Post-Hoc Power

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Introduction

Setup and Data

```
library(tidyverse)
```

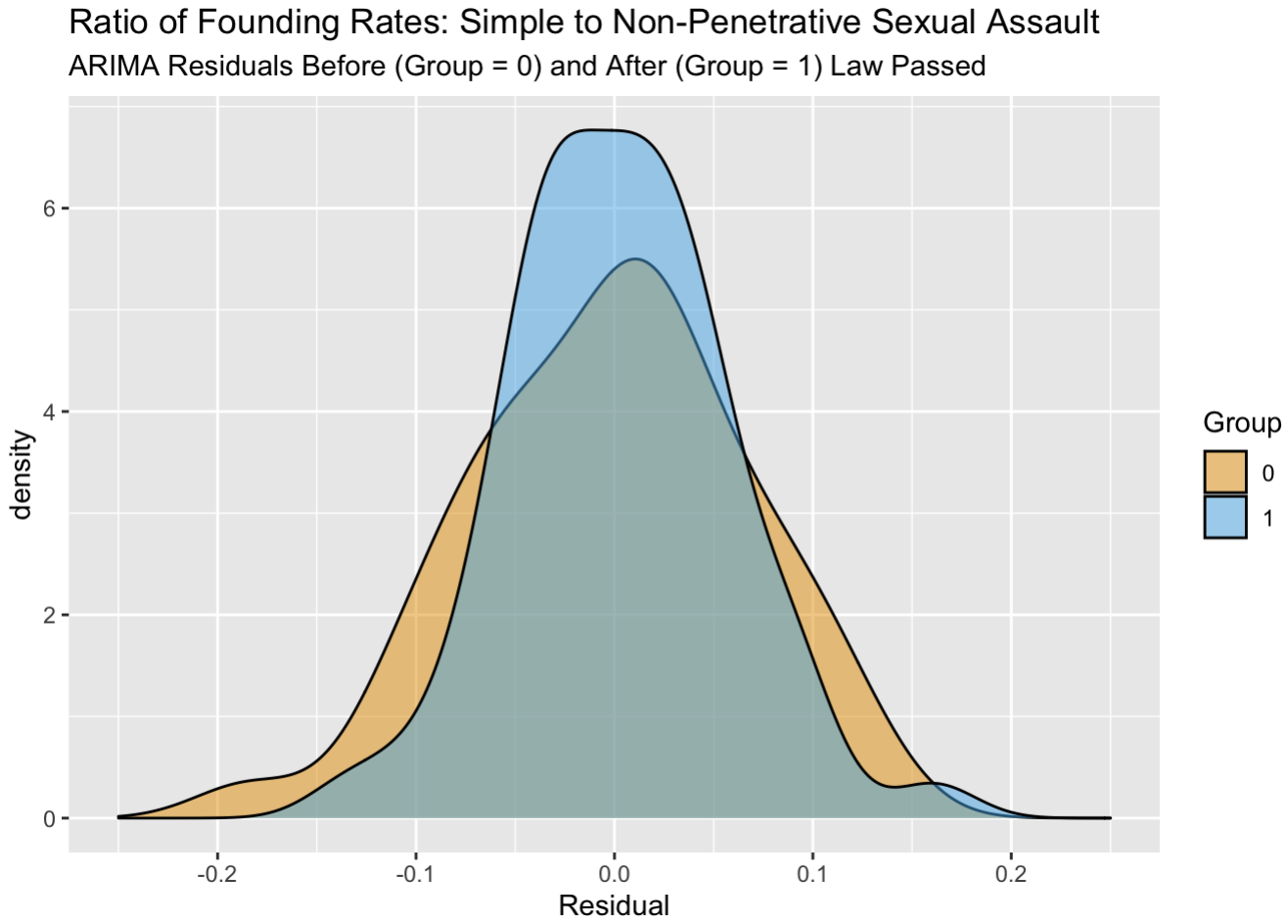
```
simpleAssault_df <-
  read_csv("data/LE-Monthly-nonpenSim_residPrePost_2022-03-23.csv") %>%
  mutate(Group = as.factor(Group))
aggAssault_df <-
  read_csv("data/LE-Monthly-penAggra_residPrePost_2022-02-16.csv") %>%
  mutate(Group = as.factor(Group))
totalAssault_df <-
  read_csv("data/LE-Monthly-SAAG_residPrePost_2022-02-16.csv") %>%
  mutate(Group = as.factor(Group))
```

