Supplementary Materials for manuscript 'Dividing up the pie: attention allocation across dual-tasks'

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Supplementary Materials for manuscript 'Dividing up the pie: attention allocation across dual-tasks'

The Supplementary Materials are organised into the below sections in order of their appearance in the main manuscript. For ease of comparison, we point to specific tables and figures for each item in the Supplementary.

1. MOT task data as function of sequential interrogation
2. Baseline Wald model that estimates boundary
    - Table 1 shows list of parameter values
    - Figure 2 compares parameter estimates against other model versions.
    - Figure 6 & 7 shows posterior predictives data for the MOT and DRT task, respectively.
3. Comparison between linearly parameterised and freely estimated models
    - Figure 2 compares parameter estimates against linear version.
    - Table 2 shows list of parameter values for fully free model.
    - Figure 3 shows the correlation matrix for all parameters in the fully free model (companion to Figure 6 in the main manuscript.
    - Figure 4 & 5 shows posterior predictive data for the MOT and DRT task, respectively.
4. Sensitivity analysis of varying capacity (G) and conversion (k) parameters
    - Figure 8 shows predicted conversion (k) and allocation (π) for varying levels of resource capacity (G).
5. Pooled resources model estimates of capacity (G)
    - Figure 9 shows estimated parameters for each emphasis condition in Exp. 2.

- Figure 10 shows posterior predictive data for the MOT and DRT task from the pooled resources model that additionally estimates capacity (G).
- Table 3 shows table of parameter values.

6. Estimates of extra-capacity (G) in the DRT-emphasis condition

- Posterior predictives for MOT-task data with additional capacity ($G = 10 : 14$), Figure 11
- Posterior predictives for DRT-task data with additional capacity values ($G = 10 : 14$), Figure 12

1. MOT task data as function of sequential interrogation

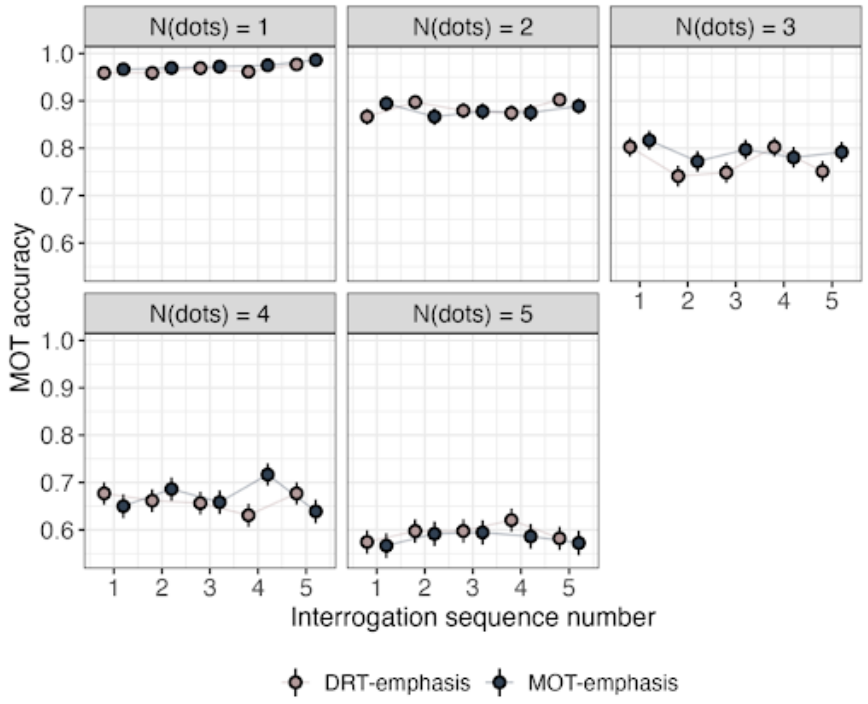

Figure 1

MOT task data as a function of interrogation sequence position. Accuracy for each emphasis condition is presented in separate colours and panels show data for each level of MOT-task difficulty. This analysis rules out the hypothesis that workload decreases as a function of sequence position (x-axis). The similar accuracy proportions from the first to last positions suggests that workload is not shed as each dot is interrogated.

