Supplemental Material for

Bi-Factor Structure of the Wechsler Preschool and Primary Scale of Intelligence-Fourth Edition

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This document provides the syntax for the analyses using all the WPPSI-IV subtests as well the subtests that comprise the Primary Index scores. All data analysis was done using the R (version 3.0.1) statistical programming language (R Development Core Team, 2013), using the lavaan (version 0.5-13) (Rosseel, 2012) package. Below, we show the WPPSI-IV models and the lavaan syntax to fit the models using effects coding. In addition, Tables C and F show the fit statistics for each model using all the subtests as well as just the subtests used for the primary indices.

1 2:6-3:11 Year-Old Group

Coalson and Raiford (2012) performed factor analysis on two sets of variables. The first was with all seven subtests (Receptive Vocabulary, Picture Naming, Block Design, Object Assembly, Picture Memory, and Zoo Locations). The second only included the subtests that contribute to a primary index score, so Picture Naming was omitted. Models using the reduced dataset have PI in the name, e.g., model1PI.model.

The full covariance matrix for the 2:6-3:11 year-old group comes from Coalson and Raiford (2012, p. 70) and is stored in an R object named WPPSI2_3.cov for the following analyses. For an example of how to input a covariance matrix into R for the purposes of a confirmatory factor analysis, see Beaujean (2013).

1.1 One first-order factor

![Diagram](image.png)

Figure 1: Model with one first-order factor for 2:6-3:11 year-old group. Residual errors not shown.
1.2 One second-order factor and two first-order factors

Information

Receptive Vocabulary

Picture Naming

Block Design

Object Assembly

Picture Memory

Zoo Locations

Figure 2: Model with one second-order factor and two first-order factors for 2:6-3:11 Year-Old Group. Residual errors not shown.
# Model 2: One second-order factor and two first-order factors

```r
model2.model<-'
F1 =~ NA*IN + a*IN + b*RV + c*PN
F2 =~ NA*BD + d*BD + e*OA + f*PM + g*Zl
g =~ NA*F1 + h*F1 + i*F2

# For effects coding
a+b+c+d+e+f+g+h+i==9
',
model2.fit<-cfa(model2.model, sample.cov=WPPSI2_3.cov, sample.nobs=600)
```

```r
model2PI.model<-'
F1 =~ NA*IN + a*IN + b*RV
F2 =~ NA*BD + c*BD + d*OA + e*PM + f*Zl
g =~ NA*F1 + g*F1 + h*F2

# For effects coding
a+b+c+d+e+f+g+h==8
',
model2PI.fit<-cfa(model2PI.model, sample.cov=WPPSI2_3.cov, sample.nobs=600)
```

1.3 One second-order factor and three first-order factors

![Diagram](image)

Figure 3: Model with one second-order factor and three first-order factors for 2:6-3:11 Year-Old Group. Residual errors not shown.
# Model 3: One second-order factor and three first-order factors

```R
# Model 3: One second-order factor and three first-order factors
model3.model<-'
VC =~ NA*IN+ a*IN+ b*RV + c*PN
VS =~ NA*BD + d*BD + e*OA
WM =~ NA*PM + f*PM + g*Zl
g=~ NA*VC + h*VC + i*VS + j*WM

# Effects Coding
a+b+c+d+e+f+g+h+i==10,

model3.fit <- cfa(model3.model, sample.cov=WPPSI2_3.cov, sample.nobs=600)

fitMeasures(model3.fit, fit.indices)

<table>
<thead>
<tr>
<th>chisq</th>
<th>df</th>
<th>pvalue</th>
<th>cfi</th>
<th>rmsea</th>
<th>srmr</th>
<th>aic</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.460</td>
<td>8.000</td>
<td>0.001</td>
<td>0.987</td>
<td>0.060</td>
<td>0.020</td>
<td>20064.669</td>
</tr>
</tbody>
</table>
```

```R
model3PI.model<-'
VC =~ NA*IN+ a*IN+ b*RV
VS =~ NA*BD + c*BD + d*OA
WM =~ NA*PM + e*PM + f*Zl
g=~ NA*VC + g*VC + h*VS + i*WM

# Effects Coding
a+b+c+d+e+f+g+h+i==9,

model3PI.fit <- cfa(model3PI.model, sample.cov=WPPSI2_3.cov, sample.nobs=600)
```
1.4 Bi-factor model