Inspect the Residuals Before and After the Law was Active

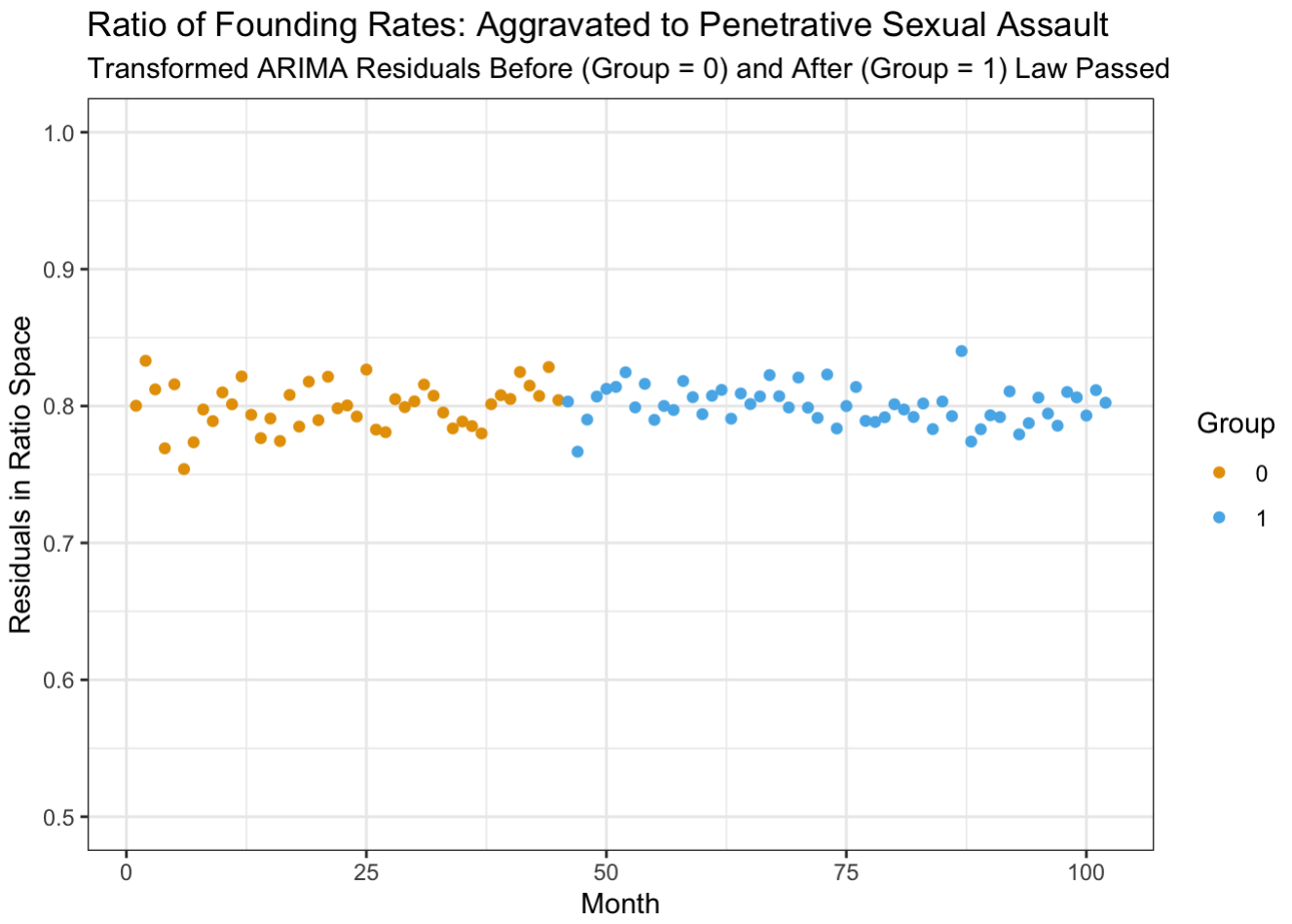
We plot the densities of the residuals before and after the law was in effect. We also plot the residuals transformed back into ratio space, shifted back to 80% (which was approximately the average ratio of the data before the ARIMA procedure was applied).

Ratio of Simple Assault to Non-Penetrative Sexual Assault

```
ggplot(data = simpleAssault_df) +
  aes(x = Residual, fill = Group) +
  labs(
    title = "Ratio of Founding Rates: Simple to Non-Penetrative Sexual Assault",
    subtitle = "ARIMA Residuals Before (Group = 0) and After (Group = 1) Law Passed"
  ) +
  scale_fill_manual(values = c("#E69F00", "#56B4E9")) +
  scale_x_continuous(limits = c(-0.25, 0.25)) +
  geom_density(alpha = 0.5)
```