2. Comparison between models

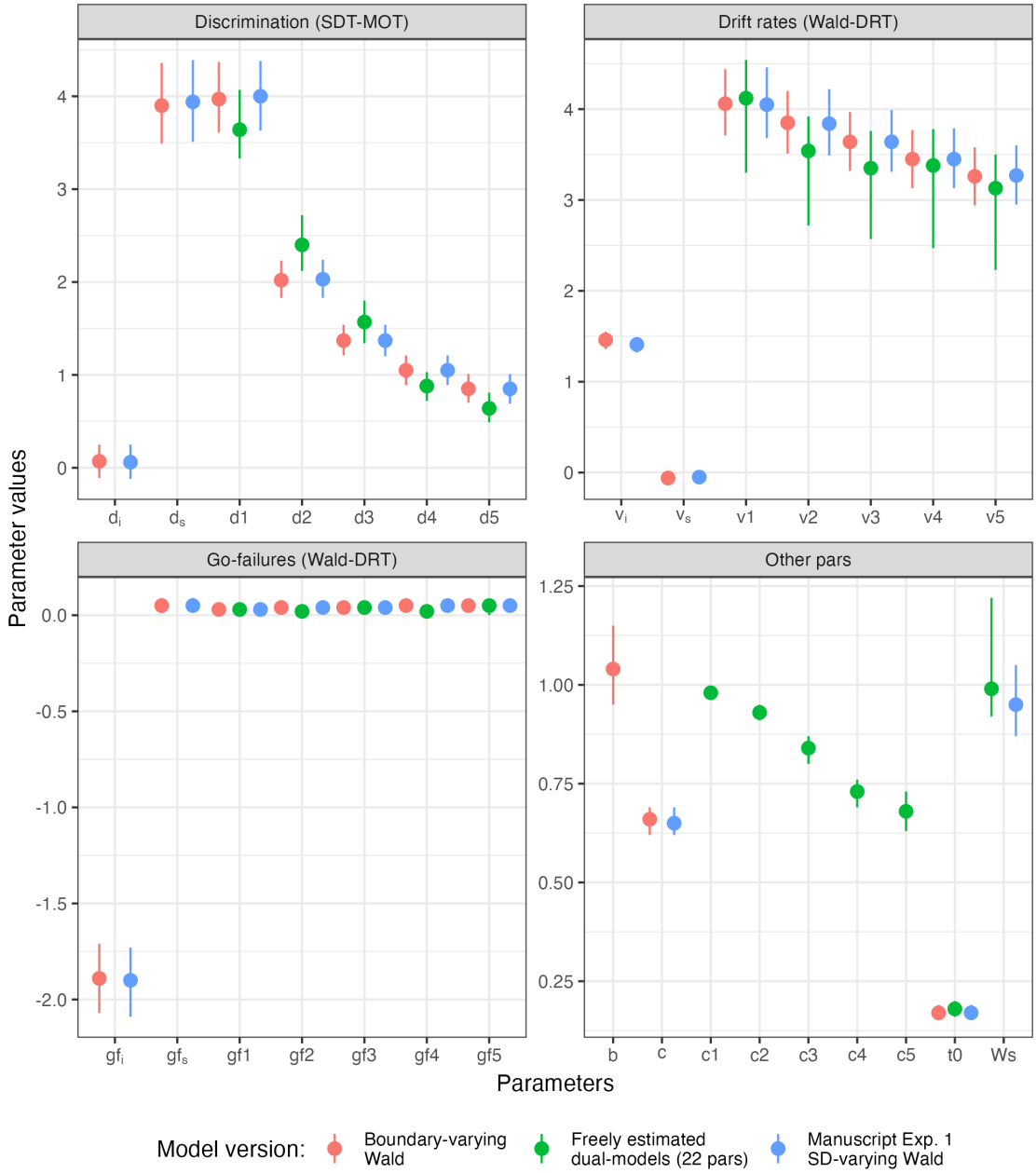

Figure 2

Estimated model parameters for three versions of the joint SDT-Wald model fitted to data from Experiment 1. Median parameter values from the posterior distribution over 5000 samples are shown with errorbars showing 95% HDI's. Across panels, parameters are grouped for interpretability. Within each panel, model version is separated by colour. Blue points represents the fixed drift rate variability Wald model ($W.s = 1$) that is used in the main manuscript. Red points show an alternate version where decision boundary is fixed ($W.b = 1$) where the general trends across parameters are similar. Both versions are linearly parameterised across stimulus difficulty levels (1:5). The last model shown in green points is the fully freely estimated joint SDT-Wald model with 22 parameters per subject.

3. Comparison between linearly parameterised & freely estimated models

Table 1

Estimated parameter values for the linearly parameterised Signal Detection Theory model and Wald model estimating boundary and fixing SD of the drift rate to 1. Values are generated from fits to data from Experiment 1. Medians and 95% HDI's are presented. Note that the 9 parameters/subject that are directly estimated by the sampler are shown in plain-text. Italicised values are extrapolated from the estimated linear components.

Parameter	2.5%	50%	97.5%	Model	Task
Boundary (b)	0.95	1.04	1.15	Wald	DRT
Non-decision time ($t0$)	0.16	0.17	0.19	"	"
Drift rate intercept (v_i)	1.36	1.46	1.55	"	"
Drift rate slope (v_s)	-0.07	-0.06	-0.04	"	"
Go-failure intercept (gf_i)	-2.07	-1.89	-1.71	"	"
Go-failure slope (gf_s)	0.01	0.05	0.09	"	"
Criterion (c)	0.62	0.66	0.69	SDT	MOT
Discrimination intercept (d_i)	-0.11	0.07	0.25	"	"
Discrimination slope (d_s)	3.49	3.9	4.36	"	"
Drift rate 1-dot	3.71	4.06	4.44	Wald	DRT
Drift rate 2-dot	3.51	3.85	4.2	"	"
Drift rate 3-dot	3.32	3.64	3.97	"	"
Drift rate 4-dot	3.13	3.45	3.77	"	"
Drift rate 5-dot	2.94	3.26	3.58	"	"
Go-failure 1-dot	0.02	0.03	0.05	"	"
Go-failure 2-dot	0.03	0.04	0.05	"	"
Go-failure 3-dot	0.03	0.04	0.06	"	"
Go-failure 4-dot	0.03	0.05	0.07	"	"
Go-failure 5-dot	0.03	0.05	0.07	"	"
Discrimination 1-dot	3.61	3.97	4.37	SDT	MOT
Discrimination 2-dot	1.83	2.02	2.23	"	"
Discrimination 3-dot	1.21	1.37	1.54	"	"
Discrimination 4-dot	0.89	1.05	1.21	"	"
Discrimination 5-dot	0.7	0.85	1.01	"	"

Table 2

Estimated parameter values for the freely estimated Signal Detection Theory and Wald models. Values are generated from fits to data from Experiment 1. Medians and 95% HDI's are presented. Note that 22 parameters/subject are estimated where in the linearly parameterised versions, e.g., boundary-estimating Wald in the Table 1 below, only 9 parameters are estimated.

Parameter	2.5%	50%	97.5%	Model	Task
Wald SD of Drift rate ($W.s$)	0.92	0.99	1.22	Wald	DRT
Non-decision time ($t0$)	0.16	0.18	0.2	"	DRT
Drift rate 1-dot	3.3	4.12	4.54	"	DRT
Drift rate 2-dot	2.72	3.54	3.92	"	DRT
Drift rate 3-dot	2.57	3.35	3.76	"	DRT
Drift rate 4-dot	2.47	3.38	3.78	"	DRT
Drift rate 5-dot	2.23	3.13	3.5	"	DRT
Go-failure 1-dot	0.01	0.03	0.05	"	DRT
Go-failure 2-dot	0	0.02	0.04	"	DRT
Go-failure 3-dot	0.01	0.04	0.06	"	DRT
Go-failure 4-dot	0	0.02	0.04	"	DRT
Go-failure 5-dot	0	0.05	0.08	"	DRT
Criterion 1-dot	0.97	0.98	0.99	SDT	MOT
Criterion 2-dot	0.91	0.93	0.95	"	MOT
Criterion 3-dot	0.8	0.84	0.87	"	MOT
Criterion 4-dot	0.69	0.73	0.76	"	MOT
Criterion 5-dot	0.63	0.68	0.73	"	MOT
Discrimination 1-dot	3.33	3.64	4.07	"	MOT
Discrimination 2-dot	2.12	2.4	2.72	"	MOT
Discrimination 3-dot	1.34	1.57	1.8	"	MOT
Discrimination 4-dot	0.72	0.88	1.03	"	MOT
Discrimination 5-dot	0.49	0.64	0.81	"	MOT