Figure 4: Bi-factor Factor Model for 2:6-3:11 year-old group. Residual errors not shown.

```
# Bi-factor
bifactor.model<-'g =~ NA*IN + a*IN + b*RV + c*PN + d*BD + e*OA + f*PM + g*Zl
VC =~ NA*IN + h*IN + i*RV + j*PN
VS =~ NA*BD + k*BD + l*OA
WM =~ NA*PM + m*PM + n*Zl'

# Effects Coding
a+b+c+d+e+f+g+h+i+j+k+l+m+n==14

bifactor.fit<-cfa(bifactor.model, sample.cov=WPPSI2_3.cov, sample.nobs=600, orthogonal=TRUE)

bifactorPI.model<-'g =~ NA*IN + a*IN + b*RV + c*BD + d*OA + e*PM + f*Zl
VC =~ NA*IN + g*IN + h*RV
```
# Constrain these to be equal for model identification
VS =~ NA*BD + i*BD + i*OA
WM =~ NA*PM + j*PM + j*Zl

# Effects Coding
a+b+c+d+e+f+g+e+i+j==10
bifactorPI.fit<-cfa(bifactorPI.model, sample.cov=WPPSI2_3.cov, sample.nobs=600, orthogonal=TRUE)

1.5 No second-order factor and three oblique first-order factors

Verbal Comp.

Information
Receptive Vocabulary
Picture Naming
Block Design
Object Assembly
Picture Memory
Zoo Locations

Visual Spatial

Working Memory

Figure 5: Model with no second-order factor and three oblique first-order factors for 2:6-3:11 year-old group. Residual errors not shown. The correlations among the factors are given in Table A and the variance explained and $\omega$ reliability estimates are given in Table B.

# Oblique Model--No second-order factor and three first-order factors
oblique.model<-' VC =~ NA*IN+ a*IN+ b*RV + c*PN
VS =~ NA*BD + d*BD + e*OA
WM =~ NA*PM + f*PM + g*Zl

# Effects Coding
a+b+c+d+e+f+g==7
oblique.fit <- cfa(oblique.model, sample.cov=WPPSI2_3.cov, sample.nobs=600)

obliquePI.model<-'
VC =~ NA*IN + a*IN+ b*RV
VS =~ NA*BD + c*BD + d*OA
WM =~ NA*PM + e*PM + f*Zl
# Effects Coding
a+b+c+d+e+f==6
''

obliquePI.fit <- cfa(obliquePI.model, sample.cov=WPPSI2_3.cov, sample.nobs=600)

Table A: Correlations among the latent variables for the oblique model for 2:6-3:11 year-old group.

<table>
<thead>
<tr>
<th>Subtest</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00 Verbal Comp.</td>
<td>1.00</td>
<td>0.81</td>
<td>0.78</td>
</tr>
<tr>
<td>2.00 Visual Spatial</td>
<td>0.81</td>
<td>1.00</td>
<td>0.85</td>
</tr>
<tr>
<td>3.00 Working Memory</td>
<td>0.78</td>
<td>0.85</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table B: Sources of Variance in the Wechsler Preschool and Primary Scale of Intelligence-Fourth Edition Among 600 Children Aged 2:6 to 3:11 Years for the Oblique Mode.

<table>
<thead>
<tr>
<th>Subtest</th>
<th>VC</th>
<th>VS</th>
<th>Ages 2:6-3:11 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Var</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IN</td>
<td>0.82</td>
<td>67.08</td>
<td></td>
</tr>
<tr>
<td>RV</td>
<td>0.72</td>
<td>52.27</td>
<td></td>
</tr>
<tr>
<td>PN</td>
<td>0.80</td>
<td>63.20</td>
<td></td>
</tr>
<tr>
<td>BD</td>
<td>0.66</td>
<td>43.69</td>
<td></td>
</tr>
<tr>
<td>OA</td>
<td>0.61</td>
<td>36.60</td>
<td></td>
</tr>
<tr>
<td>PM</td>
<td>0.70</td>
<td>49.28</td>
<td></td>
</tr>
<tr>
<td>ZL</td>
<td>0.51</td>
<td>26.32</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>26.1</td>
<td>11.5</td>
<td></td>
</tr>
<tr>
<td>ω</td>
<td>0.82</td>
<td>0.54</td>
<td></td>
</tr>
</tbody>
</table>

