Supplemental Materials

Purpose of This Document

The joint longitudinal and survival models fit in this study require considerable modeling consideration. These models combine longitudinal regression models (i.e., multilevel regression) with Cox proportional hazards survival regression, but these components are modeled jointly, not just simultaneously. This means that both components are able to affect one another, requiring the whole joint model to be estimated in order to examine results. As a consequence, modeling decisions are not as simple as fitting the longitudinal and survival components separately, examining the results, and then constructing the joint model based on those independent results.

As a result of this modeling sensitivity, many models had to be run and examined for this study to inform the final selection. This particular issue is compounded by the more exploratory nature of the study as the literature has identified multiple predictors of nursing home or assisted living (NH/AL) transition, but models for this process have not been well described, particularly not joint longitudinal and survival models. Due to the number of models and complexity of updating the models, this document aims to describe the overall process and show all the models fit as part of the study.

Readers of the above may have some specific concerns regarding the amount of models fit and the way in which models were updated based on prior fits. We intend to address these concerns before then overviewing the models fit for the study.

Considering the Modeling Process

First, we demonstrate the importance of running the joint models rather than its components separately. To do this, we compare the results of the first longitudinal regression models and survival model to the first joint model fit for the study. Since this model has not been shown yet, we will show the syntax for the two components and then the joint model. First, here is the survival model:

```
Survival.Model <- coxph(Surv(tstop, NursingHome) ~ Age + MarStat + Inf
Age + InfRel + LivSit + Sex + Race + Ethnicity + Edu,
                        data = df.surv, cluster = ID)
```

To explicate the above code some, we are fitting a Cox proportional hazards model via the `coxph()` function. The model is predicting the event (`NursingHome`) over the observed time (with time-to-event or time-to-last-visit stored as `tstop`). Predicting this risk is age (`Age`), marital status (`MarStat`), informant age (`InfAge`), informant's relationship to the participant (`InfRel`), whether the participant lives alone or not (`LivSit`), sex (`Sex`), race (`Race`), ehtnicity (`Ethnicity`), and years of education (`Edu`). We then tell the function to use the survival dataframe (`df.surv`) and that observations are clustered by the

variable `ID`. We will hold off on showing the results of this model, so the next models to fit are the longitudinal models. As with the survival model, we will show the code for these models:

```
MMSE.Model <- lme(MMSE ~ Time + AgeB + Edu,
                  random = ~ Time | ID, data = df.long)
NPIQ.Model <- lme(NPIQ ~ Time + AgeB + Edu,
                  random = ~ Time | ID, data = df.long)
FAQ.Model  <- lme(FAQ ~ Time + AgeB + Edu,
                  random = ~ Time | ID, data = df.long)
UPDRS.Model <- lme(UPDRS ~ Time + AgeB + Edu,
                   random = ~ Time | ID, data = df.long)
CAIDE.Model <- lme(CAIDE ~ Time + AgeB + Edu,
                   random = ~ Time | ID, data = df.long)
```

In this case, we are fitting seperate longitudinal regression via the `lme()` function (i.e., Linear Mixed Effects). All these models use the same population/fixed effects for years since first visit (`Time`), age at baseline (`AgeB`), and years of education (`Edu`). They also all use the same random effects, namely an implied random intercept (to drop a random intercept one would need to use `0 + ...` in the formula) and a random slope for time. The syntax of `Time | ID` indicates that this random slope for time is clustered by the ID variable. The models all use the same dataframe, namely a long-format version of the data (`df.long`). Finally, the outcome for each model is indicated by the left-hand side of the formula: e.g., `MMSE ~ ...` indicates that the model is predicting MMSE scores. Like with the survival model, we will hold off on reporting the results of this model so that the joint model syntax can be shown:

```
JointLongSurv.Model <- jm(Survival.Model, list(MMSE.Model, NPIQ,Model,
FAQ.Model, CAIDE.Model, UPDRS.Model), time_var = "Time",
                          n_iter = 15000, n_burnin = 5000, n_thin = 5)
```

In this case, the joint model is fit by the `jm()` function, which takes as input the fitted survival model (the `Survival.Model` object shown above) and then a list of the desired longitudinal models. Note that this list of longitudinal components is what makes these models multivariate joint models rather than univariate models, which could be done if the only longitudinal outcome of interest was, for example, the MMSE. We indicate that the time variable for the models is named `Time` and then pass some controls to the MCMC estimator. The MCMC estimator for the `JMbayes2` package automatically runs across 3 chains, so this indicates that we want 15000 iterations per chain, 5000 of which will be burnin (and thus removed), and then that we want to keep only every 5th iteration to help reduce autocorrelation in the chains. To put this another way, we are asking for a total of 6000 posterior samples: $(15000-5000)*35$(15000−5000)∗35. Now, with all the models estimated, we can examine the results of the models estimated separately compared to their results when estimated in the joint model. To reduce the amount of space this takes, we will show just the results from the MMSE longitudinal model.

```
## Call:
## coxph(formula = Surv(tstop, NursingHome) ~ Age + MarStat + InfAge +
##     InfRel + LivSit + Sex + Race + Ethnicity + Edu, data = df.surv,
##     cluster = ID)
##
##   n= 4421, number of events= 709
##
##                       coef exp(coef)  se(coef) robust se      z Pr
(>|z|)
## Age                0.026511  1.026865  0.005170  0.005619  4.718 2.
38e-06 ***
## MarStatSeparated   0.330071  1.391066  0.120261  0.124313  2.655  0
.00793 **
## InfAge            -0.010477  0.989577  0.003545  0.003804 -2.754  0
.00588 **
## InfRelExt         -0.265828  0.766571  0.151071  0.159309 -1.669  0
.09519 .
## InfRelImp          0.006385  1.006406  0.294614  0.313763  0.020  0
.98376
## LivSitAlone        0.322565  1.380664  0.115263  0.119851  2.691  0
.00712 **
## SexFemale          0.190130  1.209407  0.082572  0.083780  2.269  0
.02324 *
## RaceNotWhite      -0.856837  0.424503  0.141784  0.147355 -5.815 6.
07e-09 ***
## EthnicityHispanic -0.691171  0.500989  0.210348  0.220213 -3.139  0
.00170 **
## Edu               -0.001161  0.998839  0.012773  0.013142 -0.088  0
.92959
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##                   exp(coef) exp(-coef) lower .95 upper .95
## Age                  1.0269     0.9738    1.0156    1.0382
## MarStatSeparated     1.3911     0.7189    1.0903    1.7749
## InfAge               0.9896     1.0105    0.9822    0.9970
## InfRelExt            0.7666     1.3045    0.5610    1.0475
## InfRelImp            1.0064     0.9936    0.5441    1.8614
## LivSitAlone          1.3807     0.7243    1.0916    1.7462
## SexFemale            1.2094     0.8269    1.0263    1.4252
## RaceNotWhite         0.4245     2.3557    0.3180    0.5666
## EthnicityHispanic    0.5010     1.9961    0.3254    0.7714
## Edu                  0.9988     1.0012    0.9734    1.0249
##
```

```
## Concordance= 0.64  (se = 0.013 )
## Likelihood ratio test= 167.8  on 10 df,   p=<2e-16
## Wald test            = 159.7  on 10 df,   p=<2e-16
## Score (logrank) test = 179.1  on 10 df,   p=<2e-16,   Robust = 126.
6  p=<2e-16
##
##   (Note: the likelihood ratio and score tests assume independence o
f
##      observations within a cluster, the Wald and robust score tests
do not).
## Linear mixed-effects model fit by REML
##   Data: df.long
##        AIC      BIC    logLik
##   84178.95 84240.19 -42081.48
##
## Random effects:
##  Formula: ~Time | ID
##  Structure: General positive-definite, Log-Cholesky parametrization
##             StdDev   Corr
## (Intercept) 4.083737 (Intr)
## Time        1.582264 0.53
## Residual    2.058933
##
## Fixed effects:  MMSE ~ Time + AgeB + Edu
##                 Value Std.Error    DF   t-value p-value
## (Intercept) 20.620085 0.6280366 11175  32.83262  0.0000
## Time        -1.533221 0.0298022 11175 -51.44656  0.0000
## AgeB        -0.018488 0.0072919  4418  -2.53542  0.0113
## Edu          0.371380 0.0193678  4418  19.17510  0.0000
##  Correlation:
##      (Intr) Time   AgeB
## Time  0.021
## AgeB -0.881  0.009
## Edu  -0.530 -0.003  0.080
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -4.58027850 -0.42519301  0.04294972  0.43606724  4.47588585
##
## Number of Observations: 15597
## Number of Groups: 4421
##
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(MMSE, NPIQ, FAQ,
```

```
##     CAIDE, UPDRS), time_var = "Time", n_iter = 15000, n_burnin = 50
00,
##     n_thin = 5)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
##   NPIQ: 15597
##   FAQ: 15597
##   CAIDE: 15597
##   UPDRS: 15597
##
##                   DIC     WAIC      LPML
## marginal    366391.2 366310.9 -183156.1
## conditional 484741.7 471896.9 -245584.4
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
## (Intr) 4.1213 (Intr)    Time  (Intr)    Time  (Intr)    Time  (Intr
)    Time
## Time   1.5757 0.4981
## (Intr) 3.1122 -0.2606 -0.1013
## Time   0.6821 -0.1247 -0.5619 -0.0978
## (Intr) 7.6733 -0.6883 -0.3786 0.5613  0.0095
## Time   2.0718 -0.2883 -0.8198 0.0166  0.6946  0.1647
## (Intr) 1.6329 0.0396  0.1148  0.0739  -0.0274 -0.0186 -0.0996
## Time   0.1747 -0.0873 -0.0312 0.0146  0.0796  0.0799  0.0229  -0.32
48
## (Intr) 0.8920 -0.0932 -0.0332 0.1855  -0.0960 0.2007  -0.0032 0.024
9  -0.0465
## Time   0.3424 -0.3177 -0.4072 0.1606  0.1699  0.2631  0.3575  0.000
4  0.0379
##
##
## (Intr) (Intr)
## Time
## (Intr)
## Time
## (Intr)
## Time
## (Intr)
## Time
```

```
## (Intr)
## Time   0.4278
##
## Survival Outcome:
##                      Mean  StDev    2.5%   97.5%      P   Rhat
## Age                0.0272 0.0076  0.0125  0.0420 0.0000 1.0118
## MarStatSeparated   0.2128 0.1466 -0.0757  0.5001 0.1460 1.0037
## InfAge            -0.0035 0.0041 -0.0119  0.0045 0.3957 1.0032
## InfRelExt          0.1737 0.1618 -0.1465  0.4807 0.2843 1.0068
## InfRelImp          0.4576 0.3058 -0.1908  1.0206 0.1393 0.9998
## LivSitAlone        0.9936 0.1300  0.7324  1.2421 0.0000 1.0103
## SexFemale         -0.0795 0.0973 -0.2718  0.1124 0.4150 1.0026
## RaceNotWhite      -0.8756 0.1528 -1.1717 -0.5812 0.0000 1.0041
## EthnicityHispanic -1.1374 0.2227 -1.6046 -0.7297 0.0000 1.0047
## Edu                0.0131 0.0139 -0.0147  0.0402 0.3403 1.0016
## value(MMSE)        0.0068 0.0091 -0.0113  0.0241 0.4463 1.0479
## value(NPIQ)        0.0403 0.0167  0.0064  0.0728 0.0170 1.0175
## value(FAQ)         0.1072 0.0099  0.0871  0.1260 0.0000 1.0535
## value(CAIDE)       0.0061 0.0337 -0.0603  0.0710 0.8630 1.0017
## value(UPDRS)       0.0275 0.0269 -0.0260  0.0804 0.3003 1.0018
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                 Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept) 22.0138 0.8269 20.3924 23.5983 0.0000 1.0002
## Time        -1.5532 0.0557 -1.6624 -1.4430 0.0000 1.0003
## AgeB        -0.0128 0.0101 -0.0321  0.0072 0.2117 1.0007
## Edu          0.2521 0.0221  0.2081  0.2949 0.0000 0.9999
## sigma        2.0759 0.0177  2.0419  2.1109 0.0000 1.0014
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                 Mean  StDev    2.5%   97.5% P   Rhat
## (Intercept)  7.1061 0.5743  5.9880  8.2494 0 0.9999
## Time         0.3403 0.0398  0.2622  0.4199 0 1.0062
## AgeB        -0.0484 0.0078 -0.0637 -0.0331 0 0.9999
## sigma        2.6735 0.0215  2.6322  2.7158 0 1.0384
##
## Longitudinal Outcome: FAQ (family = gaussian, link = identity)
##                Mean  StDev   2.5%  97.5%     P   Rhat
## (Intercept) 2.7005 1.1564 0.5035 4.9681 0.016 0.9999
## Time        2.3051 0.0868 2.1380 2.4763 0.000 1.0014
## AgeB        0.0819 0.0158 0.0506 0.1119 0.000 0.9998
## sigma       3.2464 0.0265 3.1932 3.2990 0.000 1.0053
##
## Longitudinal Outcome: CAIDE (family = gaussian, link = identity)
```

```
##                Mean  StDev   2.5%  97.5%      P   Rhat
## (Intercept) 6.4175 0.2562 5.8976 6.9058 0.0000 1.0009
## Time        0.0419 0.0120 0.0184 0.0652 0.0000 1.0003
## AgeB        0.0066 0.0035 0.0000 0.0137 0.0517 1.0012
## sigma       0.9033 0.0070 0.8896 0.9174 0.0000 1.0070
##
## Longitudinal Outcome: UPDRS (family = gaussian, link = identity)
##                 Mean  StDev    2.5%  97.5%      P   Rhat
## (Intercept) -0.0811 0.1985 -0.4725 0.2983 0.6847 1.0002
## Time         0.2128 0.0139  0.1857 0.2400 0.0000 1.0014
## AgeB         0.0072 0.0027  0.0019 0.0126 0.0073 1.0001
## sigma        0.8063 0.0066  0.7937 0.8197 0.0000 1.0008
##
## MCMC summary:
## chains: 3
## iterations per chain: 15000
## burn-in per chain: 5000
## thinning: 5
## time: 2 hours
```

