

Supplemental Materials

Table A: Wording of the instruction in the two conditions as presented verbally in study 1

Probability framing	Proportion framing
<i>Introduction of the visualization:</i> Now put yourself in the role of a teacher. You have students in front of you who still make mistakes when comparing decimal numbers, but you don't know whether they use the SL or WN strategy. You have no other information about individual students and test them with various tasks. The blue, larger part of the bar shows the probability that a student has the 'shorter is larger' misconception – SL. The upper, green part of the bar shows the probability that a student has the 'whole number' misconception – WN. In this case, the probability for a student of having SL is rather large and the probability for a student of having WN is rather low. This can be seen from the size of the bars. If the students are from a different class, the size of the bars may also be different. In higher grades, after the introduction of decimal numbers, the proportion of students SL is greater than in lower grades. In lower grades, on the other hand, a larger proportion has WN. Now a student is given a comparison task, which he or she solves incorrectly. For example, the student says that 1.5 is smaller than 1.18. Is this a student with WN? Or could it also be a student with SL happens to make this mistake? The probability that a student has SL is high (see the blue bar again). A student with SL has a small probability of giving the wrong answer and saying that 1.5 is smaller than 1.18. This can be seen in the grey bar next to it. The probability that a student has WN is rather small (green bar). A student with WN has a large probability of giving the wrong answer and saying that 1.5 is smaller than 1.18. This can be seen in the grey bar next to it. The question now is how large the probability is that a student has WN when the answer is incorrect, and how large the probability is that a	<i>Introduction of the visualization:</i> Now put yourself in the role of a teacher. You have students in front of you who still make mistakes when comparing decimal numbers, but you don't know whether they use the SL or WN strategy. You have no other information about individual students and test them with various tasks. The blue, larger part of the bar shows the proportion of students who have the 'shorter is larger' misconception – SL. The upper, green part of the bar shows the proportion of students with 'whole number' misconception – WN. In this case, the proportion of students with SL is rather large and the proportion of students with WN is rather small. This can be seen from the size of the bars. If the students are from a different class, the size of the bars may also be different. In higher grades, after the introduction of decimal numbers, the proportion of students SL is greater than in lower grades. In lower grades, on the other hand, a larger proportion has WN. Now a student is given a comparison task, which he or she solves incorrectly. For example, the student says that 1.5 is smaller than 1.18. Is this a student with WN? Or could it also be a student with SL happens to make this mistake? The proportion of students with SL is high (see the blue bar again). Of these students with SL, a small proportion give the wrong answer and say that 1.5 is smaller than 1.18. This can be seen in the grey bar next to it. The proportion of students with WN is rather small (green bar). Of these students with the KT, a large proportion give the wrong answer and say that 1.5 is smaller than 1.18. This can be seen again in the grey bar next to it. The question now is how large the proportion of students with WN is when the answer is incorrect, and how large the proportion of students with SL

student has SL when the answer is incorrect. The dashed lines in the middle can be moved and are intended to aid your thinking. You should adjust them so that your reasoning and way of thinking are supported by the position of the lines. It should be clear how you combine the proportions of the blue and grey bars. After setting these dashed lines, set the bar on the far right to show the probability of a student for having SL or WN when the answer is incorrect <i>For each case:</i> You know how confident the students are in comparing decimals in this grade: A student's probability of having the SL misconception is high - large blue bar. A student's probability of having the WN misconception is small - small green bar. Students are asked to solve the task $4.8 > 4.63$. You do not have to memorize the task, it is only a matter of the probabilities of a student to make an error. The probability of a student making an error if he or she has the SL misconception is very small - small gray bar. The probability of a student making an error if he or she has the WN misconception is rather large - larger gray bar. Now decide, how large is the probability, that a student has the WN misconception and how large is the probability that a student has the SL misconception in case of a wrong answer. Please, first adjust the dashed lines to show how you combine the probabilities displayed with the blue and gray and with the green and gray bars. Only after you have set the dashed lines, set the probabilities for WN and SL for a student with an error at the last bar.	is when the answer is incorrect. The dashed lines in the middle can be moved and are intended to aid your thinking. You should adjust them so that your reasoning and way of thinking are supported by the position of the lines. It should be clear how you combine the proportions of the blue and grey bars. After setting these dashed lines, set the bar on the far right to show the proportion of students with SL or WN of those with an incorrect answer. <i>For each case:</i> You know how confident the students are in comparing decimals in this grade: The proportion of students with the SL misconception is large - large blue bar. The proportion of students with the WN misconception is small - small green bar. Students are asked to solve the task $4.8 > 4.63$. You do not have to memorize the task, it is only a matter of the proportions of students with an error. The proportion of students with an error of students with the SL misconception is very small - small gray bar. The proportion of students with an error of students with the WN misconception is rather large - larger gray bar. Now decide, how large is the proportion of students with the WN misconception and how large is the proportion of students with the SL misconception among students with errors. Please, first adjust the dashed lines to show how you combine the proportions displayed with the blue and gray and with the green and gray bars. Only after you have set the dashed lines, set the proportions of WN and SL for students with errors at the last bar.
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The wording is translated from German. Differences across conditions are marked in bold. The wording of all eight cases was exactly the same - only the values for the prior and conditional probabilities respectively proportions differed (very small, small, rather small, rather large, large, very large, see below). SL and WN refer to the to-be-diagnosed misconceptions (SL: shorter-is-larger misconception; WN: whole-number misconception).

