Supplemental Materials for Evaluating Local Structural-After-Measurement (LSAM) and Traditional Approaches for the Estimation of Complex Nonlinear Effects Among Latent Variables

Felipe Fontana Vieira[1], Kjell Solem Slupphaug[2], and Yves Rosseel[1]

[1]Department of Data Analysis, Faculty of Psychology and Educational Sciences, Ghent University

[2]Department of Psychology, Norwegian University of Science and Technology

Author Note

Felipe Fontana Vieira https://orcid.org/0009-0006-0949-6569

Kjell Solem Slupphaug https://orcid.org/0009-0005-8324-2834

Yves Rosseel https://orcid.org/0000-0002-4129-4477

Correspondence concerning this article should be addressed to Felipe Fontana Vieira, Department of Data Analysis, Faculty of Psychology and Educational Sciences, Ghent University, Henri Dunantlaan 2, Ghent 9000, Belgium, Email: felipe.fontanavieira@ugent.be

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This document contains the online supplemental materials mentioned throughout the manuscript. The table of contents in the next page shows all the separate sections. Specifically, in this document, we include the (i) computation time, (ii) R^2 and reliability values obtained during the calibration of the baseline population model, (iii) plots for the main results as well as (iv) additional distributional conditions, (v) SEs outliers diagnostic for condition 13 in Simulation 1, and finally (vi) information on the difference between the main results and additional distributional conditions. For the latter, we include the actual Figures with the difference uncertainty in the GitHub repository (see `supplemental_tables.qmd` and `supplemental_tables.pdf`). Moreover, for the performance measures and tables with those results, we also refer to the GitHub repository (`supplemental_tables.qmd` and `supplemental_tables.pdf`); otherwise this file would be very large.

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Computation Time, R^2 Values and Reliability

Table S2

Computation Time by Approach (in Seconds)

							Simulation 1								Simulation 2					
				Distribution			LMS		QML		UPI		LSAM		LSAM		QML		UPI	
Cond.	Model	N	Rel.	Distr.	Epsilon	Zeta	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn
1	Full	400	0.4	Normal	Normal	Normal	12.11	10.91	0.97	0.86	0.34	0.30	0.82	0.81	3.47	3.45	7.61	6.27	6.74	6.14
2	Full	1000	0.4	Normal	Normal	Normal	15.90	15.92	1.44	1.41	0.43	0.42	1.94	1.93	8.37	8.45	9.24	9.09	9.19	9.10
3	Full	400	0.6	Normal	Normal	Normal	10.50	10.44	0.88	0.87	0.29	0.28	0.84	0.83	3.63	3.66	5.91	5.88	6.02	5.89
4	Full	1000	0.6	Normal	Normal	Normal	15.62	15.53	1.53	1.50	0.41	0.40	1.94	1.92	8.40	8.45	9.92	9.85	9.18	9.07
5	Full	400	0.8	Normal	Normal	Normal	14.94	14.92	1.15	1.15	0.33	0.32	0.83	0.81	3.65	3.65	7.61	7.56	6.79	6.75
6	Full	1000	0.8	Normal	Normal	Normal	24.35	24.46	2.01	2.01	0.45	0.44	1.92	1.91	8.43	8.51	12.69	12.56	9.91	9.78
7	Full	400	0.4	Right-skewed	Normal	Normal	13.16	11.08	1.17	0.98	0.38	0.30	0.83	0.82	3.38	3.38	8.44	7.19	6.70	6.02
8	Full	1000	0.4	Right-skewed	Normal	Normal	15.59	15.50	1.69	1.68	0.43	0.42	1.93	1.93	8.10	8.14	11.89	11.66	9.26	9.16
9	Full	400	0.6	Right-skewed	Normal	Normal	11.71	11.53	1.00	0.97	0.30	0.29	0.84	0.83	3.40	3.40	7.54	7.41	6.15	6.10
10	Full	1000	0.6	Right-skewed	Normal	Normal	17.84	17.58	1.74	1.72	0.42	0.41	1.94	1.93	8.13	8.15	13.26	13.11	9.30	9.31
11	Full	400	0.8	Right-skewed	Normal	Normal	16.84	16.80	1.33	1.32	0.34	0.34	0.84	0.83	3.40	3.40	9.43	9.31	7.16	7.15
12	Full	1000	0.8	Right-skewed	Normal	Normal	27.17	27.50	2.43	2.43	0.46	0.45	1.95	1.94	8.15	8.17	17.19	16.93	10.35	10.33
13	Full	400	0.4	Uniform	Normal	Normal	16.84	11.74	1.66	0.98	0.95	0.35	0.83	0.82	3.44	3.43	12.46	9.12	11.17	8.24
14	Full	1000	0.4	Uniform	Normal	Normal	17.52	15.78	1.51	1.36	0.51	0.43	1.95	1.94	8.43	8.45	12.32	10.06	10.35	9.53
15	Full	400	0.6	Uniform	Normal	Normal	10.25	10.25	0.84	0.83	0.30	0.29	0.85	0.84	3.71	3.71	6.04	5.97	6.33	6.15
16	Full	1000	0.6	Uniform	Normal	Normal	14.90	14.84	1.51	1.50	0.41	0.40	1.97	1.96	8.61	8.63	10.30	10.19	9.33	9.30
17	Full	400	0.8	Uniform	Normal	Normal	13.81	13.83	1.24	1.24	0.32	0.32	0.84	0.82	3.63	3.61	8.59	8.57	6.90	6.91
18	Full	1000	0.8	Uniform	Normal	Normal	22.98	22.97	2.23	2.21	0.44	0.43	1.94	1.94	8.55	8.58	15.03	14.95	10.00	9.96
19	Full	400	0.4	Normal	Right-skewed	Normal	41.79	21.72	3.28	3.25	2.12	2.14	0.81	0.80	3.33	3.32	17.61	16.66	18.50	19.00
20	Full	1000	0.4	Normal	Right-skewed	Normal	54.16	30.51	4.12	3.25	2.11	2.14	1.91	1.91	8.11	8.13	23.11	21.69	20.61	20.56
21	Full	400	0.6	Normal	Right-skewed	Normal	11.06	10.66	0.92	0.89	0.30	0.28	0.85	0.84	3.71	3.69	6.31	6.20	6.36	6.24
22	Full	1000	0.6	Normal	Right-skewed	Normal	15.70	15.65	1.62	1.61	0.41	0.40	1.96	1.95	8.73	8.78	10.48	10.30	9.43	9.40
23	Full	400	0.8	Normal	Right-skewed	Normal	15.21	15.17	1.18	1.17	0.32	0.32	0.85	0.83	3.68	3.67	8.21	8.16	7.00	6.96
24	Full	1000	0.8	Normal	Right-skewed	Normal	24.35	24.44	2.06	2.04	0.44	0.43	1.95	1.94	8.62	8.64	13.82	13.64	9.98	9.93
25	Full	400	0.4	Right-skewed	Right-skewed	Normal	18.71	12.27	1.20	1.00	0.47	0.33	0.83	0.82	3.45	3.45	8.63	7.39	7.25	6.21
26	Full	1000	0.4	Right-skewed	Right-skewed	Normal	23.25	17.43	1.84	1.64	0.49	0.44	1.94	1.93	8.35	8.36	13.03	12.46	9.62	9.39
27	Full	400	0.6	Right-skewed	Right-skewed	Normal	16.75	12.00	1.20	1.00	0.30	0.29	0.84	0.83	3.47	3.47	7.76	7.49	6.20	6.17

Table S2

Computation Time by Approach (in Seconds) (continued)

							Simulation 1								Simulation 2					
				Distribution			LMS		QML		UPI		LSAM		LSAM		QML		UPI	
Cond.	Model	N	Rel.	Distr.	Epsilon	Zeta	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn
28	Full	1000	0.6	Right-skewed	Right-skewed	Normal	21.92	17.68	1.96	1.77	0.41	0.41	1.94	1.93	8.33	8.36	13.49	13.19	9.35	9.32
29	Full	400	0.8	Right-skewed	Right-skewed	Normal	16.86	16.63	1.34	1.33	0.33	0.33	0.84	0.83	3.45	3.44	9.69	9.58	7.21	7.17
30	Full	1000	0.8	Right-skewed	Right-skewed	Normal	27.12	27.03	2.43	2.41	0.45	0.44	1.95	1.94	8.31	8.33	17.23	16.97	10.31	10.28
31	Full	400	0.4	Uniform	Right-skewed	Normal	42.50	25.95	3.74	3.65	2.68	2.66	0.81	0.80	3.32	3.30	20.13	18.90	21.75	21.21
32	Full	1000	0.4	Uniform	Right-skewed	Normal	55.71	38.17	4.75	4.61	2.71	2.64	1.92	1.91	8.11	8.10	29.27	25.92	23.27	22.30
33	Full	400	0.6	Uniform	Right-skewed	Normal	13.60	10.82	1.06	0.90	0.99	0.31	0.84	0.83	3.55	3.54	7.53	6.15	10.34	6.55
34	Full	1000	0.6	Uniform	Right-skewed	Normal	17.11	15.85	1.60	1.53	1.05	0.41	1.96	1.95	8.50	8.50	10.92	10.15	13.21	9.66
35	Full	400	0.8	Uniform	Right-skewed	Normal	14.25	14.32	1.28	1.27	0.32	0.31	0.85	0.84	3.72	3.70	8.55	8.46	6.93	6.87
36	Full	1000	0.8	Uniform	Right-skewed	Normal	23.18	23.16	2.26	2.25	0.43	0.43	1.95	1.94	8.64	8.65	15.25	15.07	10.01	9.99
37	Full	400	0.4	Normal	Normal	Right-skewed	18.18	12.03	0.99	0.85	0.35	0.30	0.84	0.83	3.62	3.60	8.45	6.66	7.11	6.30
38	Full	1000	0.4	Normal	Normal	Right-skewed	22.84	16.62	1.44	1.42	0.42	0.42	1.93	1.92	8.76	8.74	10.15	9.89	9.45	9.43
39	Full	400	0.6	Normal	Normal	Right-skewed	11.18	10.67	0.89	0.89	0.29	0.28	0.85	0.84	3.70	3.70	6.37	6.34	6.17	6.17
40	Full	1000	0.6	Normal	Normal	Right-skewed	15.73	15.64	1.55	1.53	0.41	0.40	1.94	1.93	8.81	8.82	10.35	10.19	9.36	9.33
41	Full	400	0.8	Normal	Normal	Right-skewed	15.12	14.95	1.16	1.15	0.33	0.32	0.83	0.82	3.74	3.72	7.83	7.78	6.88	6.81
42	Full	1000	0.8	Normal	Normal	Right-skewed	24.10	24.01	2.00	1.99	0.45	0.44	1.93	1.93	8.72	8.73	13.66	13.50	10.08	10.03
43	Full	400	0.4	Right-skewed	Normal	Right-skewed	21.16	13.90	1.10	0.97	0.36	0.30	0.82	0.81	3.45	3.45	8.87	7.43	6.92	6.07
44	Full	1000	0.4	Right-skewed	Normal	Right-skewed	29.64	19.35	1.65	1.62	0.44	0.42	1.91	1.91	8.34	8.37	12.44	12.00	9.41	9.32
45	Full	400	0.6	Right-skewed	Normal	Right-skewed	12.90	11.88	1.01	0.98	0.31	0.29	0.83	0.82	3.46	3.44	7.63	7.51	6.21	6.15
46	Full	1000	0.6	Right-skewed	Normal	Right-skewed	17.61	17.57	1.74	1.73	0.42	0.41	1.92	1.91	8.37	8.38	13.52	13.29	9.38	9.38
47	Full	400	0.8	Right-skewed	Normal	Right-skewed	17.14	16.76	1.35	1.34	0.34	0.33	0.83	0.82	3.43	3.43	9.46	9.41	7.18	7.19
48	Full	1000	0.8	Right-skewed	Normal	Right-skewed	27.41	26.99	2.40	2.38	0.46	0.45	1.92	1.92	8.32	8.33	17.36	17.09	10.40	10.40
49	Full	400	0.4	Uniform	Normal	Right-skewed	21.22	13.19	1.70	0.98	0.89	0.34	0.82	0.81	3.44	3.42	13.00	9.93	11.62	8.08
50	Full	1000	0.4	Uniform	Normal	Right-skewed	22.50	16.49	1.52	1.35	0.49	0.43	1.93	1.92	8.60	8.59	13.17	10.43	10.38	9.61
51	Full	400	0.6	Uniform	Normal	Right-skewed	10.89	10.51	0.86	0.84	0.30	0.28	0.84	0.83	3.70	3.69	6.42	6.18	6.38	6.21
52	Full	1000	0.6	Uniform	Normal	Right-skewed	15.15	14.94	1.48	1.47	0.41	0.40	1.93	1.92	8.77	8.77	10.40	10.19	9.38	9.37
53	Full	400	0.8	Uniform	Normal	Right-skewed	14.01	14.02	1.25	1.24	0.32	0.32	0.83	0.82	3.65	3.61	8.61	8.58	6.88	6.86
54	Full	1000	0.8	Uniform	Normal	Right-skewed	22.88	22.78	2.22	2.22	0.44	0.43	1.93	1.93	8.71	8.71	14.97	14.89	10.03	9.95