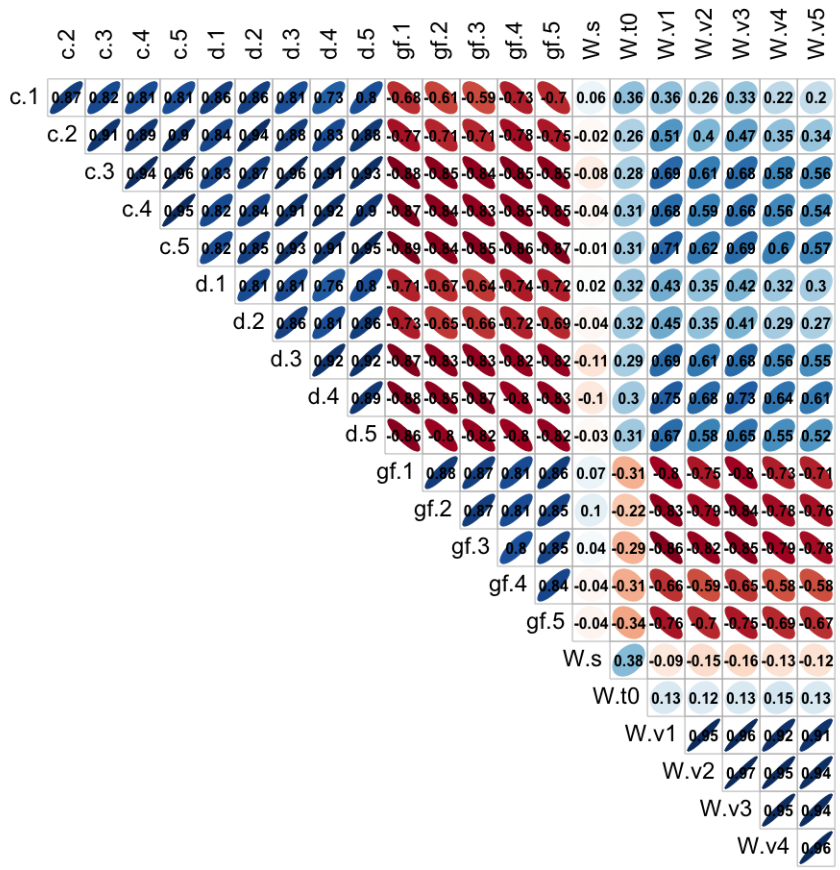

Figure 3

Posterior means of estimated correlations between parameters in the fully-free model fit to Experiment 1 data. This figure is the companion to Figure 6 in the main manuscript. Correlation values are calculated across parameters for both tasks from the posterior distribution. Colour and width of ellipses indicate the strength of correlation where red-leftward tilted ellipses are negatively correlated and blue-rightward tilted ellipses are positively correlated.

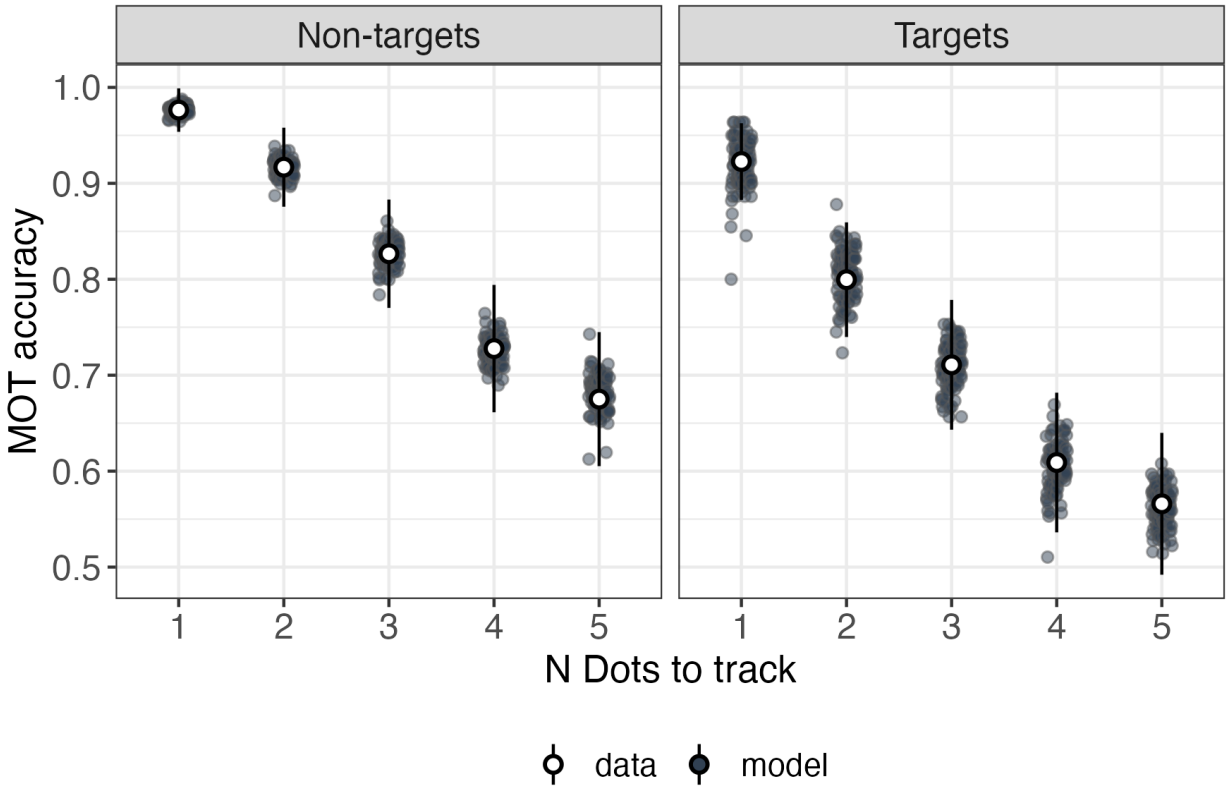

Figure 4

Experiment 1 data for Multiple Object-Tracking task (MOT) with posterior predictives generated from the fully-freely estimated SDT-Wald model. Left-panel displays mean accuracy for non-target dots as a function of MOT task difficulty (dots to track). Right panel displays mean accuracy for target dots. Black solid points show means and standard errors and transparent points show mean posterior predictive data for 100 simulations of the joint-model.

