Local Minima and Factor Rotations in Exploratory Factor Analysis

Online Supplement

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Contents

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Five Rotation Methods: Varimax, Quartimin (Oblimin), Entropy, GeominT, and GeominQ

In this section, we offer a brief review of the five rotation methods that we investigated. We first derive the varimax and quartimin functions from the more generalized Crawford-Ferguson (CF) family (Crawford & Ferguson, 1970). Next, we discuss the entropy function (McCammon, 1966) and we conclude by describing the orthogonal and oblique geomin methods (Yates, 1988).

The Crawford-Ferguson Family

This section shows a derivation of the varimax (Kaiser, 1958) and quartimin (Carroll, 1953) rotation criteria from the Crawford-Ferguson family (Crawford & Ferguson, 1970) under orthogonal rotation. Recall that Browne (2001) rewrites this family such that

$$Q(\mathbf{\Lambda}) = (p - C_2) \sum_{i}^{p} \sum_{m}^{k} \sum_{n \neq m}^{k} \lambda_{in}^2 \lambda_{im}^2 + C_2 \sum_{m}^{k} \sum_{i}^{p} \sum_{j \neq i}^{p} \lambda_{im}^2 \lambda_{jm}^2, \tag{1}$$

where Λ is an $p \times k$ factor pattern matrix, p is the number of observed variables, k is the number of common factors, C_2 ($0 \leq C_2 \leq 1$) is weight that determines the relative importance of test and factor parsimony in the rotation criterion, $Q(\mathbf{\Lambda})$, and λ_{ij} is the loading of item i on factor j. When $C_2 = 0$, (1) is equal to

$$Q(\mathbf{\Lambda}) = p \sum_{i}^{p} \sum_{m}^{k} \sum_{n \neq m}^{k} \lambda_{in}^2 \lambda_{im}^2,$$

which, by dividing both sides by p, produces

$$Q'(\mathbf{\Lambda}) = \sum_{i}^{p} \sum_{m}^{k} \sum_{n \neq m}^{k} \lambda_{in}^2 \lambda_{im}^2, \tag{2}$$

which is equal to Carroll's (1953) quartimin function.

To facilitate our derivation of the varimax criterion from (1), we note that, in an orthogonal model, the sum of squares of communalities can be written

$$H = \sum_{i}^{p} (\sum_{m}^{k} \lambda_{im}^2)^2 = \sum_{i}^{p} \sum_{m}^{k} \lambda_{im}^4 + \sum_{i}^{p} \sum_{m}^{k} \sum_{n \neq m}^{k} \lambda_{in}^2 \lambda_{im}^2 \tag{3}$$

$$= Q + T, \tag{4}$$

where Q and T are the quartimax (Neuhaus & Wrigley, 1954) and quartimin criteria, respectively. Similarly, the sum of squares of factor variances can be written

$$V = \sum_m^k (\sum_i^p \lambda_{im}^2)^2 = \sum_i^p \sum_m^k \lambda_{im}^4 + \sum_m^k \sum_i^p \sum_{j \neq i}^p \lambda_{im}^2 \lambda_{jm}^2 \tag{5}$$

$$= Q + F, \tag{6}$$

where F is a measure of factor parsimony. When $C_2 = 1$, (1) is equal to

$$\begin{aligned} Q(\mathbf{\Lambda}) &= (p-1) \sum_i^p \sum_m^k \sum_{n \neq m}^k \lambda_{in}^2 \lambda_{im}^2 + \sum_m^k \sum_i^p \sum_{j \neq i}^p \lambda_{im}^2 \lambda_{jm}^2 \\ &= (p-1)T + F \\ &= (p-1)(H - Q) + (V - Q) \\ &= (p-1)H - (pQ - V). \end{aligned}$$

Since H is constant in orthogonal rotations, minimizing $Q(\mathbf{\Lambda})$ is equivalent to maximizing $(pQ - V)$ and

$$Q'(\mathbf{\Lambda}) = \frac{(pQ - V)}{p^2} = \frac{p \sum_i^p \sum_m^k \lambda_{im}^4 - \sum_m^k (\sum_i^p \lambda_{im}^2)^2}{p^2}. \tag{7}$$

This is the familiar varimax criterion.

The Minimum Entropy (Entropy)

The minimum entropy (entropy) criterion takes the following form

$$Q(\mathbf{\Lambda}) = \frac{\Sigma_{j=1}^k \Sigma_{i=1}^k \frac{\lambda_{ji}^2}{\mathbf{\Lambda}_j} ln \frac{\lambda_{ji}^2}{\mathbf{\Lambda}_j}}{\Sigma_{j=1}^k \frac{\mathbf{\Lambda}_j}{\mathbf{\Lambda}} ln \frac{\mathbf{\Lambda}_j}{\mathbf{\Lambda}}}, \tag{8}$$

where

$$\mathbf{\Lambda}_j = \Sigma_{i=1}^p \lambda_{ji}^2$$

and

$$\mathbf{\Lambda} = \Sigma_{j=1}^k \mathbf{\Lambda}_j.$$

The numerator of (8) denotes the sum of k entropy functions, each applied to a common factor, and is minimized when each column of the factor pattern contains only one non-zero loading. On the other hand, the denominator denotes the sum of entropy functions of column sums and is maximized when column sums are equal. As a result, $Q(\mathbf{\Lambda})$ is minimized when columns of $\mathbf{\Lambda}$ satisfy Thurstone's simple structure criteria (i.e. small number of non-zero loadings) and their column sums do not differ markedly.

The Geomin Family

The geomin complexity function can be written (Yates, 1988, p. 46)

$$Q(\mathbf{\Lambda}) = \mathbf{\Sigma}_{i=1}^{p}[\textstyle\prod_{j=1}^{k}(\lambda_{ij}^2 + \epsilon)]^{1/k}, \tag{9}$$

where p denotes that number of variables, k denotes the number of factors, λ_{ij} denotes factor loadings of variable i on factor j, and ϵ controls the amount of indeterminacy of $Q(\mathbf{\Lambda})$. Here, $Q(\mathbf{\Lambda})$ is indeterminate in the sense that when $\epsilon = 0$ and any row element in $\mathbf{\Lambda}$ is zero, then other row elements do not increase the function complexity value. Yates (1987) included a small ϵ in his algorithm to reduce the indeterminacy of $Q(\mathbf{\Lambda})$.

Code to Reproduce Hattori et al.'s (2017): An Influence of ϵ on Geomin Local Solutions

In this section, we report our replication and extension of Hattori, Zhang, & Preacher (2017) with the Thurstone's (1947) Box data and show that lowering geomin's ϵ value can increase the number of LS. For these analyses, we used the three geomin's ϵ values that were used by Hattori, Zhang, & Preacher (2017): $\epsilon \in \{.001, .01, .02\}$.

To create his Box data set, Thurstone (1947) measured three dimensions (length, width, and height) for each of 30 boxes and constructed 26 score functions from these measurements. The correlation matrix for this data set was reported in (Thurstone, 1947, p. 370) and is also included in the R `fungible` package.

As the first step in our replication, we report Thurstone's (1947) original solution.

```
## Create a matrix from Thurstone's solution
## Used as a target matrix to sort columns of the estimated solution
ThurstoneSolution <- matrix(c(.95,  .01,  .01,
                              .02,  .92,  .01,
                              .02,  .05,  .91,
                              .59,  .64, -.03,
                              .60,  .00,  .62,
                             -.04,  .60,  .58,
                              .81,  .38,  .01,
                              .35,  .79,  .01,
                              .79, -.01,  .41,
                              .40, -.02,  .79,
                             -.04,  .74,  .40,
                             -.02,  .41,  .74,
                              .74, -.77,  .06,
                             -.74,  .77, -.06,
                              .74,  .02, -.73,
                             -.74, -.02,  .73,
                             -.07,  .80, -.76,
                              .07, -.80,  .76,
                              .51,  .70, -.03,
                              .56, -.04,  .69,
                             -.02,  .60,  .58,
                              .50,  .69, -.03,
                              .52, -.01,  .68,
                             -.01,  .60,  .55,
                              .43,  .46,  .45,
                              .31,  .51,  .46),
                            nrow = 26, ncol = 3, byrow=TRUE)
```

Next, we use function `faMain` to factor analyze and rotate the Box data set.

```r
## Load Thurstone's Box data from the fungible library
data(Box26, package = "fungible")
## Ignore warnings about non-positive definite sample
## correlation matrix
suppressWarnings(
  fout <- fungible::faMain(
    R             = Box26,
    numFactors    = 3,
    facMethod     = 'fapa',
    rotate        = "geominQ",
    targetMatrix  = ThurstoneSolution,
    rotateControl = list(numberStarts = 200,
                         delta = .01), ## geomin epsilon
    Seed          = 123))

## Summarize geomin rotation
summary(fout, Set = 1, print = 0)
```

```
##
## Call:
## fungible::faMain(R = Box26, numFactors = 3, facMethod = "fapa",
##     rotate = "geominQ", targetMatrix = ThurstoneSolution, Seed = 123,
##     rotateControl = list(numberStarts = 200, delta = 0.01))
##
##
##
## ====================
## Rotation Diagnostics
## ====================
##
## Rotation Method: geominQ
## Epsilon value for rotation convergence and solution set clustering: 1e-05
## Number of Rotation Random Starts: 200
## Number of Solution Sets: 4
## Solution Characteristics:
##
##                          Set 1   Set 2   Set 3   Set 4   UnSpun
## Complexity Value          3.266   3.802   3.819   4.028   3.802
## Hyperplane Count             25      15       9      12      15
## % Cases (x 100) in Set       30      34      26      10       0
## RMSD                          0   0.484   0.436   0.483   0.484
## Converged                  TRUE    TRUE    TRUE    TRUE    TRUE
##
## Factor Output for Set 1:
```

```
## Factor Pattern Matrix
##
##                               f1    f2    f3   h2
## x                           1.00 -0.01 -0.01 0.98
## y                           0.06  0.94  0.05 0.94
## z                           0.00  0.07  0.97 0.97
## xy                          0.64  0.65 -0.01 0.98
## xz                          0.60  0.01  0.64 0.99
## yz                         -0.02  0.62  0.64 0.99
## xsqy                        0.83  0.39  0.01 0.98
## xysq                        0.38  0.82  0.03 0.96
## xsqz                        0.79 -0.01  0.42 0.97
## xzsq                        0.40 -0.01  0.81 0.98
## ysqz                       -0.02  0.77  0.45 0.97
## yzsq                       -0.03  0.45  0.78 0.98
## x.over.y                    0.75 -0.82  0.01 0.99
## y.over.x                   -0.75  0.82 -0.01 0.99
## x.over.z                    0.81  0.00 -0.82 0.97
## z.over.x                   -0.81  0.00  0.82 0.97
## y.over.z                   -0.01  0.84 -0.80 1.00
## z.over.y                    0.01 -0.84  0.80 1.00
## twox.plus.twoy              0.55  0.71 -0.02 0.95
## twox.plus.twoz              0.56 -0.02  0.69 0.98
## twoy.plus.twoz             -0.01  0.62  0.63 0.99
## root.xsq.plus.ysq           0.54  0.70 -0.01 0.93
## root.xsq.plus.zsq           0.52  0.00  0.68 0.93
## root.ysq.plus.zsq           0.02  0.62  0.60 0.95
## xyz                         0.45  0.49  0.46 0.98
## root.xsq.plus.ysq.plus.zsq  0.34  0.53  0.49 0.94
##
##
## Factor Correlation Matrix
##
##      f1   f2   f3
## f1 1.00 0.20 0.28
## f2 0.20 1.00 0.26
## f3 0.28 0.26 1.00
##
##
## ====================
## Factor Indeterminacy
## ====================
##
## Observed R computationally singular. Unable to calculate FI values.
##
```

```
##
## =========
## Model Fit
## =========
##
## RMSD  (R, Rhat)   =  0.008
## RMSAD (|R|,|Rhat|) =  0.008
```

Note that Bernaards & Jennrich (2005) use `delta` to denote the geomin ϵ parameter.

Regarding our findings, note that the loading matrix with the lowest complexity value is similar to that reported in Table 2 of Hattori, Zhang, & Preacher (2017). Some minor differences between these matrices are likely due to rounding precision in the published Box correlation matrix. [R tip: The `summary` function of an `faMain` object allows users to examine other rotated solutions by changing the `Set` parameter. For example, to print the loading matrix associated with solution set 2, set `Set` = 2.]

Next, to show that lower values of ϵ can increase the number of LS, we rerun the previous R code with `delta` = .001.

```
## Ignore warnings about non-positive definite sample
## correlation matrix
suppressWarnings(
  fout <- fungible::faMain(
    R             = Box26,
    numFactors    = 3,
    facMethod     = 'fapa',
    rotate        = "geominQ",
    targetMatrix  = ThurstoneSolution,
    rotateControl = list(numberStarts = 200,
                         delta = .001), ## increase in real problem
    Seed          = 123))

## Summarize geomin rotation
summary(fout, Set = 1, print = 0)
```

```
##
## Call:
## fungible::faMain(R = Box26, numFactors = 3, facMethod = "fapa",
##     rotate = "geominQ", targetMatrix = ThurstoneSolution, Seed = 123,
##     rotateControl = list(numberStarts = 200, delta = 0.001))
##
##
##
## ====================
## Rotation Diagnostics
## ====================
```

```
##
## Rotation Method: geominQ
## Epsilon value for rotation convergence and solution set clustering: 1e-05
## Number of Rotation Random Starts: 200
## Number of Solution Sets: 9
## Solution Characteristics:
##
##                          Set 1   Set 2   Set 3   Set 4   Set 5   Set 6   Set 7
## Complexity Value          1.75   2.697   2.729   2.729   2.729   2.745   2.992
## Hyperplane Count            24      18      12      13      13      13      17
## % Cases (x 100) in Set      34      24       5       1       5      11       6
## RMSD                         0   0.498   0.437   0.448   0.443   0.444   0.492
## Converged                 TRUE    TRUE    TRUE    TRUE    TRUE    TRUE    TRUE
##                          Set 8   Set 9  UnSpun
## Complexity Value          3.02   3.049  2.992
## Hyperplane Count            15      16     17
## % Cases (x 100) in Set       5       8      0
## RMSD                     0.473   0.489  0.492
## Converged                 TRUE    TRUE   TRUE
##
## Factor Output for Set 1:
## Factor Pattern Matrix
##
##                                f1    f2    f3   h2
## x                            1.00 -0.01 -0.01 0.98
## y                            0.07  0.94  0.05 0.94
## z                            0.00  0.07  0.96 0.97
## xy                           0.64  0.64 -0.01 0.98
## xz                           0.60  0.02  0.64 0.99
## yz                          -0.02  0.62  0.64 0.99
## xsqy                         0.84  0.39  0.01 0.98
## xysq                         0.39  0.81  0.04 0.96
## xsqz                         0.79 -0.01  0.41 0.97
## xzsq                         0.41 -0.01  0.80 0.98
## ysqz                        -0.01  0.77  0.45 0.97
## yzsq                        -0.02  0.45  0.78 0.98
## x.over.y                     0.74 -0.82  0.00 0.99
## y.over.x                    -0.74  0.82  0.00 0.99
## x.over.z                     0.81  0.00 -0.82 0.97
## z.over.x                    -0.81  0.00  0.82 0.97
## y.over.z                     0.00  0.84 -0.79 1.00
## z.over.y                     0.00 -0.84  0.79 1.00
## twox.plus.twoy               0.55  0.71 -0.02 0.95
## twox.plus.twoz               0.56 -0.01  0.68 0.98
## twoy.plus.twoz               0.00  0.62  0.63 0.99
```

```
## root.xsq.plus.ysq           0.54  0.70  0.00 0.93
## root.xsq.plus.zsq           0.53  0.00  0.68 0.93
## root.ysq.plus.zsq           0.02  0.62  0.60 0.95
## xyz                         0.46  0.49  0.47 0.98
## root.xsq.plus.ysq.plus.zsq  0.35  0.53  0.49 0.94
##
##
## Factor Correlation Matrix
##
##      f1   f2   f3
## f1 1.00 0.19 0.27
## f2 0.19 1.00 0.25
## f3 0.27 0.25 1.00
##
##
## ====================
## Factor Indeterminacy
## ====================
##
## Observed R computationally singular. Unable to calculate FI values.
##
##
## =========
## Model Fit
## =========
##
## RMSD  (R, Rhat)    =  0.008
## RMSAD (|R|,|Rhat|) =  0.008
```

Although geominQ converged to more LS with `delta` = .001 than with `delta` = .01, both rotations reproduced Thurstone's preferred simple structure solution equally well.

Here, we rotate the loading matrix with geominQ and `delta` = .02.

```r
## Ignore warnings about non-positive definite sample
## correlation matrix
suppressWarnings(
  fout <- fungible::faMain(
    R             = Box26,
    numFactors    = 3,
    facMethod     = 'fapa',
    rotate        = "geominQ",
    targetMatrix  = ThurstoneSolution,
    rotateControl = list(numberStarts = 200,
                         delta = .02),
    Seed          = 123))
```

```r
## Summarize geomin rotation
summary(fout, Set = 1, print = 0)
```

```
##
## Call:
## fungible::faMain(R = Box26, numFactors = 3, facMethod = "fapa",
##     rotate = "geominQ", targetMatrix = ThurstoneSolution, Seed = 123,
##     rotateControl = list(numberStarts = 200, delta = 0.02))
##
##
##
## ====================
## Rotation Diagnostics
## ====================
##
## Rotation Method: geominQ
## Epsilon value for rotation convergence and solution set clustering: 1e-05
## Number of Rotation Random Starts: 200
## Number of Solution Sets: 4
## Solution Characteristics:
##
##                          Set 1   Set 2   Set 3   Set 4   UnSpun
## Complexity Value         4.103   4.482   4.505   4.666   4.482
## Hyperplane Count            25      13       9      10      13
## % Cases (x 100) in Set      30      36      26       9       0
## RMSD                         0   0.477   0.434   0.478   0.477
## Converged                 TRUE    TRUE    TRUE    TRUE    TRUE
##
## Factor Output for Set 1:
## Factor Pattern Matrix
##
##                                f1    f2    f3   h2
## x                            0.99 -0.01 -0.01 0.98
## y                            0.05  0.94  0.05 0.94
## z                            0.00  0.07  0.97 0.97
## xy                           0.63  0.65 -0.01 0.98
## xz                           0.59  0.01  0.65 0.99
## yz                          -0.03  0.62  0.64 0.99
## xsqy                         0.83  0.39  0.01 0.98
## xysq                         0.38  0.82  0.03 0.96
## xsqz                         0.79 -0.01  0.42 0.97
## xzsq                         0.40 -0.01  0.81 0.98
## ysqz                        -0.02  0.77  0.45 0.97
## yzsq                        -0.03  0.45  0.78 0.98
```

```
## x.over.y                     0.75 -0.83  0.01 0.99
## y.over.x                    -0.75  0.83 -0.01 0.99
## x.over.z                     0.81  0.00 -0.82 0.97
## z.over.x                    -0.81  0.00  0.82 0.97
## y.over.z                    -0.01  0.84 -0.80 1.00
## z.over.y                     0.01 -0.84  0.80 1.00
## twox.plus.twoy               0.54  0.71 -0.02 0.95
## twox.plus.twoz               0.56 -0.02  0.69 0.98
## twoy.plus.twoz              -0.01  0.62  0.63 0.99
## root.xsq.plus.ysq            0.53  0.70 -0.01 0.93
## root.xsq.plus.zsq            0.52  0.00  0.68 0.93
## root.ysq.plus.zsq            0.01  0.62  0.60 0.95
## xyz                          0.45  0.49  0.47 0.98
## root.xsq.plus.ysq.plus.zsq  0.34  0.53  0.49 0.94
##
##
## Factor Correlation Matrix
##
##      f1   f2   f3
## f1 1.00 0.20 0.27
## f2 0.20 1.00 0.26
## f3 0.27 0.26 1.00
##
##
## ====================
## Factor Indeterminacy
## ====================
##
## Observed R computationally singular. Unable to calculate FI values.
##
##
## =========
## Model Fit
## =========
##
## RMSD  (R, Rhat)   =  0.008
## RMSAD (|R|,|Rhat|) =  0.008
```

Finally, we rotated the Box structure matrix via quartimin by performing an oblimin rotation with gamma = 0.

```
## Ignore warnings about non-positive definite sample
## correlation matrix
suppressWarnings(
  fout <- fungible::faMain(
    R             = Box26,
```

```r
    numFactors    = 3,
    facMethod     = 'fapa',
    rotate        = 'oblimin',
    targetMatrix  = ThurstoneSolution,
    rotateControl = list(numberStarts = 200,
                         gamma = 0,
                         standardize  = 'none'),
    Seed          = 123))

summary(fout, Set = 1, print = 0)
```

```
##
## Call:
## fungible::faMain(R = Box26, numFactors = 3, facMethod = "fapa",
##     rotate = "oblimin", targetMatrix = ThurstoneSolution, Seed = 123,
##     rotateControl = list(numberStarts = 200, gamma = 0, standardize = "none"))
##
##
##
## ====================
## Rotation Diagnostics
## ====================
##
## Rotation Method: oblimin
## Epsilon value for rotation convergence and solution set clustering: 1e-05
## Number of Rotation Random Starts: 200
## Number of Solution Sets: 1
## Solution Characteristics:
##
##                         Set 1   UnSpun
## Complexity Value        1.978   1.978
## Hyperplane Count            6       6
## % Cases (x 100) in Set    100       0
## RMSD                        0       0
## Converged                TRUE    TRUE
##
## Factor Output for Set 1:
## Factor Pattern Matrix
##
##                             f1    f2    f3   h2
## x                         0.72 -0.28  0.60 0.98
## y                        -0.06  0.60  0.76 0.94
## z                        -0.45 -0.42  0.78 0.97
## xy                        0.41  0.27  0.86 0.98
```

```
## xz                           0.13 -0.47  0.86 0.99
## yz                          -0.37  0.12  0.92 0.99
## xsqy                         0.57  0.03  0.80 0.98
## xysq                         0.19  0.43  0.86 0.96
## xsqz                         0.38 -0.43  0.79 0.97
## xzsq                        -0.08 -0.51  0.85 0.98
## ysqz                        -0.29  0.31  0.89 0.97
## yzsq                        -0.42 -0.06  0.90 0.98
## x.over.y                     0.60 -0.77 -0.14 0.99
## y.over.x                    -0.60  0.77  0.14 0.99
## x.over.z                     0.97  0.17 -0.11 0.97
## z.over.x                    -0.97 -0.17  0.11 0.97
## y.over.z                     0.29  0.96  0.01 1.00
## z.over.y                    -0.29 -0.96 -0.01 1.00
## twox.plus.twoy               0.34  0.34  0.84 0.95
## twox.plus.twoz               0.08 -0.49  0.85 0.98
## twoy.plus.twoz              -0.35  0.12  0.93 0.99
## root.xsq.plus.ysq            0.33  0.33  0.84 0.93
## root.xsq.plus.zsq            0.06 -0.47  0.83 0.93
## root.ysq.plus.zsq           -0.32  0.13  0.92 0.95
## xyz                          0.07 -0.02  0.99 0.98
## root.xsq.plus.ysq.plus.zsq -0.02  0.04  0.97 0.94
##
##
## Factor Correlation Matrix
##
##       f1    f2    f3
## f1  1.00 -0.02  0.02
## f2 -0.02  1.00 -0.01
## f3  0.02 -0.01  1.00
##
##
## ====================
## Factor Indeterminacy
## ====================
##
## Observed R computationally singular. Unable to calculate FI values.
##
##
## =========
## Model Fit
## =========
##
## RMSD  (R, Rhat)   =  0.008
## RMSAD (|R|,|Rhat|) =  0.008
```

Although quartimin converged to only one solution, the converged solution does not faithfully reproduce Thurstone's simple structure pattern. However, we can recover Thurstone's original solution using quartimin rotations by preconditioning the factor structure matrix with row standardization via `standardize=CM`. [This is left as an exercise for the reader.]