Note. b = standardized factor loading, Var = % variance explained, h² = communality, u² = uniqueness, VC = Verbal Comprehension factor, VS = Visual-Spatial factor, WM = Working Memory factor, PS = Processing Speed factor, FR = Fluid Reasoning factor, IN = Information, RV = Receptive Vocabulary, PN = Picture Naming, BD = Block Design, OA = Object Assembly, PM = Picture Memory, ZL = Zoo Locations, ω = omega. ω estimated from the omega software program (Watkins, 2013).
2 4:0-7:7 Year-Old Group

As with the 2:6-3:11 year-old group, Coalson and Raiford (2012) performed factor analysis on two sets of variables for the 4:0-7:7 year old group. The first was with all fifteen subtests (Information, Similarities, Vocabulary, Comprehension, Receptive Vocabulary, Picture Naming, Block Design, Object Assembly, Matrix Reasoning, Picture Concepts, Picture Memory, Zoo Locations, Bug Search, Cancellation, and Animal Coding). The second only included the subtests that contribute to a primary index score, so Vocabulary, Comprehension, Receptive Vocabulary, Picture Naming, and Animal Coding were omitted. In addition, the for reduced data, models with subfactors within the Verbal Comprehension factor are not estimable. Models using the reduced dataset have PI in the name, e.g., model1PI.model.

The full covariance matrix for the 4:0-7:7 year-old group comes from Coalson and Raiford (2012, p. 71) and is stored in an R object named WPPSI4_7.cov for the following analyses. For an example of how to input a covariance matrix into R for the purposes of a confirmatory factor analysis, see Beaujean (2013).

Table C: Fit Statistics for the models used for the 2:6-3:11 year-old group data.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\chi^2$</th>
<th>df</th>
<th>$p$</th>
<th>CFI</th>
<th>RMSEA</th>
<th>SRMR</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Subtests</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>70.17</td>
<td>14.00</td>
<td>0.00</td>
<td>0.96</td>
<td>0.08</td>
<td>0.04</td>
<td>20097.37</td>
</tr>
<tr>
<td>Model 2</td>
<td>31.46</td>
<td>10.00</td>
<td>0.00</td>
<td>0.98</td>
<td>0.06</td>
<td>0.02</td>
<td>20066.66</td>
</tr>
<tr>
<td>Model 3</td>
<td>25.46</td>
<td>8.00</td>
<td>0.00</td>
<td>0.99</td>
<td>0.06</td>
<td>0.02</td>
<td>20064.67</td>
</tr>
<tr>
<td>Bifactor</td>
<td>13.73</td>
<td>4.00</td>
<td>0.01</td>
<td>0.99</td>
<td>0.06</td>
<td>0.01</td>
<td>20060.94</td>
</tr>
<tr>
<td>Oblique</td>
<td>25.46</td>
<td>9.00</td>
<td>0.00</td>
<td>0.99</td>
<td>0.06</td>
<td>0.02</td>
<td>20062.67</td>
</tr>
<tr>
<td>Primary Index Subtests</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>22.19</td>
<td>9.00</td>
<td>0.01</td>
<td>0.99</td>
<td>0.05</td>
<td>0.03</td>
<td>17467.20</td>
</tr>
<tr>
<td>Model 2</td>
<td>9.82</td>
<td>5.00</td>
<td>0.08</td>
<td>0.99</td>
<td>0.04</td>
<td>0.02</td>
<td>17462.83</td>
</tr>
<tr>
<td>Model 3</td>
<td>2.92</td>
<td>3.00</td>
<td>0.40</td>
<td>1.00</td>
<td>0.00</td>
<td>0.01</td>
<td>17459.93</td>
</tr>
<tr>
<td>Bifactor</td>
<td>2.92</td>
<td>2.00</td>
<td>0.23</td>
<td>1.00</td>
<td>0.03</td>
<td>0.01</td>
<td>17461.93</td>
</tr>
<tr>
<td>Oblique</td>
<td>2.92</td>
<td>4.00</td>
<td>0.57</td>
<td>1.00</td>
<td>0.00</td>
<td>0.01</td>
<td>17457.93</td>
</tr>
</tbody>
</table>

CFI: comparative fit index; RMSEA: root mean square error of approximation; SRMR: standardized root mean square residual; AIC: Akaike’s information criterion.
2.1 One first-order factor