We can start by comparing the results from the survival model. When estimated alone, the predictors of marital status, informant age, and sex are significant (i.e., $p < 0.05$), but in the joint model, none of these predictors emerge as significant. The statistical presence or absence of an effect is not all that can change in these models. For example, in the standalone survival model, the coefficient for ethnicity is -0.69, but this coefficient changes to -1.14 in the joint model, which exceeds the standalone model's 95% confidence interval for this coefficient: [-1.10, -0.28]. We can then examine the changes from the standalone to joint model estimates of the MMSE longitudinal regression. In this relatively simpler model, the results do not change dramatically; however, it is noted that the intercept estimate does change from 20.62 (standalone) to 22.01 (joint). While this conditional mean estimate is not an enormous raw difference, there is a clinically relevant implication to a nearly 1.5 point difference on the MMSE.

The aim of this first demonstration was to highlight the importance of the methods themselves (i.e., the association between longitudinal and survival outcomes impacts inferences about both) but also to highlight the need to iteratively fit and adjust these models. It is not possible to run a component of these models on its own and then use those results to inform the construction of the joint model as the standalone estimates can be misleading and may not carry over to the joint modeling. While helpful to have an intuitive sense for this, readers may still be concerned about this iterative process, so we want to discuss this is some greater depth.

Readers familiar with the hazards of multiple comparisons and the need to adjust *p*-values can easily be worried that fitting over twenty different models has lead to spurious results. The first response we have to this is that multiple comparisons are not a concern for Bayesian statistics as the probability of Bayes is not concerned with long-run probabilities and the chance of seeing a test statistic by chance under some

arbitrary null hypothesis. For a relatively introductory explanation of why Bayesian methods are not as concerned with multiple comparisons, we recommend Gelman et al. (2009). In short, our aim in constructing models is not to select variables that are significant or models that significantly improve some criterion; instead, we are utilizing the information function of the model to identify the most parsimonious model that incorporates all of our thoughts on the relevant data-generating processes. This use of an information criterion (namely the WAIC) is important as it approximates the leave-one-out cross-validation estimate of the model's out-of-sample performance. These information criteria guard against overfitting a model to the sample by penalizing complexity. Often, we are able to look at the posterior distribution's probability value for a coefficient to screen for predictors that are not likely adding to the model's performance and could thus be removed to improve parsimony without hurting out-of-sample predictive performance. As such, the WAIC is our ultimate standard for model selection rather than an arbitrary threshold for when an effect is "real." In this respect, we aim to have a disciplined and theoretically informed model construction process that then uses the WAIC to inform the next iteration of the model. As this is a grand goal, we wanted to ensure that this document was available to interested readers in order to follow this process. With these points made, we now turn to explicating the models and modeling process.

The Modeling Process

As already noted in the previous examples, the first joint model fit for this study was a multivariate joint model where all static and longitudinal predictors of NH/AL admission found in the literature (and available in the NACC dataset) were included. The results of this model are reported again below:

```
##
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(MMSE, NPIQ, FAQ,
##     CAIDE, UPDRS), time_var = "Time", n_iter = 15000, n_burnin = 50
00,
##     n_thin = 5)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
##   NPIQ: 15597
##   FAQ: 15597
##   CAIDE: 15597
##   UPDRS: 15597
##
##                  DIC     WAIC      LPML
## marginal    366391.2 366310.9 -183156.1
```

```
## conditional 484741.7 471896.9 -245584.4
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
## (Intr) 4.1213 (Intr)    Time  (Intr)    Time  (Intr)    Time  (Intr
)    Time
## Time   1.5757 0.4981
## (Intr) 3.1122 -0.2606 -0.1013
## Time   0.6821 -0.1247 -0.5619 -0.0978
## (Intr) 7.6733 -0.6883 -0.3786 0.5613  0.0095
## Time   2.0718 -0.2883 -0.8198 0.0166  0.6946  0.1647
## (Intr) 1.6329 0.0396  0.1148  0.0739  -0.0274 -0.0186 -0.0996
## Time   0.1747 -0.0873 -0.0312 0.0146  0.0796  0.0799  0.0229  -0.32
48
## (Intr) 0.8920 -0.0932 -0.0332 0.1855  -0.0960 0.2007  -0.0032 0.024
9  -0.0465
## Time   0.3424 -0.3177 -0.4072 0.1606  0.1699  0.2631  0.3575  0.000
4  0.0379
##
##
## (Intr) (Intr)
## Time
## (Intr)
## Time
## (Intr)
## Time
## (Intr)
## Time
## (Intr)
## Time   0.4278
##
## Survival Outcome:
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## Age                 0.0272 0.0076  0.0125  0.0420 0.0000 1.0118
## MarStatSeparated    0.2128 0.1466 -0.0757  0.5001 0.1460 1.0037
## InfAge             -0.0035 0.0041 -0.0119  0.0045 0.3957 1.0032
## InfRelExt           0.1737 0.1618 -0.1465  0.4807 0.2843 1.0068
## InfRelImp           0.4576 0.3058 -0.1908  1.0206 0.1393 0.9998
## LivSitAlone         0.9936 0.1300  0.7324  1.2421 0.0000 1.0103
## SexFemale          -0.0795 0.0973 -0.2718  0.1124 0.4150 1.0026
## RaceNotWhite       -0.8756 0.1528 -1.1717 -0.5812 0.0000 1.0041
## EthnicityHispanic -1.1374 0.2227 -1.6046 -0.7297 0.0000 1.0047
## Edu                 0.0131 0.0139 -0.0147  0.0402 0.3403 1.0016
```

```
## value(MMSE)        0.0068 0.0091 -0.0113  0.0241 0.4463 1.0479
## value(NPIQ)        0.0403 0.0167  0.0064  0.0728 0.0170 1.0175
## value(FAQ)         0.1072 0.0099  0.0871  0.1260 0.0000 1.0535
## value(CAIDE)       0.0061 0.0337 -0.0603  0.0710 0.8630 1.0017
## value(UPDRS)       0.0275 0.0269 -0.0260  0.0804 0.3003 1.0018
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept) 22.0138 0.8269 20.3924 23.5983 0.0000 1.0002
## Time        -1.5532 0.0557 -1.6624 -1.4430 0.0000 1.0003
## AgeB        -0.0128 0.0101 -0.0321  0.0072 0.2117 1.0007
## Edu          0.2521 0.0221  0.2081  0.2949 0.0000 0.9999
## sigma        2.0759 0.0177  2.0419  2.1109 0.0000 1.0014
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                Mean  StDev    2.5%   97.5% P   Rhat
## (Intercept)  7.1061 0.5743  5.9880  8.2494 0 0.9999
## Time         0.3403 0.0398  0.2622  0.4199 0 1.0062
## AgeB        -0.0484 0.0078 -0.0637 -0.0331 0 0.9999
## sigma        2.6735 0.0215  2.6322  2.7158 0 1.0384
##
## Longitudinal Outcome: FAQ (family = gaussian, link = identity)
##                Mean  StDev   2.5%  97.5%     P   Rhat
## (Intercept) 2.7005 1.1564 0.5035 4.9681 0.016 0.9999
## Time        2.3051 0.0868 2.1380 2.4763 0.000 1.0014
## AgeB        0.0819 0.0158 0.0506 0.1119 0.000 0.9998
## sigma       3.2464 0.0265 3.1932 3.2990 0.000 1.0053
##
## Longitudinal Outcome: CAIDE (family = gaussian, link = identity)
##                Mean  StDev   2.5%  97.5%      P   Rhat
## (Intercept) 6.4175 0.2562 5.8976 6.9058 0.0000 1.0009
## Time        0.0419 0.0120 0.0184 0.0652 0.0000 1.0003
## AgeB        0.0066 0.0035 0.0000 0.0137 0.0517 1.0012
## sigma       0.9033 0.0070 0.8896 0.9174 0.0000 1.0070
##
## Longitudinal Outcome: UPDRS (family = gaussian, link = identity)
##                Mean  StDev    2.5%  97.5%      P   Rhat
## (Intercept) -0.0811 0.1985 -0.4725 0.2983 0.6847 1.0002
## Time         0.2128 0.0139  0.1857 0.2400 0.0000 1.0014
## AgeB         0.0072 0.0027  0.0019 0.0126 0.0073 1.0001
## sigma        0.8063 0.0066  0.7937 0.8197 0.0000 1.0008
##
## MCMC summary:
## chains: 3
```

```
## iterations per chain: 15000
## burn-in per chain: 5000
## thinning: 5
## time: 2 hours
```

There were a few notable findings from this initial model. First, it is noted that several of the $\hat{R}$ R^ statistics exceed our desired 1.01 threshold. Examining these specific longitudinal variables that produced these estimation errors, it was suspected that correlation among these variables made the posteriors difficult to sample from. To test this, we examined the raw correlations betweens our continuous variables just at baseline:

```
##        MMSE  NPIQ   FAQ CAIDE UPDRS
## MMSE   1.00 -0.19 -0.60 -0.01 -0.07
## NPIQ  -0.19  1.00  0.40  0.05  0.11
## FAQ   -0.60  0.40  1.00 -0.02  0.14
## CAIDE -0.01  0.05 -0.02  1.00  0.03
## UPDRS -0.07  0.11  0.14  0.03  1.00
```