Table S2

Computation Time by Approach (in Seconds) (continued)

							Simulation 1								Simulation 2					
				Distribution			LMS		QML		UPI		LSAM		LSAM		QML		UPI	
Cond.	Model	N	Rel.	Distr.	Epsilon	Zeta	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn
55	Full	400	0.4	Normal	Right-skewed	Right-skewed	46.26	29.35	3.31	3.21	2.10	2.15	0.80	0.79	3.34	3.31	17.69	16.25	18.51	18.90
56	Full	1000	0.4	Normal	Right-skewed	Right-skewed	63.85	39.49	4.05	3.12	2.20	2.22	1.89	1.88	8.20	8.20	23.84	21.81	20.47	20.51
57	Full	400	0.6	Normal	Right-skewed	Right-skewed	11.53	10.82	0.94	0.90	0.30	0.28	0.84	0.83	3.64	3.63	6.60	6.39	6.29	6.29
58	Full	1000	0.6	Normal	Right-skewed	Right-skewed	15.58	15.56	1.61	1.59	0.41	0.40	1.94	1.93	8.75	8.78	10.60	10.48	9.40	9.39
59	Full	400	0.8	Normal	Right-skewed	Right-skewed	15.30	15.20	1.18	1.18	0.32	0.31	0.83	0.82	3.65	3.61	8.13	8.09	6.95	6.94
60	Full	1000	0.8	Normal	Right-skewed	Right-skewed	24.26	24.28	2.04	2.03	0.44	0.43	1.93	1.92	8.70	8.68	13.71	13.59	9.99	9.95
61	Full	400	0.4	Right-skewed	Right-skewed	Right-skewed	39.60	20.79	1.19	0.98	0.45	0.32	0.81	0.80	3.42	3.42	8.56	7.50	7.36	6.22
62	Full	1000	0.4	Right-skewed	Right-skewed	Right-skewed	49.15	21.62	1.82	1.58	0.47	0.43	1.89	1.89	8.35	8.37	12.70	12.33	9.54	9.40
63	Full	400	0.6	Right-skewed	Right-skewed	Right-skewed	17.27	12.28	1.15	1.03	0.30	0.29	0.82	0.81	3.44	3.44	7.75	7.40	6.22	6.16
64	Full	1000	0.6	Right-skewed	Right-skewed	Right-skewed	20.60	17.53	1.94	1.75	0.41	0.41	1.92	1.91	8.35	8.38	13.72	13.39	9.41	9.38
65	Full	400	0.8	Right-skewed	Right-skewed	Right-skewed	16.98	16.82	1.36	1.36	0.33	0.33	0.83	0.82	3.42	3.41	9.58	9.44	7.15	7.16
66	Full	1000	0.8	Right-skewed	Right-skewed	Right-skewed	26.88	26.60	2.40	2.40	0.45	0.44	1.93	1.92	8.33	8.34	17.19	16.90	10.30	10.29
67	Full	400	0.4	Uniform	Right-skewed	Right-skewed	45.60	28.94	3.72	3.74	2.65	2.60	0.81	0.79	3.29	3.28	20.06	18.43	22.33	21.66
68	Full	1000	0.4	Uniform	Right-skewed	Right-skewed	61.45	41.74	5.01	4.66	2.72	2.65	1.90	1.89	8.08	8.07	29.57	26.33	23.95	23.20
69	Full	400	0.6	Uniform	Right-skewed	Right-skewed	14.25	11.11	1.09	0.91	0.99	0.30	0.83	0.82	3.52	3.49	8.37	6.35	10.15	6.59
70	Full	1000	0.6	Uniform	Right-skewed	Right-skewed	16.53	15.87	1.63	1.51	1.01	0.42	1.93	1.93	8.54	8.55	11.00	10.07	13.23	9.65
71	Full	400	0.8	Uniform	Right-skewed	Right-skewed	14.48	14.45	1.29	1.28	0.32	0.31	0.83	0.82	3.67	3.67	8.56	8.55	6.91	6.87
72	Full	1000	0.8	Uniform	Right-skewed	Right-skewed	23.16	23.03	2.25	2.23	0.43	0.43	1.94	1.93	8.70	8.67	14.90	14.67	10.01	9.97
73	Linear	400	0.4	Normal	Normal	Normal	12.04	10.69	0.92	0.79	0.36	0.31	0.84	0.83	3.61	3.60	7.56	6.09	7.08	6.30
74	Linear	1000	0.4	Normal	Normal	Normal	14.13	14.04	1.16	1.14	0.43	0.42	1.95	1.94	8.75	8.75	8.92	8.58	9.47	9.47
75	Linear	400	0.6	Normal	Normal	Normal	9.43	9.41	0.69	0.68	0.29	0.29	0.86	0.85	3.72	3.70	5.19	5.12	6.23	6.23
76	Linear	1000	0.6	Normal	Normal	Normal	12.98	13.01	1.10	1.08	0.41	0.40	1.95	1.95	8.86	8.89	8.14	7.90	9.42	9.41
77	Linear	400	0.8	Normal	Normal	Normal	10.79	10.68	0.86	0.86	0.33	0.32	0.84	0.83	3.69	3.67	5.96	5.93	6.96	6.95
78	Linear	1000	0.8	Normal	Normal	Normal	15.18	15.02	1.38	1.39	0.45	0.44	1.94	1.93	8.67	8.68	9.53	9.37	10.11	10.09
79	Linear	400	0.4	Right-skewed	Normal	Normal	12.34	11.02	1.05	0.90	0.38	0.31	0.84	0.83	3.41	3.40	8.24	6.69	7.22	6.06
80	Linear	1000	0.4	Right-skewed	Normal	Normal	14.35	14.16	1.45	1.42	0.44	0.42	1.94	1.94	8.34	8.35	10.57	10.11	9.35	9.29
81	Linear	400	0.6	Right-skewed	Normal	Normal	9.92	9.86	0.82	0.81	0.30	0.29	0.85	0.84	3.44	3.43	5.86	5.74	6.13	6.06

Table S2

Computation Time by Approach (in Seconds) (continued)

							Simulation 1								Simulation 2					
				Distribution			LMS		QML		UPI		LSAM		LSAM		QML		UPI	
Cond.	Model	N	Rel.	Distr.	Epsilon	Zeta	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn
82	Linear	1000	0.6	Right-skewed	Normal	Normal	14.27	14.24	1.37	1.35	0.42	0.41	1.94	1.93	8.35	8.35	9.67	9.51	9.34	9.35
83	Linear	400	0.8	Right-skewed	Normal	Normal	11.13	10.88	0.93	0.92	0.34	0.34	0.84	0.83	3.41	3.41	6.29	6.20	7.07	7.06
84	Linear	1000	0.8	Right-skewed	Normal	Normal	16.31	16.21	1.57	1.54	0.46	0.45	1.93	1.93	8.31	8.31	10.45	10.30	10.32	10.31
85	Linear	400	0.4	Uniform	Normal	Normal	17.26	11.82	1.85	1.05	1.06	0.36	0.83	0.82	3.45	3.43	11.87	9.04	11.17	8.30
86	Linear	1000	0.4	Uniform	Normal	Normal	16.52	14.89	1.45	1.22	0.53	0.43	1.94	1.93	8.54	8.53	11.87	9.64	10.45	9.56
87	Linear	400	0.6	Uniform	Normal	Normal	9.90	9.94	0.74	0.73	0.30	0.29	0.86	0.84	3.70	3.69	5.46	5.29	6.35	6.24
88	Linear	1000	0.6	Uniform	Normal	Normal	13.44	13.43	1.17	1.15	0.41	0.40	1.95	1.94	8.71	8.69	8.42	8.22	9.38	9.36
89	Linear	400	0.8	Uniform	Normal	Normal	11.08	10.83	0.98	0.99	0.33	0.32	0.85	0.84	3.67	3.65	6.68	6.64	6.89	6.88
90	Linear	1000	0.8	Uniform	Normal	Normal	15.78	15.65	1.63	1.63	0.45	0.44	1.95	1.94	8.67	8.65	10.95	10.83	10.06	10.03
91	Linear	400	0.4	Normal	Right-skewed	Normal	42.06	24.52	3.27	3.23	1.99	2.00	0.81	0.80	3.34	3.32	17.82	16.71	17.86	17.94
92	Linear	1000	0.4	Normal	Right-skewed	Normal	52.55	32.25	4.30	3.43	1.95	1.86	1.91	1.90	8.13	8.12	23.98	22.57	20.21	20.22
93	Linear	400	0.6	Normal	Right-skewed	Normal	10.67	10.24	0.76	0.72	0.30	0.29	0.85	0.84	3.69	3.66	5.44	5.27	6.29	6.27
94	Linear	1000	0.6	Normal	Right-skewed	Normal	14.25	14.18	1.16	1.15	0.41	0.40	1.95	1.94	8.67	8.66	8.37	8.20	9.38	9.34
95	Linear	400	0.8	Normal	Right-skewed	Normal	10.99	10.87	0.88	0.87	0.33	0.32	0.85	0.84	3.67	3.64	6.01	5.95	6.98	6.99
96	Linear	1000	0.8	Normal	Right-skewed	Normal	15.08	14.71	1.42	1.40	0.44	0.43	1.95	1.94	8.67	8.66	9.57	9.40	10.03	10.01
97	Linear	400	0.4	Right-skewed	Right-skewed	Normal	22.52	13.04	1.34	1.05	0.53	0.33	0.83	0.82	3.38	3.38	9.30	7.46	8.25	6.23
98	Linear	1000	0.4	Right-skewed	Right-skewed	Normal	30.57	17.96	2.14	1.79	0.53	0.44	1.92	1.91	8.29	8.29	11.70	10.95	9.66	9.39
99	Linear	400	0.6	Right-skewed	Right-skewed	Normal	16.03	11.26	1.07	0.90	0.30	0.29	0.84	0.83	3.42	3.42	6.77	6.09	6.11	6.09
100	Linear	1000	0.6	Right-skewed	Right-skewed	Normal	20.08	15.76	1.71	1.48	0.41	0.41	1.92	1.92	8.27	8.27	9.98	9.71	9.28	9.27
101	Linear	400	0.8	Right-skewed	Right-skewed	Normal	11.20	10.96	0.96	0.94	0.34	0.33	0.85	0.84	3.40	3.40	6.29	6.19	7.02	7.01
102	Linear	1000	0.8	Right-skewed	Right-skewed	Normal	16.50	15.79	1.59	1.58	0.45	0.44	1.94	1.93	8.25	8.25	10.33	10.20	10.14	10.13
103	Linear	400	0.4	Uniform	Right-skewed	Normal	39.63	24.35	3.88	3.75	2.63	2.64	0.81	0.80	3.28	3.26	19.97	18.41	22.52	21.97
104	Linear	1000	0.4	Uniform	Right-skewed	Normal	55.23	38.73	4.95	4.67	2.69	2.61	1.91	1.90	8.01	8.00	29.78	26.99	23.73	22.99
105	Linear	400	0.6	Uniform	Right-skewed	Normal	13.60	10.78	1.10	0.81	1.02	0.30	0.84	0.83	3.50	3.48	7.28	5.76	9.77	6.48
106	Linear	1000	0.6	Uniform	Right-skewed	Normal	16.88	15.22	1.43	1.29	0.94	0.41	1.94	1.93	8.49	8.48	9.43	8.59	12.54	9.59
107	Linear	400	0.8	Uniform	Right-skewed	Normal	11.09	10.89	1.00	1.00	0.32	0.31	0.85	0.84	3.61	3.57	6.82	6.72	6.93	6.90
108	Linear	1000	0.8	Uniform	Right-skewed	Normal	15.64	15.35	1.65	1.66	0.44	0.43	1.95	1.94	8.65	8.65	10.87	10.69	10.01	9.99

Table S2

Computation Time by Approach (in Seconds) (continued)