Figure 6: Single first-order factor model for 4:0-7:7 year-old group. Residual errors not shown.
2.1 One first-order factor

# Model 1: One first-order factor
mod1.model <-'
 g =~ NA*IN + a*IN + b*SI + c*VC + d*CO + e*RV + f*PN + g*BD + h*OA + i*MR +
 j*PC + k*PM + l*ZL + m*BS + n*CA + o*AC
 # Effects Coding
 a+b+c+d+e+f+h+i+j+k+l+m+n+o==15
'
mod1.fit <- cfa(mod1.model, sample.cov=WPPSI4_7.cov,
sample.nobs=1100)

mod1PI.model <-'
 g =~ NA*IN + a*IN + b*SI + c*BD + d*OA + e*MR + f*PC + g*PM +
 h*ZL + i*BS + j*CA
 # Effects Coding
 a+b+c+d+e+f+h+i+j==10
'
mod1PI.fit <- cfa(mod1PI.model, sample.cov=WPPSI4_7.cov,
sample.nobs=1100)
2.2 One second-order factor and two first-order factors

Figure 7: Model with one second-order factor and two first-order factors for 4:0-7:7 year-old group. Residual errors not shown.
### Model 2: One second-order factor and two first-order factors

```r
# Model 2: One second-order factor and two first-order factors
model2.model<-'
VCo =~ NA*IN + a*IN+ b*SI + c*VC + d*CO + e*RV + f*PN
F2 =~ NA*BD + g*BD + h*OA + i*MR + j*PC + k*PM + l*ZL + m*BS + n*CA + o*AC
g =~ NA*VCo + p*VCo + q*F2

# Effects Coding
a+b+c+d+e+f+g+h+i+j+k+l+m+n+o+p+q==17
'
model2.fit<-cfa(model2.model, sample.cov=WPPSI4_7.cov, sample.nobs=1100)

model2PI.model<-'
VCo =~ NA*IN + a*IN+ b*SI
F2 =~ NA*BD+ c*BD + d*OA + e*MR + f*PC + g*PM + h*ZL + i*BS + j*CA
g=~NA*VCo + k*VCo + l*F2

# Effects Coding
a+b+c+d+e+f+g+h+i+j+k+l==12
'
model2PI.fit<-cfa(model2.model, sample.cov=WPPSI4_7.cov, sample.nobs=1100)
```
2.3 One second-order factor and three first-order factors

Figure 8: Model with one second-order factor and three first-order factors for 4:0-7:7 year-old group. Residual errors not shown.
2.3 One second-order factor and three first-order factors

# Model 3: One second-order factor and three first-order factors

```r
model3.model<-'
VCo=~ NA*IN + a*IN + b*SI + c*VC + d*CO + e*RV + f*PN
F2 =~ NA*BD + g*BD + h*OA + i*MR + j*PC + k*PM + l*ZL
PS =~ NA*BS + m*BS + n*CA + o*AC
g =~ NA*VCo + p*VCo + q*F2 + r*PS

# Effects Coding
a+b+c+d+e+f+g+h+i+j+k+l+m+n+p+q+r==18
'
model3.fit <- cfa(model3.model, sample.cov=WPPSI4_7.cov, sample.nobs=1100)

model3PI.model<-'
VCo=~ NA*IN + a*IN + b*SI
F2 =~ NA*BD + c*BD + d*OA + e*MR + f*PC + g*PM + h*ZL
PS =~ NA*BS + i*BS + j*CA
g =~ NA*VCo + k*VCo + l*F2 + m*PS

# Effects Coding
a+b+c+d+e+f+g+h+i+j+k+l+m==13
'
model3PI.fit<-.cfa(model3PI.model, sample.cov=WPPSI4_7.cov, sample.nobs=1100)
```
2.4 One second-order factor and four first-order factors, version 1