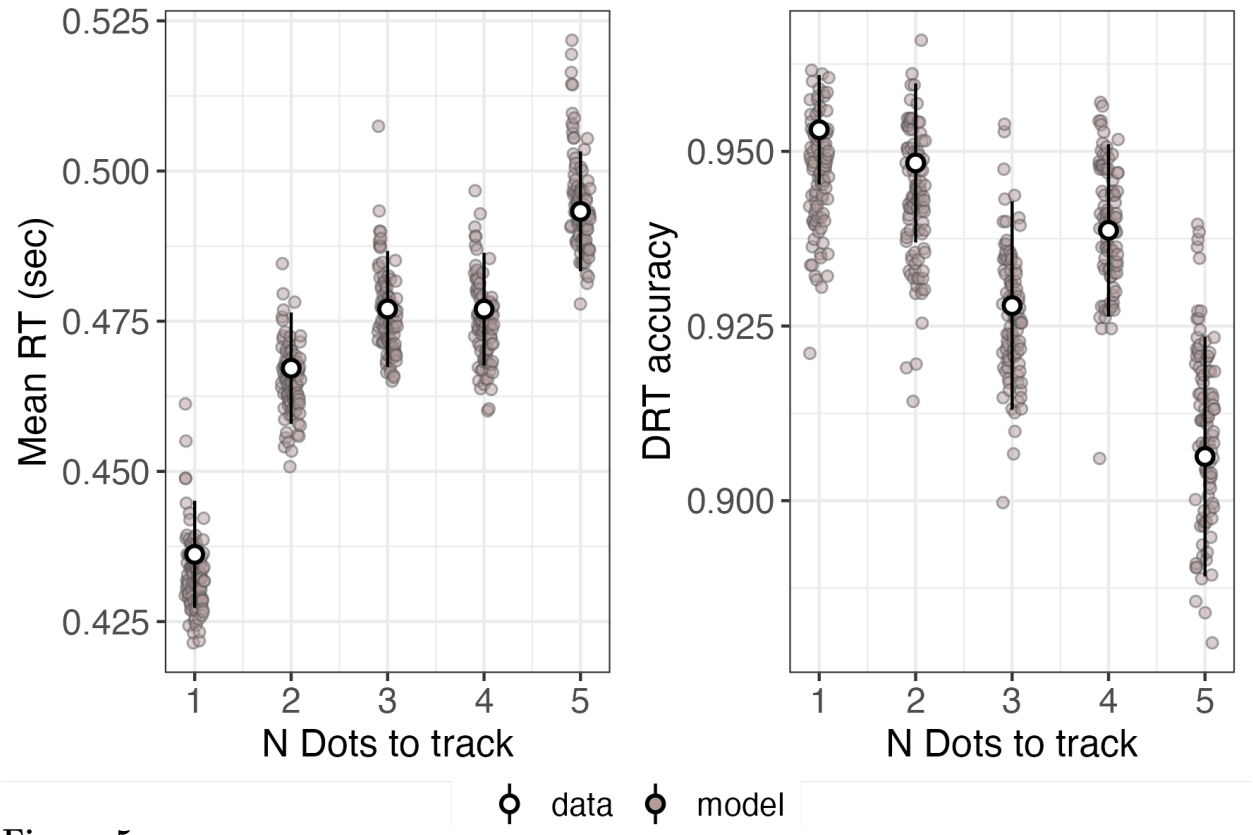

Figure 5

Experiment 1 data for Detection Response Task (DRT) with posterior predictives generated from the from the fully-freely estimated SDT-Wald model. Left-panel displays mean response times as a function of MOT task difficulty (dots to track). Right panel displays mean proportion correct as a function of task difficulty. Black solid points show means and standard errors and transparent points show mean posterior predictive data for 100 simulations of the joint-model.

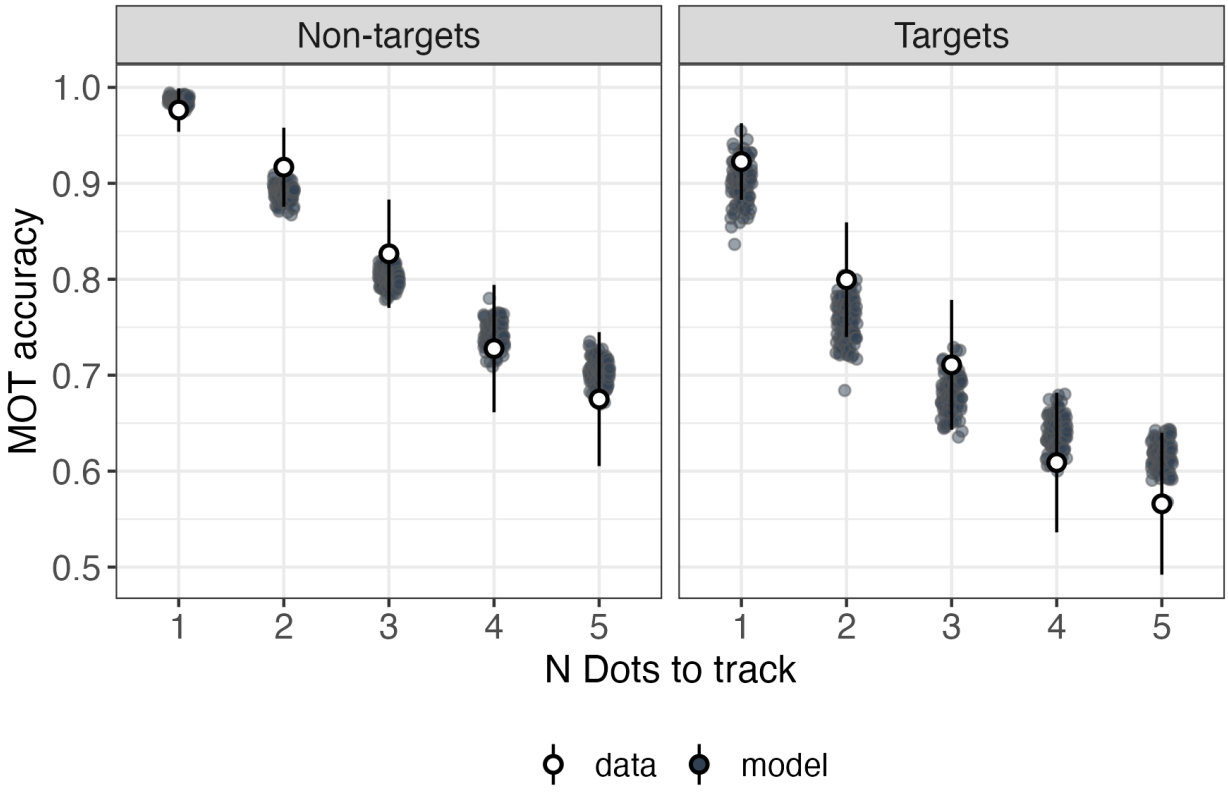

Figure 6

Experiment 1 data for Multiple Object-Tracking task (MOT) with posterior predictives generated from the boundary-estimating linearly parameterised model. Left-panel displays mean accuracy for non-target dots as a function of MOT task difficulty (dots to track). Right panel displays mean accuracy for target dots. Black solid points show means and standard errors and transparent points show mean posterior predictive data for 100 simulations of the joint-model.