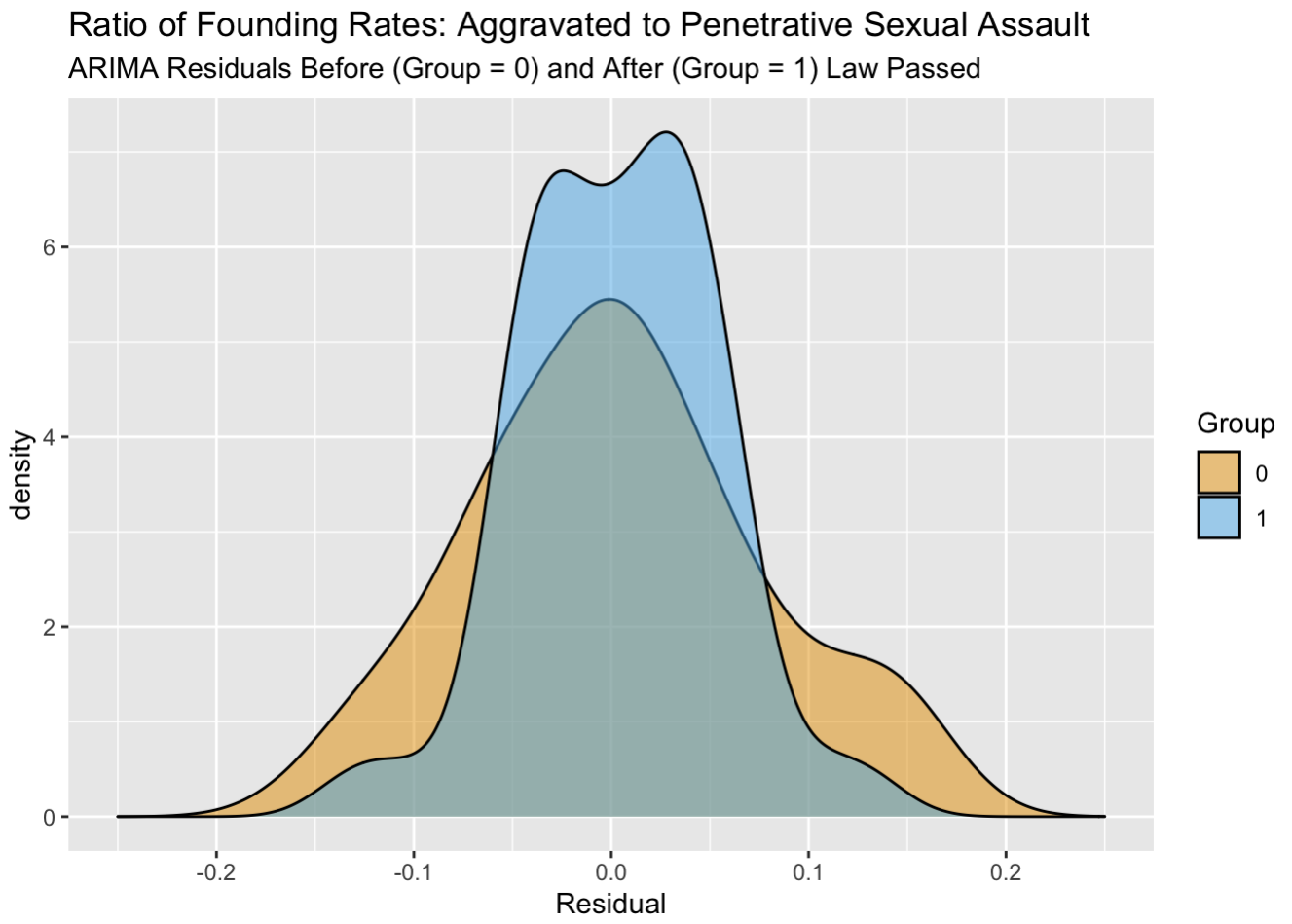

```
ggplot(data = simpleAssault_df) +
  theme_bw() +
  aes(x = Month, y = InvLogit(Residual) + 0.3, color = Group) +
  labs(
    title = "Ratio of Founding Rates: Aggravated to Penetrative Sexual Assault",
    subtitle = "Transformed ARIMA Residuals Before (Group = 0) and After (Group = 1) Law Passed"
,
    y = "Residuals in Ratio Space"
  ) +
  scale_color_manual(values = c("#E69F00", "#56B4E9")) +
  scale_y_continuous(limits = c(0.5, 1.0)) +
  geom_point()
```