							Simulation 1								Simulation 2					
				Distribution			LMS		QML		UPI		LSAM		LSAM		QML		UPI	
Cond.	Model	N	Rel.	Distr.	Epsilon	Zeta	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn
109	Linear	400	0.4	Normal	Normal	Right-skewed	15.90	11.18	0.93	0.76	0.38	0.31	0.85	0.83	3.56	3.56	8.56	6.47	7.28	6.27
110	Linear	1000	0.4	Normal	Normal	Right-skewed	19.05	14.43	1.16	1.14	0.43	0.42	1.94	1.93	8.62	8.63	8.87	8.48	9.40	9.40
111	Linear	400	0.6	Normal	Normal	Right-skewed	9.86	9.53	0.69	0.68	0.29	0.29	0.86	0.85	3.67	3.63	5.20	5.16	6.17	6.16
112	Linear	1000	0.6	Normal	Normal	Right-skewed	12.99	12.93	1.10	1.08	0.41	0.40	1.96	1.95	8.64	8.66	8.07	7.88	9.36	9.36
113	Linear	400	0.8	Normal	Normal	Right-skewed	10.87	10.72	0.85	0.86	0.33	0.32	0.85	0.84	3.63	3.63	5.96	5.92	6.98	6.99
114	Linear	1000	0.8	Normal	Normal	Right-skewed	15.05	14.93	1.39	1.38	0.45	0.44	1.95	1.94	8.58	8.57	9.45	9.34	10.06	10.06
115	Linear	400	0.4	Right-skewed	Normal	Right-skewed	18.72	11.76	1.07	0.88	0.39	0.30	0.84	0.83	3.38	3.38	9.30	6.96	7.38	6.01
116	Linear	1000	0.4	Right-skewed	Normal	Right-skewed	19.65	14.65	1.43	1.41	0.43	0.42	1.93	1.93	8.20	8.20	10.57	10.02	9.36	9.21
117	Linear	400	0.6	Right-skewed	Normal	Right-skewed	10.21	10.00	0.82	0.80	0.31	0.29	0.85	0.84	3.41	3.41	5.84	5.73	6.06	6.00
118	Linear	1000	0.6	Right-skewed	Normal	Right-skewed	14.34	14.34	1.36	1.34	0.42	0.41	1.94	1.93	8.21	8.22	9.49	9.37	9.25	9.25
119	Linear	400	0.8	Right-skewed	Normal	Right-skewed	11.20	11.05	0.93	0.91	0.34	0.34	0.85	0.84	3.40	3.40	6.20	6.12	7.07	7.08
120	Linear	1000	0.8	Right-skewed	Normal	Right-skewed	16.49	16.03	1.56	1.56	0.46	0.45	1.94	1.94	8.21	8.22	10.14	10.03	10.23	10.21
121	Linear	400	0.4	Uniform	Normal	Right-skewed	19.88	12.76	1.83	0.99	0.96	0.35	0.84	0.82	3.38	3.36	12.74	9.60	12.01	9.42
122	Linear	1000	0.4	Uniform	Normal	Right-skewed	20.72	15.39	1.38	1.22	0.51	0.44	1.94	1.93	8.41	8.42	12.37	9.56	10.54	9.51
123	Linear	400	0.6	Uniform	Normal	Right-skewed	10.31	9.97	0.74	0.71	0.30	0.29	0.86	0.85	3.62	3.60	5.88	5.33	6.28	6.19
124	Linear	1000	0.6	Uniform	Normal	Right-skewed	13.71	13.50	1.18	1.16	0.41	0.40	1.95	1.94	8.61	8.60	8.22	8.06	9.32	9.31
125	Linear	400	0.8	Uniform	Normal	Right-skewed	11.24	11.08	0.99	0.99	0.33	0.32	0.85	0.84	3.64	3.64	6.54	6.45	6.83	6.81
126	Linear	1000	0.8	Uniform	Normal	Right-skewed	15.58	15.30	1.60	1.62	0.44	0.44	1.96	1.95	8.48	8.47	10.77	10.69	9.97	9.96
127	Linear	400	0.4	Normal	Right-skewed	Right-skewed	45.05	27.55	3.39	3.30	2.07	2.03	0.82	0.80	3.28	3.26	17.97	16.62	18.67	18.99
128	Linear	1000	0.4	Normal	Right-skewed	Right-skewed	62.19	39.97	4.36	3.52	1.98	1.94	1.91	1.90	7.99	8.00	24.73	22.61	20.68	20.81
129	Linear	400	0.6	Normal	Right-skewed	Right-skewed	11.09	10.39	0.77	0.74	0.31	0.29	0.86	0.85	3.66	3.64	5.56	5.26	6.30	6.22
130	Linear	1000	0.6	Normal	Right-skewed	Right-skewed	14.46	14.45	1.18	1.16	0.41	0.40	1.96	1.95	8.59	8.61	8.24	8.08	9.37	9.37
131	Linear	400	0.8	Normal	Right-skewed	Right-skewed	11.12	11.01	0.89	0.88	0.33	0.32	0.85	0.84	3.61	3.57	6.01	5.94	6.96	6.98
132	Linear	1000	0.8	Normal	Right-skewed	Right-skewed	15.34	15.17	1.44	1.44	0.44	0.43	1.96	1.95	8.60	8.59	9.35	9.23	9.99	9.95
133	Linear	400	0.4	Right-skewed	Right-skewed	Right-skewed	35.84	14.87	1.40	1.06	0.52	0.33	0.82	0.81	3.35	3.36	9.55	7.28	8.29	6.21
134	Linear	1000	0.4	Right-skewed	Right-skewed	Right-skewed	41.67	18.56	2.13	1.78	0.51	0.44	1.92	1.91	8.16	8.17	11.79	10.80	9.66	9.30
135	Linear	400	0.6	Right-skewed	Right-skewed	Right-skewed	15.96	11.37	1.12	0.91	0.31	0.29	0.84	0.83	3.37	3.38	7.59	6.18	6.12	6.02

Table S2

Computation Time by Approach (in Seconds) (continued)

							Simulation 1								Simulation 2					
				Distribution			LMS		QML		UPI		LSAM		LSAM		QML		UPI	
Cond.	Model	N	Rel.	Distr.	Epsilon	Zeta	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn	Mean	Mdn
136	Linear	1000	0.6	Right-skewed	Right-skewed	Right-skewed	21.14	15.88	1.95	1.52	0.41	0.41	1.93	1.93	8.17	8.18	10.21	9.66	9.23	9.24
137	Linear	400	0.8	Right-skewed	Right-skewed	Right-skewed	11.15	10.86	0.95	0.94	0.34	0.33	0.85	0.84	3.38	3.39	6.25	6.15	7.01	7.00
138	Linear	1000	0.8	Right-skewed	Right-skewed	Right-skewed	15.97	15.64	1.58	1.57	0.45	0.45	1.95	1.94	8.15	8.15	10.15	9.97	10.08	10.07
139	Linear	400	0.4	Uniform	Right-skewed	Right-skewed	43.72	27.93	3.94	3.64	2.63	2.60	0.82	0.80	3.24	3.22	20.42	18.47	22.77	21.99
140	Linear	1000	0.4	Uniform	Right-skewed	Right-skewed	56.88	39.94	4.81	4.51	2.66	2.60	1.92	1.91	7.88	7.87	30.26	26.64	24.09	23.18
141	Linear	400	0.6	Uniform	Right-skewed	Right-skewed	14.37	10.98	1.04	0.81	1.04	0.31	0.85	0.84	3.43	3.42	8.44	5.94	10.03	6.40
142	Linear	1000	0.6	Uniform	Right-skewed	Right-skewed	16.48	15.09	1.37	1.28	0.99	0.41	1.95	1.94	8.30	8.32	10.52	8.69	12.62	9.46
143	Linear	400	0.8	Uniform	Right-skewed	Right-skewed	11.36	11.11	1.01	1.01	0.32	0.31	0.86	0.85	3.59	3.58	6.78	6.75	6.92	6.92
144	Linear	1000	0.8	Uniform	Right-skewed	Right-skewed	15.61	15.30	1.64	1.65	0.44	0.43	1.96	1.95	8.54	8.52	10.67	10.57	9.96	9.92

Note. Each entry is the mean or median time (in seconds) to complete a single replication, computed over 1,000 replications per condition. For example, in the first condition LMS took 12.26 seconds for one replication on average.

Table S3

Average Achieved R^2 Values Under Normal Baseline Conditions

		η_3 (Simulation 1)		η_4 (Simulation 2)		η_5 (Simulation 2)	
N	Rel.	Mean	MCSE	Mean	MCSE	Mean	MCSE
400	0.4	0.301	0.001	0.299	0.001	0.303	0.001
400	0.6	0.302	0.001	0.302	0.001	0.303	0.001
400	0.8	0.301	0.001	0.298	0.001	0.304	0.001
1000	0.4	0.299	0.001	0.301	0.001	0.303	0.001
1000	0.6	0.301	0.001	0.302	0.001	0.303	0.001
1000	0.8	0.300	0.001	0.299	0.001	0.302	0.001

Note. Mean = average observed R^2 across 1,000 replications; MCSE = Monte Carlo standard error, computed as $SD/\sqrt{K}$, where SD is the standard deviation of the observed R^2 values and K = 1,000 replications. Only baseline conditions (all distributions normal, full model) are shown.

Table S4

Average Achieved Reliability Values Under Normal Baseline Conditions

		Simulation 1						Simulation 2									
		η_1		η_2		η_3		η_1		η_2		η_3		η_4		η_5	
N	Rel.	Mean	MCSE	Mean	MCSE	Mean	MCSE	Mean	MCSE	Mean	MCSE	Mean	MCSE	Mean	MCSE	Mean	MCSE
400	0.4	0.399	0.001	0.399	0.001	0.401	0.001	0.398	0.001	0.399	0.001	0.398	0.001	0.399	0.001	0.398	0.001
400	0.6	0.599	0.001	0.599	0.001	0.600	0.001	0.599	0.001	0.599	0.001	0.599	0.001	0.598	0.001	0.600	0.001
400	0.8	0.799	0.000	0.799	0.000	0.800	0.000	0.799	0.000	0.799	0.000	0.799	0.000	0.800	0.000	0.799	0.000
1000	0.4	0.400	0.000	0.399	0.000	0.401	0.000	0.400	0.000	0.400	0.000	0.399	0.000	0.400	0.000	0.400	0.000
1000	0.6	0.599	0.000	0.600	0.000	0.600	0.000	0.600	0.000	0.599	0.000	0.600	0.000	0.599	0.000	0.600	0.000
1000	0.8	0.800	0.000	0.800	0.000	0.800	0.000	0.799	0.000	0.799	0.000	0.799	0.000	0.800	0.000	0.800	0.000

Note. Mean = average observed reliability across 1,000 replications, computed as the mean of indicator-level reliabilities within each factor; MCSE = Monte Carlo standard error, computed as $SD/\sqrt{K}$, where K = 1,000 replications. Only baseline conditions (all distributions normal, full model) are shown.