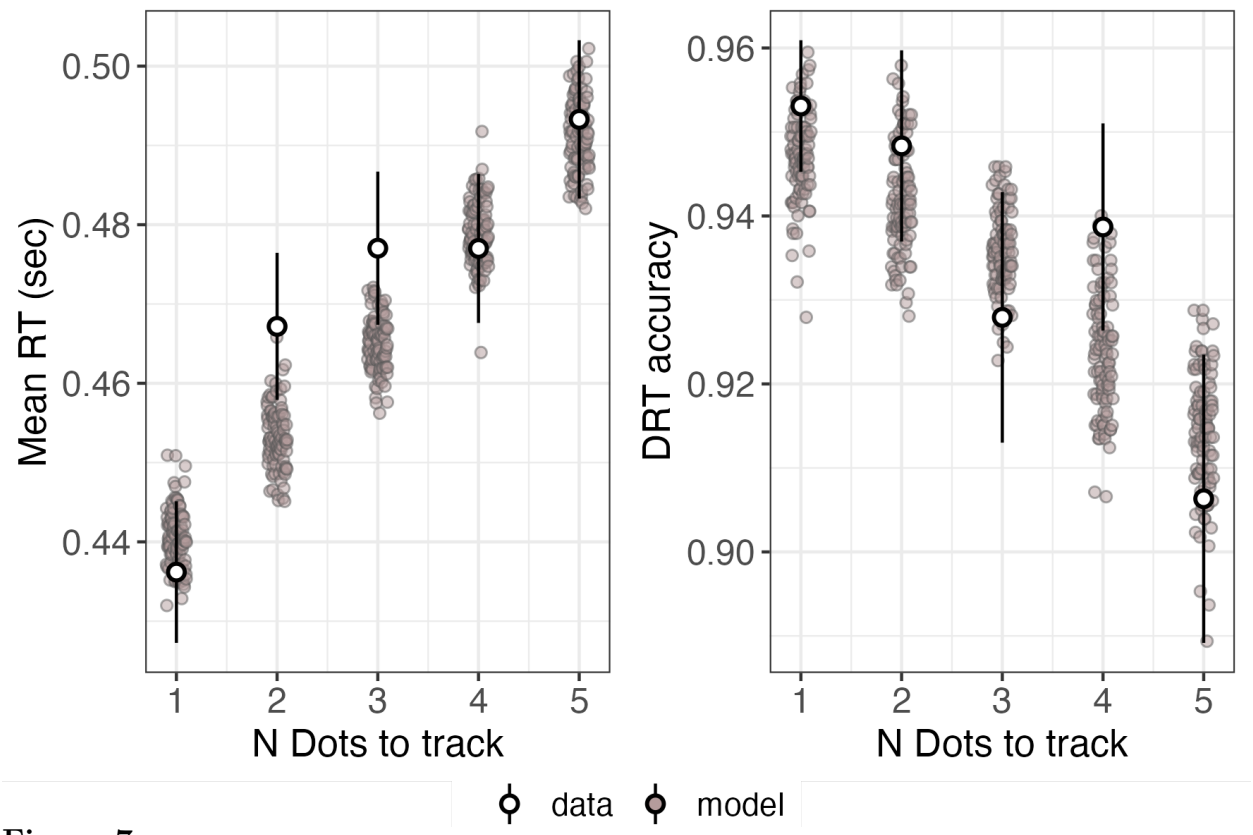

Figure 7

Experiment 1 data for Detection Response Task (DRT) with posterior predictives generated from the boundary-estimating linearly parameterised model. Left-panel displays mean response times as a function of MOT task difficulty (dots to track). Right panel displays mean proportion correct as a function of task difficulty. Black solid points show means and standard errors and transparent points show mean posterior predictive data for 100 simulations of the joint-model.