As can be seen from these correlations, there is a fairly strong negative correlation between the MMSE and FAQ scores, which is an expected observation. There was another appreciable, though smaller, correlation between the NPIQ and FAQ. To test whether these correlations were affecting the results, we ran the next several models under varying permutations of the longitudinal predictors. Additionally, we removed those predictors that were robustly non-significant in the first model. The results of these permuted models is shown below:

```
##
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(MMSE, FAQ, CAIDE,
##     UPDRS), time_var = "Time", n_iter = 15000, n_burnin = 5000,
##     n_thin = 5)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
##   FAQ: 15597
##   CAIDE: 15597
##   UPDRS: 15597
##
##                  DIC     WAIC      LPML
## marginal    284368.7 284195.1 -142097.9
## conditional 374697.4 364324.2 -190977.7
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
```

```
## (Intr) 4.1245 (Intr)    Time  (Intr)    Time  (Intr)    Time  (Intr
)
## Time   1.5774 0.4977
## (Intr) 7.6751 -0.6894 -0.3791
## Time   2.0553 -0.2895 -0.8224 0.1685
## (Intr) 1.6333 0.0388  0.1138  -0.0200 -0.0957
## Time   0.1766 -0.0790 -0.0271 0.0798  0.0141  -0.3237
## (Intr) 0.8931 -0.0933 -0.0304 0.2003  -0.0051 0.0241  -0.0449
## Time   0.3422 -0.3203 -0.4097 0.2653  0.3601  0.0001  0.0327  0.424
9
##
## Survival Outcome:
##                      Mean  StDev    2.5%   97.5%      P   Rhat
## Age                0.0237 0.0067  0.0109  0.0368 0.0007 1.0026
## LivSitAlone        1.1668 0.1076  0.9586  1.3736 0.0000 1.0025
## RaceNotWhite      -0.8339 0.1486 -1.1331 -0.5569 0.0000 1.0011
## EthnicityHispanic -1.1088 0.2162 -1.5486 -0.6953 0.0000 1.0007
## value(MMSE)        0.0141 0.0087 -0.0027  0.0315 0.1030 1.0358
## value(FAQ)         0.1187 0.0081  0.1034  0.1350 0.0000 1.0442
## value(CAIDE)       0.0146 0.0304 -0.0456  0.0745 0.6223 1.0032
## value(UPDRS)       0.0350 0.0256 -0.0157  0.0853 0.1713 1.0011
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept) 21.9921 0.8222 20.4095 23.6354 0.0000 1.0000
## Time        -1.5568 0.0559 -1.6669 -1.4475 0.0000 1.0006
## AgeB        -0.0128 0.0100 -0.0323  0.0068 0.1983 1.0000
## Edu          0.2536 0.0222  0.2096  0.2964 0.0000 0.9998
## sigma        2.0736 0.0181  2.0390  2.1098 0.0000 1.0013
##
## Longitudinal Outcome: FAQ (family = gaussian, link = identity)
##                Mean  StDev   2.5%  97.5%     P   Rhat
## (Intercept) 2.6832 1.1238 0.4853 4.8454 0.017 1.0006
## Time        2.3068 0.0860 2.1401 2.4723 0.000 1.0001
## AgeB        0.0820 0.0153 0.0523 0.1125 0.000 1.0008
## sigma       3.2543 0.0274 3.2005 3.3096 0.000 1.0012
##
## Longitudinal Outcome: CAIDE (family = gaussian, link = identity)
##                Mean  StDev    2.5%  97.5%      P   Rhat
## (Intercept) 6.4170 0.2607  5.9112 6.9303 0.0000 1.0001
## Time        0.0413 0.0122  0.0173 0.0652 0.0000 1.0079
## AgeB        0.0066 0.0035 -0.0004 0.0135 0.0647 1.0005
## sigma       0.9022 0.0070  0.8885 0.9160 0.0000 1.0009
##
```

```
## Longitudinal Outcome: UPDRS (family = gaussian, link = identity)
##                Mean  StDev    2.5%  97.5%      P   Rhat
## (Intercept) -0.0879 0.2020 -0.4826 0.3030 0.6660 1.0002
## Time         0.2123 0.0139  0.1854 0.2389 0.0000 1.0025
## AgeB         0.0073 0.0027  0.0020 0.0126 0.0077 1.0000
## sigma        0.8061 0.0066  0.7934 0.8193 0.0000 0.9997
##
## MCMC summary:
## chains: 3
## iterations per chain: 15000
## burn-in per chain: 5000
## thinning: 5
## time: 1.3 hours
##
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(MMSE, NPIQ, CAIDE,
##     UPDRS), time_var = "Time", n_iter = 15000, n_burnin = 5000,
##     n_thin = 5)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
##   NPIQ: 15597
##   CAIDE: 15597
##   UPDRS: 15597
##
##                  DIC     WAIC      LPML
## marginal    273258.5 273183.0 -136592.2
## conditional 356528.6 346153.2 -182025.1
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
## (Intr) 4.0922 (Intr)    Time  (Intr)    Time  (Intr)    Time  (Intr
)
## Time   1.5883 0.5194
## (Intr) 3.1126 -0.2515 -0.1045
## Time   0.6710 -0.1596 -0.5741 -0.0893
## (Intr) 1.6342 0.0597  0.1172  0.0756  -0.0340
## Time   0.1772 -0.0822 -0.0299 0.0198  0.0740  -0.3266
## (Intr) 0.8922 -0.0889 -0.0391 0.1836  -0.0785 0.0238  -0.0428
## Time   0.3419 -0.3306 -0.4064 0.1635  0.1670  -0.0001 0.0336  0.430
0
```

```
##
## Survival Outcome:
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## Age                 0.0442 0.0067  0.0312  0.0574 0.0000 1.0066
## LivSitAlone         1.0052 0.1043  0.7970  1.2086 0.0000 1.0062
## RaceNotWhite       -0.9552 0.1449 -1.2541 -0.6784 0.0000 1.0007
## EthnicityHispanic  -0.9228 0.2113 -1.3510 -0.5200 0.0000 1.0010
## value(MMSE)        -0.0761 0.0054 -0.0866 -0.0654 0.0000 1.0037
## value(NPIQ)         0.1310 0.0126  0.1064  0.1558 0.0000 1.0034
## value(CAIDE)       -0.0505 0.0301 -0.1086  0.0087 0.0957 1.0043
## value(UPDRS)        0.0219 0.0267 -0.0305  0.0737 0.4157 1.0023
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                 Mean  StDev    2.5%   97.5%     P   Rhat
## (Intercept) 20.4830 0.8851 18.7641 22.2289 0.000 0.9998
## Time        -1.5476 0.0555 -1.6536 -1.4383 0.000 1.0012
## AgeB        -0.0150 0.0103 -0.0354  0.0052 0.141 1.0006
## Edu          0.3642 0.0276  0.3098  0.4172 0.000 1.0001
## sigma        2.0738 0.0181  2.0389  2.1096 0.000 1.0004
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                 Mean  StDev    2.5%   97.5% P   Rhat
## (Intercept)  6.9048 0.5630  5.8136  7.9829 0 0.9997
## Time         0.3329 0.0404  0.2540  0.4128 0 1.0002
## AgeB        -0.0454 0.0076 -0.0599 -0.0304 0 0.9998
## sigma        2.6805 0.0213  2.6388  2.7223 0 1.0161
##
## Longitudinal Outcome: CAIDE (family = gaussian, link = identity)
##                Mean  StDev    2.5%  97.5%      P   Rhat
## (Intercept) 6.4581 0.2600  5.9552 6.9672 0.0000 1.0000
## Time        0.0413 0.0125  0.0168 0.0658 0.0007 1.0034
## AgeB        0.0061 0.0035 -0.0010 0.0130 0.0867 1.0003
## sigma       0.9020 0.0071  0.8884 0.9158 0.0000 1.0044
##
## Longitudinal Outcome: UPDRS (family = gaussian, link = identity)
##                 Mean  StDev    2.5%  97.5%      P   Rhat
## (Intercept) -0.0945 0.2025 -0.4882 0.3106 0.6423 1.0004
## Time         0.2121 0.0139  0.1852 0.2389 0.0000 1.0009
## AgeB         0.0074 0.0028  0.0019 0.0128 0.0050 1.0001
## sigma        0.8065 0.0066  0.7938 0.8200 0.0000 1.0147
##
## MCMC summary:
## chains: 3
## iterations per chain: 15000
```

```
## burn-in per chain: 5000
## thinning: 5
## time: 1.3 hours
##
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(NPIQ, FAQ, CAIDE,
##     UPDRS), time_var = "Time", n_iter = 15000, n_burnin = 5000,
##     n_thin = 5)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   NPIQ: 15597
##   FAQ: 15597
##   CAIDE: 15597
##   UPDRS: 15597
##
##                  DIC     WAIC      LPML
## marginal    286518.4 286471.1 -143236.0
## conditional 376971.4 366593.6 -192171.7
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
## (Intr) 3.1157 (Intr)    Time  (Intr)    Time  (Intr)    Time  (Intr
)
## Time   0.6879 -0.1024
## (Intr) 7.6934 0.5603  0.0075
## Time   2.0320 0.0127  0.6872  0.1451
## (Intr) 1.6346 0.0758  -0.0300 -0.0194 -0.0927
## Time   0.1763 0.0115  0.0718  0.0761  0.0099  -0.3268
## (Intr) 0.8931 0.1885  -0.1025 0.2028  -0.0096 0.0224  -0.0422
## Time   0.3393 0.1574  0.1595  0.2535  0.3465  0.0036  0.0328  0.429
4
##
## Survival Outcome:
##                      Mean  StdDev    2.5%   97.5%      P   Rhat
## Age                0.0240 0.0070  0.0104  0.0377 0.0003 1.0035
## LivSitAlone        1.1598 0.1120  0.9482  1.3798 0.0000 1.0055
## RaceNotWhite      -0.8340 0.1515 -1.1415 -0.5391 0.0000 1.0003
## EthnicityHispanic -1.1847 0.2184 -1.6339 -0.7706 0.0000 1.0005
## value(NPIQ)        0.0376 0.0153  0.0086  0.0683 0.0123 1.0099
## value(FAQ)         0.1040 0.0062  0.0921  0.1161 0.0000 1.0097
## value(CAIDE)       0.0055 0.0309 -0.0562  0.0655 0.8617 1.0048
```

```
## value(UPDRS)       0.0418 0.0251 -0.0075  0.0899 0.0997 1.0032
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                Mean  StDev    2.5%   97.5% P   Rhat
## (Intercept)  7.5696 0.5791  6.4198  8.7094 0 1.0009
## Time         0.3385 0.0412  0.2586  0.4204 0 1.0019
## AgeB        -0.0547 0.0079 -0.0701 -0.0392 0 1.0009
## sigma        2.6696 0.0217  2.6271  2.7114 0 1.0055
##
## Longitudinal Outcome: FAQ (family = gaussian, link = identity)
##                Mean  StDev   2.5%  97.5%      P   Rhat
## (Intercept) 6.1002 1.2357 3.6843 8.5464 0.0000 0.9995
## Time        2.2560 0.0885 2.0787 2.4250 0.0000 1.0012
## AgeB        0.0353 0.0168 0.0017 0.0680 0.0383 0.9995
## sigma       3.2382 0.0274 3.1856 3.2928 0.0000 1.0003
##
## Longitudinal Outcome: CAIDE (family = gaussian, link = identity)
##                Mean  StDev   2.5%  97.5%     P   Rhat
## (Intercept) 6.2730 0.2539 5.7860 6.7764 0.000 0.9997
## Time        0.0409 0.0123 0.0172 0.0648 0.002 1.0003
## AgeB        0.0086 0.0034 0.0017 0.0153 0.015 0.9998
## sigma       0.9023 0.0070 0.8888 0.9159 0.000 1.0075
##
## Longitudinal Outcome: UPDRS (family = gaussian, link = identity)
##                Mean  StDev    2.5%  97.5%      P   Rhat
## (Intercept) -0.0147 0.2042 -0.4247 0.3836 0.9443 1.0005
## Time         0.2070 0.0138  0.1799 0.2343 0.0000 1.0016
## AgeB         0.0063 0.0028  0.0008 0.0119 0.0247 1.0009
## sigma        0.8063 0.0067  0.7931 0.8194 0.0000 1.0021
##
## MCMC summary:
## chains: 3
## iterations per chain: 15000
## burn-in per chain: 5000
## thinning: 5
## time: 1.4 hours
```

As suspected, the estimation problem appeared to be due to the overlap of the MMSE and FAQ as the only model that produced these elevated $\hat{R}$ R^ statistics was that model, though the FAQ itself may be somewhat problematic as the model with it had a trending $\hat{R}$ R^. Now that multiple models were available, it was appropriate to compare the results. We will demonstrate the code for this here but then not repeat it later as it is rather uninteresting:

```
compare_jm(JMfit1, JMfit2, JMfit3, JMfit4, order = "WAIC")
```

```
##
##              DIC     WAIC      LPML
##  JMfit3 273258.5 273183.0 -136592.2
##  JMfit2 284368.7 284195.1 -142097.9
##  JMfit4 286518.4 286471.1 -143236.0
##  JMfit1 366391.2 366310.9 -183156.1
##
## The criteria are calculated based on the marginal log-likelihood.
```