Code to Reproduce Figure 2: An Illustration of Local Solutions in Varimax Rotation

In this section, we illustrate how varimax can converge on local minima when using the gradient projection algorithm (Bernaards & Jennrich, 2005). A graphical display of these results is shown in Figure 2 of the manuscript. The code for this figure is reported in the "mk-varimax-example.R" file (included in the "Local Minima Programs" folder).

We first construct a population factor pattern matrix using `simFA()`.

```
#### 1. Construct lambda ####
sout <- simFA(Model = list(NFac = 2,
                           NItemPerFac = 4,
                           Model = "oblique"),
              Phi = list(MaxAbsPhi = .6,
                         PhiType = "fixed"),
              Loadings = list(FacLoadDist = "fixed",
                              FacLoadRange = .4),
              Seed = 1)
```

To extract (a) the data-generating factor pattern, (b) the unrotated, population factor pattern, and (c) the factor correlation matrix, we run the following commands.

```
## True loadings
(True_A <- sout$loadings)
```

```
##     F1  F2
## V1 0.4 0.0
## V2 0.4 0.0
## V3 0.4 0.0
## V4 0.4 0.0
## V5 0.0 0.4
## V6 0.0 0.4
## V7 0.0 0.4
## V8 0.0 0.4
```

```
## Unrotated loadings
(A <- sout$urloadings)
```

```
##            [,1]       [,2]
## [1,] -0.3577709 -0.1788854
## [2,] -0.3577709 -0.1788854
## [3,] -0.3577709 -0.1788854
## [4,] -0.3577709 -0.1788854
## [5,] -0.3577709  0.1788854
## [6,] -0.3577709  0.1788854
## [7,] -0.3577709  0.1788854
```

```
## [8,] -0.3577709  0.1788854
```

```r
## Phi matrix
sout$Phi
```

```
##     F1  F2
## F1 1.0 0.6
## F2 0.6 1.0
```

Next, we rotate the unrotated, population factor pattern using `faMain`.

```r
fout <- faMain (urLoadings = A,
                numFactors = 2,
                facMethod = "fals",
                rotate= "varimax",
                rotateControl = list(numberStarts = 500))
```

Use the `summary` function to summarize the rotation results.

```r
summary(fout)
```

```
##
## Call:
## faMain(numFactors = 2, facMethod = "fals", urLoadings = A, rotate = "varimax",
##     rotateControl = list(numberStarts = 500))
##
##
##
## ====================
## Rotation Diagnostics
## ====================
##
## Rotation Method: varimax
## Epsilon value for rotation convergence and solution set clustering: 1e-05
## Number of Rotation Random Starts: 500
## Number of Solution Sets: 2
## Solution Characteristics:
##
##                         Set 1   Set 2   UnSpun
## Complexity Value        -0.016      0       0
## Hyperplane Count             0      0       0
## % Cases (x 100) in Set     100      0       0
## RMSD                         0  0.216   0.216
## Converged                 TRUE   TRUE    TRUE
##
## Factor Output for Set 1:
## Factor Pattern Matrix
##
```

```
##         f1   f2   h2
## var1 0.13 0.38 0.16
## var2 0.13 0.38 0.16
## var3 0.13 0.38 0.16
## var4 0.13 0.38 0.16
## var5 0.38 0.13 0.16
## var6 0.38 0.13 0.16
## var7 0.38 0.13 0.16
## var8 0.38 0.13 0.16
##
##
## Factor Correlation Matrix
##
##    f1 f2
## f1  1  0
## f2  0  1
```

Next, we extract factor loadings associated with local solutions:

```r
F1 <- summary(object = fout, Set = 1, digits = 4)$loadings
F2 <- summary(object = fout, Set = 2, digits = 4)$loadings
```

To facilitate factor comparison, we combine the two factor matrices.

```r
cbind(F1,F2) %>% round(7)
```

```
##             f1        f2        f1         f2
## var1 0.1264927 0.3794728 0.3577709 -0.1788854
## var2 0.1264927 0.3794728 0.3577709 -0.1788854
## var3 0.1264927 0.3794728 0.3577709 -0.1788854
## var4 0.1264927 0.3794728 0.3577709 -0.1788854
## var5 0.3794739 0.1264895 0.3577709  0.1788854
## var6 0.3794739 0.1264895 0.3577709  0.1788854
## var7 0.3794739 0.1264895 0.3577709  0.1788854
## var8 0.3794739 0.1264895 0.3577709  0.1788854
```

We can compute the varimax complexity value for F1 and F2 using the varimax algorithm described by Neudecker (1981).

```r
varimax_neudecker <- function(L) {
  C <- L*L
  k <- nrow(L)
  s <- rep(1, k)
  return((1/k) * sum(diag(t(C) %*% C)) - (1/k^2) * t(s) %*% C %*% t(C) %*% s)
}
```

The F1 complexity value equals `varimax_neudecker(F1)` $= 0.008192$, whereas the F2 complexity value equals `varimax_neudecker(F2)` $= 3.469447 \times 10^{-18}$.

Study 1: A Simulation Study of Rotation Local Minima: Model Fit Evaluated by the Comparative Fit Index

Table S1 describes the 96 EFA models that were investigated in Study 1. These EFA models were formulated by crossing six design factors: (a) two levels of factor loading size, $\lambda \in \{.4, .8\}$, (b) two levels of number of indicator per factor, $p/k \in \{5, 10\}$, (c) two levels of cross-loading probability, $\text{ProbX} \in \{0, .1\}$, (d) three levels of model approximation error, $\text{ME} \in \{\text{None, Good-Fit, Moderate-Fit}\}$, (e) two levels of factor correlation, $\phi_{\text{ij}} \in \{0, .6\}$, and (f) two levels of sample size, $\text{Sample Size} \in \{200, 500\}$. Note that in Study 1 we used the Comparative Fit Index (Bentler, 1990) to quantify model approximation error when $\text{ME} \in \{\text{Good-Fit, Moderate-Fit}\}$. For each population model, Table S1 reports two `simFA` parameters that were used to control the amount of ME. These parameters included the proportion of indicator variance due to model approximation error, V_{p}, and a parameter, ϵ_{TKL}, that controlled the expected factor loadings in successive columns of $\mathbf{W}$ (i.e., the minor-factor loadings matrix). By design, $\text{V}_{\text{p}} \in [.01, .20]$.

Table S1: Exploratory Factor Model Designs Using CFI

ID	λ	p/k	ProbX	ME	ϵ_{TKL}	V_p	ϕ_{ij}	$\mathcal{N}$
1	0.4	5	0.0	None	NA	NA	0.0	200
2	0.8	5	0.0	None	NA	NA	0.0	200
3	0.4	10	0.0	None	NA	NA	0.0	200
4	0.8	10	0.0	None	NA	NA	0.0	200
5	0.4	5	0.1	None	NA	NA	0.0	200
6	0.8	5	0.1	None	NA	NA	0.0	200
7	0.4	10	0.1	None	NA	NA	0.0	200
8	0.8	10	0.1	None	NA	NA	0.0	200
9	0.4	5	0.0	Good	0.01	0.01	0.0	200
10	0.8	5	0.0	Good	0.01	0.01	0.0	200
11	0.4	10	0.0	Good	0.01	0.01	0.0	200
12	0.8	10	0.0	Good	0.01	0.01	0.0	200
13	0.4	5	0.1	Good	0.01	0.01	0.0	200
14	0.8	5	0.1	Good	0.01	0.01	0.0	200
15	0.4	10	0.1	Good	0.01	0.01	0.0	200
16	0.8	10	0.1	Good	0.01	0.01	0.0	200
17	0.4	5	0.0	Moderate	0.04	0.17	0.0	200
18	0.8	5	0.0	Moderate	0.07	0.16	0.0	200
19	0.4	10	0.0	Moderate	0.04	0.12	0.0	200
20	0.8	10	0.0	Moderate	0.05	0.19	0.0	200
21	0.4	5	0.1	Moderate	0.06	0.12	0.0	200
22	0.8	5	0.1	Moderate	0.07	0.15	0.0	200
23	0.4	10	0.1	Moderate	0.04	0.13	0.0	200
24	0.8	10	0.1	Moderate	0.05	0.19	0.0	200
25	0.4	5	0.0	None	NA	NA	0.0	500
26	0.8	5	0.0	None	NA	NA	0.0	500
27	0.4	10	0.0	None	NA	NA	0.0	500
28	0.8	10	0.0	None	NA	NA	0.0	500
29	0.4	5	0.1	None	NA	NA	0.0	500
30	0.8	5	0.1	None	NA	NA	0.0	500
31	0.4	10	0.1	None	NA	NA	0.0	500
32	0.8	10	0.1	None	NA	NA	0.0	500
33	0.4	5	0.0	Good	0.01	0.01	0.0	500
34	0.8	5	0.0	Good	0.01	0.01	0.0	500
35	0.4	10	0.0	Good	0.01	0.01	0.0	500
36	0.8	10	0.0	Good	0.01	0.01	0.0	500
37	0.4	5	0.1	Good	0.01	0.01	0.0	500
38	0.8	5	0.1	Good	0.01	0.01	0.0	500
39	0.4	10	0.1	Good	0.01	0.01	0.0	500
40	0.8	10	0.1	Good	0.01	0.01	0.0	500
41	0.4	5	0.0	Moderate	0.04	0.18	0.0	500

42	0.8	5	0.0	Moderate	0.06	0.20	0.0	500
43	0.4	10	0.0	Moderate	0.04	0.10	0.0	500
44	0.8	10	0.0	Moderate	0.05	0.20	0.0	500
45	0.4	5	0.1	Moderate	0.06	0.20	0.0	500
46	0.8	5	0.1	Moderate	0.07	0.14	0.0	500
47	0.4	10	0.1	Moderate	0.04	0.19	0.0	500
48	0.8	10	0.1	Moderate	0.05	0.18	0.0	500
49	0.4	5	0.0	None	NA	NA	0.6	200
50	0.8	5	0.0	None	NA	NA	0.6	200
51	0.4	10	0.0	None	NA	NA	0.6	200
52	0.8	10	0.0	None	NA	NA	0.6	200
53	0.4	5	0.1	None	NA	NA	0.6	200
54	0.8	5	0.1	None	NA	NA	0.6	200
55	0.4	10	0.1	None	NA	NA	0.6	200
56	0.8	10	0.1	None	NA	NA	0.6	200
57	0.4	5	0.0	Good	0.01	0.01	0.6	200
58	0.8	5	0.0	Good	0.01	0.01	0.6	200
59	0.4	10	0.0	Good	0.01	0.01	0.6	200
60	0.8	10	0.0	Good	0.01	0.01	0.6	200
61	0.4	5	0.1	Good	0.01	0.01	0.6	200
62	0.8	5	0.1	Good	0.01	0.01	0.6	200
63	0.4	10	0.1	Good	0.01	0.01	0.6	200
64	0.8	10	0.1	Good	0.01	0.01	0.6	200
65	0.4	5	0.0	Moderate	0.07	0.18	0.6	200
66	0.8	5	0.0	Moderate	0.08	0.14	0.6	200
67	0.4	10	0.0	Moderate	0.04	0.12	0.6	200
68	0.8	10	0.0	Moderate	0.06	0.13	0.6	200
69	0.4	5	0.1	Moderate	0.14	0.07	0.6	200
70	0.8	5	0.1	Moderate	0.07	0.17	0.6	200
71	0.4	10	0.1	Moderate	0.08	0.09	0.6	200
72	0.8	10	0.1	Moderate	0.04	0.20	0.6	200
73	0.4	5	0.0	None	NA	NA	0.6	500
74	0.8	5	0.0	None	NA	NA	0.6	500
75	0.4	10	0.0	None	NA	NA	0.6	500
76	0.8	10	0.0	None	NA	NA	0.6	500
77	0.4	5	0.1	None	NA	NA	0.6	500
78	0.8	5	0.1	None	NA	NA	0.6	500
79	0.4	10	0.1	None	NA	NA	0.6	500
80	0.8	10	0.1	None	NA	NA	0.6	500
81	0.4	5	0.0	Good	0.01	0.01	0.6	500
82	0.8	5	0.0	Good	0.01	0.01	0.6	500
83	0.4	10	0.0	Good	0.01	0.01	0.6	500
84	0.8	10	0.0	Good	0.01	0.01	0.6	500
85	0.4	5	0.1	Good	0.01	0.01	0.6	500
86	0.8	5	0.1	Good	0.01	0.01	0.6	500

87	0.4	10	0.1	Good	0.01	0.01	0.6	500
88	0.8	10	0.1	Good	0.01	0.01	0.6	500
89	0.4	5	0.0	Moderate	0.10	0.05	0.6	500
90	0.8	5	0.0	Moderate	0.06	0.20	0.6	500
91	0.4	10	0.0	Moderate	0.04	0.16	0.6	500
92	0.8	10	0.0	Moderate	0.06	0.14	0.6	500
93	0.4	5	0.1	Moderate	0.14	0.15	0.6	500
94	0.8	5	0.1	Moderate	0.07	0.17	0.6	500
95	0.4	10	0.1	Moderate	0.06	0.13	0.6	500
96	0.8	10	0.1	Moderate	0.05	0.12	0.6	500

Note:
ID denotes model number, λ denotes salient loadings, p/k denotes number of variables per factor, ProbX denotes probability of cross loadings, ME denotes model approximation error, $\mathrm{V_p}$ denotes proportion of variance due to ME, ϵ_{TKL} denotes epsilon TKL, ϕ_{ij} denotes correlations between common factors, and $\mathcal{N}$ denotes sample size.

R Code to Reproduce Sample 67 of Model 26

In this section, we report the R code and sample correlation matrix (see Table S2) that were used to obtain results included in Table 2 of the main text.

```
## Load package
library(fungible)

#### 1. Generate the Population Model and 100 Monte Carlo Samples for
#### Table 2.
sout <- simFA(Model      = list(NFac = 5, NItemPerFac = 5, Model = "orthogonal"),
              Loadings   = list(FacLoadDist = "fixed", FacLoadRange = .8),
              MonteCarlo = list(NSamples = 100, SampleSize = 500),
              Seed       = 655342)
## Population EFA loadings
## (True_A <- sout$loadings)
## Population Phi matrix
## sout$Phi

## Extract sample correlation matrix
R67 = sout$Monte$MCData[[67]]
```

```
#### 2. Conduct EFA on Model 26, Sample 67 ####
## This process takes about 4 minutes
fout <- faMain (R              = R67,
                numFactors     = 5,
                targetMatrix   = sout$loadings,
                facMethod      = "fals",
                rotate         = "cfT",
                rotateControl  = list(numberStarts = 100,
                                      standardize="CM",
                                      kappa = 1/25),
                Seed           = 3366805)
summary(fout, Set = 1, DiagnosticsLevel = 2, digits=4)

#### 3. Investigate Local Solutions ####
faLocalMin(fout, Set = 1, HPthreshold = .1, digits= 5, PrintLevel = 1)

#### 4. Inspect Solutions 28 and 29 from Set 1
Lambda2 <- fout$localSolutionSets[[1]][[28]]$loadings
Lambda2 <- round(faAlign(sout$loadings, Lambda2)$F2,2)

Lambda1 <- fout$localSolutionSets[[1]][[29]]$loadings
Lambda1 <- round(faAlign(sout$loadings, Lambda1)$F2,2)

L0 <- round(sout$loadings,2)
```

```r
## Print Table 2
Table2 <- cbind(L0, Lambda1, Lambda2)
colnames(Table2) <- rep(c("F1","F2", "F3", "F4", "F5"),3)
print( Table2 )
```

Table S2: Correlation Matrix of Sample 67 of Model 26

V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14	V15	V16	V17	V18	V19	V20	V21	V22	V23	V24	V25
100	66	67	69	67	03	03	02	03	03	-02	01	-02	-02	-01	-06	01	00	-03	00	-03	-02	-02	-01	01
66	100	68	64	61	03	04	-01	03	01	01	01	-02	02	02	03	07	02	02	07	00	02	00	02	02
67	68	100	70	66	03	04	04	02	01	-06	-02	-02	-03	02	-05	-02	-03	-04	01	00	00	-02	01	00
69	64	70	100	62	01	02	04	01	03	02	05	01	01	03	-03	-01	-02	00	02	01	02	-01	02	03
67	61	66	62	100	02	00	-01	-01	-01	-05	-01	-03	-03	00	-09	-03	-09	-05	-06	-05	-05	-09	-06	-04
03	03	03	01	02	100	60	61	55	58	-07	-04	-07	-01	-03	-03	-05	-01	-04	-04	-01	-02	-02	00	-03
03	04	04	02	00	60	100	59	57	59	-01	-02	-04	02	00	-02	-02	-02	-02	-03	01	-02	-02	02	00
02	-01	04	04	-01	61	59	100	58	56	-03	-03	-05	00	-02	04	05	04	03	03	05	02	01	02	-01
03	03	02	01	-01	55	57	58	100	57	-02	-04	-05	03	02	04	03	04	03	05	02	04	-01	06	00
03	01	01	03	-01	58	59	56	57	100	02	06	02	13	07	-03	-04	-02	-02	-01	05	02	01	00	04
-02	01	-06	02	-05	-07	-01	-03	-02	02	100	69	64	68	66	-03	-02	-03	-03	-03	00	-07	-01	-09	-03
01	01	-02	05	-01	-04	-02	-03	-04	06	69	100	69	69	66	01	00	03	00	-02	02	-04	01	-05	01
-02	-02	-02	01	-03	-07	-04	-05	-05	02	64	69	100	65	66	-05	-06	00	-01	-03	02	01	02	-01	06
-02	02	-03	01	-03	-01	02	00	03	13	68	69	65	100	69	-05	-06	-04	-07	-07	01	00	04	-02	05
-01	02	02	03	00	-03	00	-02	02	07	66	66	66	69	100	-01	-01	00	-01	00	06	00	02	-02	02
-06	03	-05	-03	-09	-03	-02	04	04	-03	-03	01	-05	-05	-01	100	61	63	65	65	-01	04	-02	-04	-02
01	07	-02	-01	-03	-05	-02	05	03	-04	-02	00	-06	-06	-01	61	100	63	62	63	03	01	-01	03	01
00	02	-03	-02	-09	-01	-02	04	04	-02	-03	03	00	-04	00	63	63	100	65	64	-02	-01	00	-01	-02
-03	02	-04	00	-05	-04	-02	03	03	-02	-03	00	-01	-07	-01	65	62	65	100	63	01	02	-01	00	-01
00	07	01	02	-06	-04	-03	03	05	-01	-03	-02	-03	-07	00	65	63	64	63	100	01	04	01	-03	04
-03	00	00	01	-05	-01	01	05	02	05	00	02	02	01	06	-01	03	-02	01	01	100	65	58	64	63
-02	02	00	02	-05	-02	-02	02	04	02	-07	-04	01	00	00	04	01	-01	02	04	65	100	64	61	65
-02	00	-02	-01	-09	-02	-02	01	-01	01	-01	01	02	04	02	-02	-01	00	-01	01	58	64	100	64	64
-01	02	01	02	-06	00	02	02	06	00	-09	-05	-01	-02	-02	-04	03	-01	00	-03	64	61	64	100	66
01	02	00	03	-04	-03	00	-01	00	04	-03	01	06	05	02	-02	01	-02	-01	04	63	65	64	66	100

Note:
All coefficients are multiplied by 100.

An Example Showing Different Factor Pattens with Equal Rotation Complexity Values

This example was drawn from Sample 26 of Model 21 (see Table S1). It represents an oblique factor model with (a) salient loadings of .40, (b) five indicators per factor, (c) cross loading probability of .10, (d) moderate model fit, (e) factor obliqueness of .60, and (f) Sample Size = 200. The factor analysis was followed by oblique geomin rotations (with no row standardization) from 200 unique start values. These rotations were sorted into four solution sets (based on the geominQ complexity values).