4. Sensitivity analysis of capacity (G) & conversion (k) parameters

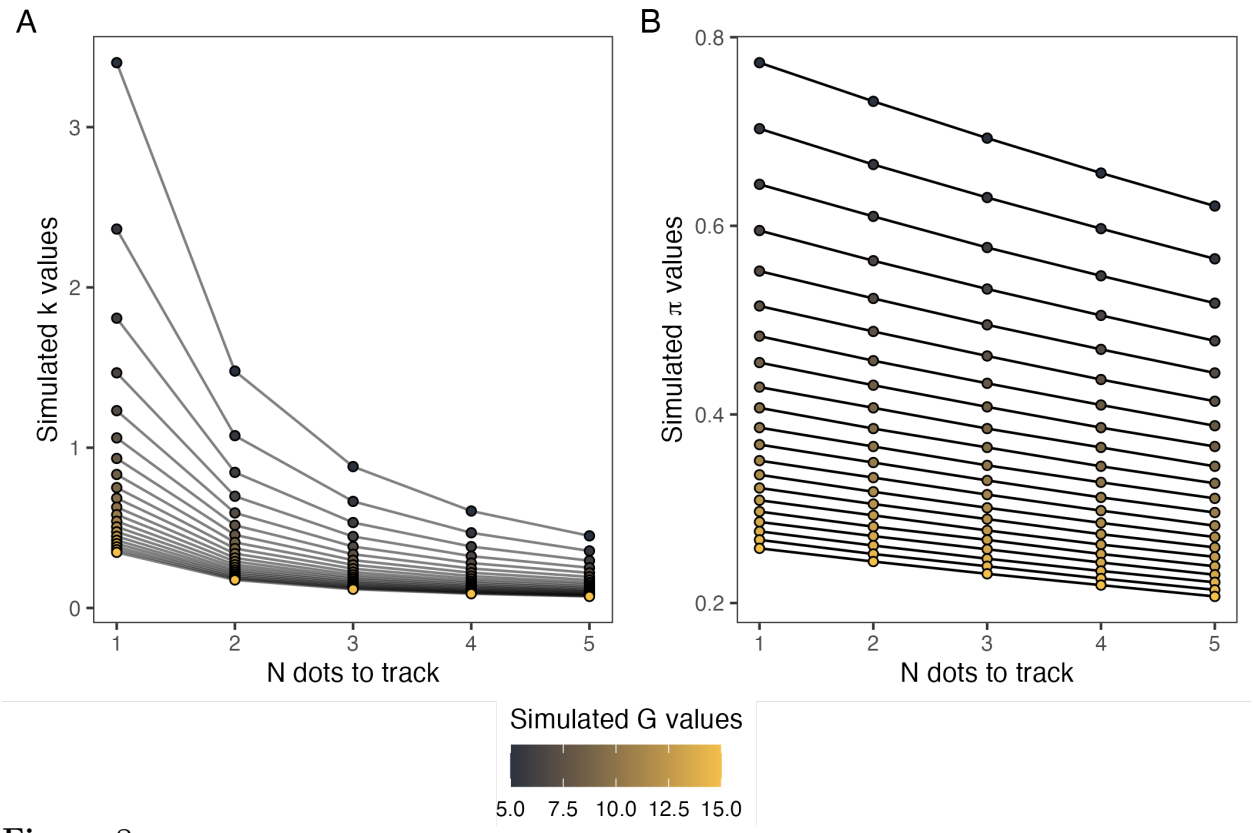

Figure 8

Predicted values of conversion (k) in panel A and allocation (π) in panel B for a given capacity (G) value based on parameter estimates in Experiment 1. The x-axis plots MOT task difficulty. Note that in the main manuscript, we fixed $G = 10$ and derived associated k values shown in panel A. Broadly, k values exponentially decreased as a function of MOT task difficulty. This reflects the pattern that a single unit of DRT-Wald-converted resources purchase less improvement in the MOT task at 5-dots difficulty than 1-dot difficulty. In panel B, π values below 0.5 indicate greater resource allocation to the MOT task and values above 0.5 indicate greater resource allocation to the DRT task. Across all simulated capacity values (G), the declining π values indicate greater allocation to the MOT task as the number of dots to track increases.

5. Pooled resources model estimates of capacity (G)

In this section, we detail the modeling analyses that make *estimates* total capacity G rather than *fixing* a capacity value ($G = 10$) as in the main manuscript.

To implement this model, we estimate an additional parameter G in addition to the 9 estimated parameters from the pooled resources model in the main manuscript. For ease, we list the parameters and their median posterior values in Table **??**, show posterior estimated values with 95% quantiles in Figure 9, and present posterior predictive data in Figure 10.

The main finding is that the DRT-emphasis condition is estimated to have greater overall capacity (G) compared to the MOT-emphasis condition (right-most panel, Figure 9). The median posterior values for overall capacity can be interpreted as relative values to capacity ($G = 10$) where other parameter values (k) were calibrated in Experiment 1. Therefore, the DRT-emphasis condition exhibits resource capacity approximately $\sim$ 10% greater than our fixed value and the MOT-emphasis condition exhibits capacity approximately $\sim$ 5% lower.

We interpret these results with caution for two main reasons. The first is experimental. We have no a priori reason to expect cognitive resource capacity to differ systematically between conditions with random allocation. Rather than that strong conclusion our intuition is that these estimates reflect strategies that mimic lower capacity (see GD section on satisficing explanation). The second reason is model-based. We suspect that this estimated capacity model (G) is unstable as other key parameters allocation (π) and scalar conversion (k) were calibrated from Experiment 1 where we assumed capacity was a fixed value ($G = 10$). There, we chose to fix capacity to identify other parameters of interest. Optimistically, this capacity-estimating model appears to converge with the predictions of the fixed model and so we chose to present this exercise together. However, removing the calibrated conversion (k) parameter would compromise the model's ability to estimate G without direct compensation. We leave it to future work to investigate further

the implications of estimating capacity limits in the pooled resources model.

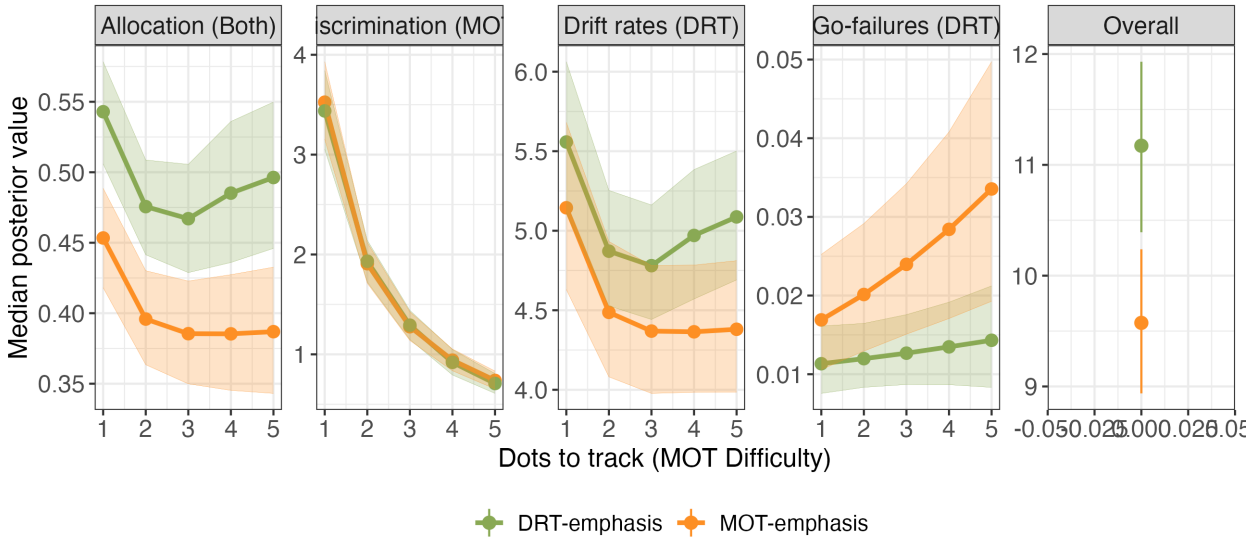

Figure 9

Median posterior paramter values with 95% quantile ribbons for the pooled resources model estimating capacity (G). Emphasis conditions are shown in separate colours with horizontal panels showing key parameters from both subordinate models as a function of MOT-task difficulty (x-axis). From left-right, the parameters are allocation π (both subordinate models), discrimination d' (MOT task, SDT model), drift rate v (DRT task, Wald model), go-failures gf (DRT task, Wald model), and Overall capacity G (both subordinate models). Right-most panel for capacity is estimated across all MOT-difficulty levels as a single value where dot represents median and error bars show 95% quantiles.