Based on the comparisons of these models, the joint model that included the MMSE, NPIQ, CAIDE, and UPDRS was most preferred by the WAIC (and the Deviance Information Criterion [DIC] and log pseudo-marginal likelihood [LPML]). Inspecting this model more closely, the UPDRS appeared to contribute none at all to the prediction while the CAIDE had a relatively probably overall effect. Note that the Bayesian credible intervals can be treated as a gross summary of the overall posterior for that parameter and have a unique interpretation compared to traditional confidence intervals. In this case, the posterior intervals indicate that there is a 95% probability that the CAIDE's coefficient is between -0.1086 and 0.0087. While the model does place a value of zero within this credible interval, there is an appreciable probability that the coefficient is negative. Based on this, the CAIDE was retained in the models. An important feature of the joint model is its association structure. By default, `JMbayes2` uses the current value of longitudinal components in the survival component; however, two other common structures are the slope (derivative of the longitudinal model) and area (integral of the longitudinal model). The models were therefore updated to test whether these alternative association structure were more appropriate.

```
##
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(MMSE, NPIQ, CAIDE),
##     time_var = "Time", functional_forms = fForms, n_iter = 15000,
##     n_burnin = 5000, n_thin = 5)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
##   NPIQ: 15597
##   CAIDE: 15597
##
##                  DIC     WAIC      LPML
## marginal    225315.0 225216.6 -112608.6
## conditional 297337.6 289552.4 -152797.6
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
## (Intr) 4.0712 (Intr)    Time  (Intr)    Time  (Intr)
```

```
## Time   1.5857 0.5436
## (Intr) 3.1124 -0.2375 -0.1394
## Time   0.6593 -0.1844 -0.5722 -0.0814
## (Intr) 1.6300 0.0634  0.1113  0.0676  -0.0322
## Time   0.1724 -0.1019 -0.0224 0.0773  0.0586  -0.3155
##
## Survival Outcome:
##                      Mean  StDev    2.5%   97.5%      P   Rhat
## Age                0.0441 0.0067  0.0312  0.0570 0.0000 1.0007
## LivSitAlone        0.9641 0.1081  0.7453  1.1726 0.0000 1.0034
## RaceNotWhite      -0.7866 0.1439 -1.0743 -0.5085 0.0000 1.0027
## EthnicityHispanic -0.5187 0.2075 -0.9402 -0.1237 0.0067 1.0005
## slope(MMSE)       -0.7145 0.0612 -0.8395 -0.5967 0.0000 1.0844
## slope(NPIQ)       -0.1812 0.2209 -0.6281  0.2436 0.3987 1.0880
## slope(CAIDE)       1.6125 0.6885  0.3020  2.9951 0.0110 1.0112
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                Mean  StDev    2.5%   97.5%     P   Rhat
## (Intercept) 20.8699 0.8957 19.1018 22.6496 0.000 1.0002
## Time        -1.5627 0.0559 -1.6718 -1.4514 0.000 1.0008
## AgeB        -0.0193 0.0104 -0.0396  0.0009 0.063 0.9998
## Edu          0.3595 0.0283  0.3051  0.4148 0.000 1.0006
## sigma        2.0840 0.0178  2.0500  2.1191 0.000 1.0007
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                Mean  StDev    2.5%   97.5% P   Rhat
## (Intercept)  6.6859 0.5661  5.5972  7.7527 0 1.0009
## Time         0.3295 0.0402  0.2500  0.4075 0 1.0018
## AgeB        -0.0423 0.0077 -0.0569 -0.0274 0 1.0004
## sigma        2.6805 0.0219  2.6384  2.7245 0 1.0087
##
## Longitudinal Outcome: CAIDE (family = gaussian, link = identity)
##                Mean  StDev    2.5%  97.5%      P   Rhat
## (Intercept) 6.4117 0.2566  5.9057 6.9214 0.0000 0.9997
## Time        0.0452 0.0121  0.0219 0.0695 0.0003 1.0005
## AgeB        0.0067 0.0035 -0.0002 0.0135 0.0573 0.9997
## sigma       0.9043 0.0071  0.8906 0.9188 0.0000 1.0095
##
## MCMC summary:
## chains: 3
## iterations per chain: 15000
## burn-in per chain: 5000
## thinning: 5
## time: 48.8 min
```

```
##
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(MMSE, NPIQ, CAIDE),
##     time_var = "Time", functional_forms = fForms, n_iter = 15000,
##     n_burnin = 5000, n_thin = 5)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
##   NPIQ: 15597
##   CAIDE: 15597
##
##                  DIC     WAIC      LPML
## marginal    225270.8 225165.4 -112582.8
## conditional 297699.9 289898.0 -152964.9
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
## (Intr) 4.0854 (Intr)    Time  (Intr)    Time  (Intr)
## Time   1.5843 0.5249
## (Intr) 3.1066 -0.2477 -0.1076
## Time   0.6675 -0.1693 -0.5736 -0.0844
## (Intr) 1.6335 0.0599  0.1181  0.0760  -0.0350
## Time   0.1750 -0.0877 -0.0320 0.0211  0.0683  -0.3262
##
## Survival Outcome:
##                      Mean  StDev    2.5%   97.5%      P   Rhat
## Age                0.0404 0.0066  0.0275  0.0529 0.0000 1.0006
## LivSitAlone        0.9861 0.1047  0.7758  1.1869 0.0000 1.0009
## RaceNotWhite      -1.0062 0.1435 -1.2952 -0.7334 0.0000 1.0009
## EthnicityHispanic -0.9656 0.2046 -1.3786 -0.5774 0.0000 0.9999
## area(MMSE)        -0.1100 0.0060 -0.1215 -0.0979 0.0000 1.0022
## area(NPIQ)         0.1336 0.0136  0.1062  0.1602 0.0000 1.0043
## area(CAIDE)       -0.0620 0.0272 -0.1149 -0.0096 0.0207 1.0003
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                 Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept) 20.8155 0.9077 19.0060 22.6046 0.0000 1.0008
## Time        -1.5414 0.0564 -1.6496 -1.4305 0.0000 1.0000
## AgeB        -0.0192 0.0105 -0.0396  0.0016 0.0707 0.9997
## Edu          0.3620 0.0285  0.3055  0.4189 0.0000 1.0024
## sigma        2.0758 0.0179  2.0410  2.1115 0.0000 1.0017
```

```
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                Mean  StDev    2.5%   97.5% P   Rhat
## (Intercept)  6.7642 0.5677  5.6616  7.8489 0 1.0004
## Time         0.3317 0.0404  0.2541  0.4099 0 0.9996
## AgeB        -0.0434 0.0077 -0.0583 -0.0287 0 1.0001
## sigma        2.6791 0.0213  2.6386  2.7214 0 1.0021
##
## Longitudinal Outcome: CAIDE (family = gaussian, link = identity)
##                Mean  StDev    2.5%  97.5%     P   Rhat
## (Intercept) 6.4294 0.2558  5.9262 6.9357 0.000 1.0000
## Time        0.0422 0.0121  0.0188 0.0661 0.001 1.0011
## AgeB        0.0065 0.0035 -0.0004 0.0132 0.062 1.0001
## sigma       0.9028 0.0070  0.8892 0.9169 0.000 1.0074
##
## MCMC summary:
## chains: 3
## iterations per chain: 15000
## burn-in per chain: 5000
## thinning: 5
## time: 49.2 min
```

It is noted that the slope estimates introduced some estimation problems. Ultimately, we did not expect that the slope association structure would be appropriate for the data as these scales are not known for having significant change over time; instead, current values or the accumulation of gradual change were thought to be more relevant. We now compare the new results:

```
##
##              DIC     WAIC      LPML
##  JMfit6 225270.8 225165.4 -112582.8
##  JMfit5 225315.0 225216.6 -112608.6
##  JMfit3 273258.5 273183.0 -136592.2
##
## The criteria are calculated based on the marginal log-likelihood.
```

Somewhat surprising with these results was that both the area (JMfit6) and slope (JMfit5) were dramatically more preferred to the current value model, though the area association structure was most preferred. One potential reason that these alternative associations performed better is that the predictors of the longitudinal models are not accounting for some important factors that we know are related to estimates of these scores. In other words, there may be spuriously dramatic change in the longitudinal models because the predictions are poor or because the prediction are omitting relevant confounders of this process. So, the next step was to add race, ethnicity, and sex as predictors to the longitudinal components:

```
##
```

```
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(MMSE, NPIQ, CAIDE),
##     time_var = "Time", functional_forms = fForms, n_iter = 15000,
##     n_burnin = 5000, n_thin = 5)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
##   NPIQ: 15597
##   CAIDE: 15597
##
##                  DIC     WAIC      LPML
## marginal    224365.0 224139.3 -112070.1
## conditional 297103.4 289424.3 -152846.4
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
## (Intr) 4.0757 (Intr)    Time  (Intr)    Time  (Intr)
## Time   1.5882 0.5383
## (Intr) 3.0713 -0.2459 -0.1178
## Time   0.6762 -0.1761 -0.5658 -0.0879
## (Intr) 1.4665 0.0731  0.1093  0.0161  -0.0297
## Time   0.1762 -0.0798 -0.0319 0.0019  0.0670  -0.3792
##
## Survival Outcome:
##                      Mean  StdDev    2.5%   97.5%      P   Rhat
## Age                0.0406 0.0066  0.0278  0.0537 0.0000 1.0035
## LivSitAlone        0.9842 0.1048  0.7809  1.1936 0.0000 1.0020
## RaceNotWhite      -0.9936 0.1411 -1.2668 -0.7163 0.0000 0.9998
## EthnicityHispanic -0.9846 0.2079 -1.4051 -0.5936 0.0000 1.0017
## area(MMSE)        -0.1100 0.0060 -0.1214 -0.0980 0.0000 1.0103
## area(NPIQ)         0.1325 0.0135  0.1064  0.1594 0.0000 1.0005
## area(CAIDE)       -0.0670 0.0280 -0.1227 -0.0141 0.0127 0.9997
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                      Mean  StdDev    2.5%   97.5%      P   Rhat
## (Intercept)       21.6208 0.9172 19.7885 23.4288 0.0000 0.9999
## Time              -1.5424 0.0553 -1.6485 -1.4332 0.0000 1.0004
## AgeB              -0.0196 0.0105 -0.0403  0.0009 0.0613 0.9996
## Edu                0.3234 0.0304  0.2622  0.3812 0.0000 1.0004
## RaceNotWhite      -0.7820 0.2732 -1.3260 -0.2353 0.0040 1.0011
## EthnicityHispanic -1.3557 0.4115 -2.1664 -0.5329 0.0010 1.0008
```

```
## sigma              2.0802 0.0181  2.0449  2.1157 0.0000 1.0016
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                      Mean  StDev    2.5%   97.5% P   Rhat
## (Intercept)        8.2229 0.6838  6.8875  9.5774 0 1.0018
## Time               0.3308 0.0407  0.2511  0.4107 0 1.0003
## AgeB              -0.0449 0.0077 -0.0600 -0.0296 0 1.0022
## Edu               -0.0756 0.0220 -0.1187 -0.0321 0 0.9997
## EthnicityHispanic  1.2092 0.2967  0.6347  1.7916 0 0.9997
## SexFemale         -0.6448 0.1375 -0.9134 -0.3774 0 1.0005
## sigma              2.6782 0.0212  2.6372  2.7201 0 1.0009
##
## Longitudinal Outcome: CAIDE (family = gaussian, link = identity)
##                      Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)        9.1674 0.2821  8.6229  9.7206 0.0000 1.0003
## Time               0.0411 0.0122  0.0179  0.0654 0.0007 1.0013
## AgeB               0.0038 0.0031 -0.0024  0.0098 0.2233 0.9998
## Edu               -0.1367 0.0092 -0.1545 -0.1185 0.0000 0.9998
## RaceNotWhite       0.2718 0.0828  0.1116  0.4380 0.0013 1.0002
## EthnicityHispanic  0.5172 0.1231  0.2767  0.7575 0.0000 0.9999
## SexFemale         -1.2333 0.0568 -1.3470 -1.1224 0.0000 0.9999
## sigma              0.9050 0.0071  0.8913  0.9191 0.0000 1.0139
##
## MCMC summary:
## chains: 3
## iterations per chain: 15000
## burn-in per chain: 5000
## thinning: 5
## time: 56.2 min
```