	$p(SL)$	$p(error SL)$	$p(error WN)$
Case 1	0.80 “large”	0.05 “very small”	0.70 “rather large”
Case 2	0.80 “large”	0.95 “very large”	0.95 “very large”
Case 3	0.80 “large”	0.05 “very small”	0.45 “medium small”
Case 4	0.80 “large”	0.95 “very large”	0.70 “rather large”
Case 5	0.20 “small”	0.70 “rather large”	0.70 “rather large”
Case 6	0.20 “small”	0.70 “rather large”	0.95 “very large”
Case 7	0.20 “small”	0.45 “medium small”	0.05 “very small”
Case 8	0.20 “small”	0.70 “rather large”	0.05 “very small”

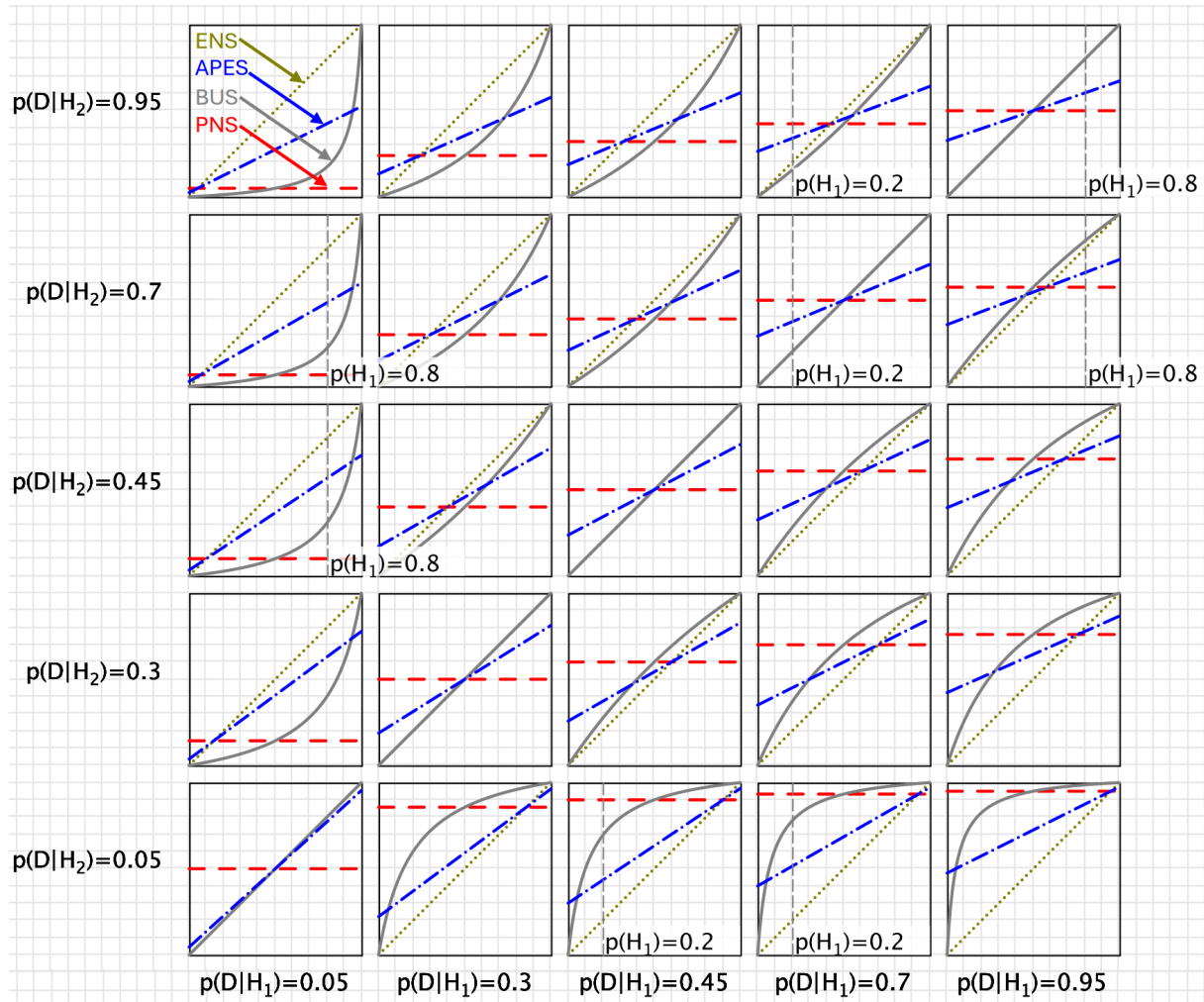


Fig. A. Construction of cases (prior and likelihood values) for the classification of reasoning strategies. Each square represents cases with the two conditional values $p(D|H_1)$ and $p(D|H_2)$. Within each square, the four graphs show the exact posterior estimation $p(H_1|D)$ (y-axis) depending on the prior value $p(H_1)$ (x-axis) for four strategies: (1) prior neglect (PNS, dashed line), (2) evidence neglect (ENS, dotted line), (3) correct Bayesian updating (BUS, solid line), and (4) averaging priors and evidence (APES, dash-dotted line). For example, for a case with conditional values $p(D|H_1) = 0.05$ and $p(D|H_2) = 0.5$ (row 2, column 1) and the prior value $p(H_1) = 0.80$, the posterior estimates are (approximately) $p(H_1|D) = 0.20$ for the prior-neglect strategy (PNS), 0.30 for the correct Bayesian update strategy (BUS), 0.50 for the averaging strategy (APES), and 0.80 for the evidence-neglect strategy (ENS). Thus, this case discriminates well between all four strategies.

Table B: Wording of the instruction in the two conditions as presented verbally in study 2 and study 3

Probability framing	Proportion framing
It is winter. You know how large the probability is that a person has or has not the flu: The probability that a person has the flu is small - small blue bar. The probability that a person does not have the flu is high - large green bar. You take a flu test. The probability for a person to have a positive test result in a person with flu is rather high - large grey bar. The probability of a person to have a positive test result in a person without flu is very low - small grey bar. Now decide, how