Table S3 reports the population factor pattern and the two geominQ rotated solutions for this example. Both rotated solutions produced complexity values equal to $Q(\boldsymbol{\Lambda}_i)$ = .82126, i=1:2. Scanning $\boldsymbol{\Lambda}_1$ and $\boldsymbol{\Lambda}_2$, it is clear that both factor patterns inaccurately approximated the known factor pattern, $\boldsymbol{\Lambda}_0$. Indeed, the two estimated factor patterns produced RMSE$(\boldsymbol{\Lambda}_0, \boldsymbol{\Lambda}_1)$ = .18630 and RMSE$(\boldsymbol{\Lambda}_0, \boldsymbol{\Lambda}_2)$ = .19089. Table S4 reports the factor correlation matrices for the two local solutions shown in Table S3.

Table S3: Two Geomin Rotated Factor Patterns with Equal Complexity Values

Population Loadings					Local Solutions									
$\mathbf{\Lambda}_0$					$\mathbf{\Lambda}_1$					$\mathbf{\Lambda}_2$				
f1	f2	f3	f4	f5	f1	f2	f3	f4	f5	f1	f2	f3	f4	f5
40	00	00	30	30	09	14	-01	26	63	11	11	-01	23	66
40	00	00	00	00	45	09	-03	-03	16	46	00	07	-04	16
40	00	00	00	30	-05	-03	26	03	58	-08	-22	11	11	59
40	00	00	00	00	52	05	00	12	00	53	00	01	13	-01
40	00	00	00	30	-05	-03	-03	16	57	-04	00	-09	10	59
00	40	00	00	00	07	-03	-10	27	23	09	09	-17	20	24
00	40	00	00	00	27	-11	10	00	19	25	-19	00	02	18
00	40	00	00	00	07	35	-01	44	-06	11	36	04	48	-03
00	40	00	00	00	-08	33	-16	05	24	-03	34	10	00	28
00	40	00	00	00	17	35	-15	01	01	22	32	14	-01	03
00	00	40	00	00	04	19	08	17	04	05	11	09	23	05
00	00	40	00	00	03	21	24	07	13	02	-01	23	19	15
30	00	40	00	00	12	04	30	35	11	10	-10	05	48	11
00	00	40	00	30	03	38	22	-12	42	04	04	40	01	45
00	00	40	00	00	-07	40	35	02	00	-08	06	41	23	02
00	00	30	40	00	-04	03	41	65	00	-06	-07	-03	83	00
00	00	00	40	00	-06	-14	-04	31	12	-06	02	-23	26	12
00	00	00	40	00	-18	-01	32	24	05	-21	-13	06	37	05
30	00	00	40	00	22	-10	11	06	48	20	-20	-01	07	48
00	30	00	40	00	00	-11	04	44	18	00	00	-22	42	19
00	00	00	00	40	-01	08	-03	-21	60	00	-04	12	-24	62
00	00	00	00	40	-20	19	00	16	26	-17	18	05	16	29
00	00	30	00	40	11	-09	06	-02	69	10	-18	-01	-04	70
00	00	00	00	40	-10	07	-41	00	45	-04	29	-16	-21	48
00	00	00	00	40	19	01	-16	11	25	21	10	-11	03	26

Note:
$\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$ represent two geomin rotated solutions with equal complexity values (.82126). After alignment to the Population Loadings ($\mathbf{\Lambda}_0$), $\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$ produced RMSE values of .18630 and .19089. When $\mathbf{\Lambda}_2$ was aligned to $\mathbf{\Lambda}_1$ the associated RMSD = .10816. All loadings are multiplied by 100.

Table S4: Factor Correlation Matrix for Two Geomin Rotated Factor Patterns with Equal Complexity Values

Population Phi Matrix					Local Phi Matrices									
$\mathbf{\Phi}_0$					$\mathbf{\Phi}_1$					$\mathbf{\Phi}_2$				
f1	f2	f3	f4	f5	f1	f2	f3	f4	f5	f1	f2	f3	f4	f5
1.00	0.60	0.60	0.60	0.60	1.00	0.10	0.05	0.12	0.26	1.00	0.04	0.04	0.12	0.26
0.60	1.00	0.60	0.60	0.60	0.10	1.00	-0.05	0.15	0.20	0.04	1.00	0.07	-0.10	0.06
0.60	0.60	1.00	0.60	0.60	0.05	-0.05	1.00	0.18	0.23	0.04	0.07	1.00	0.18	0.09
0.60	0.60	0.60	1.00	0.60	0.12	0.15	0.18	1.00	0.48	0.12	-0.10	0.18	1.00	0.53
0.60	0.60	0.60	0.60	1.00	0.26	0.20	0.23	0.48	1.00	0.26	0.06	0.09	0.53	1.00

The Probability of Obtaining Different Local Solutions with Equal Complexity Values

In the main text of our paper we showed that distinct factor patterns could produce equal rotation complexity values. These results suggested that categorizing rotated solutions into solution sets based solely on their complexity values could underestimate the true LS rates. To highlight this issue, for all solution sets we tabulated cases with distinct factor patterns and equal complexity values.

Table S5 reports the percentage of cases with different factor patterns and equal complexity values across different facets of the simulation design. To construct Table S5, after aligning all matrices to their population counterparts, we computed pairwise RMSD values among the pattern matrices with a solution set and recorded the RMSD values. If any of the RMSD values were greater than zero then the sample contained at least two distinct factor patterns with equal complexity values. Samples meeting this criterion were assigned a value of 1 and 0 otherwise. Next, for each rotation algorithm, we added these assigned values across all (orthogonal or oblique) population models. Finally, we divided this sum by 4800 (48 models with 100 samples each) to obtain the desired percentages.

The results in this table suggest that the probability of obtaining qualitatively distinct factor patterns with equal complexity values is very small and thus our method for calculating the number of local solution sets provides a very tight lower bound on the true numbers.

Table S5: The Probability of Obtaining Different Local Solutions with Equal Complexity Values

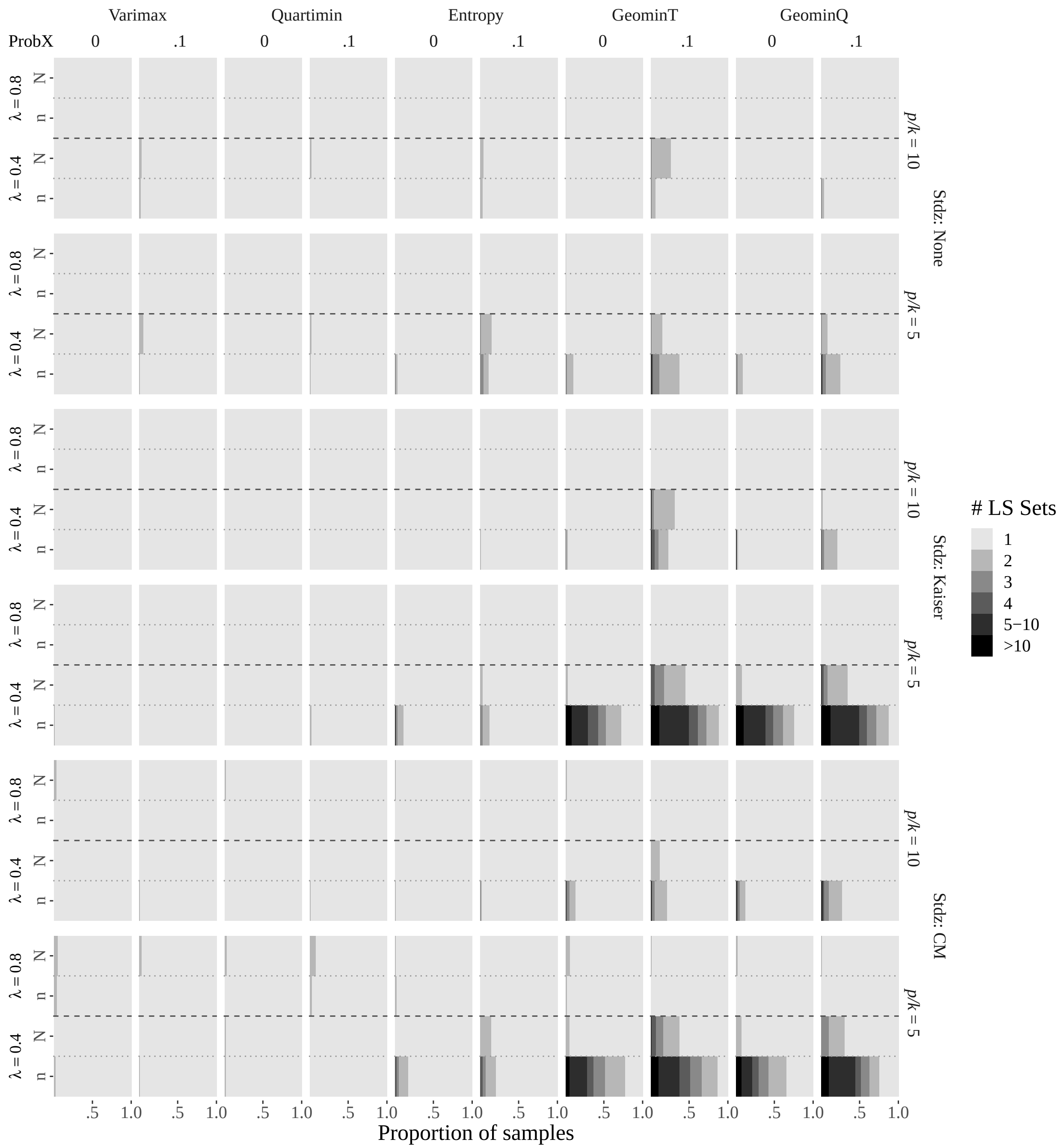
		Rotation Methods (%)				
Model	Stdz	Varimax	Quartimin	Entropy	GeominT	GeominQ
Orthogonal	None	0.02	0.04	0.00	0.00	0.02
	Kaiser	0.00	0.00	0.00	0.02	0.02
	CM	0.52	0.10	0.00	0.00	0.00
Oblique	None	0.08	0.06	0.02	0.19	0.12
	Kaiser	0.00	0.00	0.02	0.33	0.67
	CM	0.00	0.02	0.06	0.38	0.31

Note:
Stdz denotes row standardization, GeominT denotes the orthogonal geomin criterion, whereas the GeominQ denotes oblique geomin criterion.

The Number of Local Solutions in Oblique and Orthogonal Population Models with Varying Amounts of Model Approximation Error

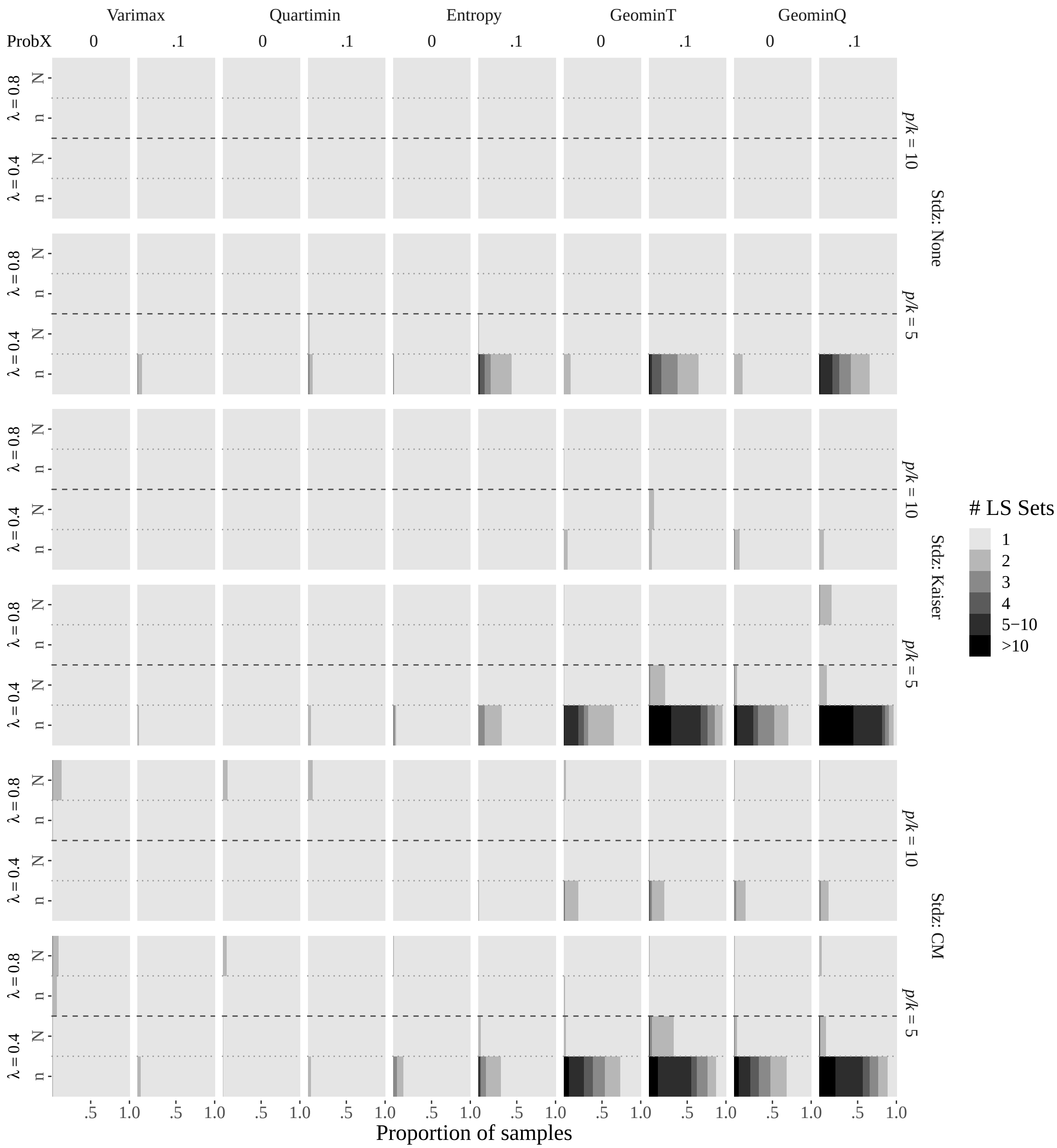
In the manuscript, we summarize LS propensity of five rotation methods, including varimax,quartimin, entropy, (orthogonal) geominT, and (oblique) geominQ under three conditions: a) error-free oblique population models (i.e., Models 49-56 and 73-80), b) oblique population models with ME (i.e., Models 49, 53, 57, 61, 65, 69 ($\lambda = .40$, $p/k = 5$, and $N = 200$); large-sample/strong-factor condition: Models 76, 80, 84, 88, 92, 96 ($\lambda = .80$, $p/k = 10$, and $N = 500$)), and c) orthogonal population models with ME (i.e., Models 1, 5, 9, 13, 17, 21 ($\lambda = .40$, $p/k = 5$, and $N = 200$)). Here, we summarize LS propensity of the aforementioned rotation algorithms in the remaining population models that are summarized in Figures S1, S2, S4,and S5.

Figure S1: Number of Local Solutions in Estimated Factor Models from Data Sets that were Generated from Orthogonal Population Factor Models; ME: None



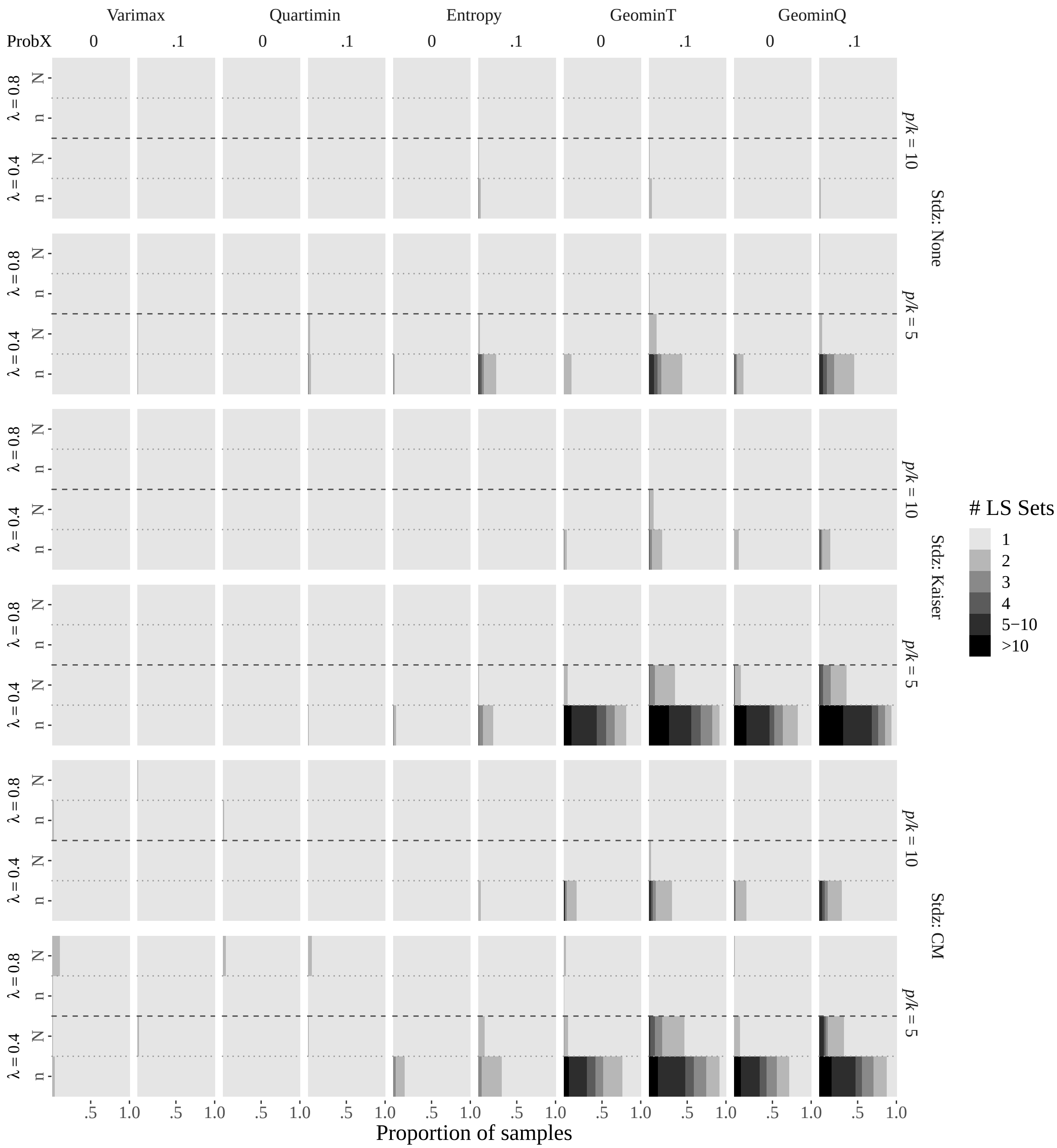
Note. λ denotes salient factor loading size; p/k denotes number of variables per factor; ProbX denotes probability of cross loadings; ME denotes model approximation error; Stdz denotes method of loading standardization; Sample size $\mathcal{N}$: $\text{n} = 200, \text{N} = 500$. LS denotes rotation Local Solutions.

Figure S2: Number of Local Solutions in Estimated Factor Models from Data Sets that were Generated from Orthogonal Population Factor Models; ME: Good-Fit



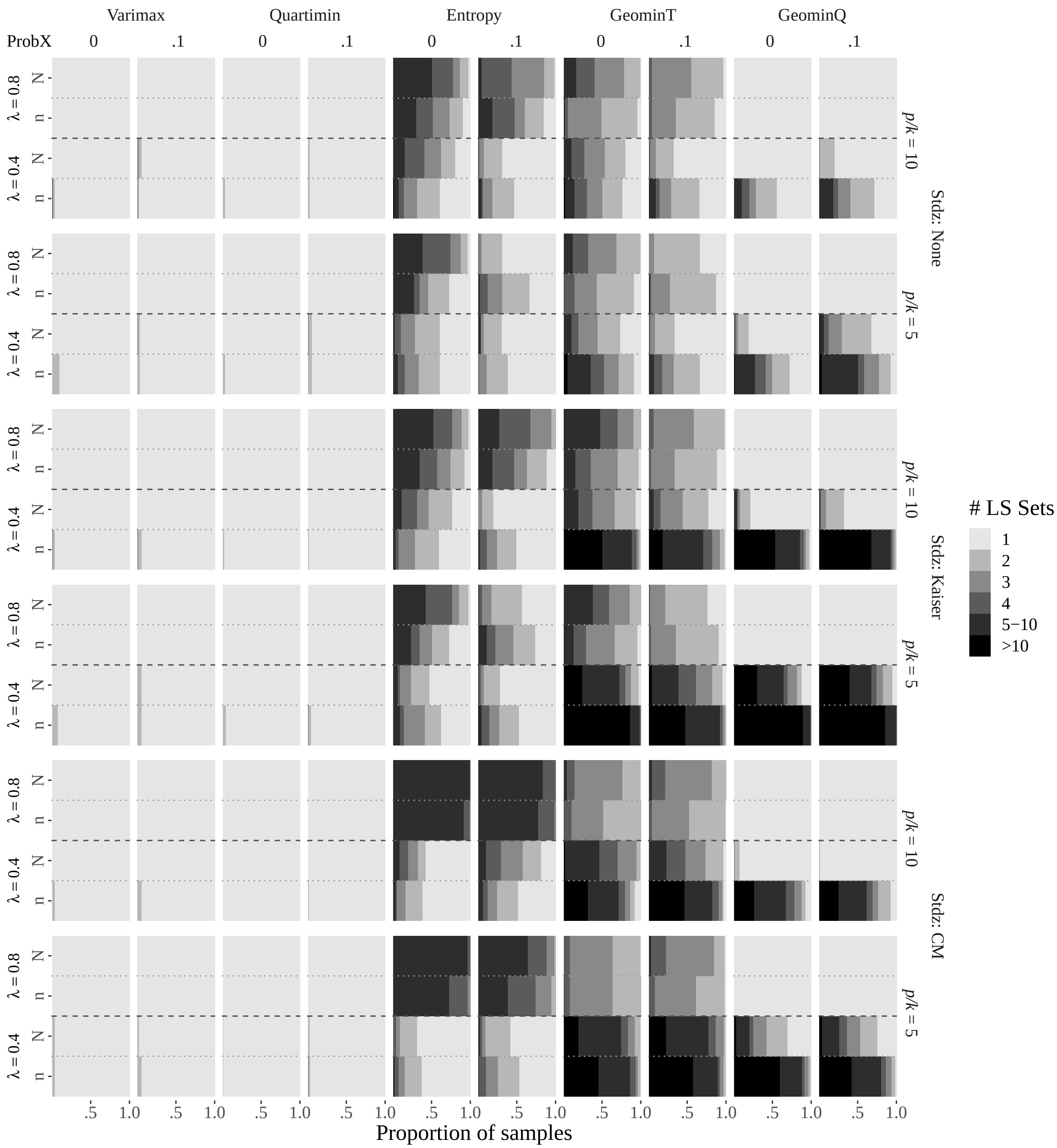
Notes. λ denotes salient factor loading size; p/k denotes number of variables per factor; ProbX denotes probability of cross loadings; ME denotes model approximation error; Stdz denotes method of loading standardization; Sample size $\mathcal{N}$: n = 150, N = 500. LS denotes rotation Local Solutions.