There is clear evidence for the effect of these additional variables as expected, and we can then confirm whether this additional complexity actually improves the model performance or not by comparing this result to our previous best fitting model:

```
##
##              DIC     WAIC      LPML
##  JMfit7 224365.0 224139.3 -112070.1
##  JMfit6 225270.8 225165.4 -112582.8
##
## The criteria are calculated based on the marginal log-likelihood.
```

In this case, we see that the extra predictors in the longitudinal components improved the model performance. With this more accurate longitudinal prediction component, we wanted to then test whether the area remained the preferred association structure, so we ran the same joint models but under the current value and slope structures. These results and comparisons are shown here:

```
##
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(MMSE, NPIQ, CAIDE),
##     time_var = "Time", functional_forms = fForms, n_iter = 15000,
##     n_burnin = 5000, n_thin = 5)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
##   NPIQ: 15597
##   CAIDE: 15597
##
##                  DIC     WAIC      LPML
## marginal    224426.0 224199.6 -112100.2
## conditional 296803.8 289116.1 -152736.9
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
## (Intr) 4.0610 (Intr)    Time  (Intr)    Time  (Intr)
## Time   1.5880 0.5563
## (Intr) 3.0794 -0.2349 -0.1503
## Time   0.6680 -0.1873 -0.5685 -0.0888
## (Intr) 1.4651 0.0763  0.1030  0.0104  -0.0304
## Time   0.1752 -0.0875 -0.0282 0.0308  0.0665  -0.3735
##
## Survival Outcome:
##                       Mean  StdDev    2.5%   97.5%      P   Rhat
## Age                 0.0442 0.0068  0.0313  0.0574 0.0000 1.0043
## LivSitAlone         0.9722 0.1057  0.7624  1.1711 0.0000 1.0007
## RaceNotWhite       -0.7490 0.1444 -1.0305 -0.4737 0.0000 1.0001
## EthnicityHispanic -0.4576 0.2042 -0.8679 -0.0774 0.0200 1.0000
## slope(MMSE)        -0.7280 0.0612 -0.8551 -0.6119 0.0000 1.0832
## slope(NPIQ)        -0.2268 0.2158 -0.6770  0.1626 0.2773 1.0941
## slope(CAIDE)        0.8728 0.6994 -0.5714  2.2659 0.2040 1.0076
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                       Mean  StdDev    2.5%   97.5%      P   Rhat
## (Intercept)        21.6885 0.9095 19.9034 23.5011 0.0000 1.0009
## Time               -1.5636 0.0543 -1.6696 -1.4542 0.0000 1.0008
## AgeB               -0.0196 0.0105 -0.0402  0.0010 0.0613 1.0002
## Edu                 0.3199 0.0299  0.2609  0.3777 0.0000 1.0006
## RaceNotWhite       -0.7899 0.2725 -1.3246 -0.2681 0.0040 1.0011
```

```
## EthnicityHispanic -1.3776 0.4085 -2.1886 -0.5748 0.0007 1.0002
## sigma              2.0895 0.0185  2.0542  2.1258 0.0000 1.0029
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%     P   Rhat
## (Intercept)         8.1649 0.6800  6.8071  9.4837 0e+00 1.0017
## Time                0.3264 0.0406  0.2462  0.4065 0e+00 1.0011
## AgeB               -0.0438 0.0076 -0.0586 -0.0286 0e+00 1.0023
## Edu                -0.0766 0.0217 -0.1194 -0.0339 3e-04 1.0000
## EthnicityHispanic   1.2211 0.2929  0.6432  1.7949 0e+00 1.0004
## SexFemale          -0.6553 0.1380 -0.9199 -0.3817 0e+00 1.0006
## sigma               2.6801 0.0215  2.6389  2.7227 0e+00 1.0056
##
## Longitudinal Outcome: CAIDE (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)         9.1565 0.2830  8.6089  9.7126 0.0000 1.0003
## Time                0.0427 0.0123  0.0186  0.0675 0.0003 1.0007
## AgeB                0.0039 0.0031 -0.0024  0.0100 0.2157 1.0000
## Edu                -0.1368 0.0093 -0.1546 -0.1186 0.0000 0.9998
## RaceNotWhite        0.2723 0.0837  0.1111  0.4374 0.0007 1.0007
## EthnicityHispanic   0.5169 0.1231  0.2772  0.7604 0.0000 0.9998
## SexFemale          -1.2317 0.0568 -1.3456 -1.1220 0.0000 0.9998
## sigma               0.9054 0.0070  0.8918  0.9198 0.0000 1.0149
##
## MCMC summary:
## chains: 3
## iterations per chain: 15000
## burn-in per chain: 5000
## thinning: 5
## time: 1 hours
##
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(MMSE, NPIQ, CAIDE),
##     time_var = "Time", functional_forms = fForms, n_iter = 15000,
##     n_burnin = 5000, n_thin = 5)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
##   NPIQ: 15597
##   CAIDE: 15597
##
##                  DIC     WAIC      LPML
```

```
## marginal    224344.3 224114.4 -112057.7
## conditional 296973.5 289287.5 -152774.0
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
## (Intr) 4.0795 (Intr)    Time  (Intr)    Time  (Intr)
## Time   1.5887 0.5372
## (Intr) 3.0758 -0.2500 -0.1171
## Time   0.6739 -0.1680 -0.5678 -0.0890
## (Intr) 1.4659 0.0743  0.1081  0.0154  -0.0289
## Time   0.1760 -0.0829 -0.0342 0.0004  0.0724  -0.3773
##
## Survival Outcome:
##                      Mean  StdDev    2.5%   97.5%    P   Rhat
## Age                0.0450 0.0069  0.0317  0.0588 0.00 1.0014
## LivSitAlone        1.0036 0.1049  0.7907  1.2054 0.00 0.9999
## RaceNotWhite      -0.9276 0.1426 -1.2101 -0.6496 0.00 1.0001
## EthnicityHispanic -0.9307 0.2137 -1.3646 -0.5215 0.00 1.0010
## value(MMSE)       -0.0786 0.0050 -0.0883 -0.0689 0.00 1.0171
## value(NPIQ)        0.1273 0.0127  0.1020  0.1525 0.00 1.0052
## value(CAIDE)      -0.0524 0.0303 -0.1128  0.0065 0.08 1.0016
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                      Mean  StdDev    2.5%   97.5%      P   Rhat
## (Intercept)       21.6222 0.9169 19.8066 23.4052 0.0000 1.0012
## Time              -1.5465 0.0553 -1.6524 -1.4354 0.0000 1.0017
## AgeB              -0.0197 0.0105 -0.0408  0.0011 0.0587 1.0006
## Edu                0.3242 0.0304  0.2638  0.3838 0.0000 1.0001
## RaceNotWhite      -0.7843 0.2702 -1.3236 -0.2522 0.0043 1.0013
## EthnicityHispanic -1.3528 0.4133 -2.1467 -0.5515 0.0013 1.0010
## sigma              2.0802 0.0183  2.0454  2.1163 0.0000 1.0005
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                      Mean  StdDev    2.5%   97.5% P   Rhat
## (Intercept)        8.2383 0.6864  6.8925  9.5849 0 1.0024
## Time               0.3341 0.0406  0.2534  0.4128 0 0.9999
## AgeB              -0.0450 0.0077 -0.0600 -0.0297 0 1.0022
## Edu               -0.0762 0.0221 -0.1197 -0.0327 0 1.0000
## EthnicityHispanic  1.2062 0.2981  0.6159  1.7909 0 1.0001
## SexFemale         -0.6462 0.1381 -0.9171 -0.3751 0 1.0005
## sigma              2.6810 0.0214  2.6402  2.7238 0 0.9998
##
## Longitudinal Outcome: CAIDE (family = gaussian, link = identity)
```

```
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)         9.1658 0.2829  8.6265  9.7181 0.0000 1.0002
## Time                0.0410 0.0121  0.0175  0.0647 0.0003 1.0005
## AgeB                0.0038 0.0031 -0.0022  0.0098 0.2363 0.9996
## Edu                -0.1367 0.0093 -0.1547 -0.1186 0.0000 1.0001
## RaceNotWhite        0.2725 0.0830  0.1111  0.4370 0.0007 1.0001
## EthnicityHispanic   0.5182 0.1226  0.2810  0.7625 0.0000 1.0002
## SexFemale          -1.2336 0.0565 -1.3463 -1.1242 0.0000 0.9999
## sigma               0.9052 0.0071  0.8915  0.9196 0.0000 1.0123
##
## MCMC summary:
## chains: 3
## iterations per chain: 15000
## burn-in per chain: 5000
## thinning: 5
## time: 1 hours
##
##              DIC     WAIC      LPML
##  JMfit9 224344.3 224114.4 -112057.7
##  JMfit7 224365.0 224139.3 -112070.1
##  JMfit8 224426.0 224199.6 -112100.2
##
## The criteria are calculated based on the marginal log-likelihood.
```

With these new changes, the current value model is again preferred. Another factor to consider in the longitudinal models is the potential for an interaction with the time variable, meaning that individuals with certain other traits may have unique trajectories in their longitudinal change. To test this potential, saturated longitudinal components were each predictor interacted with time was fit. The results are shown here:

```
##
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(MMSE, NPIQ, CAIDE),
##     time_var = "Time", functional_forms = fForms, n_iter = 15000,
##     n_burnin = 5000, n_thin = 5)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
##   NPIQ: 15597
##   CAIDE: 15597
##
##                  DIC     WAIC      LPML
## marginal    224592.8 224228.7 -112190.4
## conditional 297961.6 290639.5 -154048.2
```

```
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
## (Intr) 4.0796 (Intr)    Time  (Intr)    Time  (Intr)
## Time   1.5658 0.5353
## (Intr) 3.0759 -0.2488 -0.1157
## Time   0.6741 -0.1704 -0.5528 -0.0896
## (Intr) 1.4653 0.0748  0.1036  0.0151  -0.0232
## Time   0.1765 -0.0899 -0.0251 0.0009  0.0622  -0.3751
##
## Survival Outcome:
##                       Mean  StDev    2.5%   97.5%     P   Rhat
## Age                 0.0459 0.0071  0.0322  0.0598 0.000 1.0062
## LivSitAlone         1.0015 0.1077  0.7906  1.2077 0.000 1.0016
## RaceNotWhite       -0.9182 0.1512 -1.2241 -0.6322 0.000 0.9999
## EthnicityHispanic -0.8999 0.2358 -1.3821 -0.4548 0.000 1.0004
## value(MMSE)        -0.0788 0.0050 -0.0890 -0.0691 0.000 1.0028
## value(NPIQ)         0.1267 0.0125  0.1027  0.1513 0.000 1.0014
## value(CAIDE)       -0.0537 0.0304 -0.1137  0.0053 0.078 1.0011
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)        20.4974 1.0510 18.4904 22.5313 0.0000 1.0007
## Time               -3.5266 0.5427 -4.6023 -2.4821 0.0000 1.0006
## AgeB               -0.0048 0.0119 -0.0279  0.0182 0.6827 1.0009
## Edu                 0.3242 0.0349  0.2548  0.3921 0.0000 1.0005
## RaceNotWhite       -0.5944 0.3007 -1.1874 -0.0156 0.0443 1.0002
## EthnicityHispanic -1.1623 0.4854 -2.1071 -0.2059 0.0170 0.9999
## Time:AgeB           0.0262 0.0061  0.0145  0.0384 0.0000 1.0006
## Time:Edu           -0.0005 0.0177 -0.0356  0.0339 0.9827 1.0005
## Time:RaceNotWhite   0.3677 0.1417  0.0862  0.6421 0.0103 0.9999
## T:EH                0.3250 0.2389 -0.1384  0.8059 0.1673 0.9997
## sigma               2.0999 0.0192  2.0641  2.1385 0.0000 0.9997
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)         8.8220 0.8871  7.0884 10.5853 0.0000 0.9999
## Time                0.4086 0.3910 -0.3576  1.1619 0.2877 1.0012
## AgeB               -0.0525 0.0100 -0.0722 -0.0331 0.0000 1.0002
## Edu                -0.0802 0.0297 -0.1374 -0.0218 0.0070 1.0007
## EthnicityHispanic  1.1947 0.3997  0.4132  1.9709 0.0020 1.0005
## SexFemale          -0.5974 0.1836 -0.9542 -0.2380 0.0013 1.0008
## Time:AgeB          -0.0017 0.0045 -0.0104  0.0071 0.7060 1.0029
```

```
## Time:Edu           0.0052 0.0127 -0.0196  0.0306 0.6820 1.0014
## T:EH              -0.0959 0.1715 -0.4389  0.2351 0.5763 1.0018
## Time:SexFemale    -0.0467 0.0729 -0.1907  0.0951 0.5167 0.9998
## sigma              2.6819 0.0214  2.6413  2.7248 0.0000 1.0033
##
## Longitudinal Outcome: CAIDE (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)         8.8179 0.3638  8.0880  9.5204 0.0000 1.0014
## Time                0.1675 0.1254 -0.0770  0.4197 0.1757 1.0017
## AgeB                0.0083 0.0041  0.0004  0.0163 0.0400 1.0005
## Edu                -0.1361 0.0120 -0.1596 -0.1121 0.0000 1.0021
## RaceNotWhite        0.3262 0.1066  0.1137  0.5318 0.0020 1.0003
## EthnicityHispanic   0.5109 0.1550  0.2124  0.8160 0.0013 1.0013
## SexFemale          -1.2203 0.0753 -1.3685 -1.0735 0.0000 1.0006
## Time:AgeB          -0.0016 0.0014 -0.0045  0.0011 0.2457 1.0000
## Time:Edu           -0.0001 0.0039 -0.0081  0.0074 0.9830 1.0068
## Time:RaceNotWhite -0.0158 0.0340 -0.0815  0.0515 0.6313 1.0005
## T:EH                0.0158 0.0512 -0.0831  0.1154 0.7683 1.0020
## Time:SexFemale     -0.0070 0.0251 -0.0563  0.0413 0.7780 1.0021
## sigma               0.9061 0.0069  0.8923  0.9193 0.0000 1.0017
##
## MCMC summary:
## chains: 3
## iterations per chain: 15000
## burn-in per chain: 5000
## thinning: 5
## time: 1.4 hours
```