Plots — Simulation 1

Figure S1

Absolute Bias for Simulation 1

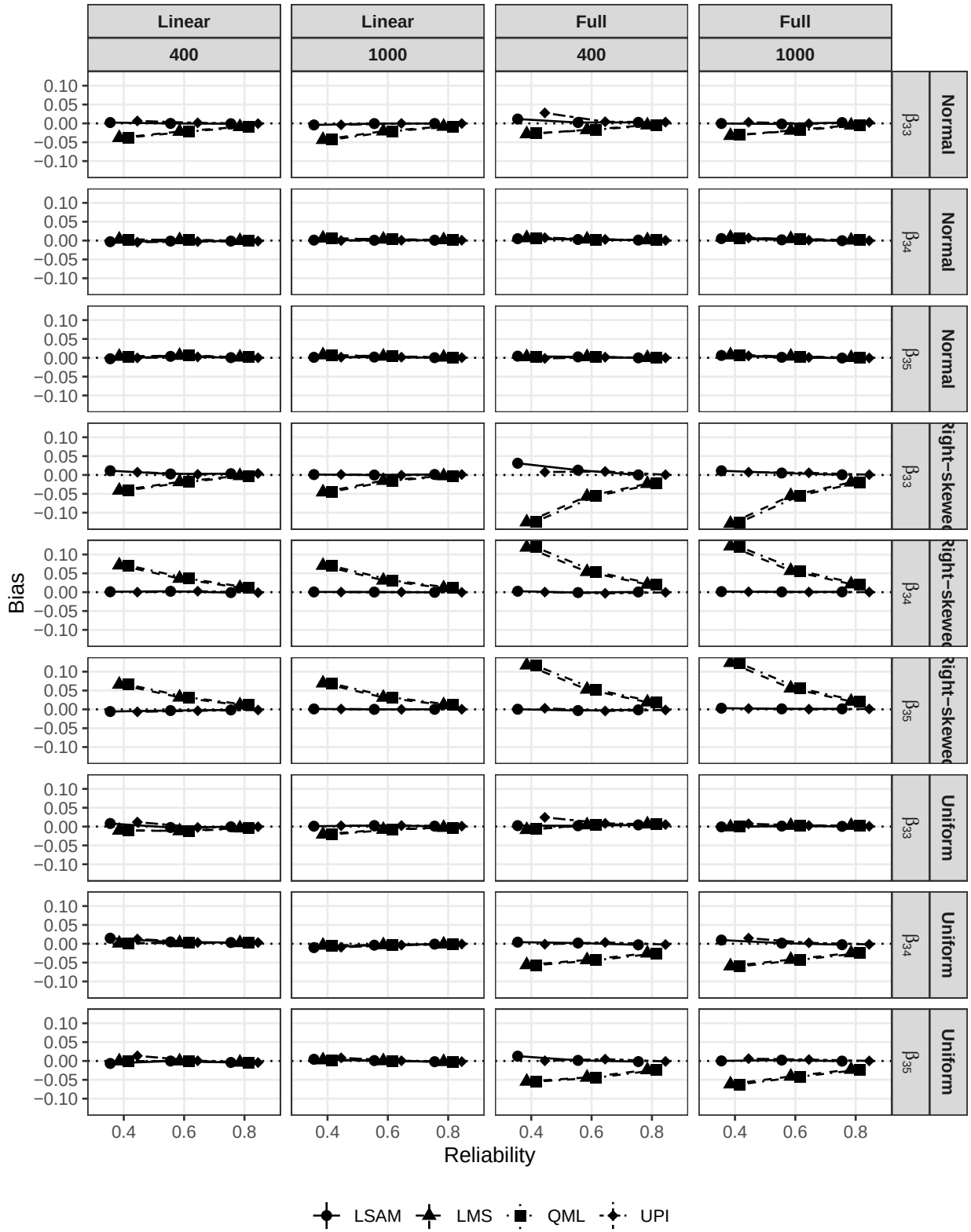


Figure S2

Relative Bias for Simulation 1

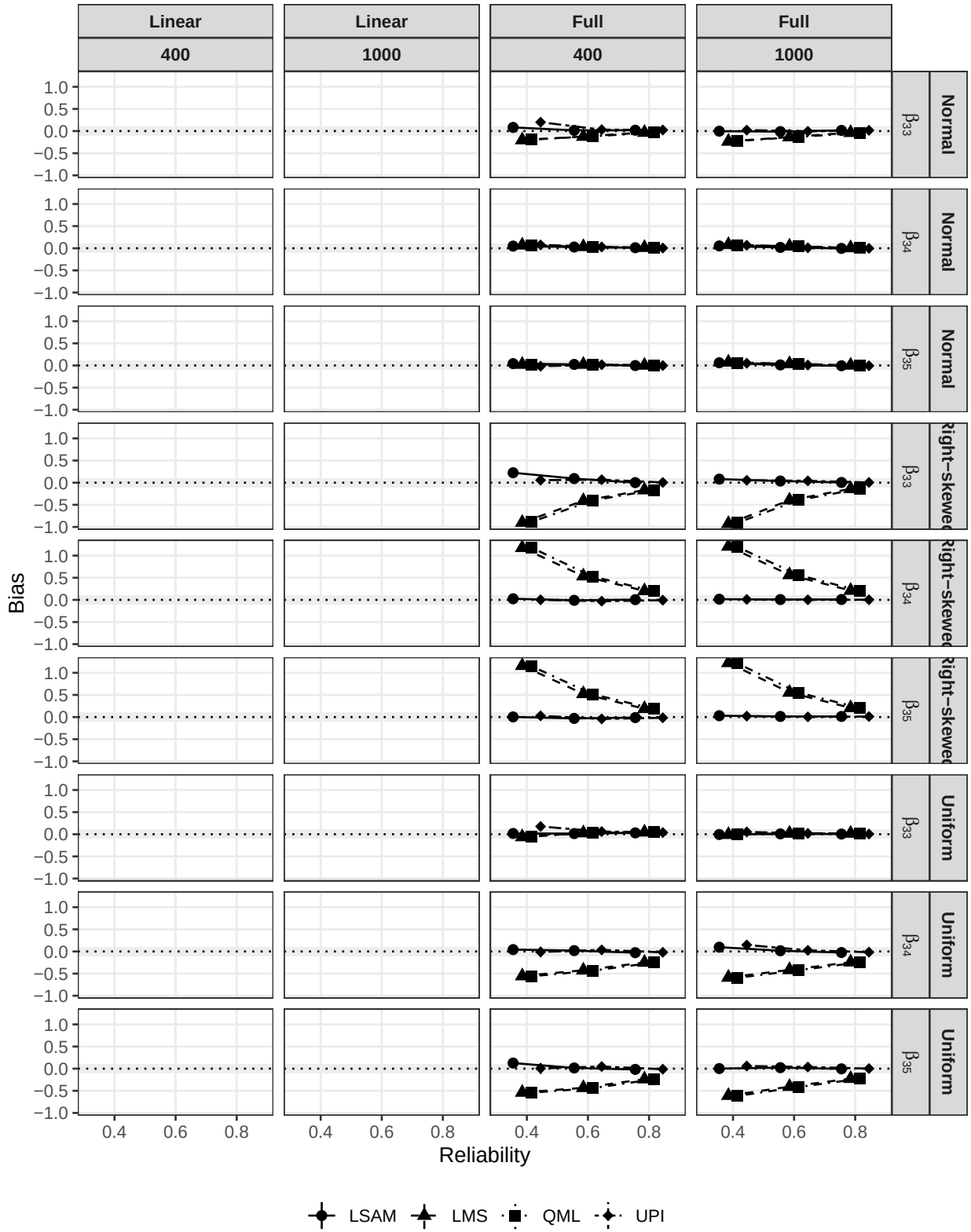


Figure S3

RMSE for Simulation 1

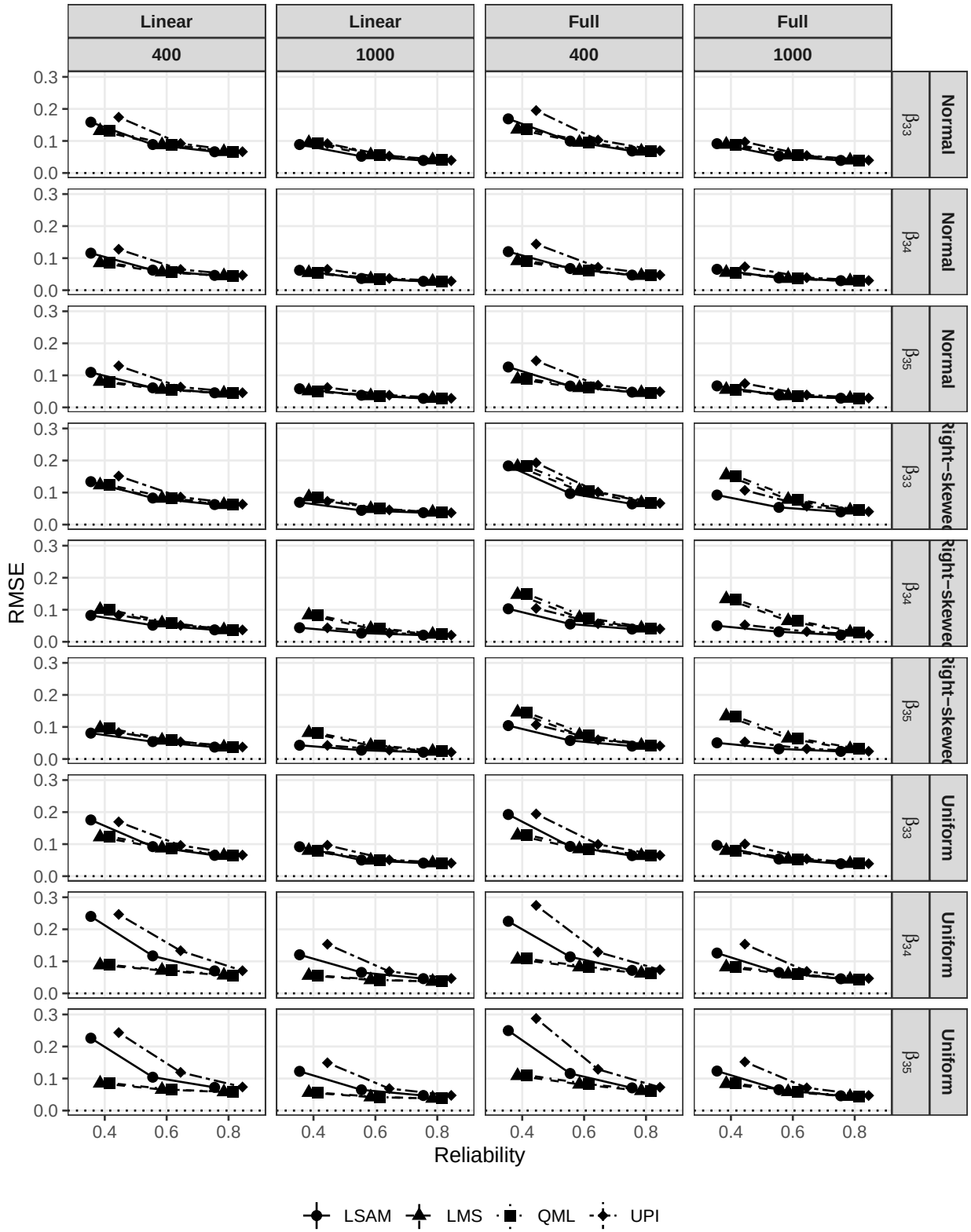


Figure S4

Relative RMSE for Simulation 1

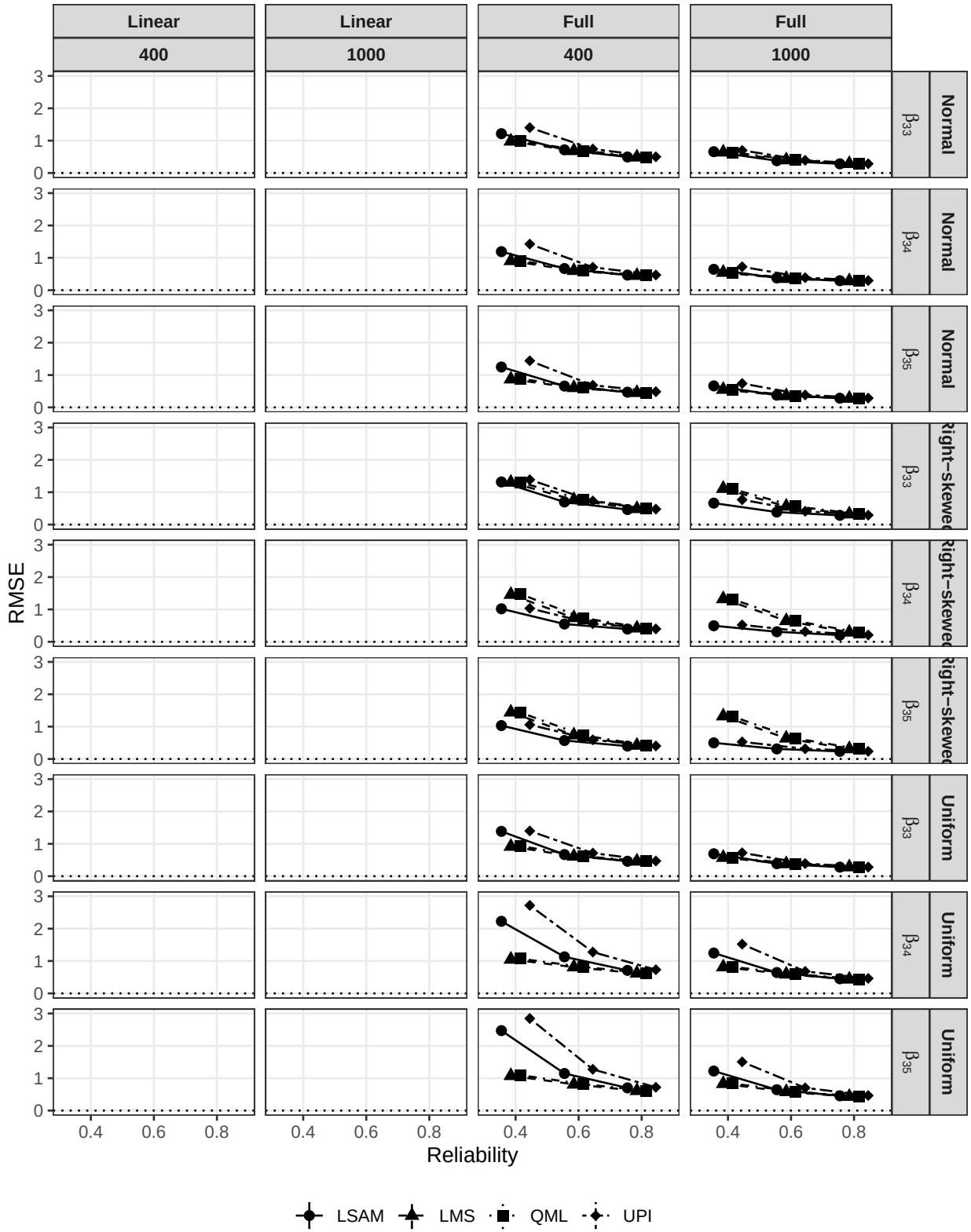


Figure S5