Figure S3: Number of Local Solutions in Estimated Factor Models from Data Sets that were Generated from Orthogonal Population Factor Models; ME: Moderate-Fit



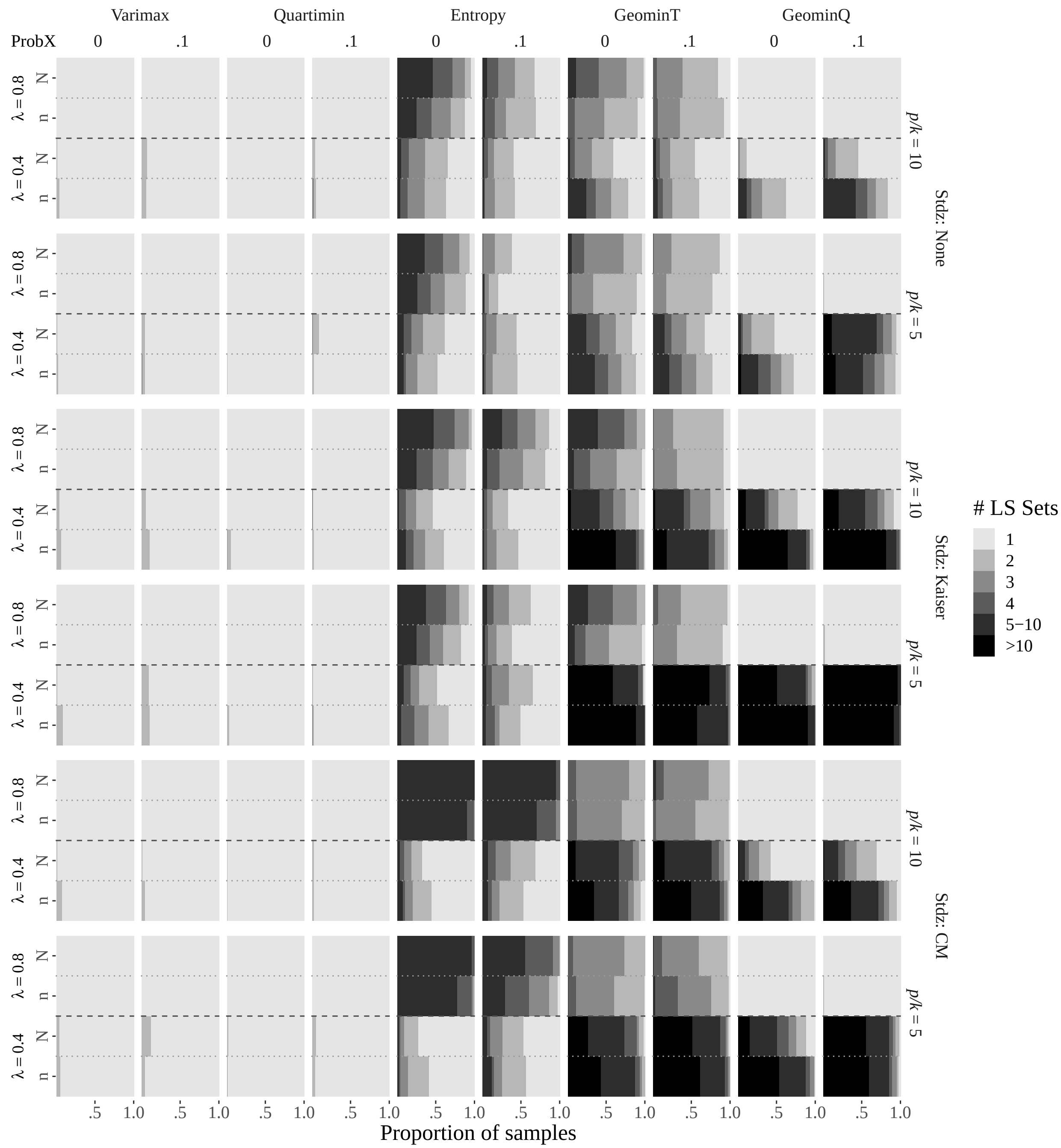
Notes. λ denotes salient factor loading size; p/k denotes number of variables per factor; ProbX denotes probability of cross loadings; ME denotes model approximation error; Stdz denotes method of loading standardization; Sample size $\mathcal{N}$: n = 150, N = 500. LS denotes rotation Local Solutions.

Figure S4: Number of Local Solutions in Estimated Factor Models from Data Sets that were Generated from Oblique Population Factor Models; ME: Good-Fit



Notes. λ denotes salient factor loading size; p/k denotes number of variables per factor; ProbX denotes probability of cross loadings; ME denotes model approximation error; Stdz denotes method of loading standardization; Sample size $\mathcal{N}$: n = 150, N = 500. LS denotes rotation Local Solutions.

Figure S5: Number of Local Solutions in Estimated Factor Models from Data Sets that were Generated from Oblique Population Factor Models; ME: Moderate-Fit



Notes. λ denotes salient factor loading size; p/k denotes number of variables per factor; ProbX denotes probability of cross loadings; ME denotes model approximation error; Stdz denotes method of loading standardization; Sample size $\mathcal{N}$: n = 150, N = 500. LS denotes rotation Local Solutions.

The Proprotion of Solutions with the Lowest Complexity Value

For each sample with LS, we computed the proportion of solutions that converged to the putative global solution. Next, for each loading-standardization/rotation-method pair, we tabulated the mean and standard deviation of these relative frequencies. Table S6 summarizes these results.

Table S6: Frequency of Solution with the Lowest Complexity Value

	Rotation Methods				
	Varimax	Quartimin	Entropy	GeominT	GeominQ
None	.69† (.17‡)	.55 (.22)	.59 (.19)	.68 (.22)	.59 (.23)
Kaiser	.64 (.13)	.54 (.22)	.53 (.22)	.62 (.13)	.46 (.27)
CM	.66 (.23)	.42 (.24)	.49 (.21)	.56 (.30)	.51 (.25)

† Expected frequency of solution with lowest complexity value. ‡ Standard deviation of frequency of solution with lowest complexity value. Kaiser denotes Kaiser standardization; CM denotes Cureton-Mulaik standardization.

Sample Level Model Recovery

In this section we summarize the model recovery properties of five rotation criteria in sample (estimated) factor patterns. For each population model and rotation criterion combination, we computed the average minimum Root Mean Squared Error (RMSE) between the sample local solutions and their corresponding population factor patterns (for major common factors only). Table S7 summarizes these results.

Table S7: The Minimum Root Mean Squared Error Between Sample Local Solutions and Major Common Factor Matrix

	Stdz: None					Stdz: Kaiser					Stdz: CM				
Model ID	V	Q	E	GT	GQ	V	Q	E	GT	GQ	V	Q	E	GT	GQ
M1	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.11	0.12	0.11	0.11	0.11
M2	0.05	0.04	0.05	0.05	0.04	0.05	0.04	0.05	0.05	0.04	0.05	0.05	0.05	0.05	0.04
M3	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.09	0.09	0.08	0.09	0.09
M4	0.05	0.04	0.05	0.05	0.04	0.05	0.04	0.05	0.05	0.04	0.05	0.05	0.05	0.05	0.04
M5	0.11	0.11	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.11	0.12	0.11	0.11	0.11
M6	0.05	0.05	0.05	0.05	0.04	0.05	0.04	0.05	0.05	0.04	0.06	0.07	0.05	0.05	0.05
M7	0.09	0.09	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.09	0.09	0.08	0.08	0.08
M8	0.05	0.05	0.05	0.05	0.04	0.05	0.05	0.05	0.05	0.04	0.05	0.06	0.05	0.05	0.05
M9	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.11	0.12	0.10	0.11	0.11
M10	0.05	0.04	0.05	0.05	0.04	0.05	0.04	0.05	0.05	0.04	0.05	0.06	0.05	0.05	0.05
M11	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.09	0.09	0.08	0.09	0.09
M12	0.05	0.04	0.05	0.05	0.04	0.05	0.04	0.05	0.05	0.04	0.05	0.05	0.05	0.05	0.04
M13	0.13	0.13	0.11	0.10	0.11	0.10	0.11	0.10	0.10	0.10	0.12	0.12	0.11	0.10	0.11
M14	0.05	0.05	0.05	0.05	0.04	0.05	0.05	0.05	0.05	0.04	0.06	0.08	0.05	0.05	0.05
M15	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.09	0.09	0.08	0.08	0.08
M16	0.05	0.05	0.05	0.05	0.04	0.05	0.05	0.05	0.05	0.04	0.05	0.06	0.05	0.05	0.05
M17	0.11	0.10	0.10	0.11	0.10	0.10	0.10	0.10	0.11	0.11	0.11	0.12	0.11	0.11	0.12
M18	0.05	0.04	0.05	0.05	0.04	0.05	0.04	0.05	0.05	0.04	0.05	0.06	0.05	0.05	0.05
M19	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.09	0.09	0.09	0.09	0.09	0.09	0.09
M20	0.05	0.04	0.05	0.05	0.04	0.05	0.04	0.05	0.05	0.04	0.05	0.05	0.05	0.05	0.05
M21	0.12	0.13	0.11	0.11	0.12	0.11	0.11	0.11	0.11	0.11	0.13	0.13	0.12	0.12	0.12
M22	0.05	0.06	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.04	0.06	0.08	0.05	0.05	0.06
M23	0.09	0.09	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.09	0.09	0.08	0.08	0.09
M24	0.05	0.05	0.05	0.05	0.04	0.05	0.04	0.05	0.05	0.04	0.05	0.06	0.05	0.05	0.05
M25	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.07	0.06	0.06	0.06
M26	0.03	0.02	0.03	0.03	0.02	0.03	0.02	0.03	0.03	0.02	0.03	0.04	0.03	0.03	0.03
M27	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.05

M28	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
M29	0.09	0.09	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.07	0.08	0.06	0.06	0.06
M30	0.03	0.03	0.03	0.03	0.02	0.03	0.03	0.03	0.03	0.02	0.04	0.08	0.03	0.03	0.04
M31	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.05
M32	0.04	0.04	0.03	0.03	0.03	0.03	0.04	0.03	0.03	0.02	0.03	0.04	0.03	0.03	0.03
M33	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.07	0.06	0.06	0.06
M34	0.03	0.02	0.03	0.03	0.02	0.03	0.02	0.03	0.03	0.02	0.03	0.04	0.03	0.03	0.03
M35	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.05
M36	0.03	0.02	0.03	0.03	0.02	0.03	0.02	0.03	0.03	0.02	0.03	0.04	0.03	0.03	0.03
M37	0.06	0.07	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.07	0.08	0.06	0.06	0.06
M38	0.05	0.06	0.03	0.03	0.05	0.04	0.05	0.03	0.03	0.05	0.04	0.05	0.03	0.03	0.04
M39	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
M40	0.03	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.02	0.04	0.06	0.03	0.03	0.03
M41	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.07	0.07	0.07	0.07	0.07
M42	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.05	0.03	0.03	0.04
M43	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.05
M44	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.03	0.03	0.03
M45	0.08	0.08	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.08	0.10	0.07	0.07	0.08
M46	0.04	0.06	0.03	0.03	0.04	0.04	0.05	0.03	0.03	0.03	0.04	0.06	0.03	0.03	0.04
M47	0.06	0.07	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.05
M48	0.03	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.07	0.03	0.03	0.04
M49	0.17	0.16	0.17	0.17	0.16	0.17	0.16	0.18	0.17	0.16	0.17	0.16	0.17	0.17	0.16
M50	0.18	0.06	0.26	0.27	0.07	0.18	0.06	0.26	0.27	0.07	0.19	0.07	0.24	0.27	0.07
M51	0.15	0.14	0.17	0.17	0.14	0.15	0.14	0.17	0.17	0.15	0.15	0.14	0.15	0.15	0.14
M52	0.18	0.06	0.26	0.27	0.06	0.18	0.06	0.26	0.27	0.06	0.19	0.07	0.25	0.27	0.07
M53	0.19	0.17	0.21	0.20	0.17	0.17	0.16	0.20	0.19	0.17	0.17	0.17	0.18	0.17	0.16
M54	0.19	0.06	0.29	0.29	0.06	0.19	0.06	0.29	0.29	0.06	0.20	0.07	0.26	0.28	0.06
M55	0.19	0.15	0.21	0.21	0.15	0.16	0.14	0.21	0.20	0.14	0.16	0.15	0.16	0.16	0.14
M56	0.19	0.06	0.29	0.29	0.05	0.19	0.06	0.29	0.29	0.05	0.20	0.06	0.28	0.29	0.06
M57	0.17	0.17	0.17	0.17	0.16	0.17	0.16	0.18	0.17	0.16	0.17	0.17	0.17	0.17	0.16
M58	0.19	0.07	0.26	0.27	0.07	0.19	0.07	0.26	0.27	0.07	0.19	0.07	0.24	0.27	0.07
M59	0.15	0.14	0.17	0.17	0.14	0.15	0.14	0.17	0.17	0.15	0.15	0.15	0.15	0.15	0.14

M60	0.18	0.06	0.26	0.27	0.06	0.18	0.06	0.26	0.27	0.06	0.19	0.07	0.25	0.27	0.07
M61	0.21	0.18	0.22	0.22	0.18	0.18	0.17	0.21	0.20	0.17	0.18	0.17	0.18	0.17	0.16
M62	0.19	0.06	0.29	0.29	0.06	0.19	0.06	0.29	0.29	0.06	0.20	0.07	0.26	0.28	0.07
M63	0.18	0.15	0.21	0.21	0.15	0.16	0.14	0.21	0.20	0.14	0.16	0.15	0.16	0.16	0.14
M64	0.19	0.06	0.29	0.29	0.06	0.19	0.06	0.29	0.29	0.06	0.20	0.06	0.27	0.28	0.06
M65	0.18	0.17	0.18	0.18	0.17	0.18	0.17	0.18	0.18	0.17	0.18	0.17	0.18	0.17	0.17
M66	0.19	0.08	0.26	0.27	0.08	0.19	0.08	0.26	0.27	0.08	0.19	0.08	0.24	0.27	0.08
M67	0.16	0.15	0.17	0.17	0.15	0.16	0.15	0.18	0.17	0.15	0.16	0.15	0.16	0.16	0.15
M68	0.19	0.07	0.26	0.27	0.07	0.19	0.07	0.26	0.27	0.07	0.19	0.07	0.25	0.27	0.07
M69	0.21	0.20	0.24	0.23	0.20	0.20	0.19	0.23	0.21	0.19	0.20	0.19	0.20	0.20	0.19
M70	0.19	0.08	0.28	0.28	0.08	0.19	0.08	0.28	0.28	0.08	0.20	0.09	0.25	0.27	0.08
M71	0.21	0.16	0.23	0.23	0.17	0.17	0.15	0.23	0.22	0.16	0.17	0.16	0.18	0.17	0.16
M72	0.19	0.07	0.30	0.29	0.06	0.19	0.06	0.30	0.29	0.06	0.20	0.07	0.28	0.29	0.07
M73	0.13	0.12	0.15	0.15	0.12	0.13	0.12	0.16	0.15	0.12	0.13	0.12	0.14	0.14	0.12
M74	0.18	0.04	0.26	0.27	0.04	0.18	0.04	0.26	0.27	0.04	0.18	0.04	0.26	0.27	0.05
M75	0.11	0.09	0.14	0.14	0.08	0.11	0.08	0.14	0.15	0.09	0.11	0.09	0.12	0.13	0.09
M76	0.18	0.04	0.26	0.27	0.04	0.18	0.04	0.26	0.27	0.04	0.18	0.04	0.26	0.27	0.05
M77	0.19	0.14	0.20	0.20	0.14	0.15	0.12	0.20	0.20	0.12	0.15	0.13	0.16	0.16	0.12
M78	0.19	0.04	0.28	0.28	0.04	0.19	0.04	0.28	0.28	0.04	0.19	0.04	0.27	0.28	0.04
M79	0.16	0.09	0.17	0.17	0.09	0.12	0.09	0.16	0.16	0.09	0.12	0.09	0.13	0.15	0.09
M80	0.19	0.04	0.28	0.28	0.04	0.19	0.04	0.28	0.28	0.04	0.19	0.04	0.27	0.28	0.04
M81	0.13	0.11	0.14	0.15	0.12	0.13	0.11	0.15	0.15	0.12	0.13	0.12	0.13	0.13	0.12
M82	0.18	0.04	0.26	0.27	0.04	0.18	0.04	0.26	0.27	0.04	0.18	0.04	0.26	0.27	0.05
M83	0.11	0.09	0.14	0.14	0.09	0.11	0.09	0.14	0.15	0.09	0.11	0.09	0.12	0.13	0.09
M84	0.18	0.04	0.26	0.27	0.04	0.18	0.04	0.26	0.27	0.04	0.18	0.04	0.25	0.26	0.05
M85	0.20	0.14	0.21	0.21	0.14	0.15	0.12	0.20	0.19	0.13	0.15	0.13	0.16	0.16	0.12
M86	0.19	0.05	0.29	0.28	0.04	0.19	0.04	0.28	0.28	0.04	0.20	0.04	0.27	0.28	0.04
M87	0.21	0.09	0.21	0.20	0.08	0.13	0.08	0.20	0.19	0.08	0.13	0.09	0.15	0.17	0.09
M88	0.19	0.05	0.28	0.28	0.04	0.19	0.04	0.28	0.29	0.04	0.19	0.04	0.27	0.28	0.04
M89	0.15	0.14	0.15	0.16	0.14	0.15	0.14	0.16	0.16	0.14	0.15	0.14	0.15	0.15	0.14
M90	0.18	0.05	0.26	0.27	0.05	0.18	0.05	0.26	0.27	0.05	0.19	0.05	0.26	0.27	0.06
M91	0.13	0.11	0.15	0.15	0.11	0.13	0.11	0.16	0.16	0.11	0.13	0.11	0.13	0.13	0.11

M92	0.18	0.05	0.26	0.27	0.05	0.18	0.05	0.26	0.27	0.05	0.18	0.05	0.26	0.27	0.05
M93	0.21	0.20	0.23	0.22	0.20	0.20	0.19	0.22	0.21	0.19	0.20	0.20	0.20	0.20	0.19
M94	0.19	0.07	0.30	0.29	0.06	0.19	0.06	0.29	0.29	0.06	0.19	0.06	0.27	0.28	0.06
M95	0.17	0.13	0.21	0.20	0.13	0.15	0.12	0.20	0.20	0.13	0.15	0.13	0.17	0.16	0.13
M96	0.19	0.05	0.29	0.29	0.04	0.19	0.05	0.29	0.29	0.04	0.19	0.05	0.28	0.28	0.05
Mean	0.12	0.08	0.14	0.14	0.08	0.11	0.08	0.14	0.14	0.07	0.12	0.09	0.13	0.14	0.08
SD	0.06	0.04	0.10	0.10	0.05	0.06	0.04	0.09	0.10	0.04	0.06	0.04	0.09	0.09	0.04

Note:

Model IDs correspond to those specified in Table S1. Five rotation criteria includes Varimax (V), Quartimin (Q), Entropy (E), orthogonal Geomin (GT), and oblique Geomin (GQ). Three loading standardization (Stdz) methods include None, Kaiser, and Cureton-Mulaik (CM).

The Number of Local Solutions at the Population Level

In this section we report the number of local solutions obtained from the five rotation criteria at the population level. Specifically, for each population model, we factor analyzed the population correlation matrix and rotated (without loading standardization) the extracted factors by each rotation method. Table S8 summarizes these results.

Figure S6 contains 60 panels (i.e., 6 rows and 10 columns), each of which is divided into two subpanels. Each subpanel represents the average number of LS for one population model design characterized by (a) salient-loading sizes, $\lambda \in \{.40, .80\}$, (b) numbers of indicators per factor, $p/k \in \{5, 10\}$, (c) numbers of factor cross loadings, $\text{ProbX} \in \{0, .10\}$, and (d) amounts of model approximation error, $\text{ME} \in \{\text{None}, \text{Good-Fit}, \text{Moderate-Fit}\}$. Note that each subpanel represents an average statistic because each population model has two replicates, each corresponds to a sample size level (i.e., $\mathcal{N} \in \{200,\ 500\}$).

Scanning Figure S6 shows that varimax, quartimin and geominQ very rarely converged to population LS, whereas entropy and geominT frequently converged to many LS. Interestingly, Figure S6 also demonstrates that entropy and geominT were more likely to produce many population LS in EFA models with large loadings ($\lambda = .8$) and few cross-loadings, suggesting that imposing orthogonal rotations on strong oblique models can distort simple structure that is inherent in the data-generating loading matrix, which makes it difficult for these rotation criteria to recover the true model. Delving deeper into these results, we found that the number of population LS does not set an “upper bound” to how often sample LS occur. On the contrary, we found that in general sample LS occurred more often than population LS (i.e., in some model, population LS count was larger than all corresponding sample LS counts).