Most of these interactions are clearly not additive, so it was not expected that this extra complexity would improve the model. We can confirm this by comparing the model to the previous best fitting model:

```
##
##               DIC     WAIC      LPML
##   JMfit9 224344.3 224114.4 -112057.7
##  JMfit10 224592.8 224228.7 -112190.4
##
## The criteria are calculated based on the marginal log-likelihood.
```

As expected, the complexity added by the interactions was not helpful; however, there was a signal for an interaction of time by race on the MMSE, so we next simplified the interactions to just time by race on the MMSE. This result and its WAIC comparison is shown here:

```
##
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(MMSE, NPIQ, CAIDE),
```

```
##     time_var = "Time", functional_forms = fForms, n_iter = 15000,
##     n_burnin = 5000, n_thin = 5)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
##   NPIQ: 15597
##   CAIDE: 15597
##
##                  DIC     WAIC      LPML
## marginal    224352.9 224068.6 -112042.1
## conditional 297281.1 289705.2 -153330.8
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
## (Intr) 4.0771 (Intr)    Time  (Intr)    Time  (Intr)
## Time   1.5671 0.5372
## (Intr) 3.0762 -0.2492 -0.1165
## Time   0.6748 -0.1657 -0.5546 -0.0890
## (Intr) 1.4663 0.0737  0.1028  0.0149  -0.0242
## Time   0.1769 -0.0822 -0.0154 0.0036  0.0617  -0.3785
##
## Survival Outcome:
##                      Mean  StDev    2.5%   97.5%      P   Rhat
## Age                0.0458 0.0070  0.0323  0.0593 0.0000 1.0011
## LivSitAlone        0.9998 0.1062  0.7906  1.2090 0.0000 1.0003
## RaceNotWhite      -0.9190 0.1480 -1.2114 -0.6393 0.0000 1.0044
## EthnicityHispanic -0.9269 0.2193 -1.3720 -0.5144 0.0000 1.0025
## value(MMSE)       -0.0790 0.0049 -0.0886 -0.0694 0.0000 1.0089
## value(NPIQ)        0.1270 0.0125  0.1022  0.1514 0.0000 1.0037
## value(CAIDE)      -0.0527 0.0292 -0.1104  0.0049 0.0733 1.0006
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                      Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)       20.5411 0.9757 18.6237 22.4702 0.0000 1.0003
## Time              -3.4688 0.4177 -4.2950 -2.6618 0.0000 0.9999
## AgeB              -0.0054 0.0113 -0.0274  0.0164 0.6397 1.0004
## Edu                0.3248 0.0308  0.2648  0.3847 0.0000 1.0006
## RaceNotWhite      -0.5861 0.2947 -1.1587 -0.0083 0.0473 1.0000
## EthnicityHispanic -1.3354 0.4212 -2.1650 -0.5123 0.0013 1.0006
## Time:AgeB          0.0256 0.0057  0.0146  0.0368 0.0000 0.9999
## Time:RaceNotWhite  0.3804 0.1361  0.1114  0.6448 0.0060 1.0002
```

```
## sigma              2.0874 0.0186  2.0520  2.1241 0.0000 1.0036
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)         8.8788 0.6859  7.5601 10.1868 0.0000 0.9999
## Time                0.3332 0.0403  0.2525  0.4106 0.0000 1.0005
## AgeB               -0.0542 0.0078 -0.0691 -0.0390 0.0000 1.0006
## Edu                -0.0743 0.0223 -0.1184 -0.0302 0.0013 1.0006
## EthnicityHispanic   1.1921 0.2981  0.5921  1.7652 0.0000 0.9996
## SexFemale          -0.6543 0.1393 -0.9336 -0.3825 0.0000 1.0017
## sigma               2.6791 0.0212  2.6384  2.7211 0.0000 1.0043
##
## Longitudinal Outcome: CAIDE (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)         9.0316 0.2801  8.4848  9.5833 0.0000 1.0005
## Time                0.0404 0.0122  0.0158  0.0645 0.0007 1.0027
## AgeB                0.0056 0.0032 -0.0007  0.0118 0.0820 1.0013
## Edu                -0.1365 0.0090 -0.1541 -0.1188 0.0000 0.9996
## RaceNotWhite        0.3002 0.0820  0.1402  0.4594 0.0000 1.0003
## EthnicityHispanic   0.5208 0.1200  0.2858  0.7575 0.0000 1.0000
## SexFemale          -1.2326 0.0572 -1.3428 -1.1188 0.0000 0.9996
## sigma               0.9047 0.0069  0.8911  0.9185 0.0000 1.0019
##
## MCMC summary:
## chains: 3
## iterations per chain: 15000
## burn-in per chain: 5000
## thinning: 5
## time: 56.8 min
##
##               DIC     WAIC      LPML
##  JMfit11 224352.9 224068.6 -112042.1
##   JMfit9 224344.3 224114.4 -112057.7
##
## The criteria are calculated based on the marginal log-likelihood.
```

In this case, the added complexity helps the model's performance; however, the interaction models can still be extended. Aside from the association structure, it also possible to specify interactions between the predictors of the longitudinal components with the predictors of the survival component. A fully saturated interaction model (i.e., current value of longitudinal outcome with each survival model predictor) was then fit. Here are those results:

```
##
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(MMSE, NPIQ, CAIDE),
```

```
##     time_var = "Time", functional_forms = fForms, n_iter = 25000,
##     n_burnin = 5000, n_thin = 10)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
##   NPIQ: 15597
##   CAIDE: 15597
##
##                  DIC     WAIC      LPML
## marginal    224338.1 224061.5 -112041.0
## conditional 297222.7 289648.5 -153908.6
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
## (Intr) 4.0753 (Intr)    Time  (Intr)    Time  (Intr)
## Time   1.5663 0.5390
## (Intr) 3.0756 -0.2501 -0.1152
## Time   0.6739 -0.1651 -0.5565 -0.0899
## (Intr) 1.4656 0.0736  0.1032  0.0164  -0.0269
## Time   0.1763 -0.0808 -0.0189 0.0030  0.0673  -0.3770
##
## Survival Outcome:
##                                   Mean  StdDev    2.5%   97.5%
P   Rhat
## Age                              0.0466 0.0069  0.0332  0.0602 0.000
0 1.0017
## LivSitAlone                      1.0458 0.6016 -0.0611  2.2498 0.066
3 1.1055
## RaceNotWhite                    -1.7281 0.7153 -3.1584 -0.3250 0.018
3 1.0049
## EthnicityHispanic               -0.6618 1.0733 -2.7664  1.4091 0.550
7 1.0148
## value(MMSE)                     -0.0818 0.0055 -0.0928 -0.0714 0.000
0 1.0017
## value(MMSE):RaceNotWhite         0.0325 0.0149  0.0039  0.0627 0.026
0 1.0026
## value(MMSE):EthnicityHispanic    0.0033 0.0264 -0.0496  0.0534 0.902
7 1.0177
## value(MMSE):LivSitAlone         -0.0094 0.0152 -0.0397  0.0199 0.530
0 1.0465
```

```
## value(NPIQ)                      0.1382 0.0150  0.1082  0.1676 0.000
0 1.0057
## value(NPIQ):RaceNotWhite         0.0494 0.0406 -0.0313  0.1280 0.231
3 1.0015
## value(NPIQ):EthnicityHispanic  -0.0907 0.0589 -0.2132  0.0191 0.108
7 1.0198
## value(NPIQ):LivSitAlone        -0.0421 0.0328 -0.1080  0.0213 0.196
3 1.0335
## value(CAIDE)                   -0.0709 0.0354 -0.1402 -0.0020 0.042
0 1.0194
## value(CAIDE):RaceNotWhite       0.0008 0.0869 -0.1749  0.1730 0.975
7 1.0073
## value(CAIDE):EthnicityHispanic  0.0400 0.1101 -0.1681  0.2560 0.723
7 1.0414
## value(CAIDE):LivSitAlone        0.0504 0.0681 -0.0823  0.1801 0.465
7 1.0505
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)       20.5465 0.9502 18.6734 22.4289 0.0000 1.0002
## Time              -3.4715 0.4000 -4.2554 -2.6898 0.0000 1.0002
## AgeB              -0.0055 0.0110 -0.0271  0.0159 0.6233 1.0002
## Edu                0.3249 0.0310  0.2640  0.3874 0.0000 1.0000
## RaceNotWhite      -0.5823 0.3005 -1.1711  0.0072 0.0527 0.9997
## EthnicityHispanic -1.3434 0.4133 -2.1624 -0.5354 0.0017 0.9999
## Time:AgeB          0.0256 0.0054  0.0150  0.0362 0.0000 1.0003
## Time:RaceNotWhite  0.3830 0.1375  0.1202  0.6569 0.0043 0.9999
## sigma              2.0890 0.0181  2.0538  2.1254 0.0000 1.0008
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)        8.8488 0.6914  7.5243 10.2005 0.0000 1.0000
## Time               0.3353 0.0403  0.2550  0.4134 0.0000 0.9997
## AgeB              -0.0539 0.0079 -0.0699 -0.0385 0.0000 0.9997
## Edu               -0.0736 0.0221 -0.1181 -0.0310 0.0023 1.0011
## EthnicityHispanic  1.1904 0.2886  0.6268  1.7606 0.0000 0.9998
## SexFemale         -0.6559 0.1388 -0.9290 -0.3799 0.0000 1.0004
## sigma              2.6791 0.0210  2.6390  2.7211 0.0000 0.9996
##
## Longitudinal Outcome: CAIDE (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)        9.0375 0.2821  8.4720  9.5928 0.0000 1.0001
## Time               0.0406 0.0121  0.0165  0.0642 0.0007 0.9998
## AgeB               0.0055 0.0032 -0.0007  0.0117 0.0837 1.0007
```

```
## Edu               -0.1365 0.0091 -0.1543 -0.1186 0.0000 1.0001
## RaceNotWhite       0.3009 0.0836  0.1386  0.4625 0.0003 1.0026
## EthnicityHispanic  0.5199 0.1189  0.2845  0.7455 0.0000 1.0008
## SexFemale         -1.2313 0.0571 -1.3418 -1.1201 0.0000 0.9998
## sigma              0.9050 0.0070  0.8916  0.9192 0.0000 1.0028
##
## MCMC summary:
## chains: 3
## iterations per chain: 25000
## burn-in per chain: 5000
## thinning: 10
## time: 2.6 hours
```