large is the probability of a person to have the flu with a positive test result. And how large is the probability of a person to not have the flu with a positive test result. To do this, first adjust the dashed lines above the central bars so that your reasoning and way of thinking is supported by the position of the dashed lines. The position of the dashed lines should show how you combine the probabilities displayed by the blue and grey bar, as well as the green and grey bar. Finally, set the probabilities of a person with the flu or without the flu for a person with a positive test result on the last bar.	It is winter. You know how large the proportion of persons is who have or do not have the flu: The proportion of persons with flu is small - small blue bar. The proportion of persons without flu is large - large green bar. You take flu tests. The proportion of persons with a positive test result out of the persons with flu is rather large - large grey bar. The proportion of persons with a positive test result out of the persons without flu is very small - small grey bar. Now decide, how large is the proportion of persons with the flu out of the persons with a positive test result. And how large is the proportion of persons without the flu out of the persons with a positive test result. To do this, first adjust the dashed lines above the central bars so that your reasoning and way of thinking is supported by the position of the dashed lines. The position of the dashed lines should show how you combine the proportions displayed by the blue and grey bar, as well as well as the green and grey bar. Finally, set the proportions of persons with flu or without flu out of the persons with a positive test result on the last bar.

	$p(\text{flu})$	$p(\text{positive} \text{flu})$	$p(\text{positive} \text{not flu})$
Case 1	0.20 “small”	0.70 “rather large”	0.05 “very small”
Case 2	0.20 “small”	0.95 “very large”	0.05 “very small”
Case 3	0.20 “small”	0.80 “large”	0.05 “very small”
Case 4	0.05 “very small”	0.80 “large”	0.35 “rather small”
Case 5	0.05 “very small”	0.70 “rather large”	0.35 “rather small”
Case 6	0.05 “very small”	0.95 “very large”	0.35 “rather small”