SE/SD Ratio for Simulation 1

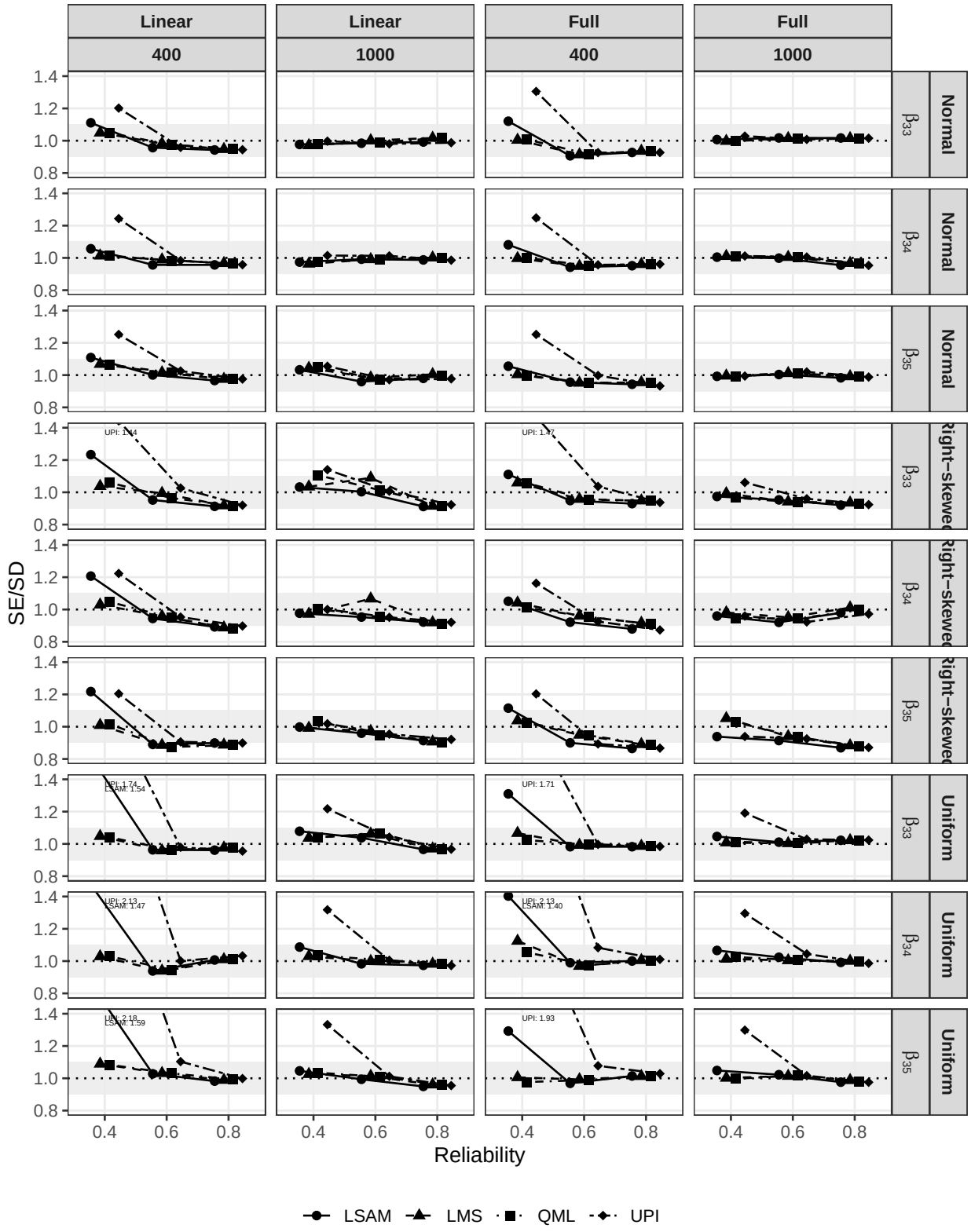


Figure S6

Coverage for Simulation 1

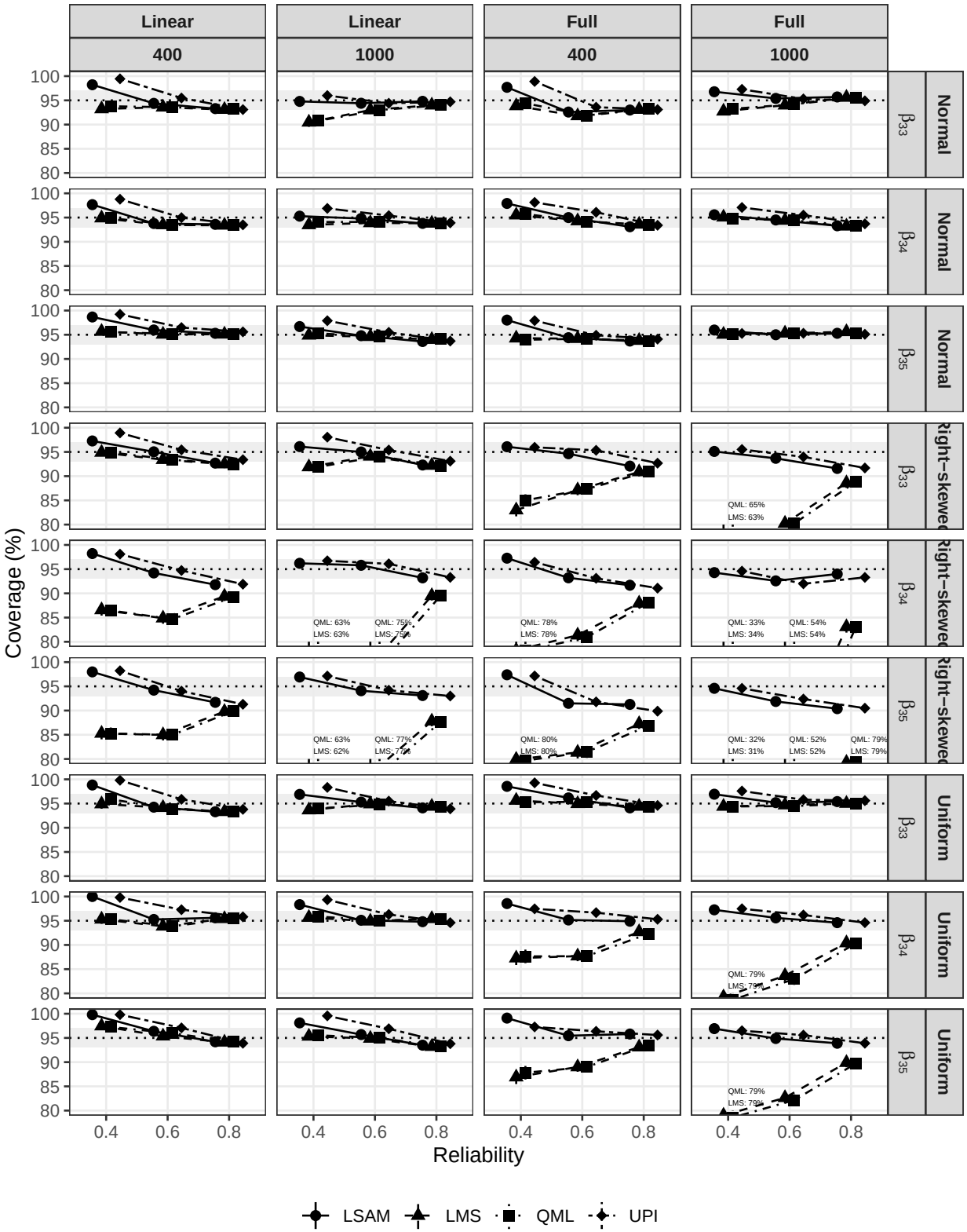


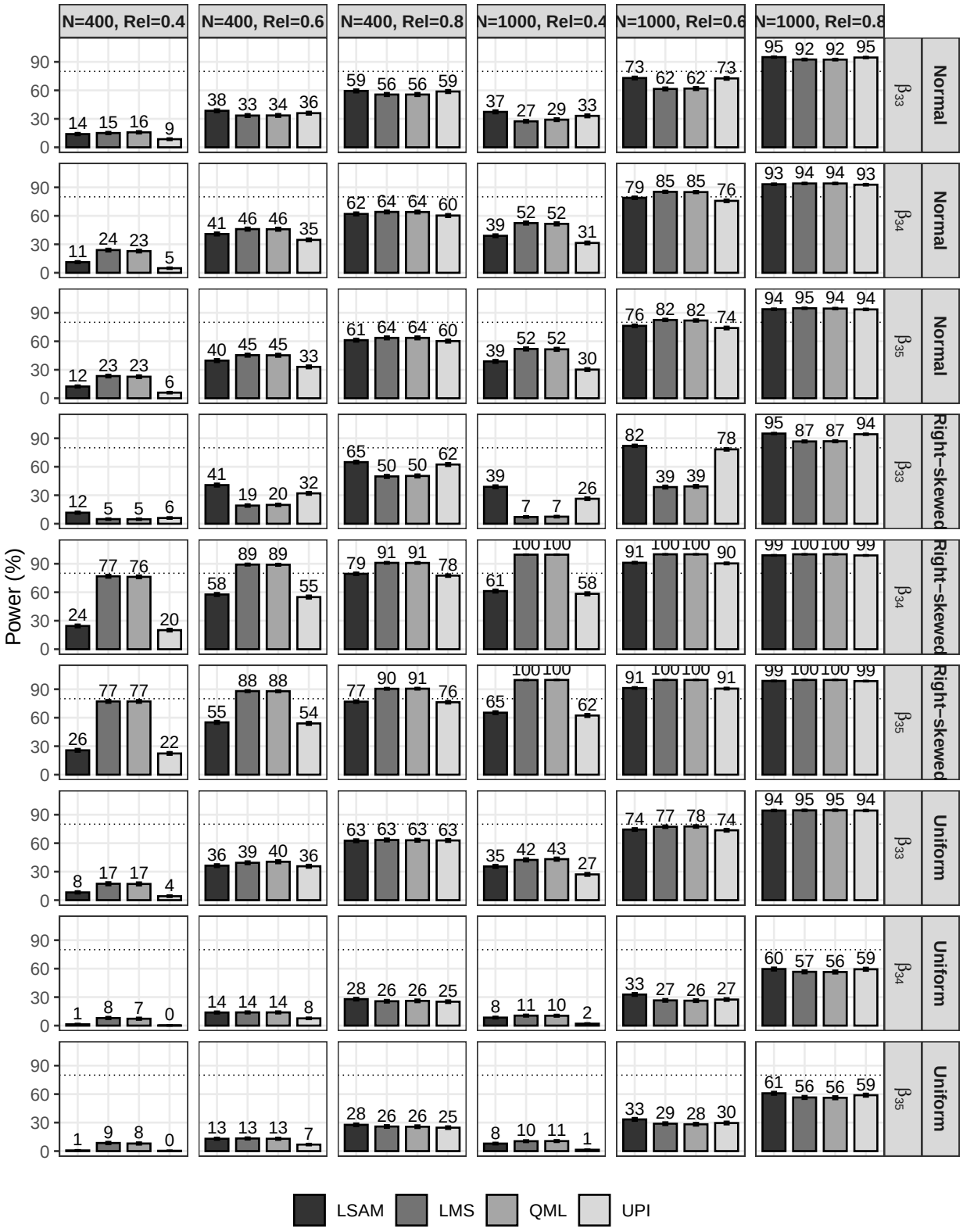
Figure S7

Type I Error Rate for Simulation 1



Figure S8

Empirical Power for Simulation 1



Plots — Simulation 2

Normal

Figure S9

Absolute Bias for Simulation 2 (Normal)