Table S8: Number of Local Solutions at the Population Level

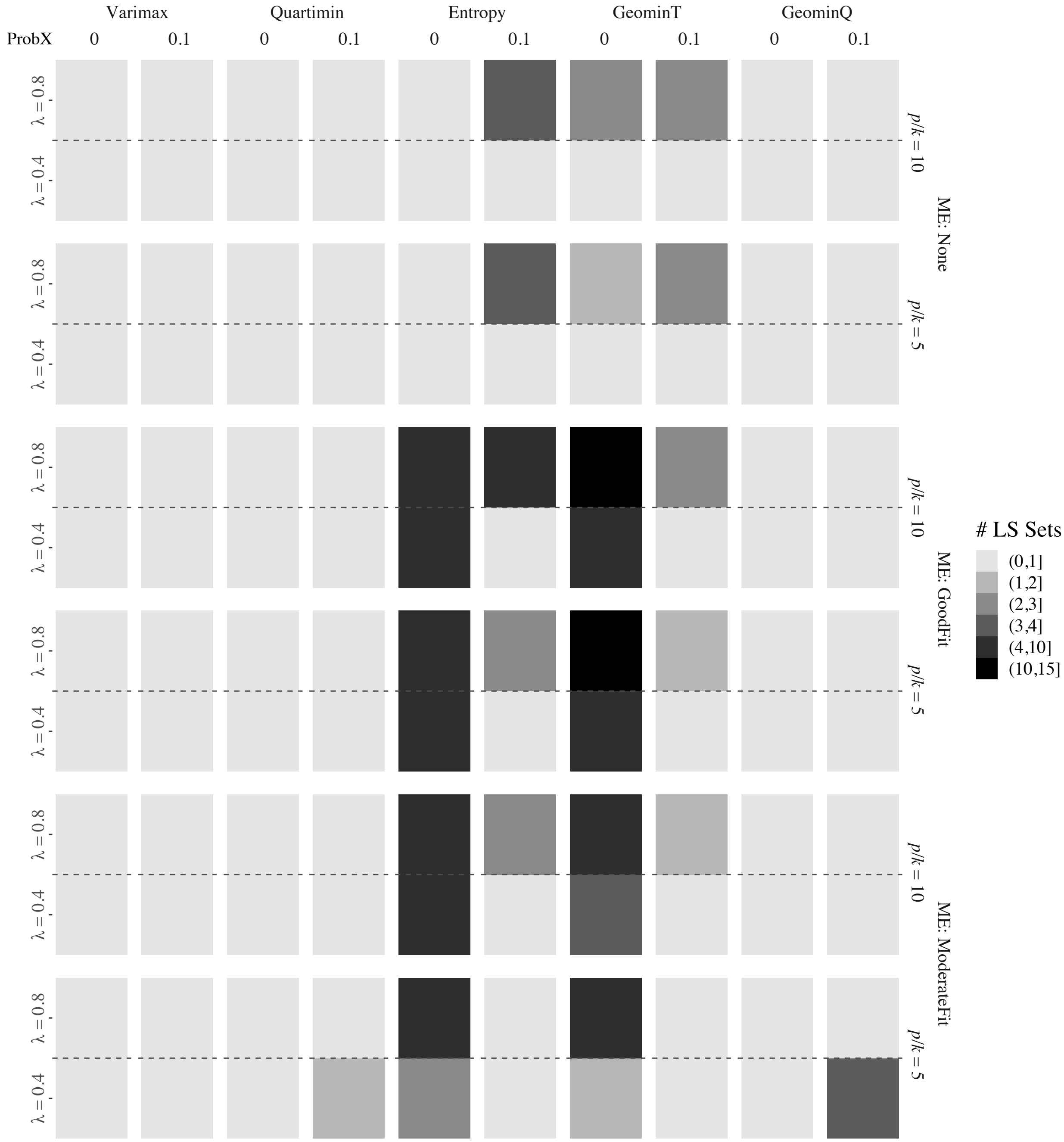
Model ID	Varimax	Quartimin	Entropy	GeominT	GeominQ
M1	1	1	1	1	1
M2	1	1	1	1	1
M3	1	1	1	1	1
M4	1	1	1	1	1
M5	1	1	1	1	1
M6	1	1	1	1	1
M7	1	1	1	1	1
M8	1	1	1	1	1
M9	1	1	1	1	1
M10	1	1	1	1	1
M11	1	1	1	1	1
M12	1	1	1	1	1
M13	1	1	1	1	1
M14	1	1	1	1	1
M15	1	1	1	1	1
M16	1	1	1	1	1
M17	1	1	1	1	1
M18	1	1	1	1	1
M19	1	1	1	1	1
M20	1	1	1	1	1
M21	1	1	1	1	1
M22	1	1	1	1	1
M23	1	1	1	1	1
M24	1	1	1	1	1
M25	1	1	1	1	1
M26	1	1	1	1	1
M27	1	1	1	1	1
M28	1	1	1	1	1
M29	1	1	1	1	1
M30	1	1	1	1	1
M31	1	1	1	2	1
M32	1	1	1	1	1
M33	1	1	1	1	1
M34	1	1	1	1	1
M35	1	1	1	1	1
M36	1	1	1	1	1
M37	1	1	1	1	1
M38	1	1	1	1	1
M39	1	1	1	1	1
M40	1	1	1	1	1
M41	1	1	1	1	1

M42	1	1	1	1	1
M43	1	1	1	1	1
M44	1	1	1	1	1
M45	1	1	1	1	1
M46	1	1	1	1	1
M47	1	1	1	1	1
M48	1	1	1	1	1
M49	1	1	1	1	1
M50	1	1	1	2	1
M51	1	1	1	1	1
M52	1	1	1	3	1
M53	1	1	1	1	1
M54	1	1	4	4	1
M55	1	1	2	1	1
M56	1	1	5	4	1
M57	1	1	5	5	1
M58	1	1	5	12	1
M59	1	1	5	5	1
M60	1	1	5	15	1
M61	1	1	1	2	1
M62	1	1	5	3	1
M63	1	1	1	1	1
M64	1	1	5	3	1
M65	1	1	1	1	1
M66	1	1	5	6	1
M67	1	1	5	5	1
M68	1	1	5	9	1
M69	1	1	1	1	1
M70	1	1	1	1	1
M71	1	1	1	1	1
M72	1	1	4	2	1
M73	1	1	1	1	1
M74	1	1	1	2	1
M75	1	1	1	1	1
M76	1	1	1	3	1
M77	1	1	1	1	1
M78	1	1	4	2	1
M79	1	1	1	2	1
M80	1	1	3	2	1
M81	1	1	5	5	1
M82	1	1	5	15	1
M83	1	1	5	5	1
M84	1	1	5	15	1
M85	1	1	1	1	1
M86	1	1	1	2	1

M87	1	1	1	1	1
M88	1	1	5	3	1
M89	1	1	5	4	1
M90	1	1	5	5	1
M91	1	1	5	4	1
M92	1	1	5	10	1
M93	1	3	1	1	8
M94	1	1	1	1	1
M95	1	1	1	1	1
M96	1	1	2	3	1

Note:
Model IDs correspond to those specified in Table S1

Figure S6: Average Number of Population Local Solutions In Oblique Models



Note. λ denotes factor loading size; p/k denotes number of indicators per factor; ProbX denotes probability of cross loadings; ME denotes model approximation error; Good-Fit denotes $.95 \leq$ CFI < 1.00; and Moderate-Fit denotes $.90 \leq$ CFI $< .95$.

The Model Recovery at the Population Level

In this section we report the model recovery of five rotation criteria at the population level. Specifically, for each population model and rotation criterion combination, we computed the RMSE between population local solutions and their corresponding data-generating major common factor matrix. Note that when multiple population local solutions existed, we reported the minimum RMSE value. During these analyses, no loading standardization method was used. Table S9 summarizes this aspect of our results.

Table S9: The Minimum Root Mean Squared Error Between Population Local Solutions and the Data-Generating Common Factor Pattern Matrix

Model ID	Varimax	Quartimin	Entropy	GeominT	GeominQ
M1	0.00	0.00	0.00	0.00	0.00
M2	0.00	0.00	0.00	0.00	0.00
M3	0.00	0.00	0.00	0.00	0.00
M4	0.00	0.00	0.00	0.00	0.00
M5	0.02	0.03	0.00	0.00	0.01
M6	0.01	0.02	0.00	0.00	0.01
M7	0.01	0.03	0.00	0.00	0.01
M8	0.02	0.04	0.01	0.00	0.01
M9	0.00	0.00	0.00	0.00	0.00
M10	0.00	0.00	0.00	0.00	0.00
M11	0.00	0.00	0.00	0.00	0.00
M12	0.00	0.00	0.00	0.00	0.00
M13	0.04	0.04	0.00	0.00	0.01
M14	0.02	0.03	0.01	0.00	0.01
M15	0.01	0.02	0.00	0.00	0.01
M16	0.02	0.03	0.00	0.00	0.01
M17	0.02	0.02	0.02	0.02	0.02
M18	0.01	0.01	0.01	0.01	0.01
M19	0.01	0.01	0.01	0.01	0.01
M20	0.01	0.01	0.01	0.01	0.01
M21	0.05	0.06	0.03	0.03	0.03
M22	0.03	0.05	0.02	0.02	0.02
M23	0.02	0.03	0.01	0.01	0.01
M24	0.01	0.03	0.01	0.01	0.01
M25	0.00	0.00	0.00	0.00	0.00
M26	0.00	0.00	0.00	0.00	0.00
M27	0.00	0.00	0.00	0.00	0.00
M28	0.00	0.00	0.00	0.00	0.00
M29	0.05	0.05	0.00	0.00	0.01
M30	0.01	0.02	0.00	0.00	0.01
M31	0.02	0.04	0.00	0.00	0.01
M32	0.02	0.03	0.01	0.00	0.01
M33	0.00	0.00	0.00	0.00	0.00
M34	0.00	0.00	0.00	0.00	0.00
M35	0.00	0.00	0.00	0.00	0.00
M36	0.00	0.00	0.00	0.00	0.00
M37	0.02	0.03	0.00	0.00	0.01
M38	0.04	0.06	0.01	0.01	0.05
M39	0.03	0.04	0.00	0.00	0.01
M40	0.01	0.03	0.00	0.00	0.01

M41	0.02	0.02	0.02	0.02	0.02
M42	0.01	0.01	0.01	0.01	0.01
M43	0.01	0.01	0.01	0.01	0.01
M44	0.01	0.01	0.01	0.01	0.01
M45	0.04	0.05	0.03	0.03	0.04
M46	0.03	0.05	0.02	0.02	0.02
M47	0.04	0.04	0.01	0.01	0.02
M48	0.01	0.03	0.01	0.01	0.01
M49	0.09	0.00	0.13	0.13	0.02
M50	0.18	0.00	0.27	0.27	0.01
M51	0.09	0.00	0.13	0.13	0.02
M52	0.18	0.00	0.27	0.27	0.01
M53	0.16	0.05	0.17	0.16	0.01
M54	0.19	0.02	0.29	0.28	0.01
M55	0.15	0.03	0.17	0.17	0.01
M56	0.19	0.03	0.29	0.28	0.01
M57	0.09	0.00	0.13	0.13	0.02
M58	0.18	0.00	0.27	0.27	0.01
M59	0.09	0.00	0.13	0.13	0.02
M60	0.18	0.00	0.27	0.27	0.01
M61	0.18	0.05	0.18	0.17	0.01
M62	0.18	0.02	0.29	0.28	0.01
M63	0.14	0.03	0.17	0.17	0.01
M64	0.19	0.02	0.29	0.29	0.01
M65	0.11	0.09	0.15	0.15	0.09
M66	0.18	0.04	0.26	0.27	0.04
M67	0.09	0.03	0.13	0.13	0.04
M68	0.18	0.02	0.27	0.28	0.02
M69	0.20	0.17	0.22	0.22	0.17
M70	0.19	0.05	0.27	0.27	0.04
M71	0.18	0.09	0.20	0.20	0.10
M72	0.19	0.03	0.29	0.29	0.02
M73	0.09	0.00	0.13	0.13	0.02
M74	0.18	0.00	0.27	0.27	0.01
M75	0.09	0.00	0.13	0.13	0.02
M76	0.18	0.00	0.27	0.27	0.01
M77	0.17	0.05	0.18	0.18	0.01
M78	0.18	0.02	0.28	0.28	0.01
M79	0.15	0.03	0.15	0.15	0.01
M80	0.19	0.02	0.28	0.28	0.01
M81	0.09	0.00	0.13	0.13	0.02
M82	0.18	0.00	0.27	0.27	0.01
M83	0.09	0.00	0.13	0.13	0.02
M84	0.18	0.00	0.27	0.27	0.01
M85	0.18	0.06	0.19	0.18	0.01

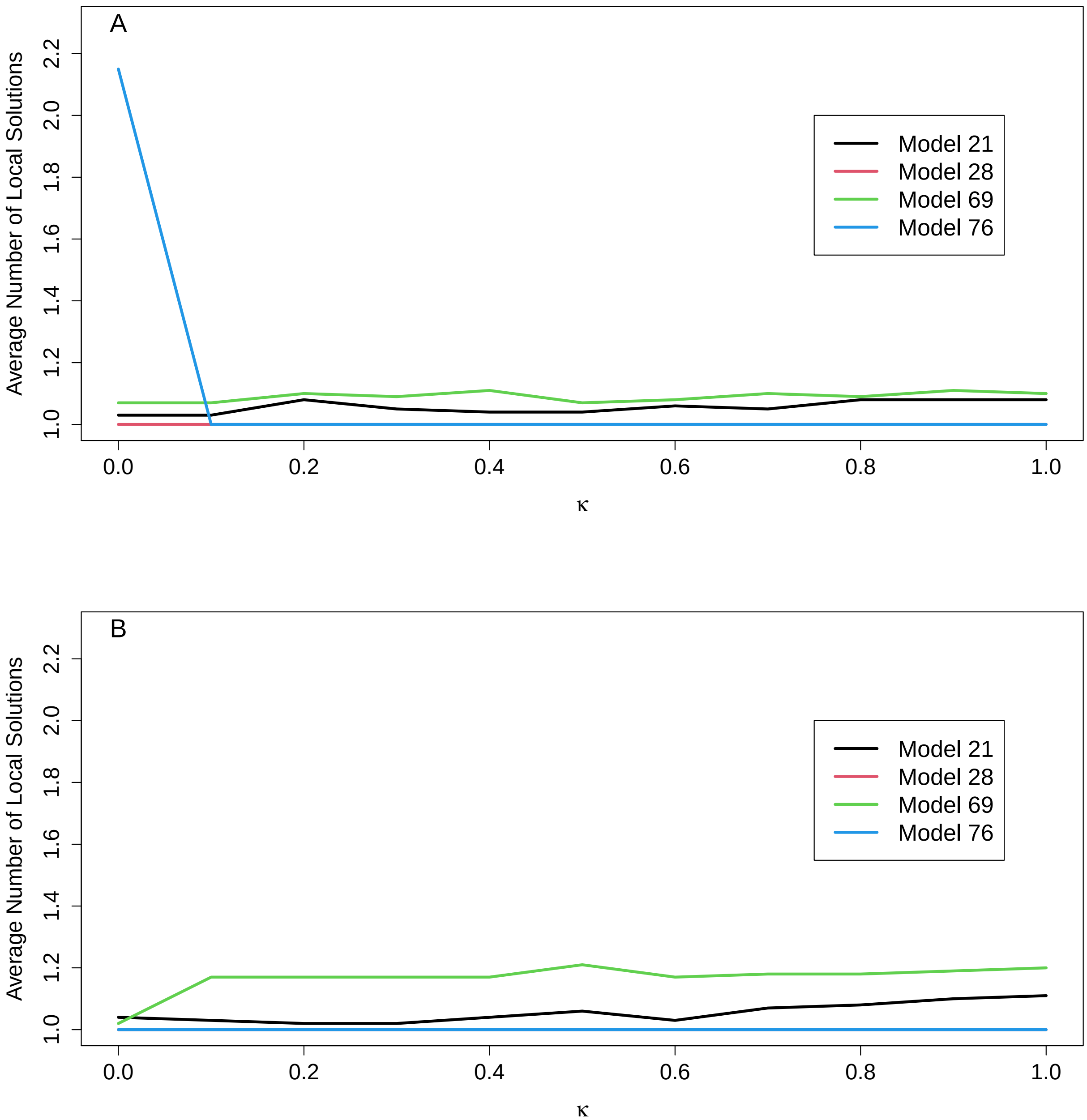
M86	0.19	0.03	0.29	0.28	0.01
M87	0.20	0.04	0.19	0.19	0.01
M88	0.19	0.03	0.28	0.28	0.01
M89	0.11	0.07	0.13	0.14	0.08
M90	0.18	0.02	0.27	0.28	0.03
M91	0.10	0.05	0.13	0.14	0.05
M92	0.18	0.02	0.27	0.28	0.02
M93	0.21	0.18	0.23	0.23	0.19
M94	0.19	0.05	0.30	0.29	0.05
M95	0.13	0.08	0.18	0.17	0.07
M96	0.19	0.04	0.29	0.29	0.03
$Mean$	0.09	0.03	0.11	0.11	0.02
SD	0.08	0.03	0.12	0.12	0.03

Note:
Model IDs correspond to those specified in Table S1. SD denotes standard deviation.

The Number of Local Solutions as a Function of the Crawford-Ferguson Kappa Parameter

In this section we investigate the number of local solutions as a function the CF κ parameter. For these analyses we selected Models 21, 28, 69, and 76 (see Table S1). Model 21 represents a worst case orthogonal model; Model 28 represents a best case orthogonal model. Models 69 and 76 are the oblique counterparts of Models 21 and 28, respectively. We used 200 random starting points and did not precondition the factor loading matrix via row standardization.

Figure S7: Crawford-Ferguson Family Rotations: Number of Local Solutions as a Function of κ



Notes. Panel A summarizes the results for orthogonal Crawford-Ferguson (CF) rotations, whereas Panel B summarizes the results for oblique CF rotations. Model 21 has small loading size ($\lambda = .4$), five indicators per factor ($p/k = 5$), few cross loadings ($ProbX = .1$), and moderate model approximation error (ME = Moderate). Model 28 has large loading size ($\lambda = .8$), ten indicators per factor ($p/k = 10$), no cross loadings ($ProbX = 0$), and no model approximation error (ME = None). Models 69 and 76 are oblique counterparts of Models 21 and 28, respectively.

The Number of Iterations Required to Reach Function Convergence in Factor Rotations with the Gradient Projection Algorithm

In this section we summarize the number of iterations that were required to reach function convergence with the Gradient Projection Algorithm. Our analyses were conducted as follows. For each of the 144,000 sample factor analyses (96 models $\times$ 100 samples $\times$ 3 loading standardization methods $\times$ 5 rotation methods) we averaged the total number of iterations required to reach convergence over the 200 rotation trials from different starting values. Next, for each loading-standardization/rotation-method combination, we tabulated the: (1) minimum, (2) 1st quartile, (3) median, (4) mean, (5) 3rd quartile, and (6) maximum of the averaged (across the 200 starting values) iteration totals. Table S10 summarizes these results.

Table S10: The Number of Iterations Required to Reach Function Convergence in the Gradient Projection Algorithm

Stdz	Rotation	Min	1st Qu.	Median	Mean	3rd Qu.	Max
	Varimax	7	12	16	30	23	1916
	Quartimin	12	18	47	81	77	6108
None	Entropy	12	17	41	78	114	1866
	GeominT	12	22	47	99	154	1905
	GeominQ	18	35	66	95	94	1414
	Varimax	9	15	17	24	21	559
	Quartimin	17	21	47	95	80	5995
Kaiser	Entropy	12	18	46	73	111	1374
	GeominT	18	26	88	108	159	1538
	GeominQ	27	46	91	130	160	1395
	Varimax	10	17	22	90	35	13438
	Quartimin	16	51	77	213	143	14967
CM	Entropy	13	24	44	62	74	11996
	GeominT	18	32	80	108	129	10017
	GeominQ	30	57	87	165	196	14876

Note:
Stdz denotes row standardization; GeominT denotes the orthogonal geomin criterion; GeominQ denotes the oblique geomin criterion.

Study 1R: A Simulation Study of Rotation Local Minima: Model Fit Evaluated by the RMSEA

In this section, we report the results of Study 1R (using the RMSEA rather than the CFI to quantify population model fit). Using published guidelines (Browne & Cudeck, 1992), we required that RMSEA values fall between .03-.05 for the Good-Fit condition and between .07-.089 for the Moderate-Fit condition.

Table S11 describes 96 EFA models investigated in Study 1R. These EFA models were formulated by crossing six design factors of our simulations: (a) two levels of factor loading size ($\lambda \in \{.4, .8\}$), (b) two levels of number of indicator per factor ($p/k \in \{5, 10\}$), (c) two levels of cross-loading probability ($\text{ProbX} \in \{0, .1\}$), (d) three levels of model approximation error ($\text{ME} \in \{\text{None, Good-Fit, Moderate-Fit}\}$), (e) two levels of factor correlation ($\phi_{\text{ij}} \in \{0, .6\}$), and (f) two levels of sample size ($\text{Sample Size} \in \{200, 500\}$). For each population model, Table S11 reports two `simFA` parameters that were used to control the amount of ME. These parameters included the proportion of indicator variance due to model approximation error, V_p, and a parameter, ϵ_{TKL}, that controlled the expected factor loadings in successive columns of $\mathbf{W}$ (i.e., the minor-factor loadings matrix). By design, $\text{V}_\text{p} \in [.01, .20]$.