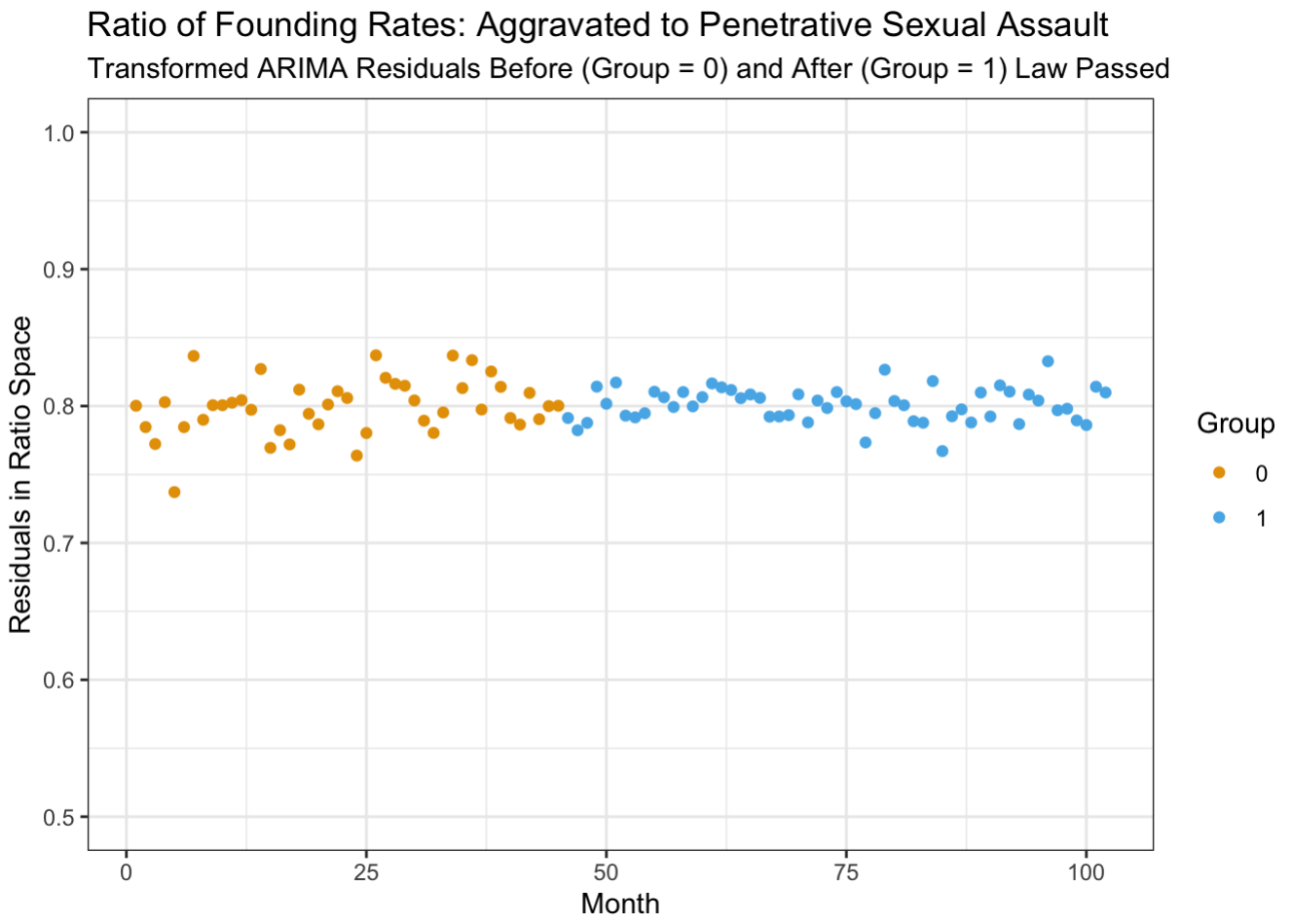
Ratio of Aggravated Assault to Penetrative Sexual Assault

```
ggplot(aggAssault_df) +
  aes(x = Residual, fill = Group) +
  labs(
    title = "Ratio of Founding Rates: Aggravated to Penetrative Sexual Assault",
    subtitle = "ARIMA Residuals Before (Group = 0) and After (Group = 1) Law Passed"
  ) +
  scale_fill_manual(values = c("#E69F00", "#56B4E9")) +
  scale_x_continuous(limits = c(-0.25, 0.25)) +
  geom_density(alpha = 0.5)
```

```
## Warning: Removed 1 rows containing non-finite values (stat_density).
```