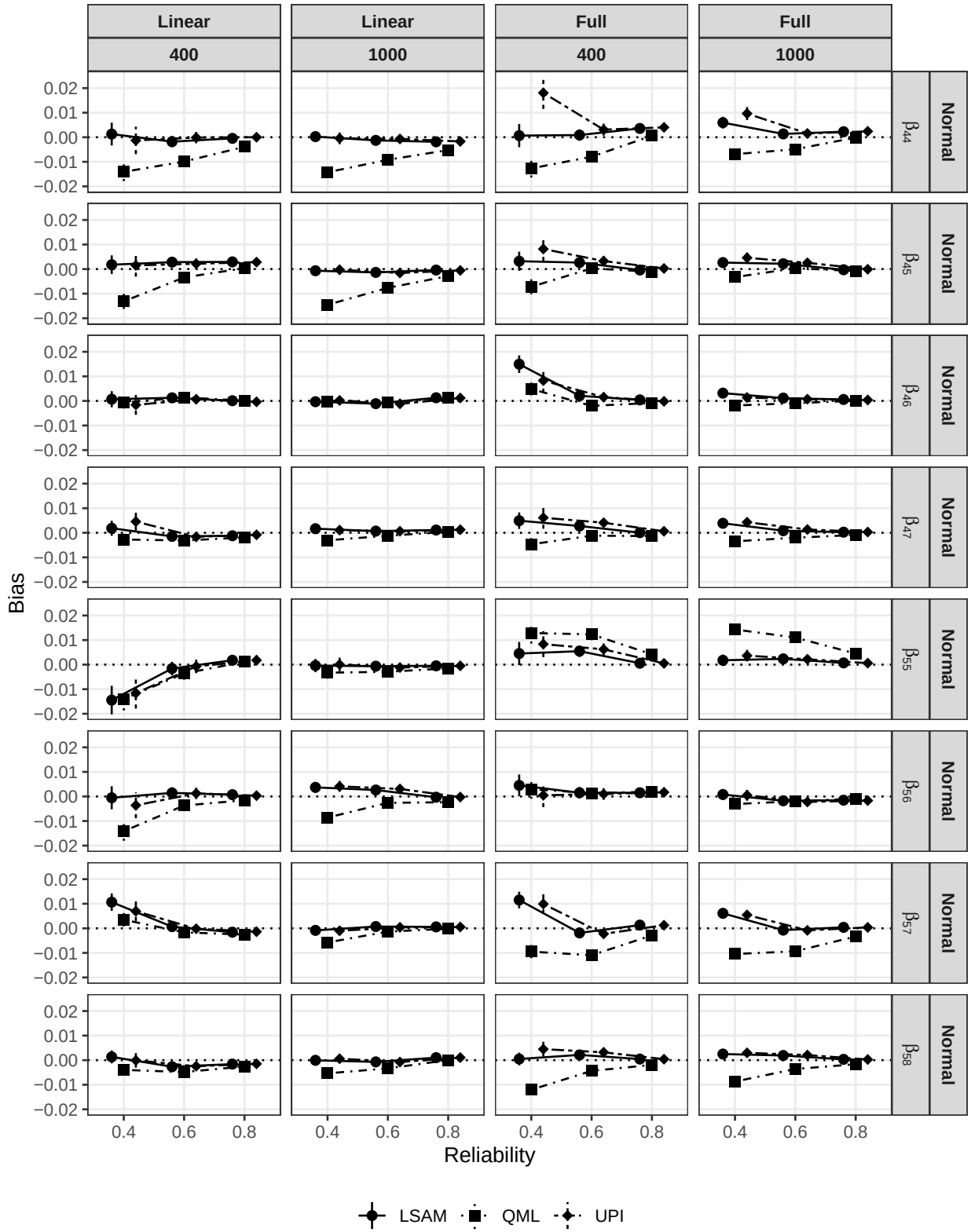


Figure S10

Relative Bias for Simulation 2 (Normal)

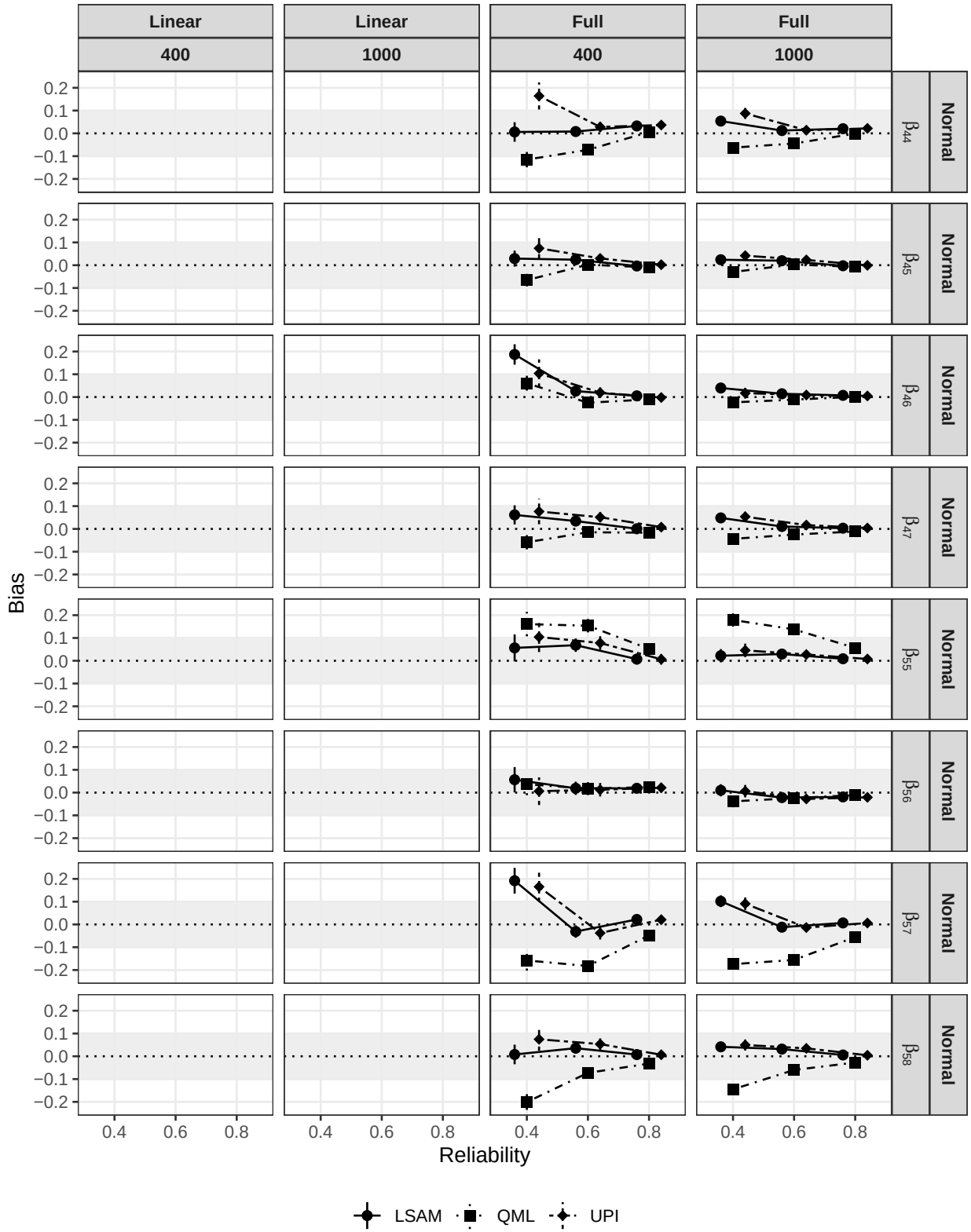


Figure S11

RMSE for Simulation 2 (Normal)

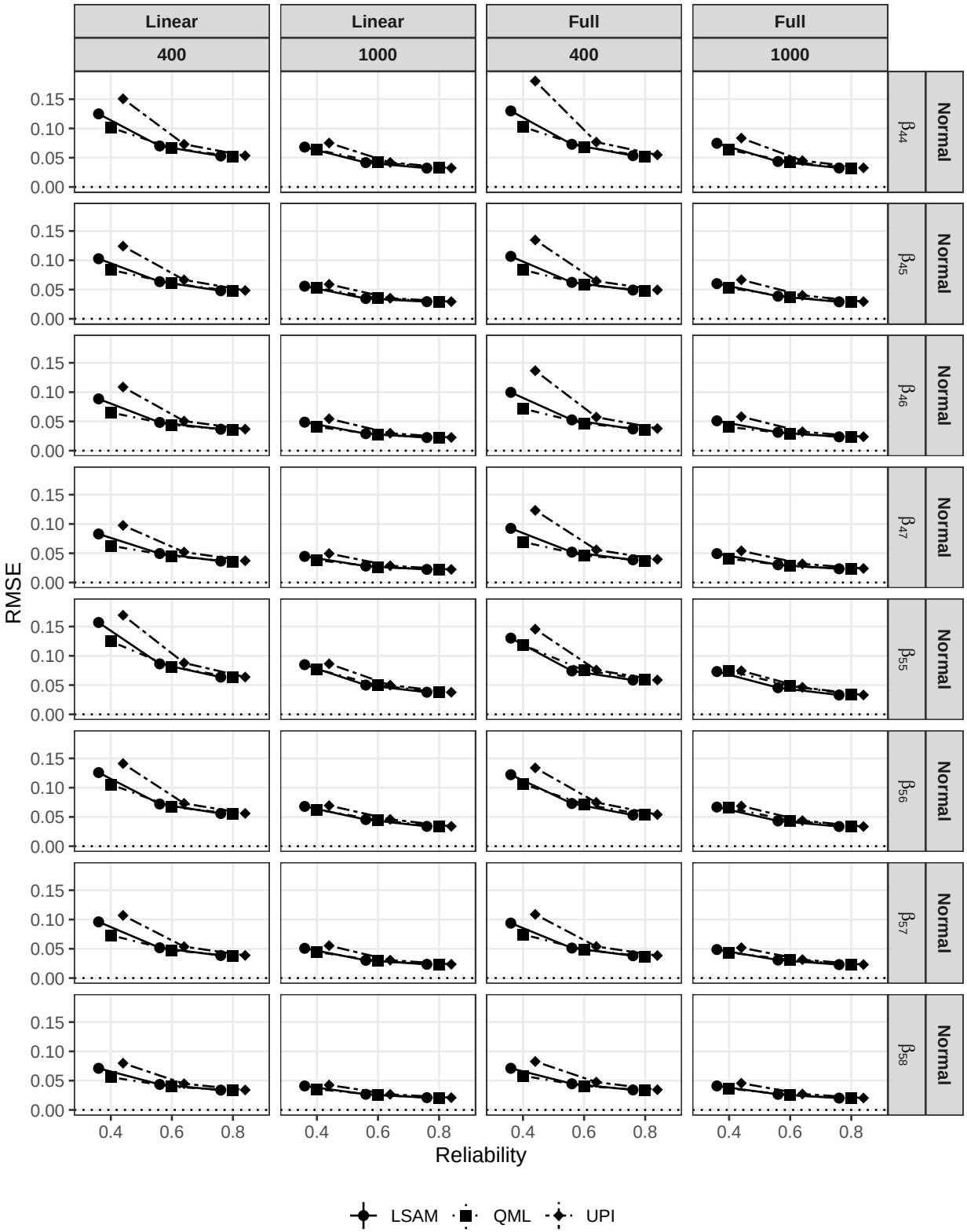


Figure S12

Relative RMSE for Simulation 2 (Normal)