Table S11: Exploratory Factor Model Designs Using RMSEA

ID	λ	p/k	ProbX	ME	ϵ_{TKL}	V_p	ϕ_{ij}	$\mathcal{N}$
1	0.4	5	0.0	None	NA	NA	0.0	200
2	0.8	5	0.0	None	NA	NA	0.0	200
3	0.4	10	0.0	None	NA	NA	0.0	200
4	0.8	10	0.0	None	NA	NA	0.0	200
5	0.4	5	0.1	None	NA	NA	0.0	200
6	0.8	5	0.1	None	NA	NA	0.0	200
7	0.4	10	0.1	None	NA	NA	0.0	200
8	0.8	10	0.1	None	NA	NA	0.0	200
9	0.4	5	0.0	Good	0.10	0.15	0.0	200
10	0.8	5	0.0	Good	0.03	0.18	0.0	200
11	0.4	10	0.0	Good	0.07	0.20	0.0	200
12	0.8	10	0.0	Good	0.04	0.12	0.0	200
13	0.4	5	0.1	Good	0.06	0.20	0.0	200
14	0.8	5	0.1	Good	0.03	0.14	0.0	200
15	0.4	10	0.1	Good	0.08	0.15	0.0	200
16	0.8	10	0.1	Good	0.03	0.18	0.0	200
17	0.4	5	0.0	Moderate	0.28	0.10	0.0	200
18	0.8	5	0.0	Moderate	0.08	0.15	0.0	200
19	0.4	10	0.0	Moderate	0.35	0.05	0.0	200
20	0.8	10	0.0	Moderate	0.09	0.18	0.0	200
21	0.4	5	0.1	Moderate	0.27	0.08	0.0	200
22	0.8	5	0.1	Moderate	0.07	0.15	0.0	200
23	0.4	10	0.1	Moderate	0.34	0.05	0.0	200
24	0.8	10	0.1	Moderate	0.09	0.15	0.0	200
25	0.4	5	0.0	None	NA	NA	0.0	500
26	0.8	5	0.0	None	NA	NA	0.0	500
27	0.4	10	0.0	None	NA	NA	0.0	500
28	0.8	10	0.0	None	NA	NA	0.0	500
29	0.4	5	0.1	None	NA	NA	0.0	500
30	0.8	5	0.1	None	NA	NA	0.0	500
31	0.4	10	0.1	None	NA	NA	0.0	500
32	0.8	10	0.1	None	NA	NA	0.0	500
33	0.4	5	0.0	Good	0.08	0.18	0.0	500
34	0.8	5	0.0	Good	0.03	0.19	0.0	500
35	0.4	10	0.0	Good	0.08	0.16	0.0	500
36	0.8	10	0.0	Good	0.04	0.10	0.0	500
37	0.4	5	0.1	Good	0.08	0.16	0.0	500
38	0.8	5	0.1	Good	0.03	0.14	0.0	500
39	0.4	10	0.1	Good	0.08	0.16	0.0	500
40	0.8	10	0.1	Good	0.04	0.10	0.0	500
41	0.4	5	0.0	Moderate	0.30	0.08	0.0	500

42	0.8	5	0.0	Moderate	0.07	0.18	0.0	500
43	0.4	10	0.0	Moderate	0.38	0.04	0.0	500
44	0.8	10	0.0	Moderate	0.09	0.19	0.0	500
45	0.4	5	0.1	Moderate	0.26	0.11	0.0	500
46	0.8	5	0.1	Moderate	0.07	0.14	0.0	500
47	0.4	10	0.1	Moderate	0.30	0.06	0.0	500
48	0.8	10	0.1	Moderate	0.09	0.14	0.0	500
49	0.4	5	0.0	None	NA	NA	0.6	200
50	0.8	5	0.0	None	NA	NA	0.6	200
51	0.4	10	0.0	None	NA	NA	0.6	200
52	0.8	10	0.0	None	NA	NA	0.6	200
53	0.4	5	0.1	None	NA	NA	0.6	200
54	0.8	5	0.1	None	NA	NA	0.6	200
55	0.4	10	0.1	None	NA	NA	0.6	200
56	0.8	10	0.1	None	NA	NA	0.6	200
57	0.4	5	0.0	Good	0.12	0.09	0.6	200
58	0.8	5	0.0	Good	0.03	0.18	0.6	200
59	0.4	10	0.0	Good	0.13	0.08	0.6	200
60	0.8	10	0.0	Good	0.04	0.11	0.6	200
61	0.4	5	0.1	Good	0.11	0.17	0.6	200
62	0.8	5	0.1	Good	0.03	0.11	0.6	200
63	0.4	10	0.1	Good	0.11	0.07	0.6	200
64	0.8	10	0.1	Good	0.03	0.08	0.6	200
65	0.4	5	0.0	Moderate	0.39	0.06	0.6	200
66	0.8	5	0.0	Moderate	0.08	0.16	0.6	200
67	0.4	10	0.0	Moderate	0.43	0.04	0.6	200
68	0.8	10	0.0	Moderate	0.09	0.18	0.6	200
69	0.4	5	0.1	Moderate	0.29	0.09	0.6	200
70	0.8	5	0.1	Moderate	0.06	0.19	0.6	200
71	0.4	10	0.1	Moderate	0.30	0.06	0.6	200
72	0.8	10	0.1	Moderate	0.06	0.14	0.6	200
73	0.4	5	0.0	None	NA	NA	0.6	500
74	0.8	5	0.0	None	NA	NA	0.6	500
75	0.4	10	0.0	None	NA	NA	0.6	500
76	0.8	10	0.0	None	NA	NA	0.6	500
77	0.4	5	0.1	None	NA	NA	0.6	500
78	0.8	5	0.1	None	NA	NA	0.6	500
79	0.4	10	0.1	None	NA	NA	0.6	500
80	0.8	10	0.1	None	NA	NA	0.6	500
81	0.4	5	0.0	Good	0.15	0.08	0.6	500
82	0.8	5	0.0	Good	0.04	0.11	0.6	500
83	0.4	10	0.0	Good	0.13	0.09	0.6	500
84	0.8	10	0.0	Good	0.04	0.12	0.6	500
85	0.4	5	0.1	Good	0.09	0.18	0.6	500
86	0.8	5	0.1	Good	0.03	0.09	0.6	500

87	0.4	10	0.1	Good	0.09	0.10	0.6	500
88	0.8	10	0.1	Good	0.03	0.11	0.6	500
89	0.4	5	0.0	Moderate	0.41	0.06	0.6	500
90	0.8	5	0.0	Moderate	0.07	0.16	0.6	500
91	0.4	10	0.0	Moderate	0.45	0.03	0.6	500
92	0.8	10	0.0	Moderate	0.10	0.14	0.6	500
93	0.4	5	0.1	Moderate	0.29	0.08	0.6	500
94	0.8	5	0.1	Moderate	0.06	0.15	0.6	500
95	0.4	10	0.1	Moderate	0.36	0.04	0.6	500
96	0.8	10	0.1	Moderate	0.07	0.11	0.6	500

Note:
ID denotes model number, λ denotes salient loadings, p/k denotes number of variables per factor, ProbX denotes probability of cross loadings, ME denotes model approximation error, $\mathrm{V_p}$ denotes proportion of variance due to ME, ϵ_{TKL} denotes epsilon TKL, ϕ_{ij} denotes correlations between common factors, and $\mathcal{N}$ denotes sample size.

The Probability of Obtaining Different Local Solutions with Equal Complexity Values

We have proved (by example) that substantively different LS can have equal complexity values. These results suggest that complexity values may not be a reliable index for categorizing LS into distinct solution sets. To determine the usefulness of this index, we counted the number of instances in which different local solutions had equal complexity values.

Table S12 reports the probability of obtaining different LS with equal complexity values for various facets of our simulation design. This table was constructed in a manner similar to that of Table S5.

These results suggest that the probability of obtaining qualitatively distinct factor patterns with equal complexity values is small. Thus, our method for calculating the number of local solution sets provides a tight lower bound on the true numbers.

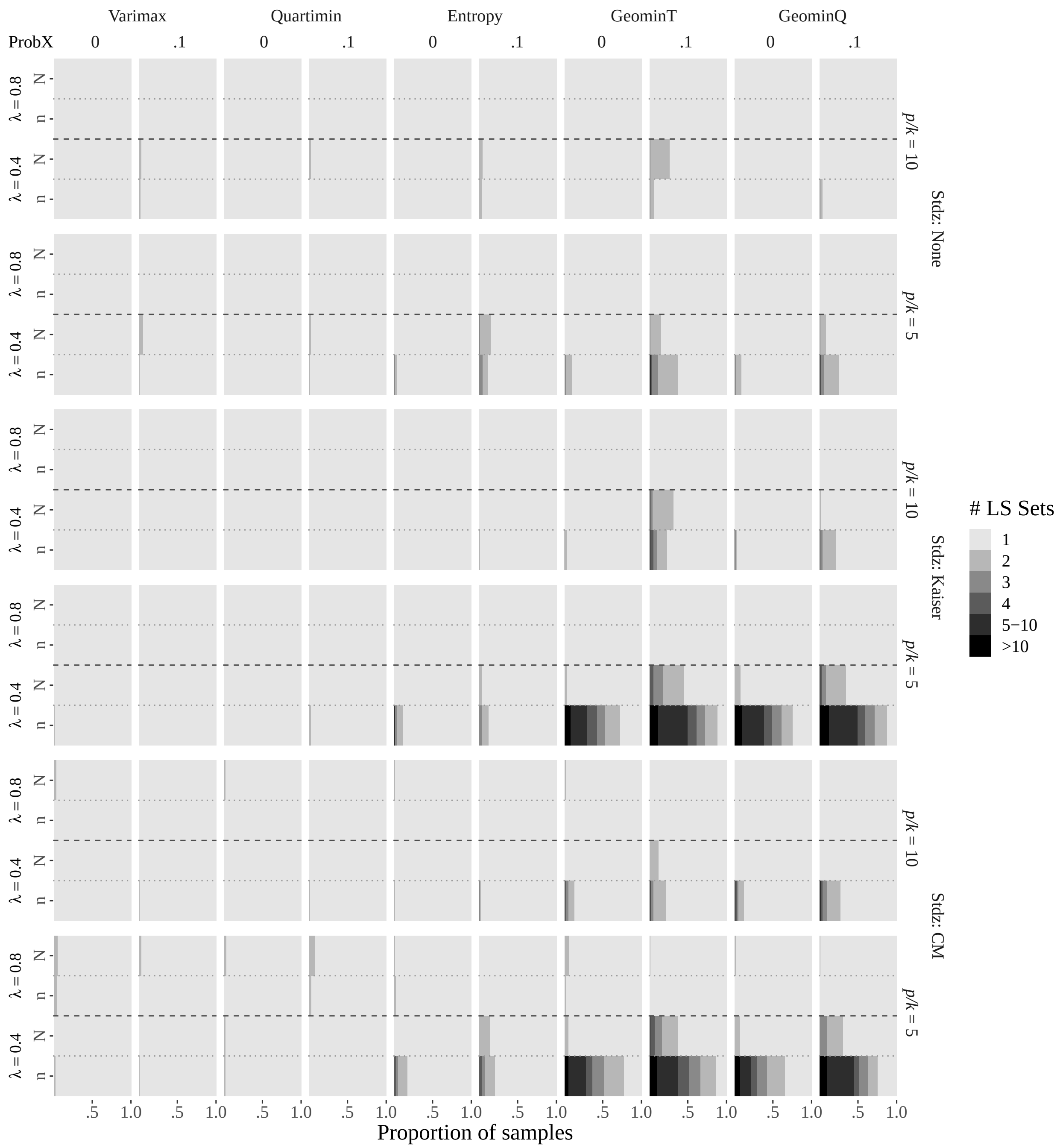
Table S12: Probability of Obtaining Different Local Solutions with Equal Complexity Values in Data Sets from Oblique or Orthogonal Population Factor Anlysis Models

		Rotation Methods (%)				
Model	Stdz	Varimax	Quartimin	Entropy	GeominT	GeominQ
	none	0.02	0.02	0.00	0.02	0.06
orth	kaiser	0.00	0.02	0.00	0.06	0.15
	cm	0.48	0.04	0.00	0.17	0.15
	none	0.08	0.06	0.06	0.33	0.35
oblq	kaiser	0.00	0.00	0.02	0.33	1.17
	cm	0.00	0.02	0.04	0.65	0.60

Note:
Stdz denotes row standardization; GeominT denotes the orthogonal geomin algorithm; GeominQ denotes the oblique geomin algorithm.

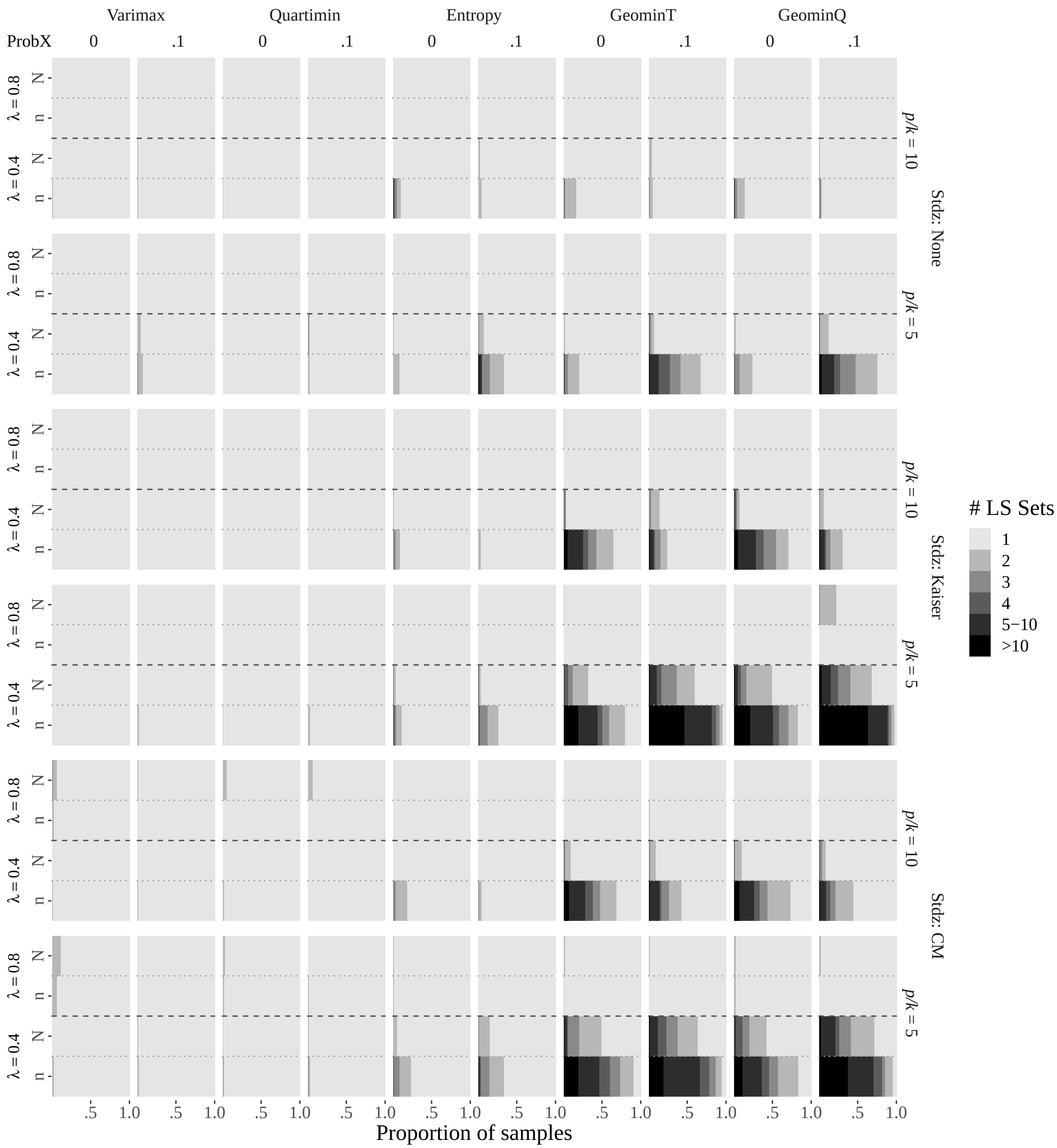
Number of Local Solutions Under All Conditions

Figure S8: Number of Local Solutions in Estimated Factor Models from Data Sets that were Generated from Orthogonal Population Factor Models; ME:None



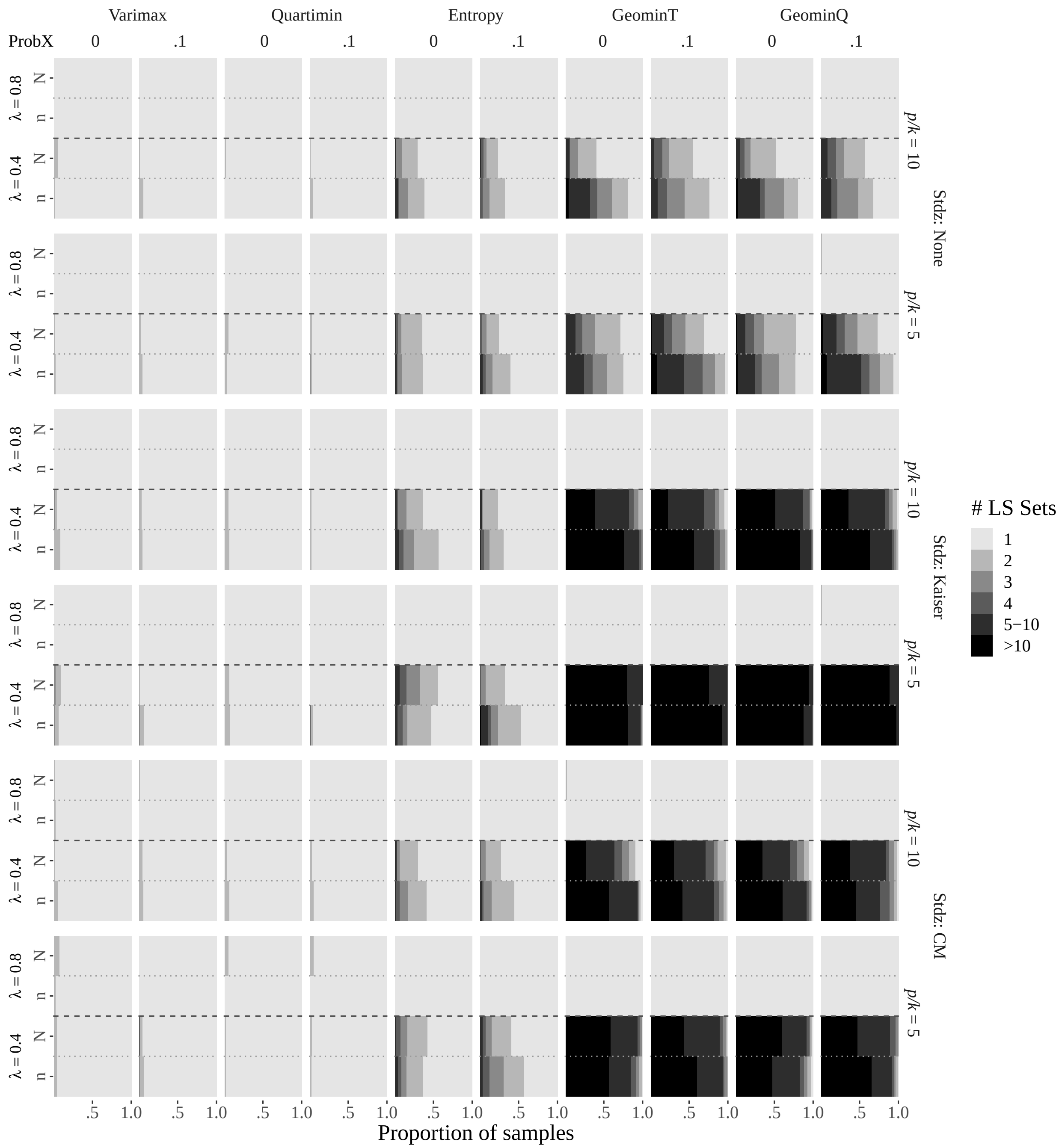
Notes. λ denotes salient factor loading size; p/k denotes number of variables per factor; ProbX denotes probability of cross loadings; ME denotes model approximation error; Stdz denotes method of loading standardization; Sample size $\mathcal{N}$: n = 150, N = 500. LS denotes rotation Local Solutions.