As with the other saturated interaction model, there is clearly a lot of unhelpful complexity in this model, which could be easily verified by the WAIC comparisons:

```
##
##               DIC     WAIC      LPML
##  JMfit12 224338.1 224061.5 -112041.0
##  JMfit11 224352.9 224068.6 -112042.1
##
## The criteria are calculated based on the marginal log-likelihood.
```

Surprisingly, the complex model does seem to produce better fit; however, we expect that simplifying these interactions could produce even better results, so a model with just an interaction between the MMSE value and race was run.

```
##
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(MMSE, NPIQ, CAIDE),
##     time_var = "Time", functional_forms = fForms, n_iter = 25000,
##     n_burnin = 5000, n_thin = 10)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
##   NPIQ: 15597
##   CAIDE: 15597
##
##                  DIC     WAIC      LPML
## marginal    224336.6 224050.5 -112031.8
## conditional 297169.0 289588.4 -153801.4
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
```

```
## (Intr) 4.0777 (Intr)    Time  (Intr)    Time  (Intr)
## Time   1.5674 0.5374
## (Intr) 3.0715 -0.2502 -0.1158
## Time   0.6699 -0.1643 -0.5562 -0.0843
## (Intr) 1.4654 0.0728  0.1034  0.0154  -0.0259
## Time   0.1756 -0.0762 -0.0153 0.0019  0.0668  -0.3763
##
## Survival Outcome:
##                              Mean  StDev    2.5%   97.5%      P   Rh
at
## Age                        0.0462 0.0068  0.0330  0.0595 0.0000 1.00
10
## LivSitAlone                0.9965 0.1076  0.7821  1.2064 0.0000 1.00
04
## RaceNotWhite              -1.3183 0.2423 -1.7964 -0.8459 0.0000 1.00
54
## EthnicityHispanic         -0.9078 0.2118 -1.3384 -0.5111 0.0000 1.00
06
## value(MMSE)               -0.0819 0.0051 -0.0919 -0.0721 0.0000 1.00
06
## value(MMSE):RaceNotWhite  0.0264 0.0127  0.0010  0.0514 0.0413 1.00
21
## value(NPIQ)                0.1286 0.0122  0.1054  0.1526 0.0000 1.00
12
## value(CAIDE)              -0.0529 0.0294 -0.1108  0.0043 0.0723 1.00
03
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)       20.5367 0.9595 18.6245 22.4252 0.0000 1.0001
## Time              -3.4667 0.4111 -4.2581 -2.6508 0.0000 1.0005
## AgeB              -0.0055 0.0111 -0.0272  0.0160 0.6160 1.0001
## Edu                0.3255 0.0306  0.2674  0.3855 0.0000 1.0001
## RaceNotWhite      -0.5815 0.3015 -1.1720  0.0045 0.0523 1.0003
## EthnicityHispanic -1.3315 0.4193 -2.1622 -0.5020 0.0003 0.9998
## Time:AgeB          0.0256 0.0056  0.0145  0.0365 0.0000 1.0004
## Time:RaceNotWhite  0.3803 0.1398  0.1087  0.6516 0.0063 1.0009
## sigma              2.0874 0.0189  2.0513  2.1250 0.0000 1.0019
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%     P   Rhat
## (Intercept)        8.8670 0.6984  7.4732 10.2523 0e+00 0.9996
## Time               0.3329 0.0406  0.2531  0.4116 0e+00 1.0012
## AgeB              -0.0541 0.0078 -0.0694 -0.0382 0e+00 0.9998
```

```
## Edu                -0.0740 0.0223 -0.1182 -0.0302 3e-04 0.9998
## EthnicityHispanic   1.1914 0.2972  0.6113  1.7728 3e-04 0.9997
## SexFemale          -0.6547 0.1391 -0.9264 -0.3873 0e+00 1.0005
## sigma               2.6797 0.0212  2.6381  2.7217 0e+00 1.0073
##
## Longitudinal Outcome: CAIDE (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%     P   Rhat
## (Intercept)         9.0328 0.2819  8.4897  9.5804 0.000 0.9996
## Time                0.0400 0.0120  0.0167  0.0642 0.001 0.9999
## AgeB                0.0056 0.0032 -0.0006  0.0116 0.080 0.9997
## Edu                -0.1365 0.0091 -0.1541 -0.1188 0.000 0.9999
## RaceNotWhite        0.3011 0.0824  0.1423  0.4624 0.000 1.0009
## EthnicityHispanic   0.5192 0.1199  0.2796  0.7557 0.000 1.0013
## SexFemale          -1.2324 0.0574 -1.3429 -1.1194 0.000 1.0000
## sigma               0.9050 0.0071  0.8914  0.9193 0.000 1.0020
##
## MCMC summary:
## chains: 3
## iterations per chain: 25000
## burn-in per chain: 5000
## thinning: 10
## time: 1.7 hours
##
##               DIC     WAIC      LPML
##  JMfit13 224336.6 224050.5 -112031.8
##  JMfit12 224338.1 224061.5 -112041.0
##
## The criteria are calculated based on the marginal log-likelihood.
```

As expected, the simplified interaction model does fit slightly better. Still, however, there are other interactions worth considering. When some predictors have large coefficients, it can be worth checking for interaction effects (e.g., Gelman et al., 2020). Inspecting the coefficients of each longitudinal component, interactions were added to those that were relatively large in relation to other coefficients. The result of this saturated interaction model is shown here:

```
##
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(MMSE, NPIQ, CAIDE),
##     time_var = "Time", functional_forms = fForms, n_iter = 25000,
##     n_burnin = 5000, n_thin = 10)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
```

```
##   NPIQ: 15597
##   CAIDE: 15597
##
##                   DIC     WAIC    LPML
## marginal    224456.3 224074.3 -112073
## conditional 297901.1 290569.2 -154915
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
## (Intr) 4.0738 (Intr)    Time  (Intr)    Time  (Intr)
## Time   1.5668 0.5355
## (Intr) 3.0786 -0.2509 -0.1167
## Time   0.6744 -0.1623 -0.5511 -0.0920
## (Intr) 1.4541 0.0809  0.1043  0.0173  -0.0356
## Time   0.1777 -0.0743 -0.0151 0.0025  0.0668  -0.3749
##
## Survival Outcome:
##                              Mean  StDev    2.5%   97.5%      P   Rh
at
## Age                        0.0464 0.0070  0.0326  0.0598 0.0000 1.00
08
## LivSitAlone                0.9957 0.1058  0.7903  1.2072 0.0000 1.00
03
## RaceNotWhite              -1.3086 0.2502 -1.8131 -0.8341 0.0000 1.01
50
## EthnicityHispanic         -0.9029 0.2197 -1.3564 -0.4962 0.0000 1.00
47
## value(MMSE)               -0.0820 0.0052 -0.0922 -0.0718 0.0000 1.01
27
## value(MMSE):RaceNotWhite   0.0260 0.0130  0.0008  0.0519 0.0437 1.01
31
## value(NPIQ)                0.1290 0.0125  0.1038  0.1534 0.0000 1.00
13
## value(CAIDE)              -0.0515 0.0298 -0.1088  0.0071 0.0843 0.99
97
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)       21.1382 1.0018 19.1332 23.1297 0.0000 1.0004
## Time              -3.3700 0.4858 -4.3376 -2.4142 0.0000 1.0002
## AgeB              -0.0048 0.0112 -0.0265  0.0174 0.6643 1.0007
## Edu                0.2826 0.0365  0.2117  0.3535 0.0000 0.9997
## RaceNotWhite      -2.3976 1.0168 -4.4091 -0.4473 0.0180 1.0002
```

```
## EthnicityHispanic -2.2138 1.0288 -4.2662 -0.1948 0.0297 1.0000
## Time:AgeB          0.0254 0.0057  0.0142  0.0369 0.0000 1.0009
## Time:RaceNotWhite  0.3741 0.1409  0.1003  0.6478 0.0080 1.0002
## Edu:RaceNotWhite   0.1307 0.0708 -0.0061  0.2704 0.0613 1.0001
## E:EH               0.0831 0.0829 -0.0767  0.2469 0.3147 0.9999
## Time:Edu          -0.0054 0.0152 -0.0356  0.0245 0.7283 0.9996
## sigma              2.0974 0.0188  2.0610  2.1344 0.0000 0.9998
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)        8.8277 0.7096  7.4622 10.2275 0.0000 1.0010
## Time               0.3376 0.0415  0.2558  0.4204 0.0000 0.9998
## AgeB              -0.0541 0.0078 -0.0694 -0.0387 0.0000 1.0014
## Edu               -0.0720 0.0246 -0.1209 -0.0246 0.0037 0.9997
## EthnicityHispanic  1.5328 0.8555 -0.1190  3.2258 0.0690 1.0000
## SexFemale         -0.6341 0.1433 -0.9152 -0.3531 0.0003 1.0006
## E:EH              -0.0021 0.0587 -0.1174  0.1146 0.9723 1.0004
## T:EH              -0.0599 0.1469 -0.3452  0.2269 0.6733 1.0006
## EH:S              -0.4116 0.5511 -1.4733  0.6564 0.4540 1.0002
## sigma              2.6814 0.0211  2.6404  2.7225 0.0000 1.0008
##
## Longitudinal Outcome: CAIDE (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)        8.3538 0.3032  7.7620  8.9457 0.0000 0.9999
## Time               0.0402 0.0123  0.0162  0.0641 0.0013 1.0027
## AgeB               0.0046 0.0031 -0.0015  0.0109 0.1327 0.9997
## Edu               -0.0891 0.0126 -0.1135 -0.0639 0.0000 1.0007
## RaceNotWhite       0.2773 0.0814  0.1153  0.4371 0.0007 1.0007
## EthnicityHispanic  2.0742 0.3452  1.3930  2.7516 0.0000 1.0004
## SexFemale         -0.3323 0.2802 -0.8792  0.2173 0.2373 1.0007
## EH:S              -0.2506 0.2455 -0.7325  0.2332 0.3097 1.0022
## Edu:SexFemale     -0.0583 0.0181 -0.0935 -0.0228 0.0027 1.0009
## E:EH              -0.1268 0.0244 -0.1741 -0.0782 0.0000 1.0003
## sigma              0.9064 0.0071  0.8928  0.9203 0.0000 1.0071
##
## MCMC summary:
## chains: 3
## iterations per chain: 25000
## burn-in per chain: 5000
## thinning: 10
## time: 2.1 hours
```

As with other saturated interaction models so far, these results do suggest that a more simplified model could be preferred. Still, we want to check how this model compares to our current best fitting model.

```
##
##               DIC     WAIC      LPML
##  JMfit13 224336.6 224050.5 -112031.8
##  JMfit14 224456.3 224074.3 -112073.0
##
## The criteria are calculated based on the marginal log-likelihood.
```