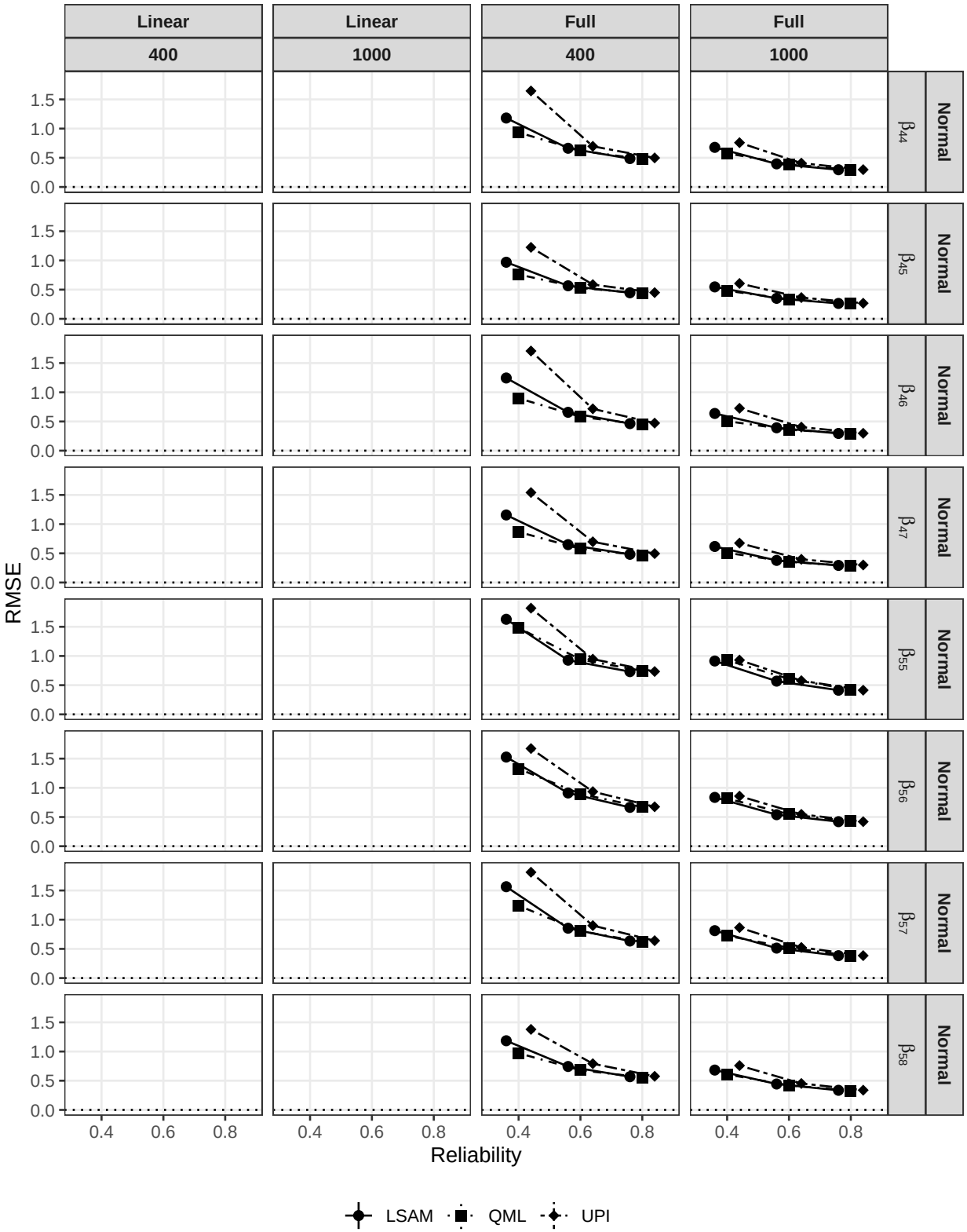


Figure S13

SE/SD Ratio for Simulation 2 (Normal)

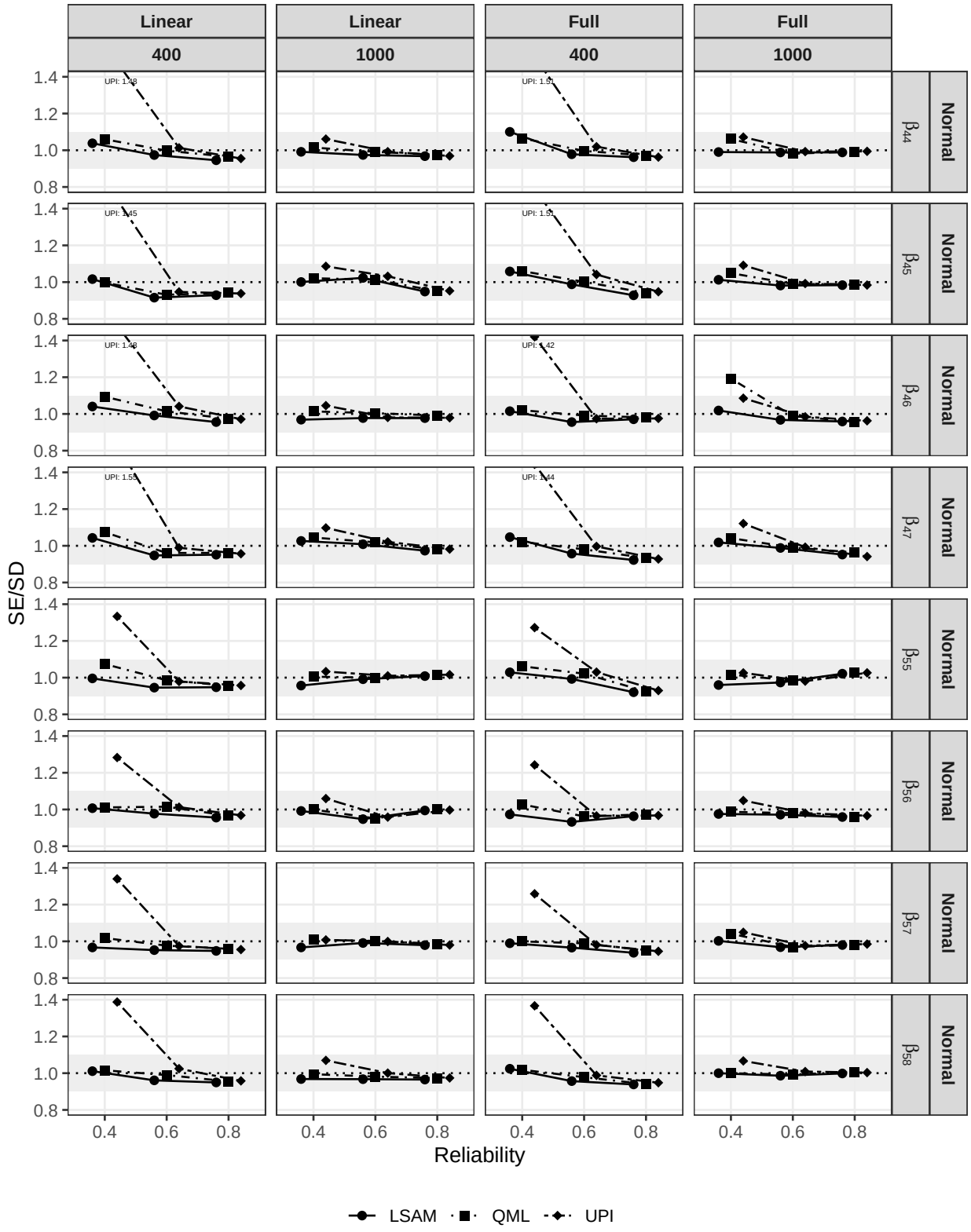


Figure S14

Coverage for Simulation 2 (Normal)

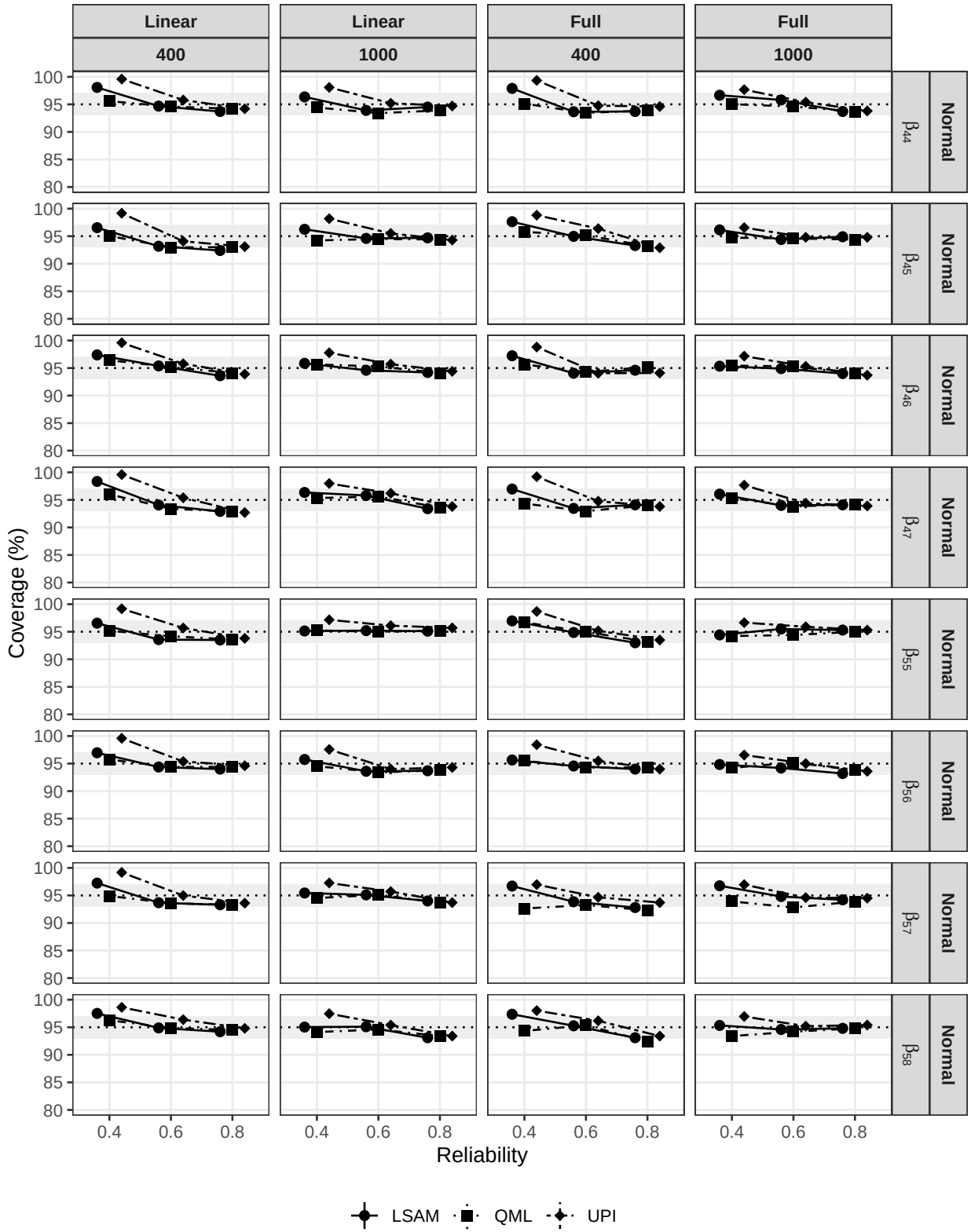


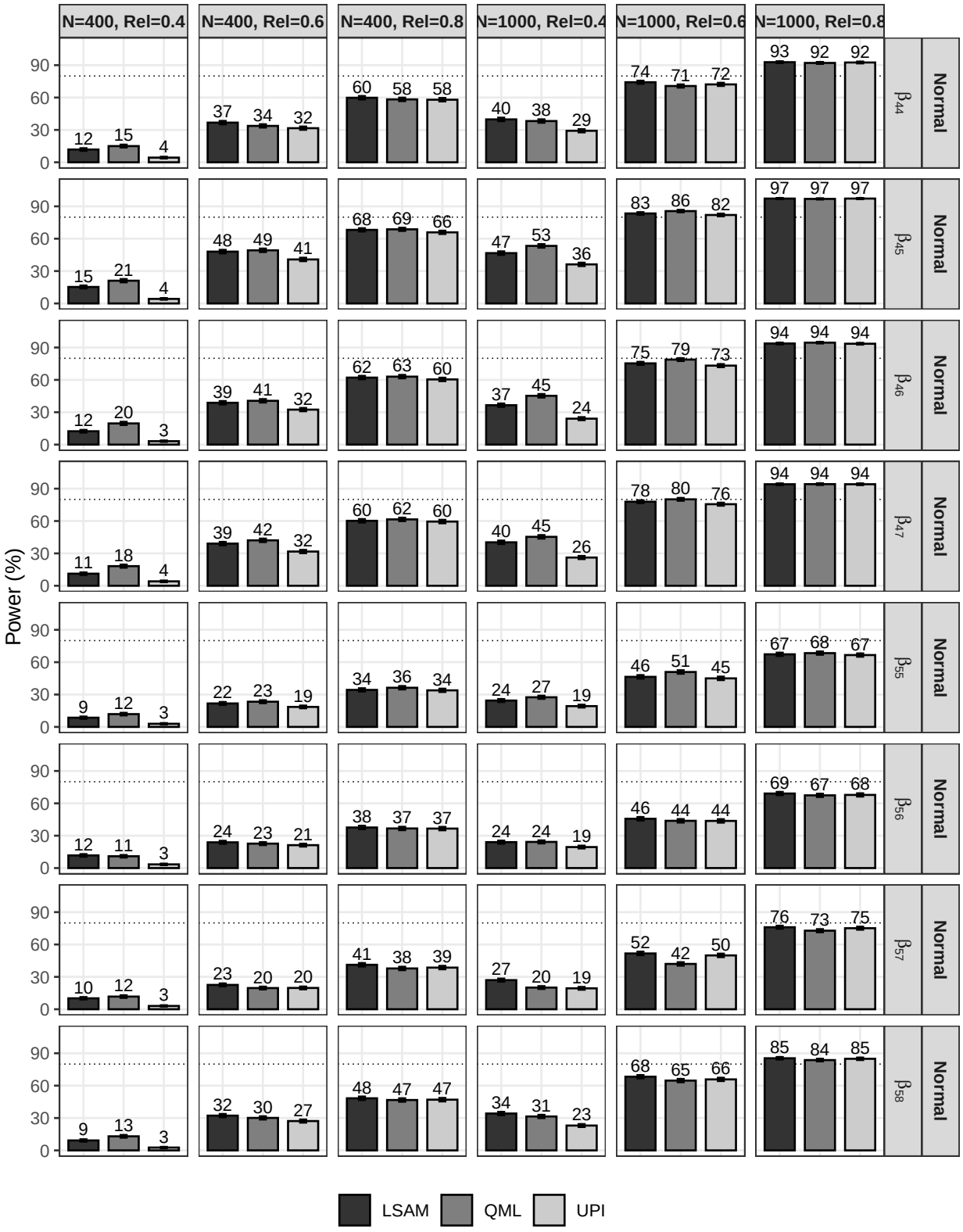
Figure S15

Type I Error Rate for Simulation 2 (Normal)