Figure S9: Number of Local Solutions in Estimated Factor Models from Data Sets that were Generated from Orthogonal Population Factor Models; ME:Good-Fit



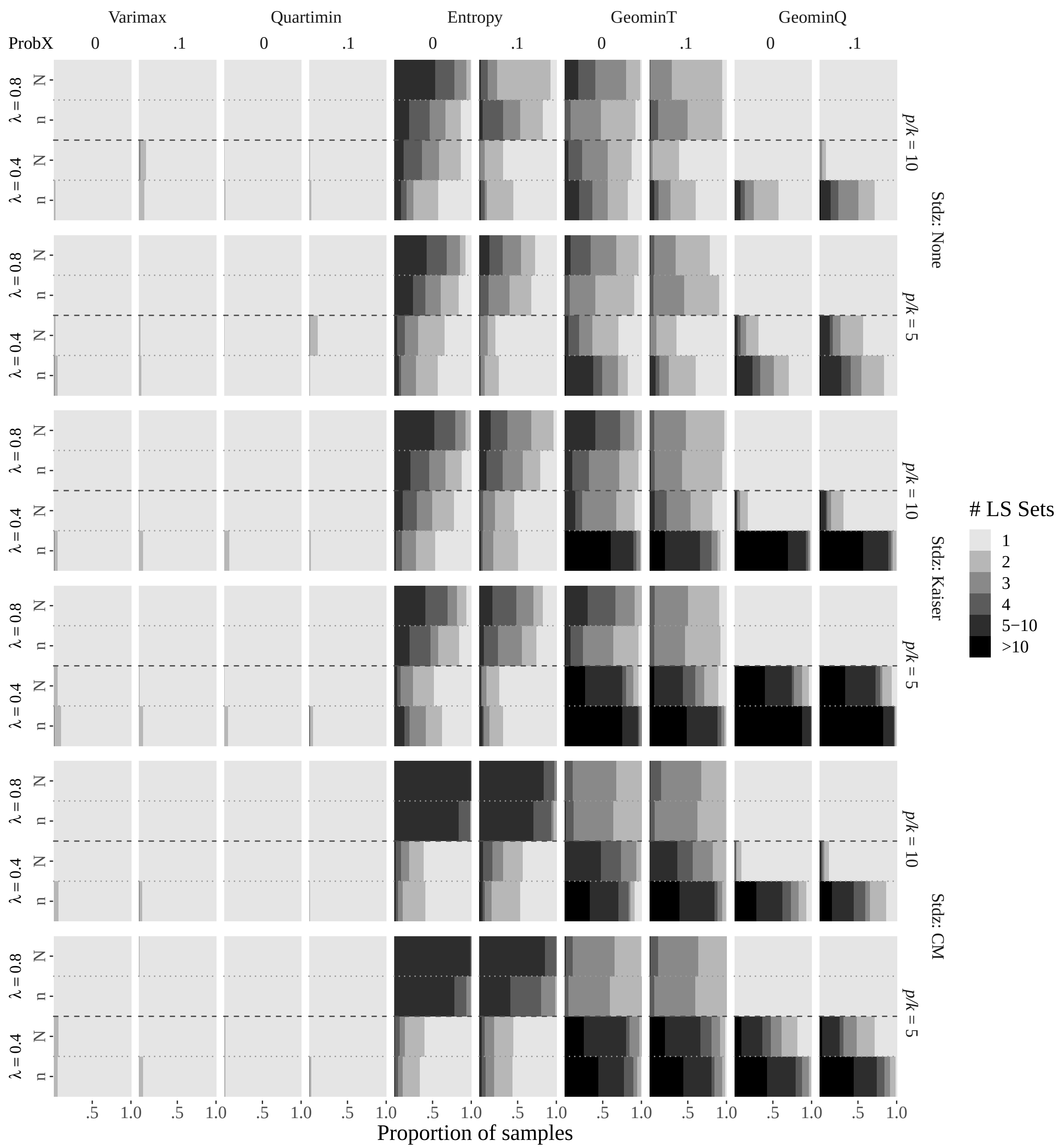
Notes. λ denotes salient factor loading size; p/k denotes number of variables per factor; ProbX denotes probability of cross loadings; ME denotes model approximation error; Stdz denotes method of loading standardization; Sample size $\mathcal{N}$: n = 150, N = 500. LS denotes rotation Local Solutions.

Figure S10: Number of Local Solutions in Estimated Factor Models from Data Sets that were Generated from Orthogonal Population Factor Models; ME:Moderate-Fit



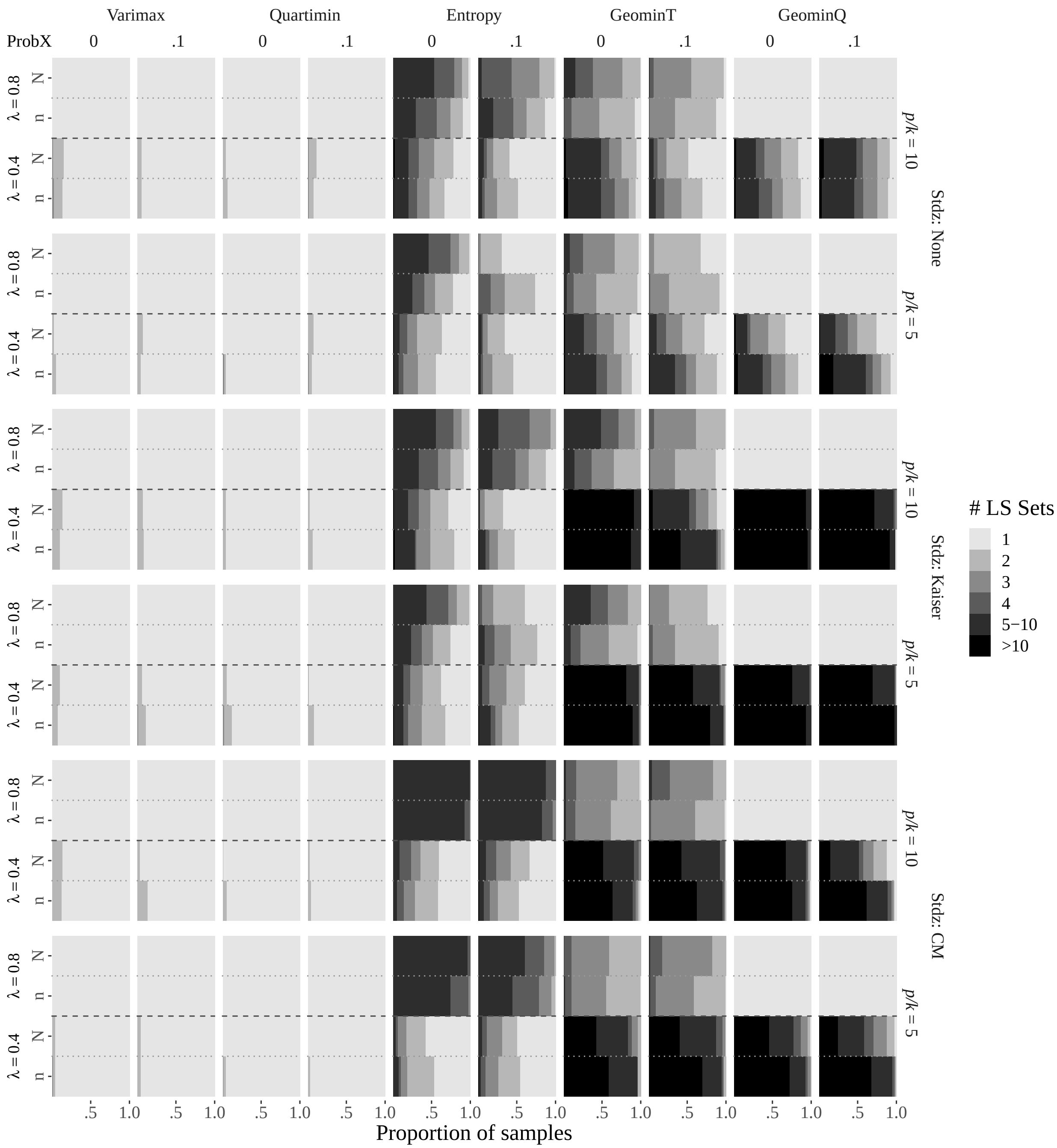
Notes. λ denotes salient factor loading size; p/k denotes number of variables per factor; ProbX denotes probability of cross loadings; ME denotes model approximation error; Stdz denotes method of loading standardization; Sample size $\mathcal{N}$: n = 150, N = 500. LS denotes rotation Local Solutions.

Figure S11: Number of Local Solutions in Estimated Factor Models from Data Sets that were Generated from Oblique Population Factor Models; ME:None



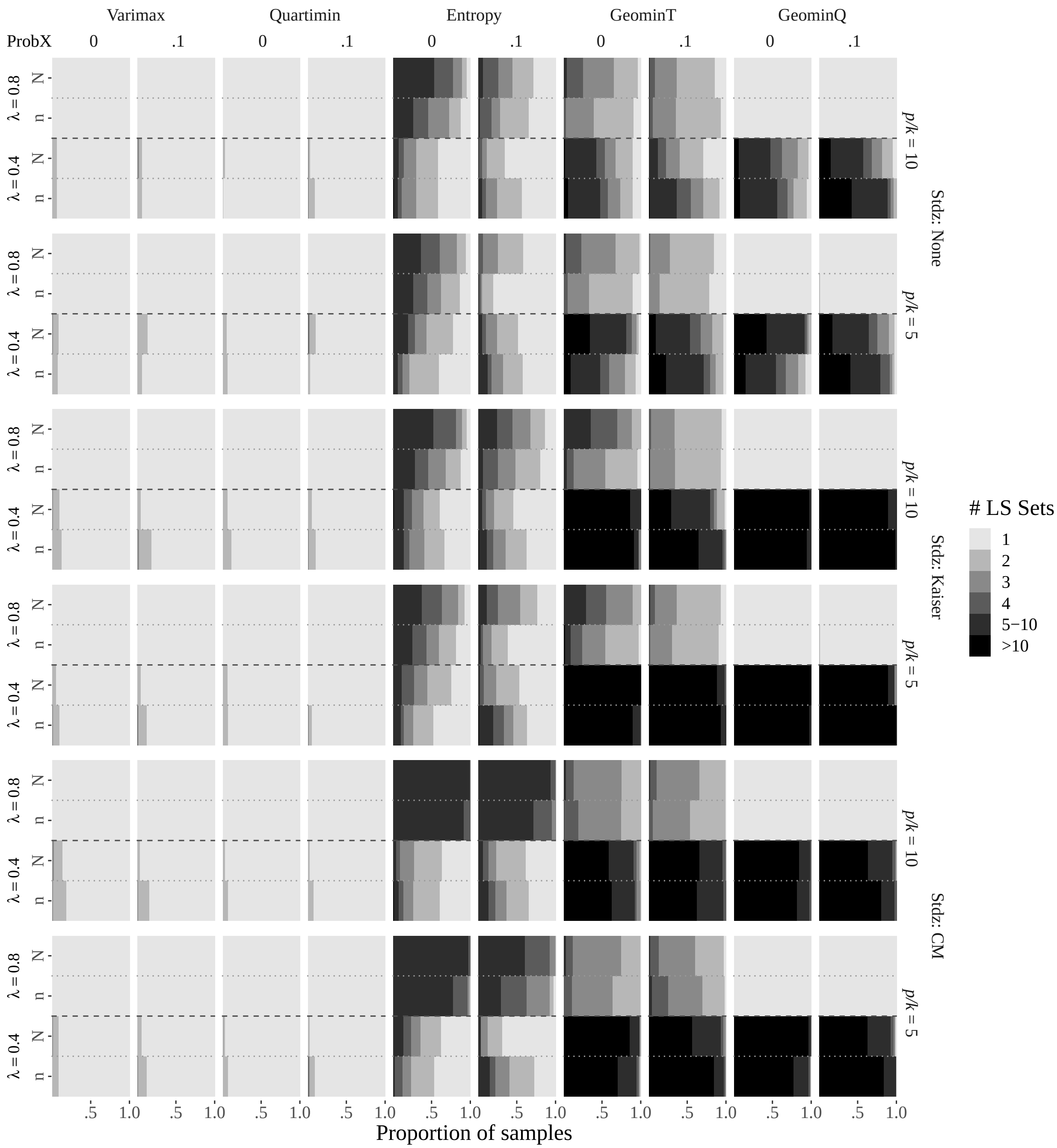
Notes. λ denotes salient factor loading size; p/k denotes number of variables per factor; ProbX denotes probability of cross loadings; ME denotes model approximation error; Stdz denotes method of loading standardization; Sample size $\mathcal{N}$: n = 150, N = 500. LS denotes rotation Local Solutions.

Figure S12: Number of Local Solutions in Estimated Factor Models from Data Sets that were Generated from Oblique Population Factor Models; ME:Good-Fit



Notes. λ denotes salient factor loading size; p/k denotes number of variables per factor; ProbX denotes probability of cross loadings; ME denotes model approximation error; Stdz denotes method of loading standardization; Sample size $\mathcal{N}$: n = 150, N = 500. LS denotes rotation Local Solutions.

Figure S13: Number of Local Solutions in Estimated Factor Models from Data Sets that were Generated from Oblique Population Factor Models; ME:Moderate-Fit



Notes. λ denotes salient factor loading size; p/k denotes number of variables per factor; ProbX denotes probability of cross loadings; ME denotes model approximation error; Stdz denotes method of loading standardization; Sample size $\mathcal{N}$: n = 150, N = 500. LS denotes rotation Local Solutions.

Choosing Between Local Solutions with (a) the Lowest Complexity Value or (b) the Highest Hyperplane Count

The findings in Table S13 show that, across all factor models, the local solution with the highest hyperplane count (rather than the lowest complexity value) was most likely to be the solution closest in mean squared error to the known population loading matrix. These results replicate findings summarized in Study 1.

Table S13: The Probability that the Solution with the Minimum Complexity Value or the Maximum Hyperplane Count had the Lowest RMSE with the Population Factor Pattern

	Varimax	Quartimin	Entropy	GeominT	GeominQ
None	61 (69)	58 (72)	55 (77)	52 (73)	45 (56)
Kaiser	58 (70)	57 (73)	54 (77)	39 (58)	30 (39)
CM	64 (69)	52 (75)	45 (51)	39 (56)	36 (46)

Note:
Numbers outside of the parentheses report the proportion of times that a smallest complexity value solution had the lowest RMSE with the population factor pattern. Numbers inside the parentheses report the proportion of times that a highest hyperplane solution had the lowest RMSE with the population factor pattern. CM denotes Cureton-Mulaik loading standardization; Kaiser denotes Kaiser loading standardization; GeominT denotes the orthogonal geomin rotation; GeominQ denotes the oblique geomin rotation.

A Real Data Example of Rotational Local Solutions

The Personality Item Set

Table S14 reports the 54 personality items (out of the original 57) that were used in our empirical example. For our analyses, we excluded the three items that were scored on a different scoring format from the others. Specifically, the items "Timekeeping," "Do you lie to others?" and "How much time do you spend online?" were scored using a binary response format whereas the remaining items were scored using a 1-to-5 Likert scale.

Table S14: Young People Survey Personality and Attitude Items

Item	Full Item	Abbreviation
1	I take notice of what goes on around me.	Daily events
2	I try to do tasks as soon as possible and not leave them until last minute.	Prioritising workload
3	I always make a list so I don't forget anything.	Writing notes
4	I often study or work even in my spare time.	Workaholism
5	I look at things from all different angles before I go ahead.	Thinking ahead
6	I believe that bad people will suffer one day and good people will be rewarded.	Final judgement
7	I am reliable at work and always complete all tasks given to me.	Reliability
8	I always keep my promises.	Keeping promises
9	I can fall for someone very quickly and then completely lose interest.	Loss of interest
10	I would rather have lots of friends than lots of money.	Friends versus money
11	I always try to be the funniest one.	Funniness
12	I can be two faced sometimes.	Fake
13	I damaged things in the past when angry.	Criminal damage
14	I take my time to make decisions.	Decision making
15	I always try to vote in elections.	Elections
16	I often think about and regret the decisions I make.	Self-criticism
17	I can tell if people listen to me or not when I talk to them.	Judgment calls
18	I am a hypochondriac.	Hypochondria
19	I am emphatetic person.	Empathy
20	I eat because I have to. I don't enjoy food and eat as fast as I can.	Eating to survive
21	I try to give as much as I can to other people at Christmas.	Giving
22	I don't like seeing animals suffering.	Compassion to animals
23	I look after things I have borrowed from others.	Borrowed stuff
24	I feel lonely in life.	Loneliness
25	I used to cheat at school.	Cheating in school
26	I worry about my health.	Health
27	I wish I could change the past because of the things I have done.	Changing the past
28	I believe in God.	God
29	I always have good dreams.	Dreams

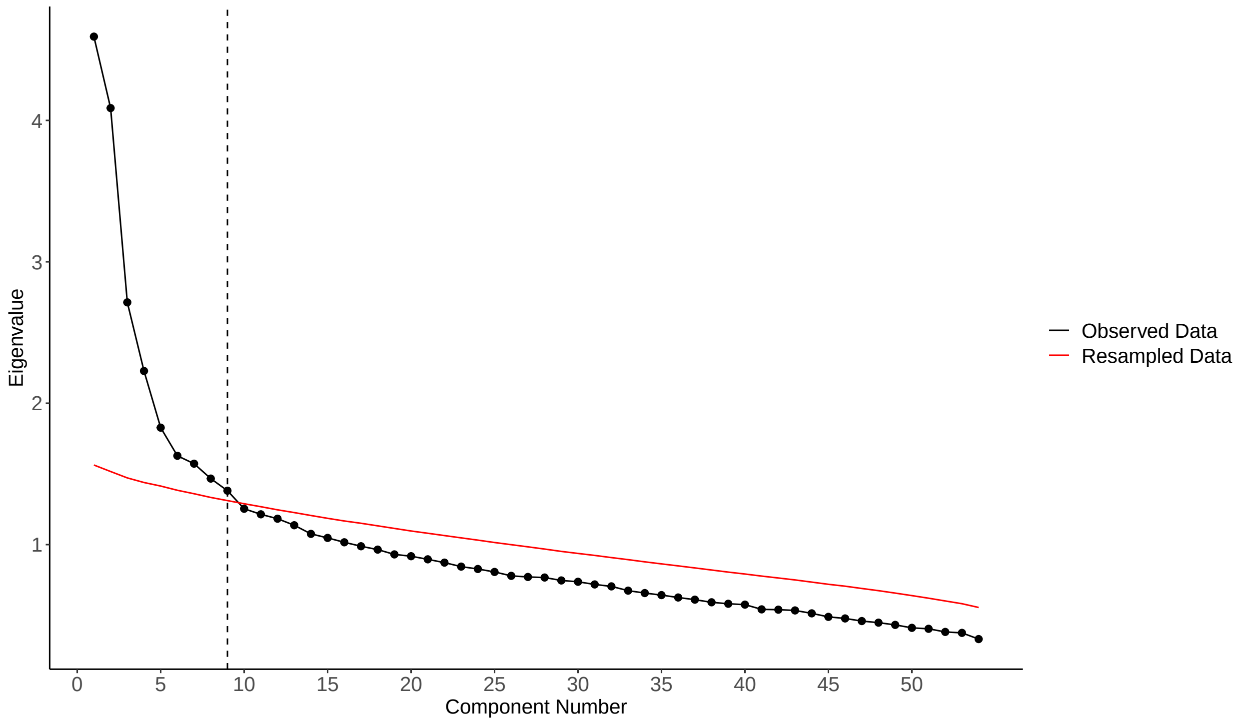
30	I always give to charity.	Charity
31	I have lots of friends.	Number of friends
32	I am very patient.	Waiting
33	I can quickly adapt to a new environment.	New environment
34	My moods change quickly.	Mood swings
35	I am well mannered and I look after my appearance.	Appearence and gestures
36	I enjoy meeting new people.	Socializing
37	I always let other people know about my achievements.	Achievements
38	I think carefully before answering any important letters.	Responding to a serious letter
39	I enjoy childrens' company.	Children
40	I am not afraid to give my opinion if I feel strongly about something.	Assertiveness
41	I can get angry very easily.	Getting angry
42	I always make sure I connect with the right people.	Knowing the right people
43	I have to be well prepared before public speaking.	Public speaking
44	I will find a fault in myself if people don't like me.	Unpopularity
45	I cry when I feel down or things don't go the right way.	Life struggles
46	I am 100 percent happy with my life.	Happiness in life
47	I am always full of life and energy.	Energy levels
48	I prefer big dangerous dogs to smaller, calmer dogs.	Small - big dogs
49	I believe all my personality traits are positive.	Personality
50	If I find something the doesn't belong to me I will hand it in.	Finding lost valuables
51	I find it very difficult to get up in the morning.	Getting up
52	I have many different hobbies and interests.	Interests or hobbies
53	I always listen to my parents' advice.	Parents' advice
54	I enjoy taking part in surveys.	Questionnaires or polls

Note:
The data set was obtained from https://www.kaggle.com/miroslavsabo/young-people-survey

Determining Number of Common Factors in the Personality Data with Parallel Analysis

We used the `fa.parallel` function from the R `psych` package (Revelle, 2018) to estimate the number of common factors in the personality data set. `fa.parallel` compares the eigenvalues of the observed correlation matrix to the associated eigenvalues from bootstrap generated correlation matrices as follows. First, `fa.parallel` extracted 54 eigenvalues—one for each variable—from the data correlation matrix (with ones on the diagonal). Next, `fa.parallel` simulated 100 data sets (with sample size of 741) by resampling (with replacement) the observed data. Next, for each resampled correlation matrix, `fa.parallel` extracted 54 eigenvalues. This resulted in 100 sets of ordered eigenvalues from the bootstrap correlation matrices. Finally, the number of common factors was estimated as the number of observed eigenvalues greater than the 95th percentile points of the eigenvalue distributions from the bootstrap samples. Figure S14 summarizes the parallel analysis results.

Figure S14: A Parallel Analysis of the Personality Data



Notes. The vertical dashed line denotes the suggested number of common factors to retain.

An Example of Two Geomin-Rotated Local Solutions for the Personality Data

Table S15 reports two local solutions obtained by rotating the personality data using geominQ and 1000 random starts. The first solution produced the lowest complexity value ($Q(\Lambda) = 1.44153$) whereas the second solution generated a complexity value that was slightly larger ($Q(\Lambda) = 1.47257$).

After aligning one solution to the other (using the `faAlign` function in the R `fungible` package (Waller, 2020) to minimize the RMSD between the aligned loadings), we computed congruence coefficients (Tucker, 1951) between aligned factors. These coefficients equaled (1.00, 1.00, .93, .56, .88, and .85) and clearly indicated that the two solutions are not identical. Furthermore, approximately 20% of factor loadings differed (i.e., their difference was larger than .30 in absolute value) and their RMSD was equal to .215.