As expected, the fully saturated model does not produce superior fit, so we next fit a simplified version of this model by including just the most probable coefficients. The results of this model are shown next:

```
##
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(MMSE, NPIQ, CAIDE),
##     time_var = "Time", functional_forms = fForms, n_iter = 25000,
##     n_burnin = 5000, n_thin = 10)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
##   NPIQ: 15597
##   CAIDE: 15597
##
##                  DIC     WAIC      LPML
## marginal    224324.3 224003.4 -112012.8
## conditional 297379.9 289874.3 -154191.5
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
## (Intr) 4.0736 (Intr)    Time  (Intr)    Time  (Intr)
## Time   1.5655 0.5372
## (Intr) 3.0719 -0.2507 -0.1163
## Time   0.6712 -0.1649 -0.5545 -0.0850
## (Intr) 1.4533 0.0820  0.1042  0.0172  -0.0350
## Time   0.1774 -0.0765 -0.0151 0.0008  0.0756  -0.3731
##
## Survival Outcome:
##                             Mean  StDev    2.5%   97.5%      P   Rh
at
## Age                       0.0463 0.0068  0.0331  0.0595 0.0000 1.00
00
## LivSitAlone               0.9987 0.1037  0.7914  1.1991 0.0000 0.99
98
```

```
## RaceNotWhite              -1.3100 0.2429 -1.7902 -0.8461 0.0000 1.00
03
## EthnicityHispanic         -0.9153 0.2154 -1.3493 -0.5107 0.0000 1.00
02
## value(MMSE)               -0.0820 0.0051 -0.0919 -0.0721 0.0000 1.00
20
## value(MMSE):RaceNotWhite  0.0261 0.0127  0.0018  0.0512 0.0380 1.00
00
## value(NPIQ)                0.1285 0.0124  0.1042  0.1529 0.0000 1.00
04
## value(CAIDE)              -0.0522 0.0299 -0.1109  0.0062 0.0797 1.00
05
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)       20.9276 0.9790 18.9814 22.8581 0.0000 1.0014
## Time              -3.4655 0.4094 -4.2888 -2.6547 0.0000 1.0006
## AgeB              -0.0049 0.0111 -0.0267  0.0169 0.6610 1.0002
## Edu                0.2966 0.0332  0.2313  0.3611 0.0000 1.0006
## RaceNotWhite      -2.5708 0.9879 -4.5259 -0.6479 0.0103 0.9999
## EthnicityHispanic -1.2569 0.4186 -2.0840 -0.4240 0.0023 1.0001
## Time:AgeB          0.0256 0.0055  0.0146  0.0367 0.0000 1.0005
## Time:RaceNotWhite  0.3817 0.1391  0.1221  0.6579 0.0043 1.0000
## Edu:RaceNotWhite   0.1438 0.0688  0.0112  0.2813 0.0367 1.0001
## sigma              2.0899 0.0188  2.0540  2.1268 0.0000 1.0026
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%     P   Rhat
## (Intercept)        8.8637 0.7068  7.4755 10.2315 0e+00 1.0006
## Time               0.3333 0.0404  0.2523  0.4129 0e+00 0.9995
## AgeB              -0.0541 0.0080 -0.0696 -0.0387 0e+00 0.9997
## Edu               -0.0735 0.0227 -0.1182 -0.0291 7e-04 1.0010
## EthnicityHispanic  1.1919 0.2964  0.6140  1.7724 0e+00 1.0007
## SexFemale         -0.6553 0.1387 -0.9247 -0.3861 0e+00 0.9998
## sigma              2.6800 0.0217  2.6382  2.7224 0e+00 1.0018
##
## Longitudinal Outcome: CAIDE (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)        8.4004 0.3001  7.8126  8.9858 0.0000 0.9999
## Time               0.0401 0.0123  0.0160  0.0639 0.0010 1.0004
## AgeB               0.0047 0.0032 -0.0015  0.0108 0.1450 1.0000
## Edu               -0.0917 0.0122 -0.1152 -0.0675 0.0000 0.9998
## RaceNotWhite       0.2823 0.0816  0.1237  0.4419 0.0007 1.0009
## EthnicityHispanic  1.9092 0.2934  1.3348  2.4872 0.0000 1.0001
```

```
## SexFemale         -0.4359 0.2619 -0.9443  0.0893 0.0977 1.0016
## Edu:SexFemale     -0.0524 0.0172 -0.0865 -0.0186 0.0023 1.0010
## E:EH              -0.1250 0.0235 -0.1709 -0.0791 0.0000 0.9998
## sigma              0.9058 0.0072  0.8916  0.9197 0.0000 1.0039
##
## MCMC summary:
## chains: 3
## iterations per chain: 25000
## burn-in per chain: 5000
## thinning: 10
## time: 1.9 hours
```

These results do appear to appropriately simplified, so we can compare this to the previous best fitting result:

```
##
##               DIC     WAIC      LPML
##  JMfit15 224324.3 224003.4 -112012.8
##  JMfit13 224336.6 224050.5 -112031.8
##
## The criteria are calculated based on the marginal log-likelihood.
```

As would be expected, the simplified model does indeed improve the model's performance. So far, the CAIDE has remained unconvincing in its effect. Rather than drop it due to non-"significance," we next check whether dropping it would improve the joint model's performance. Here the resulting model where just the CAIDE is omitted:

```
##
## Call:
## jm(Surv_object = Surv, Mixed_objects = list(MMSE, NPIQ), time_var =
"Time",
##     functional_forms = fForms, n_iter = 25000, n_burnin = 5000,
##     n_thin = 10)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
##   NPIQ: 15597
##
##                  DIC     WAIC       LPML
## marginal    172815.4 172550.9  -86284.43
## conditional 234958.4 229960.2 -122638.34
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
```

```
## (Intr) 4.0726 (Intr)    Time  (Intr)
## Time   1.5666 0.5365
## (Intr) 3.0750 -0.2503 -0.1169
## Time   0.6734 -0.1652 -0.5538 -0.0896
##
## Survival Outcome:
##                             Mean  StDev    2.5%   97.5%      P   Rh
at
## Age                       0.0456 0.0070  0.0318  0.0592 0.0000 1.00
20
## LivSitAlone               1.0206 0.1062  0.8125  1.2263 0.0000 1.00
23
## RaceNotWhite             -1.3189 0.2494 -1.8129 -0.8385 0.0000 1.00
29
## EthnicityHispanic        -0.9580 0.2170 -1.4107 -0.5577 0.0000 1.00
17
## value(MMSE)              -0.0835 0.0051 -0.0933 -0.0738 0.0000 1.00
07
## value(MMSE):RaceNotWhite  0.0257 0.0130  0.0007  0.0510 0.0457 1.00
21
## value(NPIQ)               0.1261 0.0123  0.1012  0.1500 0.0000 1.00
14
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                      Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)       20.8834 0.9794 18.9534 22.8056 0.0000 1.0016
## Time              -3.4629 0.4109 -4.2808 -2.6446 0.0000 1.0002
## AgeB              -0.0046 0.0112 -0.0267  0.0175 0.6790 1.0006
## Edu                0.2982 0.0339  0.2304  0.3657 0.0000 1.0012
## RaceNotWhite      -2.4669 0.9884 -4.3682 -0.5451 0.0110 1.0011
## EthnicityHispanic -1.2619 0.4269 -2.0979 -0.4234 0.0027 1.0001
## Time:AgeB          0.0255 0.0056  0.0143  0.0368 0.0000 1.0003
## Time:RaceNotWhite  0.3843 0.1384  0.1151  0.6563 0.0047 1.0010
## Edu:RaceNotWhite   0.1360 0.0687  0.0052  0.2701 0.0433 1.0013
## sigma              2.0896 0.0187  2.0536  2.1267 0.0000 1.0020
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                      Mean  StDev    2.5%   97.5%     P   Rhat
## (Intercept)        8.8770 0.6925  7.5217 10.2282 0.000 1.0001
## Time               0.3337 0.0403  0.2550  0.4115 0.000 0.9997
## AgeB              -0.0543 0.0078 -0.0693 -0.0390 0.000 1.0006
## Edu               -0.0736 0.0225 -0.1178 -0.0286 0.001 1.0008
## EthnicityHispanic  1.2020 0.2965  0.6192  1.7830 0.000 1.0001
## SexFemale         -0.6539 0.1382 -0.9209 -0.3804 0.000 1.0016
```

```
## sigma              2.6777 0.0212  2.6358  2.7202 0.000 1.0051
##
## MCMC summary:
## chains: 3
## iterations per chain: 25000
## burn-in per chain: 5000
## thinning: 10
## time: 1.1 hours
```

The omission of the CAIDE does not impact the estimates too much, so we now compare the new model's fit:

```
##
##               DIC     WAIC       LPML
##  JMfit16 172815.4 172550.9  -86284.43
##  JMfit15 224324.3 224003.4 -112012.81
##
## The criteria are calculated based on the marginal log-likelihood.
```

The relatively dramatic improvement in WAIC is somewhat spurious because an entire predictor is dropped, which dramatically reduces the model complexity; however, it does illustrate that the CAIDE was not an integral part of the model's performance. At this point, we have tested but then omitted the longitudinal effects of the UPDRS, CAIDE, and FAQ. The FAQ was omitted due to its correlation with the MMSE and the impact that it had on the model estimation; however, the UPDRS and CAIDE were dropped because the modeling demonstrated that their longitudinal component did not improve the model's performance. One reason for this, perhaps, is that these variables change relatively little from baseline, reflecting more stable factors that would be better modeled in the survival component rather than a longitudinal change model. To test this, the survival model was updated to include the UPDRS and CAIDE while keeping the longitudinal components the same. The results of this model are shown here:

```
##
## Call:
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##     n_thin = 10)
##
## Data Descriptives:
## Number of Groups: 4421       Number of events: 709 (16%)
## Number of Observations:
##   MMSE: 15597
##   NPIQ: 15597
##
##                  DIC     WAIC       LPML
## marginal    172794.4 172530.4  -86275.88
```

```
## conditional 234923.9 229929.0 -122662.56
##
## Random-effects covariance matrix:
##
##        StdDev   Corr
## (Intr) 4.0728 (Intr)    Time  (Intr)
## Time   1.5660 0.5364
## (Intr) 3.0758 -0.2503 -0.1163
## Time   0.6737 -0.1634 -0.5546 -0.0892
##
## Survival Outcome:
##                             Mean  StDev    2.5%   97.5%      P   Rh
at
## Age                       0.0444 0.0069  0.0311  0.0580 0.0000 1.00
00
## LivSitAlone               1.0130 0.1049  0.8066  1.2140 0.0000 1.00
27
## RaceNotWhite             -1.3021 0.2434 -1.7924 -0.8379 0.0000 1.00
66
## EthnicityHispanic        -0.9068 0.2107 -1.3403 -0.5227 0.0000 1.00
04
## UPDRS                     0.1448 0.0311  0.0823  0.2044 0.0000 1.00
22
## CAIDE                    -0.0576 0.0217 -0.1005 -0.0155 0.0077 1.00
01
## value(MMSE)              -0.0814 0.0051 -0.0916 -0.0715 0.0000 0.99
99
## value(MMSE):RaceNotWhite  0.0260 0.0126  0.0014  0.0513 0.0380 1.00
81
## value(NPIQ)               0.1267 0.0125  0.1025  0.1515 0.0000 1.00
22
##
## Longitudinal Outcome: MMSE (family = gaussian, link = identity)
##                       Mean  StDev    2.5%   97.5%      P   Rhat
## (Intercept)       20.8777 0.9567 19.0116 22.7423 0.0000 0.9998
## Time              -3.4613 0.4019 -4.2563 -2.6865 0.0000 1.0003
## AgeB              -0.0045 0.0109 -0.0258  0.0166 0.6823 1.0001
## Edu                0.2981 0.0330  0.2335  0.3632 0.0000 0.9996
## RaceNotWhite      -2.4609 1.0007 -4.3983 -0.4725 0.0147 1.0009
## EthnicityHispanic -1.2497 0.4200 -2.0824 -0.4268 0.0027 1.0012
## Time:AgeB          0.0255 0.0054  0.0150  0.0362 0.0000 1.0001
## Time:RaceNotWhite  0.3876 0.1376  0.1186  0.6589 0.0053 1.0002
## Edu:RaceNotWhite   0.1359 0.0697 -0.0012  0.2730 0.0523 1.0006
## sigma              2.0892 0.0187  2.0533  2.1260 0.0000 1.0004
```

```
##
## Longitudinal Outcome: NPIQ (family = gaussian, link = identity)
##                      Mean  StDev    2.5%   97.5%     P   Rhat
## (Intercept)        8.8798 0.6923  7.5371 10.2339 0e+00 1.0002
## Time               0.3325 0.0405  0.2529  0.4130 0e+00 1.0020
## AgeB              -0.0543 0.0079 -0.0702 -0.0392 0e+00 1.0004
## Edu               -0.0735 0.0220 -0.1175 -0.0302 7e-04 1.0002
## EthnicityHispanic  1.1910 0.2971  0.6252  1.7832 3e-04 1.0010
## SexFemale         -0.6536 0.1370 -0.9205 -0.3793 0e+00 0.9997
## sigma              2.6787 0.0215  2.6371  2.7216 0e+00 1.0013
##
## MCMC summary:
## chains: 3
## iterations per chain: 25000
## burn-in per chain: 5000
## thinning: 10
## time: 1.4 hours
```

As expected, the UPDRS and CAIDE are significant predictors in the survival function, suggesting that they are more relevant as baseline predictors and not as longitudinal indicators. Regardless of this result, we still want to confirm whether these additions improve the model:

```
##
##               DIC     WAIC      LPML
##  JMfinal 172794.4 172530.4 -86275.88
##  JMfit16 172815.4 172550.9 -86284.43
##
## The criteria are calculated based on the marginal log-likelihood.
```

This final joint model does indeed improve upon the previous best fitting model. With this result, we have no further revisions for the model and consider this to be the final model based on current literature and aims of the study.