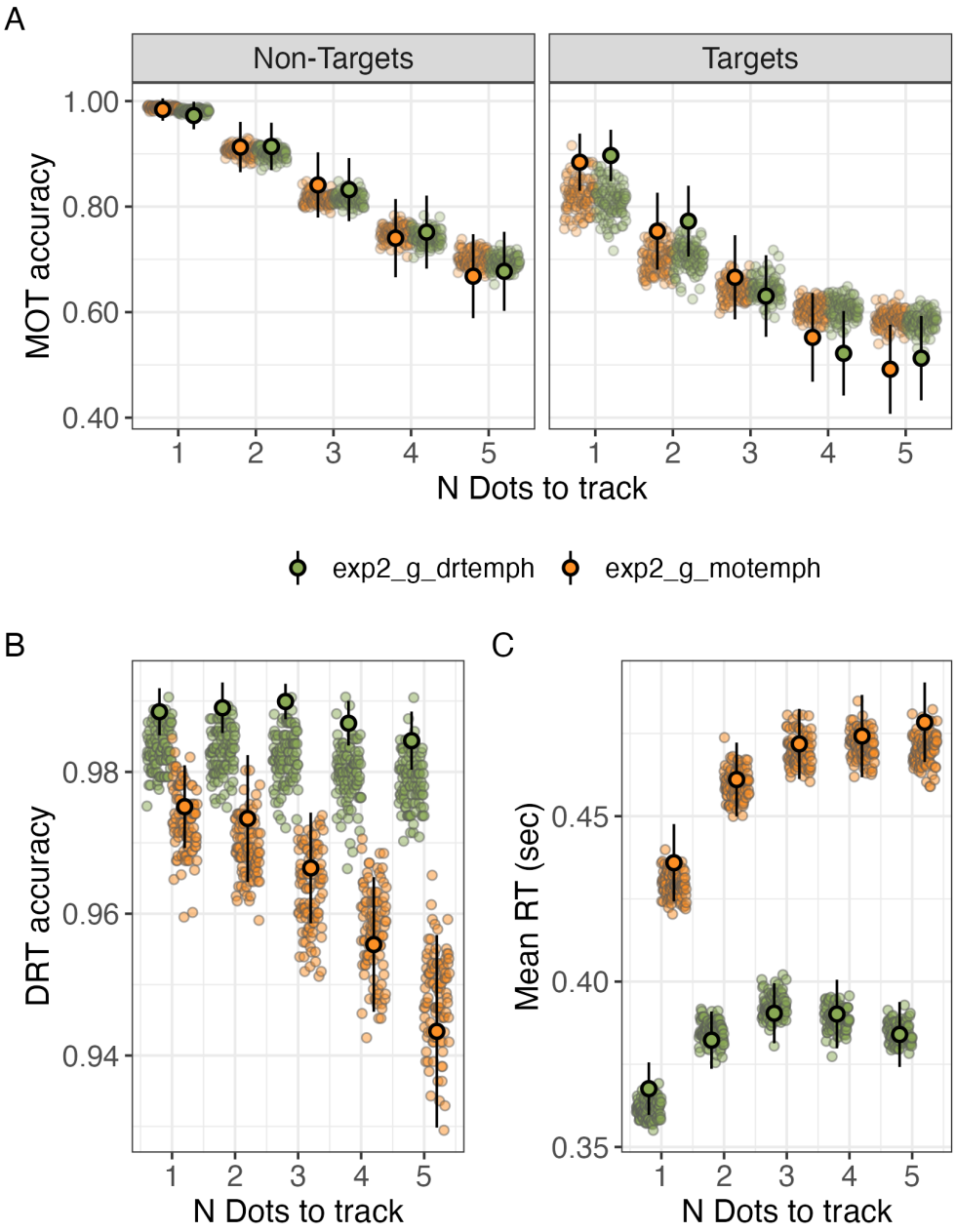

Figure 10

Posterior predictive data for the pooled resources model estimating capacity (G). Data for Experiment 2 from both the Multiple Object Tracking task in panel A and the Detection Response Task (DRT) in panels B & C. Posterior predictives are shown in transparent coloured dots generated from 100 simulations of the capacity-estimating pooled resources model. Colours correspond to task-emphasis conditions where green points show the DRT-emphasis condition, and orange points the MOT-emphasis condition. Black-rimmed dots show means and standard errors. Panel A shows MOT accuracy separated across panels for Targets and Non-target stimuli, as a function of MOT task difficulty. Panel B displays mean response times for the DRT as a function of MOT task difficulty. Right panel C displays mean proportion correct for the DRT.

Table 3

Median parameter values for pooled-resources model estimating capacity (G). Fitted values are from Experiment 2 data with task emphasis manipulations shown in separate columns. Italicised parameter values are extrapolated from their linear components and their allocated resources (π). Note that in this version of the model, we estimated total capacity (G) but retained the same conversion units (k) when total capacity is assumed to be 10. Therefore, the estimated capacity values are to be interpreted relative to this implicit benchmark value of G = 10.

Parameter	DRT-emphasis	MOT-emphasis	Model	Task
DRT-Allocation 1-dot	0.54	0.45	Both	Both
DRT-Allocation 2-dot	0.48	0.4	"	"
DRT-Allocation 3-dot	0.47	0.39	"	"
DRT-Allocation 4-dot	0.49	0.39	"	"
DRT-Allocation 5-dot	0.5	0.39	"	"
Resource Capacity	11.17	9.58	"	"
Wald SD of Drift rate (W.s)	1.09	0.85	Wald	DRT
Non-decision time (W.t0)	0.18	0.18	"	"
Drift rate 1-dot	5.56	5.14	"	"
Drift rate 2-dot	4.87	4.49	"	"
Drift rate 3-dot	4.78	4.37	"	"
Drift rate 4-dot	4.97	4.37	"	"
Drift rate 5-dot	5.09	4.39	"	"
Go-failure 1-dot	0.01	0.02	"	"
Go-failure 2-dot	0.01	0.02	"	"
Go-failure 3-dot	0.01	0.02	"	"
Go-failure 4-dot	0.01	0.03	"	"
Go-failure 5-dot	0.01	0.03	"	"
Criterion	0.73	0.73	SDT	MOT
Discrimination 1-dot	3.44	3.53	"	"
Discrimination 2-dot	1.93	1.91	"	"
Discrimination 3-dot	1.29	1.28	"	"
Discrimination 4-dot	0.92	0.94	"	"
Discrimination 5-dot	0.71	0.74	"	"

