

# A Latent Markov Model for the Analysis of Longitudinal Data collected in Continuous Time: States, Durations, and Transitions

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## Supplemental Material for Psychological Methods

### Website

### SAS Proc IML Program for CT-LM Model

The following Proc IML code (SAS Institute, 2004) defines the log-likelihood function for the CT-LM model presented in Application I. The log-likelihood function can be maximized with any non-linear SAS optimization routines. For example, “NLPQN”-subroutine is well-suited for estimating the parameters of the CT-LM model.

```
start LM(x); dt= {1 1 1 1 487,  
 1 1 1 2 35,  
 1 1 2 1 27,  
 1 1 2 2 6,  
 1 2 1 1 39,  
 1 2 1 2 11,  
 1 2 2 1 9,  
 1 2 2 2 7,
```

```

2 1 1 1 50,
2 1 1 2 11,
2 1 2 1 9,
2 1 2 2 9,
2 2 1 1 16,
2 2 1 2 9,
2 2 2 1 8,
2 2 2 2 19};
sb=dt[,1:4]; /* data matrix */
nt=ncol(sb); rn=nrow(sb); /* number of columns and rows of data matrix */
nn=dt[,5]; /* frequency vector of binary observations*/
d={1,1,1,1}; /* time differences between observation; here
               the observations are spaced equally in time*/
pnn=J(rn,1,0); /* initialize probability vector*/
cl=2; /* number of states*/

/* response probabilities of states */ ps=exp(J(cl,1,0)||x[{1
2}]); ps=ps/(ps[,+] # J(cl,cl,1));

/* initial state-specific probabilities */ w=exp(0//x[5]);
w=w/w[+];

/* transition rate matrix */ y=exp(x[{3 4}]);
q=(-y[1]||y[1])/(y[2]||-y[2]);

/*eigenvalue and eigenvector decomposition */ call eigen(ea,ev,q);

/* compute probability of observations under the CT-LM model*/ DO
j=1 to rn; ss=ps[,sb[j,1]] # I(cl); DO i=2 TO nt; eea=exp(ea #
d[i-1]) # I(cl); p = ev * eea * inv(ev); ss = ss * p *
(ps[,sb[j,i]] # I(cl)); end; pnn[j,1]=w' * ss * J(cl,1,1); end;

/* log-likelihood function to be maximized */ f=nn # log(pnn);
f=f[+]; return(f); finish LM;

```