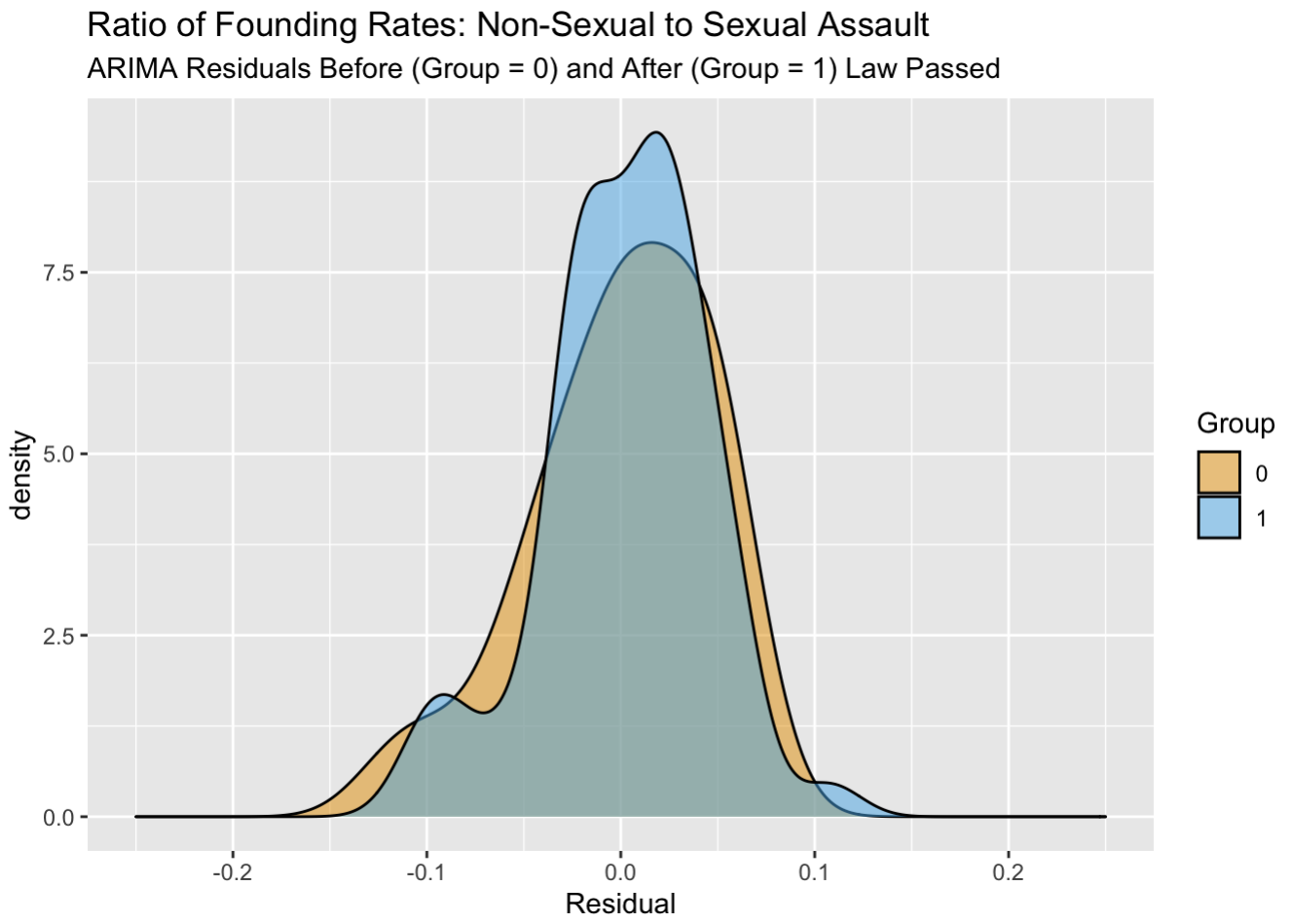



```
ggplot(data = aggAssault_df) +
  theme_bw() +
  aes(x = Month, y = InvLogit(Residual) + 0.3, color = Group) +
  labs(
    title = "Ratio of Founding Rates: Aggravated to Penetrative Sexual Assault",
    subtitle = "Transformed ARIMA Residuals Before (Group = 0) and After (Group = 1) Law Passed"
,
    y = "Residuals in Ratio Space"
  ) +
  scale_color_manual(values = c("#E69F00", "#56B4E9")) +
  scale_y_continuous(limits = c(0.5, 1.0)) +
  geom_point()
```