6. Estimates of extra-capacity (G) in the DRT-emphasis condition

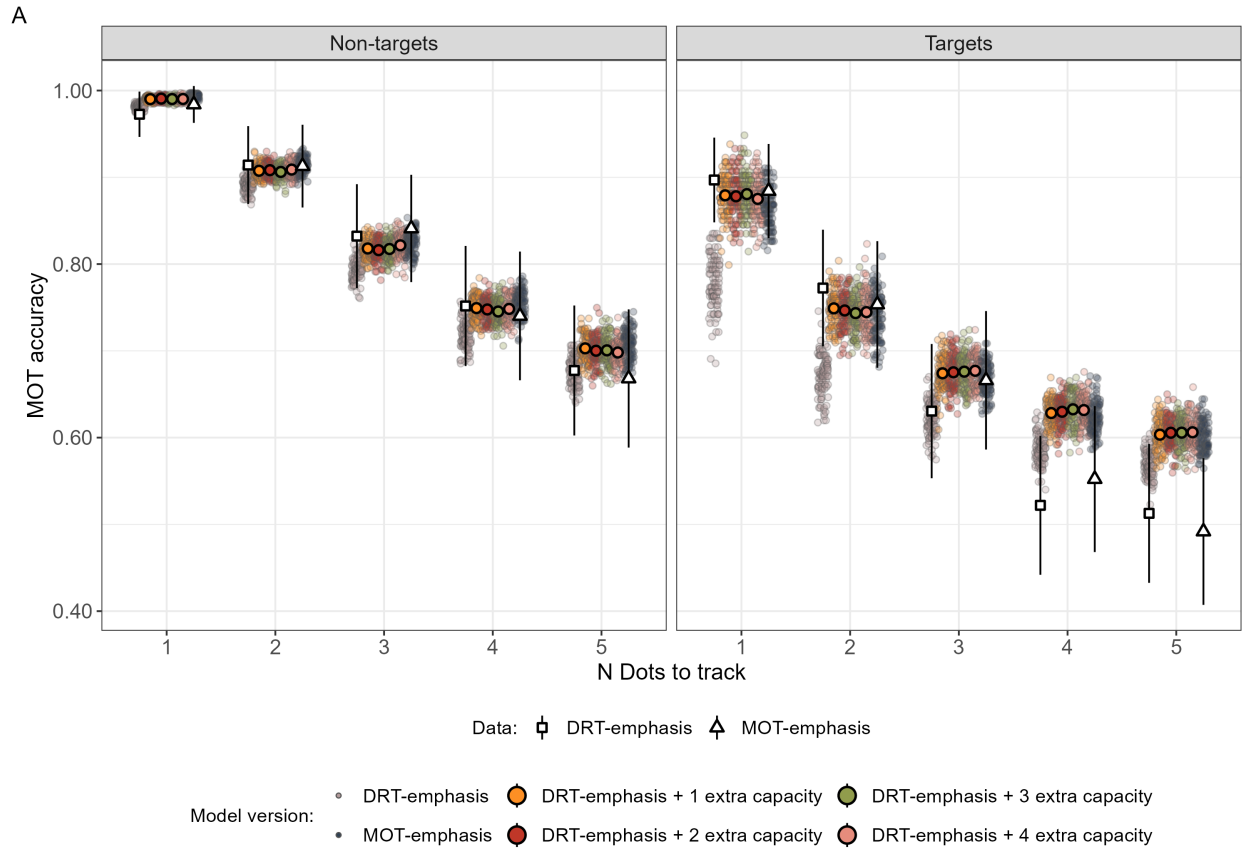

Figure 11

Reproduced figure of Multiple Object Tracking (MOT) task data from Experiment 2 equivalent to panel A Figure 8 in the main manuscript with additional satisficed DRT-emphasis conditions, shown in coloured points. Shaded points represent 100 mean posterior predictive data points for each condition. Orange-colour filled points shows means of the posterior predictives of the satisficed model from the main manuscript where additional capacity is estimated ($G = 11$). Additional versions of the satisficed model with capacity values of $G = 12, 13, \&14$ are shown in coloured-points. White-unfilled points show means and standard errors of behavioural data. Note, that the DRT- & MOT-emphasis conditions, shown in tan- & navy-coloured points, are the same data and model fits from the main manuscript where capacity (G) is fixed at 10.

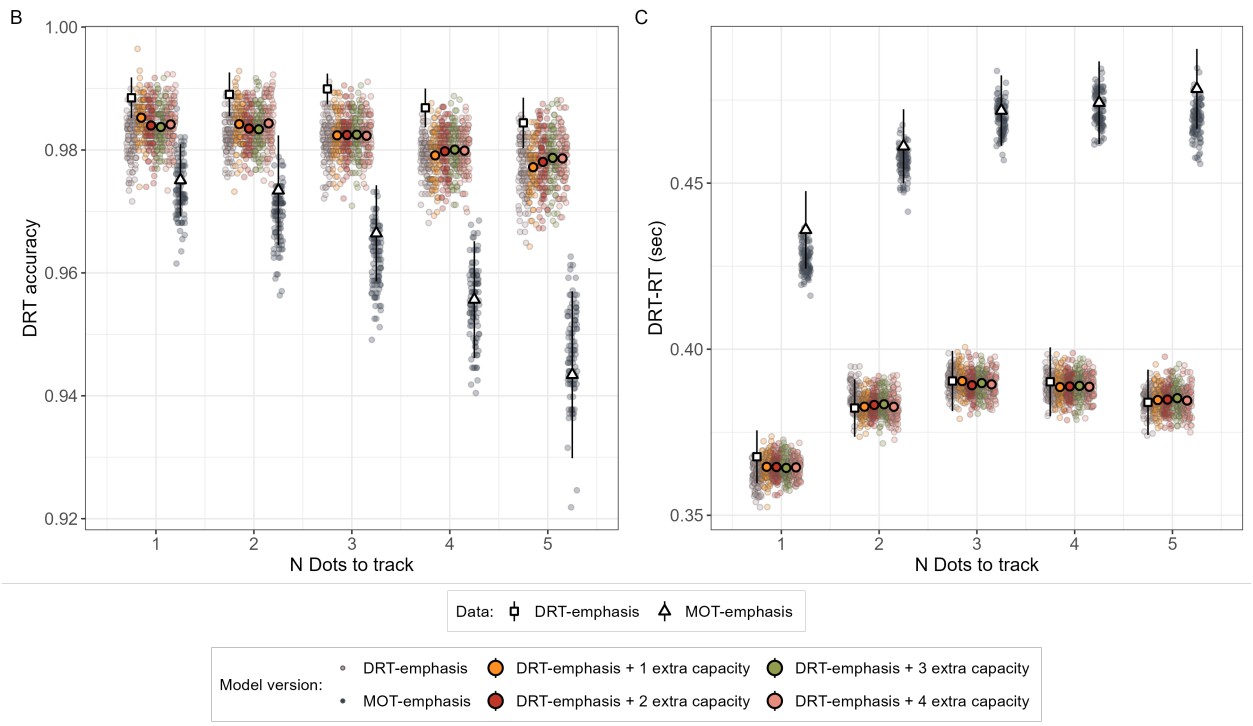

Figure 12

Reproduced figure of Detection Response Task (DRT) task data from Experiment 2 equivalent to panel A Figure 8 in the main manuscript with additional satisficed DRT-emphasis conditions, shown in coloured points. Shaded points represent 100 mean posterior predictive data points for each condition. Orange-colour filled points shows means of the posterior predictives of the satisficed model from the main manuscript where additional capacity is estimated ($G = 11$). Additional versions of the satisficed model with capacity values of $G = 12, 13, \& 14$ are shown in coloured-points. White-unfilled points show means and standard errors of behavioural data. Note, that the DRT- & MOT-emphasis conditions, shown in tan- & navy-coloured points, are the same data and model fits from the main manuscript where capacity (G) is fixed at 10.