Figure 9: Model with one second-order factor and four first-order factors (version 1) for 4:0-7:7 year-old group. Residual errors not shown.
# Model 4a: One second-order factor and four first-order factors, version 1

```r
model4a.model <- 'VCo =~ NA*IN + a*IN + b*SI + c*VC + d*CO + e*RV + f*PN
F2 =~ NA*BD + g*BD + h*OA + i*MR + j*PC
WM =~ NA*PM + k*PM + l*ZL
PS =~ NA*BS + m*BS + n*CA + o*AC
g =~ NA*VCo + p*VCo + q*F2 + r*WM + s*PS

# Effects Coding
a+b+c+d+e+f+g+h+i+j+k+l+m+n+o+p+q+r+s==19
'

model4a.fit <- cfa(model4a.model, sample.cov=WPPSI4_7.cov, sample.nobs=1100)

model4aPI.model <- 'VCo =~ NA*IN + a*IN + b*SI
F2 =~ NA*BD + c*BD + d*OA + e*MR + f*PC
WM =~ NA*PM + g*PM + h*ZL
PS =~ NA*BS + i*BS + j*CA
g =~ NA*VCo + k*VCo + l*F2 + m*WM + n*PS

# Effects Coding
a+b+c+d+e+f+g+h+i+j+k+l+m+n==14
'

model4aPI.fit <- cfa(model4aPI.model, sample.cov=WPPSI4_7.cov, sample.nobs=1100)
```
2.5 One second-order factor and four first-order factors, version 2

Figure 10: Model with one second-order factor and four first-order factors (version 2) for 4:0-7:7 year-old group. Residual errors not shown.
# Model 4b: One second-order factor and four first-order factors, version 2

model4b.model<-' VCo =~ NA*IN + a*IN + b*SI + c*VC + d*CO + e*RV + f*PN VS =~ NA*BD + g*BD + h*OA F3 =~ NA*MR + i*MR + j*PC + k*PM + l*ZL PS =~ NA*BS + m*BS + n*CA + o*AC g =~ NA*VCo + p*VCo + q*VS + r*F3 + s*PS

# Effects Coding
a+b+c+d+e+f+g+h+i+j+k+l+m+n+o+p+q+r+s==19
model4b.fit<-cfa(model4b.model, sample.cov=WPPSI4_7.cov, sample.nobs=1100)

model4bPI.model<-' VCo =~ NA*IN + a*IN + b*SI VS =~ NA*BD + c*BD + d*OA F3 =~ NA*MR + e*MR + f*PC + g*PM + h*ZL PS =~ NA*BS + i*BS + j*CA g =~ NA*VCo + k*VCo + l*VS + m*F3 + n*PS

# Effects Coding
a+b+c+d+e+f+g+h+i+j+k+l+m+n==14
model4bPI.fit<-cfa(model4bPI.model, sample.cov=WPPSI4_7.cov, sample.nobs=1100)
2.6 One second-order factor and five first-order factors

Figure 11: Model with one second-order factor and five first-order factors for 4:0-7:7 year-old group. Residual errors not shown.
Model 5a: One second-order factor and five first-order factors

model5a.model<-'
VCo =~ NA*IN + a*IN + b*SI + c*VC + d*CO + e*RV + f*PN
VS =~ NA*BD + g*BD + h*OA
FR =~ NA*MR + i*MR + j*PC
WM =~ NA*PM + k*PM + l*ZL
PS =~ NA*BS + m*BS + n*CA + o*AC
g =~ NA*VCo + p*VCo + q*VS + r*FR + s*WM + t*PS

# Constraint to prevent negative variance
VCo~~u*VCo
u >0

# Effects Coding
a+b+c+d+e+f+g+h+i+j+k+l+m+n+o+p+q+r+s+t==20

model5a.fit<-cfa(model5a.model, sample.cov=WPPSI4_7.cov,
sample.nobs=1100)

model5aPI.model<-'
VCo =~ NA*IN + a*IN + b*SI
VS =~ NA*BD + c*BD + d*OA
FR =~ NA*MR + e*MR + f*PC
WM =~ NA*PM + g*PM + h*ZL
PS =~ NA*BS + i*BS + j*CA
g =~ NA*VCo + k*VCo + l*VS + m*FR + n*WM + o*PS

# Effects Coding
a+b+c+d+e+f+g+h+i+j+k+l+m+n+o==15

model5aPI.fit<-cfa(model5aPI.model, sample.cov=WPPSI4_7.cov,
sample.nobs=1100)
2.7 One second-order factor, five first-order factors, and two subfactors nested within Verbal Comprehension

Figure 12: Model with one second-order factor, five first-order factors, and two subfactors nested within Verbal Comprehension for 4:0-7:7 year-old group. Residual errors not shown.
2.7 One second-order factor, five first-order factors, and two subfactors nested within Verbal Comprehension