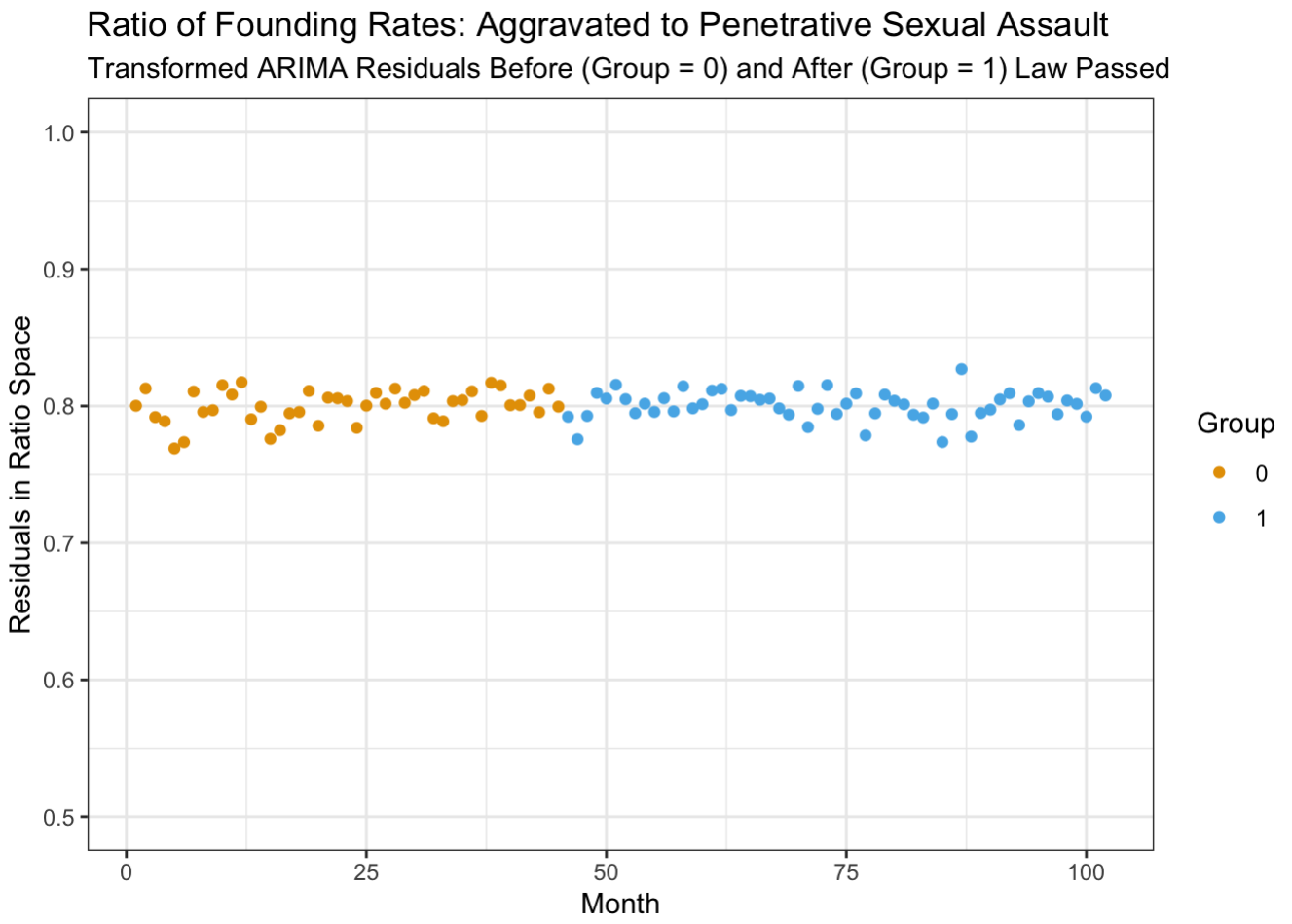


Ratio of All Assault to All Sexual Assault

```
ggplot(totalAssault_df) +
  aes(x = Residual, fill = Group) +
  labs(
    title = "Ratio of Founding Rates: Non-Sexual to Sexual Assault",
    subtitle = "ARIMA Residuals Before (Group = 0) and After (Group = 1) Law Passed"
  ) +
  scale_fill_manual(values = c("#E69F00", "#56B4E9")) +
  scale_x_continuous(limits = c(-0.25, 0.25)) +
  geom_density(alpha = 0.5)
```




```
ggplot(data = totalAssault_df) +
  theme_bw() +
  aes(x = Month, y = InvLogit(Residual) + 0.3, color = Group) +
  labs(
    title = "Ratio of Founding Rates: Aggravated to Penetrative Sexual Assault",
    subtitle = "Transformed ARIMA Residuals Before (Group = 0) and After (Group = 1) Law Passed"
,
    y = "Residuals in Ratio Space"
  ) +
  scale_color_manual(values = c("#E69F00", "#56B4E9")) +
  scale_y_continuous(limits = c(0.5, 1.0)) +
  geom_point()
```



Here, group 0 means no “treatment”; that is, the law had not yet been passed. As we can see across all figures, the only change before and after the law passing was that the variability of the residuals may have decreased. My recurring hypothesis is that the mandatory training / re-training period after the law changed reduced how arbitrary some decision making related to sexual assault cases had been previously.

Ratio Differences Necessary to Reject

Ratio of Simple Assault to Non-Penetrative Sexual Assault

Here are the results, and the inverse power analysis:

```
t.test(Residual ~ Group, data = simpleAssault_df)
```

```
##
##  Welch Two Sample t-test
##
## data:  Residual by Group
## t = -0.39789, df = 81.844, p-value = 0.6917
## alternative hypothesis: true difference in means between group 0 and group 1 is not equal to
0
## 95 percent confidence interval:
##  -0.02992263  0.01994816
## sample estimates:
## mean in group 0 mean in group 1
##    -0.002302598     0.002684638
```

```
DetectableDelta(simpleAssault_df)
```

```
## # A tibble: 1 × 5
##       n  delta     sd sig.level power
##   <dbl>  <dbl>  <dbl>     <dbl> <dbl>
## 1    51 0.0347 0.0620      0.05   0.8
```

It shows that we would be statistically powered to detect a change of 0.035 in the ratio residual space, but we don't know what that means in terms of the original ratio space. Let's find out.