Table S15: An Example of Two Geomin-Rotated Factor Patterns

	Global Solution									Unspun Solution								
	f1	f2	f3	f4	f5	f6	f7	f8	f9	f1	f2	f3	f4	f5	f6	f7	f8	f9
Daily events	48	-11	28	30	-09	-06	-05	09	-08	49	-07	26	29	-06	-21	-05	18	-09
Prioritising workload	46	12	-02	03	-28	00	04	00	-02	48	04	-01	01	-02	00	04	32	-02
Writing notes	46	-10	-01	-07	07	08	-02	-11	02	48	-17	-02	-06	14	02	-01	00	-01
Workaholism	39	01	09	01	02	-01	-03	00	35	43	01	11	01	-03	11	-04	-02	21
Thinking ahead	31	00	15	-27	-07	01	20	-01	11	33	-04	16	-24	04	03	21	06	10
Final judgement	31	11	-20	05	-21	09	-12	-15	13	34	00	-19	01	-05	25	-13	16	-02
Reliability	29	27	05	16	-08	11	-10	-19	10	32	05	04	11	12	29	-11	17	-10
Keeping promises	22	-03	-11	06	09	05	01	-18	20	25	-05	-11	05	02	20	00	-13	06
Loss of interest	22	-52	01	25	-08	00	-01	00	-02	22	-30	00	24	-34	-13	00	-14	-06
Friends versus money	01	48	02	03	18	-10	06	03	00	01	35	03	04	36	00	05	13	07
Funniness	-15	46	-01	-17	19	02	11	-07	10	-14	26	00	-16	36	21	10	02	10
Fake	-05	35	-02	00	02	00	23	-29	05	-03	23	-03	-02	14	35	22	01	-10
Criminal damage	-30	34	-02	18	22	-02	-02	-03	00	-31	24	-02	17	23	12	-04	-03	-01
Decision making	30	-34	34	20	00	05	-04	03	-01	31	-28	32	19	-14	-13	-04	-05	-09
Elections	16	-33	03	25	-04	-03	02	08	-03	15	-18	03	25	-22	-17	02	-05	-01
Self-criticism	01	26	01	00	00	17	-02	12	-12	-01	-02	01	00	27	01	-03	27	00
Judgment calls	09	24	00	-05	-05	-02	01	-18	24	13	17	00	-08	02	29	00	00	04
Hypochondria	-03	01	53	23	-06	-01	05	15	21	-03	01	53	21	-18	03	04	03	07
Empathy	17	03	48	25	10	02	-02	05	20	18	-03	48	24	00	05	-03	-02	03
Eating to survive	-03	01	-43	-06	09	-05	13	09	05	-04	08	-39	-03	05	-07	13	-04	24
Giving	26	-09	39	14	-05	13	-08	-04	-02	27	-20	36	12	00	04	-08	07	-17
Compassion to animals	01	-09	37	02	13	03	-01	-09	-05	02	-10	34	02	06	01	00	-12	-16
Borrowed stuff	05	01	-37	-03	01	04	29	-06	00	05	00	-35	00	04	08	29	00	11
Loneliness	31	45	-23	58	-12	11	28	49	23	30	18	-14	59	10	00	23	57	48
Cheating in school	26	00	04	-31	02	-04	09	-11	01	28	00	03	-29	11	-01	11	-03	00
Health	-02	-02	00	30	22	-14	-01	-01	-01	-02	12	00	30	03	-10	-01	-15	00
Changing the past	04	-16	-02	29	-05	-08	-05	00	11	04	00	-01	27	-25	-01	-06	-08	02

God	00	14	10	25	00	-17	03	-14	-11	01	22	08	22	-01	00	03	00	-20
Dreams	03	10	00	00	49	-04	-02	-04	-13	03	05	-02	04	47	-14	-01	-18	-01
Charity	00	29	-03	-07	41	06	04	00	00	00	09	-03	-03	50	03	03	-05	09
Number of friends	06	-08	09	17	39	-05	16	-01	23	07	00	10	20	11	02	15	-30	20
Waiting	-01	-03	25	35	38	02	10	-04	02	-01	-05	23	36	18	00	09	-21	-03
New environment	24	-02	-17	03	32	06	14	-04	-06	25	-09	-18	08	32	-06	14	-09	07
Mood swings	02	23	-07	-01	25	09	-12	01	-04	02	03	-08	00	37	02	-13	04	03
Appearence and gestures	-19	20	16	24	25	00	13	06	03	-21	13	17	25	18	03	12	-05	05
Socializing	04	-18	-08	-03	19	66	03	-05	02	03	-68	-09	-02	28	36	02	00	01
Achievements	-02	02	01	06	11	54	-05	-27	07	-01	-44	-02	02	22	54	-07	-03	-15
Responding to a serious letter	18	-02	-09	-02	22	41	07	-13	08	19	-38	-09	-01	30	31	06	-03	04
Children	11	-10	19	00	14	-37	01	10	-12	11	22	18	04	00	-43	02	-12	-01
Assertiveness	12	12	-03	-02	00	-36	-01	00	-09	13	36	-03	-01	01	-25	00	01	-02
Getting angry	-03	00	-21	01	01	-26	-01	-20	09	00	26	-20	00	-15	04	-01	-20	-01
Knowing the right people	08	04	-02	02	-23	25	45	-05	-15	07	-19	-02	02	00	20	45	24	-09
Public speaking	00	-12	-02	03	-10	28	41	-14	-02	-01	-28	-02	03	-07	29	41	02	-06
Unpopularity	01	00	30	-06	01	-05	39	02	01	01	03	30	-03	-02	-02	40	-02	02
Life struggles	11	-04	-03	-17	-08	16	36	-09	-01	11	-16	-03	-15	02	16	37	05	01
Happiness in life	-08	10	-11	09	00	32	35	-15	-05	-09	-19	-12	09	12	36	34	07	-07
Energy levels	40	35	-16	44	-03	08	10	45	34	40	13	-07	46	12	-02	06	43	52
Small - big dogs	01	02	10	16	06	02	01	-35	-03	04	00	06	12	00	25	01	-15	-27
Personality	10	-01	-01	06	-19	01	01	-33	00	13	01	-04	01	-16	26	02	-02	-24
Finding lost valuables	32	04	-05	-27	31	-01	00	03	33	35	00	-01	-22	27	01	-01	-15	35
Getting up	-09	11	13	-07	26	-01	00	01	-30	-11	03	10	-04	36	-19	01	-01	-16
Interests or hobbies	-13	06	22	-01	-09	-02	-05	00	-28	-15	03	18	-02	03	-11	-03	12	-26
Parents' advice	-01	13	00	-01	00	09	14	01	28	01	02	03	-01	-01	24	13	01	20
Questionnaires or polls	-03	-01	19	-03	-22	01	-10	-05	25	-01	01	20	-07	-26	20	-11	00	02

Note:
Notes. The complexity value of the Global and Unspun Solutions were 1.44153 and 1.47257, respectively. These solutions produced congruence coefficients (1.00, .61, .99, .99, .76, .71, 1.00, .47, and .69) between the corresponding aligned common factors. Approximately 20

An Example of Two Varimax-Rotated Local Solutions for the Personality Data

Table S16 reports two local solutions obtained by rotating the personality data using varimax and 1000 random starts. The first solution produced the lowest complexity value ($Q(\Lambda) = -.38535$) whereas the second solution generated a complexity value that was slightly larger ($Q(\Lambda) = -.38060$) (recall that the GPA function minimizes the negative of the varimax function).

After aligning one solution to the other (using the `faAlign` function in the R `fungible` package (Waller, 2020) to minimize the RMSD between the aligned loadings), we computed congruence coefficients (Tucker, 1951) between aligned factors. These coefficients equaled (1.00, 1.00, .93, .56, .88, and .85) and clearly indicated that the two solutions are not identical.

Table S16: Two Varimax-Rotated Local Solutions of Personality Data

	Lowest CV Solution						A Different Local Solution					
	f1	f2	f3	f4	f5	f6	f1	f2	f3	f4	f5	f6
Energy levels	71	05	-01	02	-02	03	71	04	01	-02	01	05
Number of friends	63	-07	05	06	18	01	62	-07	05	-10	19	01
Happiness in life	59	16	-18	05	-11	-08	61	15	-11	17	-03	-01
Interests or hobbies	53	06	04	-07	11	08	52	06	00	-18	06	02
Loneliness	-52	-02	26	05	14	16	-53	-01	21	-19	10	10
Socializing	49	-04	15	-09	18	14	47	-04	07	-29	09	05
Personality	46	07	-08	01	-08	-06	47	06	-04	09	-03	-02
New environment	43	-13	-01	-25	07	19	42	-12	-14	-30	-07	06
Public speaking	-37	07	-03	26	-05	-10	-36	07	08	26	09	03
Assertiveness	32	-01	19	-10	-07	14	31	-01	13	-18	-13	08
Dreams	27	04	-08	-09	05	-08	28	03	-10	-03	02	-10
Responding to a serious letter	-17	-05	00	07	04	01	-17	-04	02	03	06	03
Eating to survive	-14	00	09	-06	09	-05	-14	01	05	-09	03	-10
Prioritising workload	10	62	-01	11	-03	-10	11	62	06	15	04	-05
Workaholism	06	58	06	-07	14	-02	06	58	02	-11	07	-09
Writing notes	03	50	12	25	10	-01	03	49	20	08	20	05
Thinking ahead	-11	50	05	-03	05	22	-12	50	01	-13	00	16
Reliability	09	48	-11	04	-04	30	09	49	-10	-03	-02	29
Cheating in school	20	-39	21	09	-15	05	20	-40	24	00	-10	10
Keeping promises	03	37	-11	03	04	31	02	38	-12	-08	04	29
Getting up	-09	-32	12	11	04	03	-10	-32	14	-02	07	06
Decision making	-26	27	-02	26	01	08	-26	27	07	16	13	17
Daily events	16	20	17	-11	09	13	14	21	09	-24	00	03
Elections	10	19	-08	-04	15	13	09	20	-12	-12	10	08
Getting angry	-09	-02	53	16	-09	-07	-10	-03	56	-03	-03	-04
Mood swings	-29	03	43	17	05	-01	-30	03	45	-07	08	00
Knowing the right people	33	12	43	-08	-02	09	32	12	36	-26	-10	00
Criminal damage	-07	-13	37	-06	-17	00	-07	-13	34	-11	-21	-03
Changing the past	-32	-16	35	03	21	09	-35	-15	28	-25	15	01
Loss of interest	-01	-01	32	-19	01	10	-02	00	21	-30	-13	-02
Fake	-28	-03	32	-26	07	-03	-30	-02	18	-30	-11	-18
Appearence and gestures	15	12	28	18	00	10	14	11	32	-04	06	13
Hypochondria	-20	02	27	06	10	-15	-20	01	27	-04	09	-15
Waiting	11	-04	-25	-13	13	08	11	-04	-31	-09	06	02
Achievements	14	-03	25	-06	-01	-06	14	-03	21	-12	-06	-10
Funniness	11	-06	20	-18	10	10	10	-05	09	-30	-03	-02
Life struggles	-15	01	13	56	14	01	-15	00	31	24	39	20
Giving	21	16	05	39	15	01	20	16	18	15	32	14
Health	-01	22	29	36	01	-02	-01	21	41	13	17	10
Empathy	05	02	-08	36	09	28	04	02	02	10	25	39

Compassion to animals	-07	-01	02	35	03	17	-07	-01	13	13	19	29
Children	19	04	-05	33	29	06	18	04	03	06	42	14
Small - big dogs	08	-17	07	-28	-11	05	08	-16	-04	-19	-25	-06
Parents' advice	05	24	03	24	18	01	04	24	10	05	28	07
God	06	06	-02	22	44	-08	04	06	01	-02	50	-06
Charity	09	16	07	02	41	04	07	17	01	-23	35	-04
Final judgement	00	13	-02	11	40	00	-02	14	-03	-11	40	-03
Finding lost valuables	-01	19	-17	15	35	-07	-02	19	-14	04	40	-05
Friends versus money	14	02	-13	21	30	08	12	03	-09	00	37	12
Unpopularity	-17	06	-01	10	28	14	-18	07	-02	-12	27	11
Questionnaires or polls	-12	13	-06	09	16	14	-12	14	-06	-04	17	13
Judgment calls	12	00	08	06	-09	43	11	01	06	-15	-08	42
Borrowed stuff	-06	29	-09	23	-02	30	-06	30	-02	07	09	36
Self-criticism	-29	18	18	-01	22	29	-31	20	10	-29	13	19

Note:
CV denotes complexity value. All loadings are multiplied by 100.

An Empirical Example of Different Local Solutions with the Same Complexity Value

Table S17 reports different local solutions with equal complexity values (up to five decimal place) for the personality data. We obtained these solutions by rotating the 9-factor model by geominQ from 1000 random starts. These rotations produced 108 solution sets. Table S17 reports loading patterns from Set 49 with complexity values of $Q(\Lambda) = 1.47211$. We divided the 17 loading patterns in this set into two subsets of qualitatively distinct factor patterns (i.e., one group had four pattern matrices and the other group had 13 pattern matrices). Table S17 presents a representative loading matrix from each group.

Table S17: An Example of Two Obliquely Rotated Factor Patterns with the Same Complexity Value Using Real Data

	Solution 1									Solution 2								
	f1	f2	f3	f4	f5	f6	f7	f8	f9	f1	f2	f3	f4	f5	f6	f7	f8	f9
Number of friends	48	34	29	-05	01	16	01	-07	-06	49	28	16	-09	-06	-07	02	-26	-01
Giving	47	-06	-01	09	15	00	07	03	-03	48	-08	-06	07	05	-03	08	-03	-05
Friends versus money	45	01	-01	-25	-03	21	12	00	02	46	01	-01	-31	00	00	13	-01	01
Children	39	01	09	02	-01	03	-03	35	-03	42	02	13	02	-01	-15	-03	26	00
Life struggles	34	-28	-05	28	-01	-02	-20	31	01	37	-23	-01	16	-03	06	-19	34	00
Empathy	33	-25	15	-09	05	01	-05	11	20	34	-24	06	-05	02	-03	-03	07	23
Fake	-32	00	-02	25	-17	10	13	02	-03	-31	17	00	08	-03	32	08	21	-03
Finding lost valuables	29	01	-20	-16	04	10	25	16	-13	31	01	-06	-17	09	-08	23	17	-18
Parents' advice	22	03	-12	13	05	-08	16	19	00	23	03	-11	12	04	-12	13	21	-03
Happiness in life	23	45	01	-07	19	-18	-04	-04	-01	22	22	-09	09	01	-46	-03	-29	-01
Personality	17	39	04	-02	10	-06	-08	-05	02	16	24	-07	03	-01	-27	-08	-22	02
Loneliness	-16	-38	-01	19	-16	12	10	12	11	-14	-15	01	02	00	40	07	37	10
Dreams	03	35	-01	-03	-04	-06	03	09	-05	04	27	00	02	-06	-26	01	-01	-05
Compassion to animals	28	-32	03	00	00	-07	01	-01	09	28	-29	-01	02	-07	02	03	03	09
Achievements	-02	27	01	24	-14	-03	04	-03	-01	-02	29	-07	13	-16	02	01	01	-01
New environment	-04	22	54	-06	-02	07	-10	23	06	-02	24	44	05	04	-10	-09	01	21
Socializing	16	20	50	12	02	08	00	22	-02	19	26	38	15	04	-02	00	02	10
Public speaking	-03	-02	-44	07	-08	00	-11	02	13	-04	-05	-39	-10	-05	01	-15	20	02
Interests or hobbies	25	13	40	-01	18	05	09	02	-08	27	12	28	10	13	-04	11	-21	01
Assertiveness	02	-01	38	15	09	-09	05	-04	00	03	02	24	25	02	02	07	-17	08
Decision making	05	-02	-38	02	02	00	08	-02	28	04	-03	-45	-09	04	00	04	16	17
Energy levels	31	31	35	02	20	-05	-06	00	-04	32	19	19	15	05	-24	-03	-31	04
Getting angry	04	-10	00	50	-05	00	-01	-14	-02	04	01	-11	28	-09	36	-03	00	-04
Knowing the right people	-01	27	26	43	02	-02	07	02	10	01	34	02	34	01	11	03	-02	14
Mood swings	-01	-22	-04	41	-04	12	01	01	03	01	-05	-09	17	03	43	-02	20	00
Appearence and gestures	07	15	09	39	-03	-10	-06	20	16	08	19	-07	31	-05	-03	-09	20	18
Health	25	01	-18	34	08	03	03	-06	13	25	04	-33	14	03	16	-01	03	05

Loss of interest	-21	12	17	26	-10	10	03	04	13	-19	25	04	12	00	24	-02	11	16
Hypochondria	01	-12	-08	26	00	14	03	-01	-13	03	-01	-04	08	06	33	01	10	-16
Prioritising workload	02	-02	-10	23	68	05	06	12	02	04	-05	-19	29	67	06	03	-04	-05
Workaholism	-05	-05	00	18	49	01	33	16	-06	-01	00	-03	29	54	08	29	08	-10
Writing notes	17	-07	-10	26	40	04	14	14	06	20	-04	-18	23	40	10	10	11	00
Getting up	14	-05	-03	-02	-36	01	-01	-15	-02	12	-01	-01	-14	-39	08	00	00	-01
Cheating in school	14	06	21	11	-28	-08	-19	-19	01	12	04	10	04	-40	01	-16	-20	08
Changing the past	00	-17	02	19	-27	24	08	01	05	01	05	01	-09	-13	45	05	25	05
Responding to a serious letter	-02	-01	-21	01	-26	-20	14	02	-01	-03	-01	-13	02	-28	-16	12	17	-04
Final judgement	26	51	-22	-08	-13	69	-08	30	25	30	59	-27	-55	18	24	-18	40	16
God	35	40	-15	-01	-10	57	-17	39	08	40	46	-12	-40	13	16	-23	42	02
Eating to survive	-02	-09	01	02	07	28	00	-06	-04	-01	00	01	-13	17	34	-01	-01	-05
Daily events	01	05	11	12	02	-14	39	-02	00	02	12	02	21	00	-03	35	-01	02
Self-criticism	-06	-20	-02	06	-13	02	39	06	22	-05	-01	-09	00	00	19	33	29	20
Elections	10	01	-01	-13	01	-12	37	00	00	10	01	-01	00	00	-16	36	01	00
Charity	26	01	06	-01	01	17	34	15	-12	30	12	10	-06	10	11	31	16	-11
Funniness	00	14	12	04	-21	01	25	-12	02	00	22	03	-01	-18	08	22	-04	05
Criminal damage	-08	-15	14	26	-02	05	-02	-30	-01	-09	-06	01	13	-06	38	-01	-21	01
Questionnaires or polls	-01	-06	00	00	01	05	01	29	14	01	00	01	-01	12	-03	-01	32	14
Small - big dogs	-13	-04	23	-09	-02	05	05	-26	-05	-14	-01	17	-05	-03	16	07	-29	01
Waiting	-04	-02	20	-23	00	-06	03	26	-09	-02	-04	31	-02	05	-23	05	12	-01
Unpopularity	09	-17	00	-03	-11	01	21	24	00	11	-07	08	-02	-02	01	19	33	01
Keeping promises	07	-01	-01	-19	23	14	21	-10	43	06	01	-25	-18	29	05	17	-07	38
Reliability	-01	04	-02	-07	30	-04	20	02	40	-01	02	-25	04	31	-11	16	-01	36
Judgment calls	03	-08	31	00	-03	-03	-03	00	40	03	-03	05	03	-03	02	-03	-01	46
Borrowed stuff	11	-16	-04	-07	18	-01	09	01	36	11	-16	-20	-03	18	-03	07	04	32
Thinking ahead	-10	00	-12	05	25	07	29	00	33	-09	07	-30	03	34	11	22	10	26

Note:

Notes. The complexity value of Solutions 1 and 2 were 1.472106 and 1.472109, respectively. These solutions produced congruence coefficients (1.00, .89, .83, .69, .88, .55, .98, .74, and .93) between the corresponding aligned common factors. All factor loadings are multiplied by 100.

Varying Number of Random Starts

In this section we discuss the topic of the "proper'' number of random starts when using rotation methods with GPA optimization. To introduce this discussion, we reanalyzed the personality data with geominQ rotations and varying numbers of random starts. Table S18 summarizes this aspect of our results and shows that when Stdz = None, the random-start number did not influence the LS counts. However, under row standardization with Kaiser or CM normalization, the LS rates increased as the random-start number increased. The results in Table S18 suggest that factor analysts should analyze their data multiple times using many numbers of random starts (e.g., 200, 500, 1000). Although running the GPA algorithm from 200 random starts worked well in our simulation studies, the current results suggest that models with a larger number of common factors will require a larger number of random starts to sufficiently cover the function landscape.

Table S18: Number of Local Solutions for Oblique Geomin Rotation in a Real-Data Example with 54 Personality Items

	Number of Local Solutions								
# Factors	5			6			9		
# Starts	500	1000	2000	500	1000	2000	500	1000	2000
None	3	3	3	1	1	1	1	1	1
Kaiser	3	3	3	12	15	15	62	85	127
CM	7	7	7	22	25	26	87	108	136

Note:
Kaiser denotes Kaiser loading standardization; CM denotes Cureton-Mulaik loading standardization.

Additional Considerations

Local Solutions in a Five- and Ten-factor Model (simulated data)

In this section, we present a preliminary simulation study that examines the rotation local solution rates for five rotation algorithms (varimax, quartimin, entropy, geominT, and geominQ) in a 10-factor model. For these analyses, we modified Model 1 (see Table S1) to obtain a 10-factor model with (a) small salient loadings ($\lambda = .40$), (b) 5 indicators per factor ($p/k = 5$), (c) no cross loadings (ProbX = 0), (d) no model approximation error (ME = None), (e) orthogonal factors $\phi_{ij} = 0$, and (f) a small sample size $\mathcal{N} = 200$.

Using this modified factor model, we generated and factor analyzed 100 sample correlation matrices and obtained 100 unrotated factor structure matrices. Row standardization was not used in these analyses. Table S19 summarizes the average local solution rates for the original 5-factor model (from Model 1) and the 10-factor model (from the modified Model 1).

Our preliminary results suggest that the probability of obtaining local solutions during factor rotations will increase for factor models with larger numbers of major common factors. We suggest that this result merits further study in future work on rotation local minima.

Table S19: Average Number of Local Solutions for a Five- and Ten-factor Model

	Varimax	Quartimin	Entropy	GeominT	geominQ
5-factor	1.00	1.00	1.04	1.12	1.11
10-factor	1.14	1.11	1.72	4.87	5.42

Note:
GeominT denotes orthogonal geomin rotation and GeominQ denotes oblique geomin rotation.

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