Model 5b: One second-order factor, five first-order factors, and two subfactors nested within Verbal Comprehension

```r
model5b.model<-'

VCo1 =~ NA*IN + a*IN + b*SI + c*VC + d*CO
VCo2 =~ NA*RV + e*RV + f*PN
VCo =~ NA*VCo1 + g*VCo1 + h*VCo2
VS =~ NA*BD + i*BD + j*OA
FR =~ NA*MR + k*MR + l*PC
WM =~ NA*PM + m*PM + n*ZL
PS =~ NA*BS + o*BS + p*CA + q*AC
g =~ NA*VCo + r*VCo + s*VS + t*FR + u*WM + v*PS

# Effects Coding
a+b+c+d+e+f+g+h+i+j+k+l+m+n+o+p+q+r+s+t+u+v==22
'

model5b.fit<-cfa(model5b.model, sample.cov=WPPSI4_7.cov, sample.nobs=1100)
```
2.8 Bi-factor model with one Verbal Comprehension factor

Figure 13: Bi-factor model 1 for 4:0-7:7 year-old group. Residual errors not shown.
# Bi-factor model with VCo, PS, VS, FR, and WM domain factors
# No VCo subfactors
biFactor1.model<-'
g =~ NA*IN + a*IN + b*SI + c*VC + d*CO + e*RV + f*PN + g*BD + h*OA + i*MR + 
j*PC + k*PM + l*ZL + m*BS + n*CA + o*AC
VCo =~ NA*IN + p*IN + q*SI + r*VC + s*CO + t*RV + u*PN
PS =~ NA*BS + w*BS + y*CA + z*AC
VS =~ NA*BD + aa*BD + bb*OA
FR =~ NA*MR + cc*MR + dd*PC
WM =~ NA*PM + ee*PM + ff*ZL

# Effects Coding
a+b+c+d+e+f+g+h+i+j+k+l+m+n+o==15
p+q+r+s+t+u==6
w+y+z==3
aa+bb==2
cce+dd==2
eef==2
'

biFactor1.fit<cfa(biFactor1.model, sample.cov=WPPSI4_7.cov, 
sample.nobs=1100, orthogonal=TRUE)

biFactor1PI.model<-'
g =~ NA*IN + a*IN + b*SI + c*BD + d*OA + e*MR + f*PC + g*PM + h*ZL + 
i*BS + j*CA
VCo =~ NA*IN + k*IN + l*SI
PS =~ NA*BS + m*BS + n*CA
VS =~ NA*BD + o*BD + p*OA
FR =~ NA*MR + q*MR + r*PC
WM =~ NA*PM + s*PM + t*ZL

# Effects Coding
a+b+c+d+e+f+g+h+i+j==10
k+l==2
m+n==2
o+o==2
q+r==2
s+t==2
'

biFactor1PI.fit<cfa(biFactor1PI.model, sample.cov=WPPSI4_7.cov, 
sample.nobs=1100, orthogonal=TRUE)
2.9 Bi-factor model with two Verbal Comprehension subfactors

Figure 14: Bi-factor model 2 for 4:0-7:7 year-old group. Residual errors not shown.
2.9 Bi-factor model with two Verbal Comprehension subfactors

# Bi-factor model with PS, WM, PS and 2 oblique VCo domain factors that are allowed to correlate
biFactor2.model<-

```
g =~ NA*IN + a*IN + b*SI + c*VC + d*CO + e*RV + f*PN + g*BD + h*OA + i*MR + j*PC + k*PM + l*ZL + m*BS + n*CA + o*AC
VCo1 =~ NA*IN + p*IN + q*SI + r*VC + s*CO
VCo2 =~ NA*RV + t*RV + u*PN
PS =~ NA*BS + v*BS + w*CA + y*AC
VS =~ NA*BD + aa*BD + bb*OA
FR =~ NA*MR + cc*MR + dd*PC
WM =~ NA*PM + ee*PM + ff*ZL
```

```
g ~~ 0*PS + 0*VCo1 + 0*VCo2 + 0*VS + 0*FR + 0*WM
PS ~~ 0*VCo1 + 0*VCo2 + 0*VS + 0*FR + 0*WM
VCo1 ~~ 0*VS + 0*FR + 0*WM
VCo2 ~~ 0*VS + 0*FR + 0*WM
VS ~~ 0*FR + 0*WM
FR ~~ 0*WM
```