Vector of Possible Ratios

And here are the percentage point changes we would be powered to detect at various levels of the original residual ratio. We first transform from ratio space to residual space via the logit function.

```
# Possible ratios
ratios_num <- seq(0.1, 0.9, by = 0.05)
ratios_num
```

```
##  [1] 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80
## [16] 0.85 0.90
```

```
# Transform to residual space
residualThresholds_num <- Logit(ratios_num)
residualThresholds_num
```

```
##  [1] -2.1972246 -1.7346011 -1.3862944 -1.0986123 -0.8472979 -0.6190392
##  [7] -0.4054651 -0.2006707  0.0000000  0.2006707  0.4054651  0.6190392
## [13]  0.8472979  1.0986123  1.3862944  1.7346011  2.1972246
```

Note that these are not the actual ratios between simple assault and non-penetrative sexual assault, but potential ratios after temporal autocorrelations have been removed. Remember that our first analysis step was to calculate the ARIMA residuals to remove the dependence of these data points on themselves.

Ratio to Residual Transformation

At each of these ratios, we create pairs of “before and after” changes in the ratio of case founding rates.

```
# Matrix of possible residual ranges
logits1_mat <- matrix(
  c(
    residualThresholds_num,
    residualThresholds_num + DetectableDelta(simpleAssault_df)$delta
  ),
  nrow = length(residualThresholds_num),
  ncol = 2
)

logits1_mat
```

```
##             [,1]       [,2]
##  [1,] -2.1972246 -2.16249303
##  [2,] -1.7346011 -1.69986950
##  [3,] -1.3862944 -1.35156281
##  [4,] -1.0986123 -1.06388074
##  [5,] -0.8472979 -0.81256631
##  [6,] -0.6190392 -0.58430766
##  [7,] -0.4054651 -0.37073356
##  [8,] -0.2006707 -0.16593914
##  [9,]  0.0000000  0.03473155
## [10,]  0.2006707  0.23540225
## [11,]  0.4054651  0.44019666
## [12,]  0.6190392  0.65377076
## [13,]  0.8472979  0.88202941
## [14,]  1.0986123  1.13334384
## [15,]  1.3862944  1.42102591
## [16,]  1.7346011  1.76933261
## [17,]  2.1972246  2.23195613
```

Recall that these are in the residual space, not the ratio space. In the ninth row, if the original founding ratio is 50% (that half of the non-penetrative sexual assault cases were founded compared to simple assault cases), then we would be able to statistically detect if the ratio residuals increased from 0 to 0.035 in the ARIMA residuals. In the last row, if the original founding ratio is 90% (nearly the same ratio of founded non-penetrative sexual assault cases to simple assault cases), then we would be able to statistically detect if the ratio residuals increased from 2.20 to 2.23.

Detectable Differences

If these simple assault vs non-penetrative sexual assault founding ratios ranged from 10% to 90%, then would be powered to detect the following differences:

```
# Transform to Ratios
round(
  diff(t(InvLogit(logits1_mat))) * 100,
  digits = 2
)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13] [,14]
## [1,] 0.32 0.45 0.56 0.66 0.73 0.79 0.84 0.86 0.87  0.86  0.83  0.79  0.72  0.65
##      [,15] [,16] [,17]
## [1,]  0.55  0.44  0.31
```

The units here are ratio percentage points. They are **all below 1**, which means that no matter what the true ratio is, we would be statistically powered to detect even a 1 point difference in the ratios (i.e. an increase in founding ratios from 80% to 81% after the law went into effect, after temporal effects are removed).

Ratio of Aggravated Assault to Penetrative Sexual Assault

I'll repeat the same calculations as above, skipping the middle points.

Here's the power analysis.

```
t.test(Residual ~ Group, data = aggAssault_df)
```

```
##
##  Welch Two Sample t-test
##
## data:  Residual by Group
## t = -0.4357, df = 68.289, p-value = 0.6644
## alternative hypothesis: true difference in means between group 0 and group 1 is not equal to
0
## 95 percent confidence interval:
##  -0.03366056  0.02159500
## sample estimates:
## mean in group 0 mean in group 1
##    -0.002248447     0.003784331
```

```
DetectableDelta(aggAssault_df)
```

```
## # A tibble: 1 × 5
##       n  delta     sd sig.level power
##   <dbl>  <dbl>  <dbl>     <dbl> <dbl>
## 1    51 0.0379 0.0676      0.05   0.8
```

It shows that we'd be able to detect a 0.038 difference in ratio residual space.

```
logits2_mat <- matrix(
  c(
    residualThresholds_num,
    residualThresholds_num + DetectableDelta(aggAssault_df)$delta
  ),
  nrow = length(residualThresholds_num),
  ncol = 2
)

round(
  diff(t(InvLogit(logits2_mat))) * 100,
  digits = 2
)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13] [,14]
## [1,] 0.35 0.49 0.61 0.72  0.8 0.87 0.91 0.94 0.95  0.94  0.91  0.86  0.79   0.7
##      [,15] [,16] [,17]
## [1,]   0.6  0.48  0.34
```

As we can see here, this 0.038 ratio residual change again corresponds to the ability to detect smaller than 1 percentage point changes in the original ratio.

