ normal(0, 1e+07);
a11 ~ normal(0, 1e+07);
a12 ~ normal(0, 1e+07);
a21 ~ normal(0, 1e+07);
a22 ~ normal(0, 1e+07);
a23 ~ normal(0, 1e+07);
a24 ~ normal(0, 1e+07);
a25 ~ normal(0, 1e+07);
a26 ~ normal(0, 1e+07);
a27 ~ normal(0, 1e+07);

// priors for item loadings

g8 ~ normal(0, 1e+07);
g9 ~ normal(0, 1e+07);
g10 ~ normal(0, 1e+07);
g11 ~ normal(0, 1e+07);
g12 ~ normal(0, 1e+07);
g21 ~ normal(0, 1e+07);
g22 ~ normal(0, 1e+07);
g23 ~ normal(0, 1e+07);
g24 ~ normal(0, 1e+07);
g25 ~ normal(0, 1e+07);
g26 ~ normal(0, 1e+07);
g27 ~ normal(0, 1e+07);

// growth factors
for (i in 1:nobs){
eta[i, 1:3] ~ multi_normal(eta_means[1:3], sigma2_eta);

// level-1 latent factor disturbances

for(t in 1:8){
u[i, t] ~ normal(0, sqrt(exp(eta[i, 3]))); // depends on log variance
factor for each person
}

// observed indicators

y7[i, 1:8] ~ multi_normal(0 + 1*pa[i, 1:8], sigma2_7);
y8[i, 1:8] ~ multi_normal(a8 + g8*pa[i, 1:8], sigma2_8);
y9[i, 1:8] ~ multi_normal(a9 + g9*pa[i, 1:8], sigma2_9);
y10[i, 1:8] ~ multi_normal(a10 + g10*pa[i, 1:8], sigma2_10);
y11[i, 1:8] ~ multi_normal(a11 + g11*pa[i, 1:8], sigma2_11);
y12[i, 1:8] ~ multi_normal(a12 + g12*pa[i, 1:8], sigma2_12);

```

```
y21[i, 1:8] ~ multi_normal(a21 + g21*pa[i, 1:8], sigma2_21);
y22[i, 1:8] ~ multi_normal(a22 + g22*pa[i, 1:8], sigma2_22);
y23[i, 1:8] ~ multi_normal(a23 + g23*pa[i, 1:8], sigma2_23);
y24[i, 1:8] ~ multi_normal(a24 + g24*pa[i, 1:8], sigma2_24);
y25[i, 1:8] ~ multi_normal(a25 + g25*pa[i, 1:8], sigma2_25);
y26[i, 1:8] ~ multi_normal(a26 + g26*pa[i, 1:8], sigma2_26);
y27[i, 1:8] ~ multi_normal(a27 + g27*pa[i, 1:8], sigma2_27);

}
} // end of model block
```