# Effects Coding
```
a+b+c+d+e+f+g+h+i+j+k+l+m+n+o==15
p+q+r+s==4
t+u==2
v+w+y==3
aa+bb==2
cc+dd==2
ee+ff==2
```

biFactor2.fit<-cfa(biFactor2.model, sample.cov=WPPSI4_7.cov, sample.nobs=1100, control=list(rel.tol=.000001))
2.10 No second-order factor and five oblique first-order factors

Figure 15: Model with no second-order factors and five oblique first-order factors for 4:0-7:7 year-old group. Residual errors not shown. Correlations among the factors are given in Table D and the variance explained and $\omega$ reliability estimates are given in Table E.
No second-order factor and five oblique first-order factors

# Oblique model: No second-order factor and five correlated first-order factors

```r
oblique.model<-'
VCo =~ NA*IN + a*IN + b*SI + c*VC + d*CO + e*RV + f*PN
VS =~ NA*BD + g*BD + h*OA
FR =~ NA*MR + i*MR + j*PC
WM =~ NA*PM + k*PM + l*ZL
PS =~ NA*BS + m*BS + n*CA + o*AC

# Effects Coding
a+b+c+d+e+f+g+h+i+j+k+l+m+n+o==15
',
```

```r
oblique.fit<-cfa(oblique.model, sample.cov=WPPSI4_7.cov, sample.nobs=1100)
```

```r
obliquePI.model<-'
VCo =~ NA*IN + a*IN + b*SI
VS =~ NA*BD + c*BD + d*OA
FR =~ NA*MR + e*MR + f*PC
WM =~ NA*PM + g*PM + h*ZL
PS =~ NA*BS + i*BS + j*CA

# Effects Coding
a+b+c+d+e+f+g+h+i+j==10
',
```

```r
obliquePI.fit<-cfa(obliquePI.model, sample.cov=WPPSI4_7.cov, sample.nobs=1100)
```

Table D: Correlations among the latent variables for the oblique model for 4:0-7:7 year-old group.

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<th>Subtest</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
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<tbody>
<tr>
<td>All Subtests</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00 Verbal Comp.</td>
<td>1.00</td>
<td>0.77</td>
<td>0.82</td>
<td>0.72</td>
<td>0.60</td>
</tr>
<tr>
<td>2.00 Visual Spatial</td>
<td>0.77</td>
<td>1.00</td>
<td>0.88</td>
<td>0.81</td>
<td>0.67</td>
</tr>
<tr>
<td>3.00 Fluid Reasoning</td>
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<td>0.88</td>
<td>1.00</td>
<td>0.87</td>
<td>0.69</td>
</tr>
<tr>
<td>4.00 Working Memory</td>
<td>0.72</td>
<td>0.81</td>
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<td>0.76</td>
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<tr>
<td>5.00 Processing Speed</td>
<td>0.60</td>
<td>0.67</td>
<td>0.69</td>
<td>0.76</td>
<td>1.00</td>
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<th>3</th>
<th>4</th>
<th>5</th>
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<td>0.87</td>
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<tr>
<td>3.00 Fluid Reasoning</td>
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<td>0.76</td>
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<td>5.00 Processing Speed</td>
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<td>0.68</td>
<td>0.69</td>
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Table E: Sources of Variance in the Wechsler Preschool and Primary Scale of Intelligence-Fourth Edition Among 1,100 Children Aged 4:00-7:7 Years for the Oblique Mode.

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<tr>
<th>Subtest</th>
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Table F: Fit Statistics for the models used for the 4:0-7:7 year-old group data.

<table>
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<tr>
<th>Model</th>
<th>$\chi^2$</th>
<th>df</th>
<th>$p$</th>
<th>CFI</th>
<th>RMSEA</th>
<th>SRMR</th>
<th>AIC</th>
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<td>0.07</td>
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<td>0.97</td>
<td>0.05</td>
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<td>0.03</td>
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<th>$p$</th>
<th>CFI</th>
<th>RMSEA</th>
<th>SRMR</th>
<th>AIC</th>
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CFI: comparative fit index; RMSEA: root mean square error of approximation; SRMR: standardized root mean square residual; AIC: Akaike’s information criterion.
References