Ratio of Assault to Sexual Assault

Here's the power analysis.

```
t.test(Residual ~ Group, data = totalAssault_df)
```

```
##
##  Welch Two Sample t-test
##
## data:  Residual by Group
## t = -0.23125, df = 89.137, p-value = 0.8176
## alternative hypothesis: true difference in means between group 0 and group 1 is not equal to
0
## 95 percent confidence interval:
##  -0.01982530  0.01569159
## sample estimates:
## mean in group 0 mean in group 1
##    7.475187e-05    2.141611e-03
```

```
DetectableDelta(totalAssault_df)
```

```
## # A tibble: 1 × 5
##       n  delta     sd sig.level power
##   <dbl>  <dbl>  <dbl>     <dbl> <dbl>
## 1    51 0.0249 0.0445      0.05   0.8
```

It shows that we'd be able to detect a 0.025 difference in ratio residual space.

```
logits3_mat <- matrix(
  c(
    residualThresholds_num,
    residualThresholds_num + DetectableDelta(totalAssault_df)$delta
  ),
  nrow = length(residualThresholds_num),
  ncol = 2
)

round(
  diff(t(InvLogit(logits3_mat))) * 100,
  digits = 2
)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13] [,14]
## [1,] 0.23 0.32  0.4 0.47 0.53 0.57  0.6 0.62 0.62  0.62   0.6  0.57  0.52  0.46
##      [,15] [,16] [,17]
## [1,]   0.4  0.32  0.22
```

As we can see here, this 0.025 ratio residual change again corresponds to the ability to detect smaller than 1 percentage point changes in the original ratio.

Summary

Our methods were well powered to detect even a single percentage point change in the case founding ratios (after autocorrelative effects are removed).

```
sessionInfo()
```

```
## R version 4.2.0 (2022-04-22)
## Platform: x86_64-apple-darwin17.0 (64-bit)
## Running under: macOS Big Sur/Monterey 10.16
##
## Matrix products: default
## BLAS:   /Library/Frameworks/R.framework/Versions/4.2/Resources/lib/libRblas.0.dylib
## LAPACK: /Library/Frameworks/R.framework/Versions/4.2/Resources/lib/libRlapack.dylib
##
## locale:
## [1] en_US.UTF-8/en_US.UTF-8/en_US.UTF-8/C/en_US.UTF-8/en_US.UTF-8
##
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
##
## other attached packages:
## [1] forcats_0.5.1   stringr_1.4.0   dplyr_1.0.9     purrr_0.3.4
## [5] readr_2.1.2     tidyr_1.2.0     tibble_3.1.7    ggplot2_3.3.6
## [9] tidyverse_1.3.1
##
## loaded via a namespace (and not attached):
##  [1] lubridate_1.8.0  assertthat_0.2.1 digest_0.6.29    utf8_1.2.2
##  [5] R6_2.5.1         cellranger_1.1.0 backports_1.4.1  reprex_2.0.1
##  [9] evaluate_0.15    httr_1.4.3       highr_0.9        pillar_1.7.0
## [13] rlang_1.0.2      readxl_1.4.0     rstudioapi_0.13  jquerylib_0.1.4
## [17] rmarkdown_2.14   labeling_0.4.2   bit_4.0.4        munsell_0.5.0
## [21] broom_0.8.0      compiler_4.2.0   modelr_0.1.8     xfun_0.31
## [25] pkgconfig_2.0.3  htmltools_0.5.2  tidyselect_1.1.2 fansi_1.0.3
## [29] crayon_1.5.1     tzdb_0.3.0       dbplyr_2.1.1     withr_2.5.0
## [33] grid_4.2.0       jsonlite_1.8.0   gtable_0.3.0     lifecycle_1.0.1
## [37] DBI_1.1.2        magrittr_2.0.3   scales_1.2.0     cli_3.3.0
## [41] stringi_1.7.6    vroom_1.5.7      farver_2.1.0     fs_1.5.2
## [45] xml2_1.3.3       bslib_0.3.1      ellipsis_0.3.2   generics_0.1.2
## [49] vctrs_0.4.1      tools_4.2.0      bit64_4.0.5      glue_1.6.2
## [53] hms_1.1.1        parallel_4.2.0   fastmap_1.1.0    yaml_2.3.5
## [57] colorspace_2.0-3 rvest_1.0.2      knitr_1.39       haven_2.5.0
## [61] sass_0.4.1
```