Figure S16

Empirical Power for Simulation 2 (Normal)



Right-Skewed

Figure S17

Absolute Bias for Simulation 2 (Right-skewed)

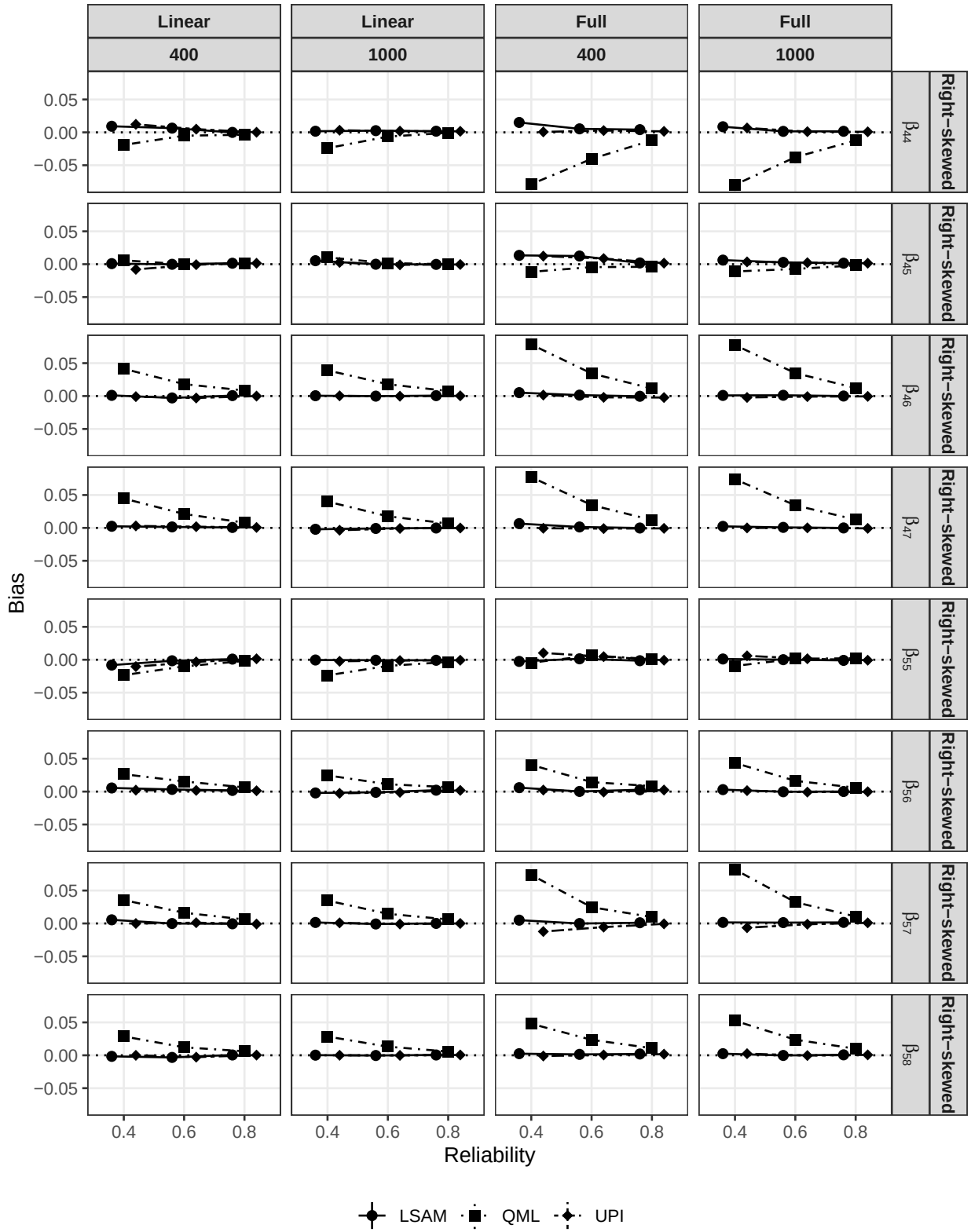


Figure S18

Relative Bias for Simulation 2 (Right-skewed)

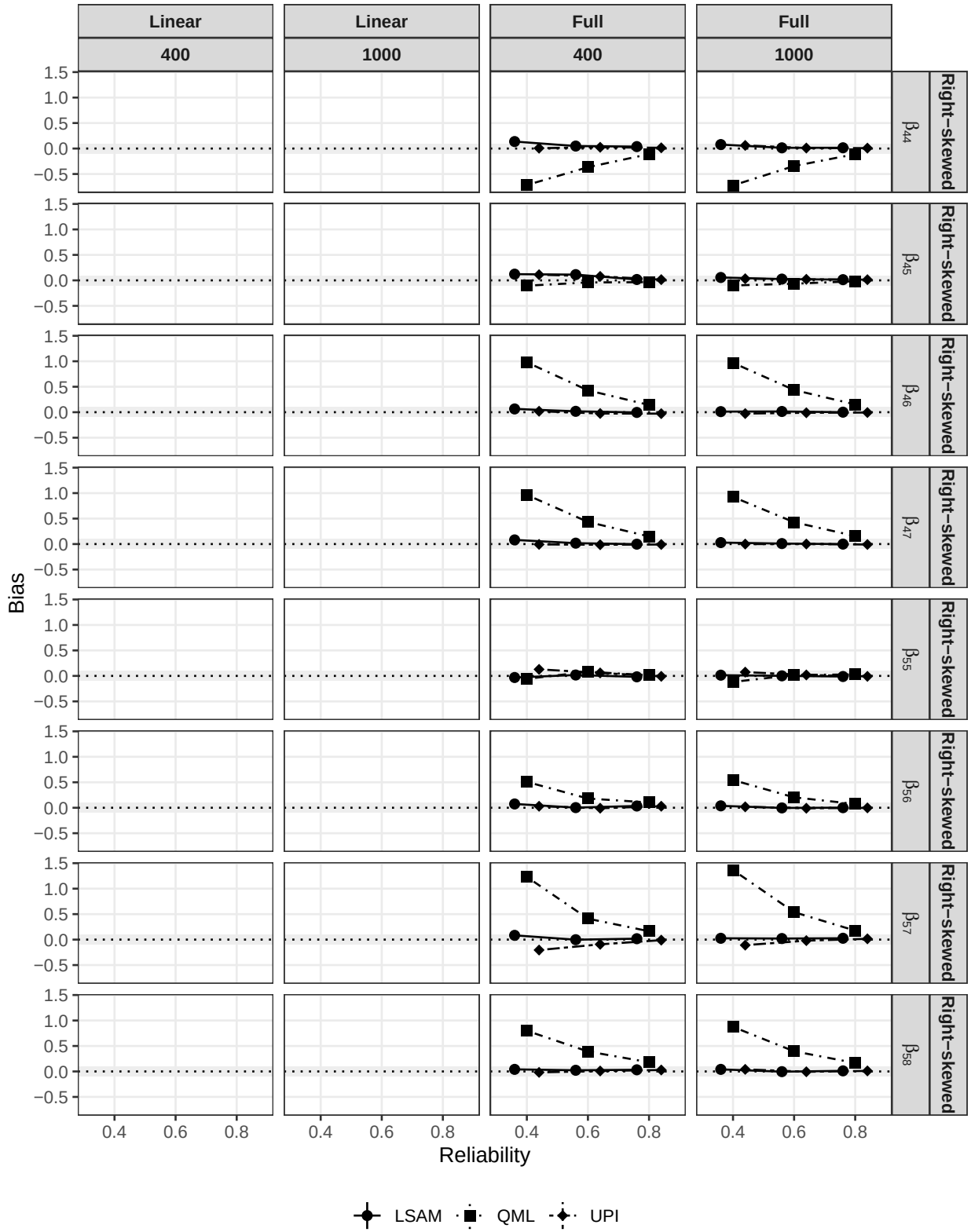


Figure S19

RMSE for Simulation 2 (Right-skewed)

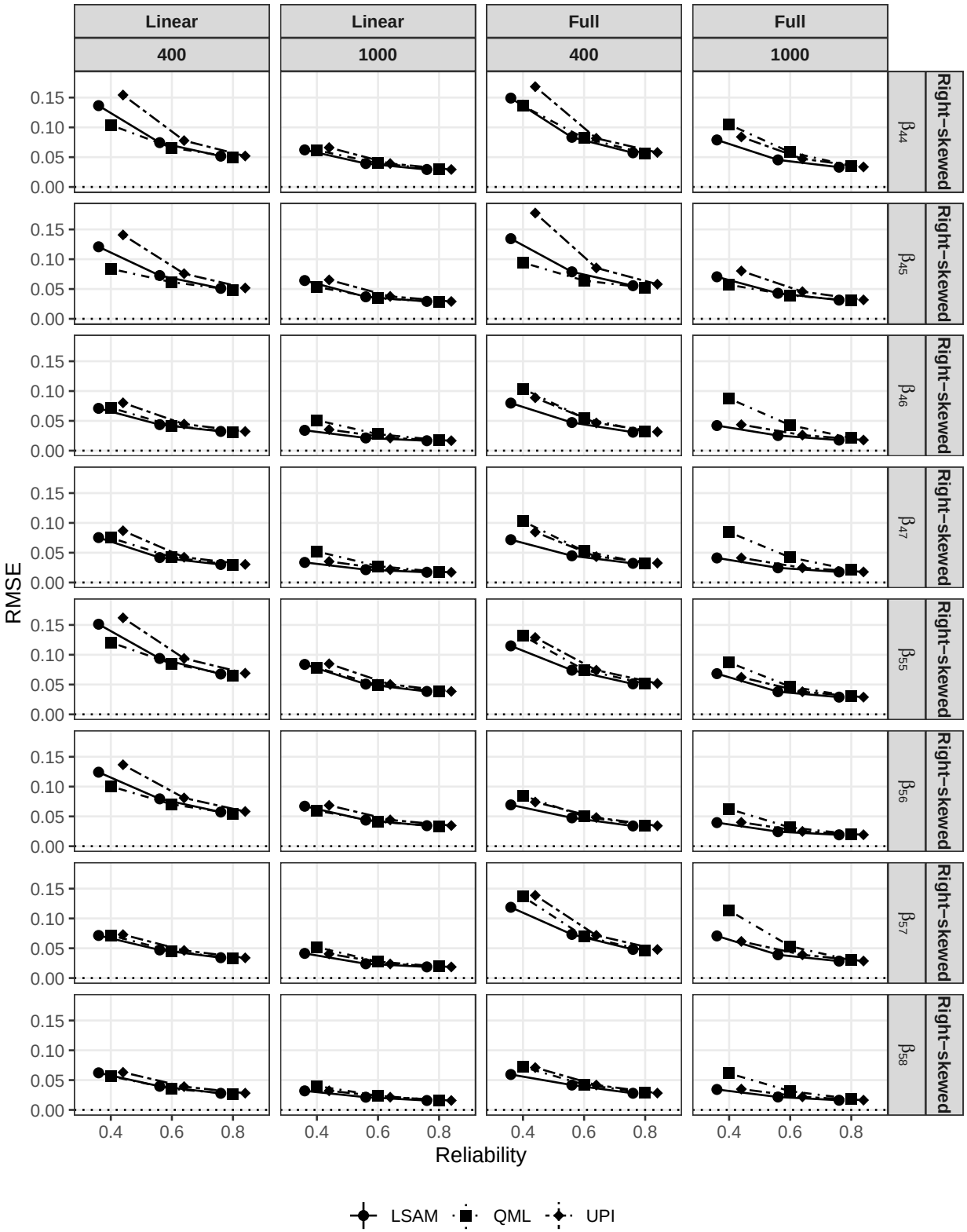


Figure S20

Relative RMSE for Simulation 2 (Right-skewed)

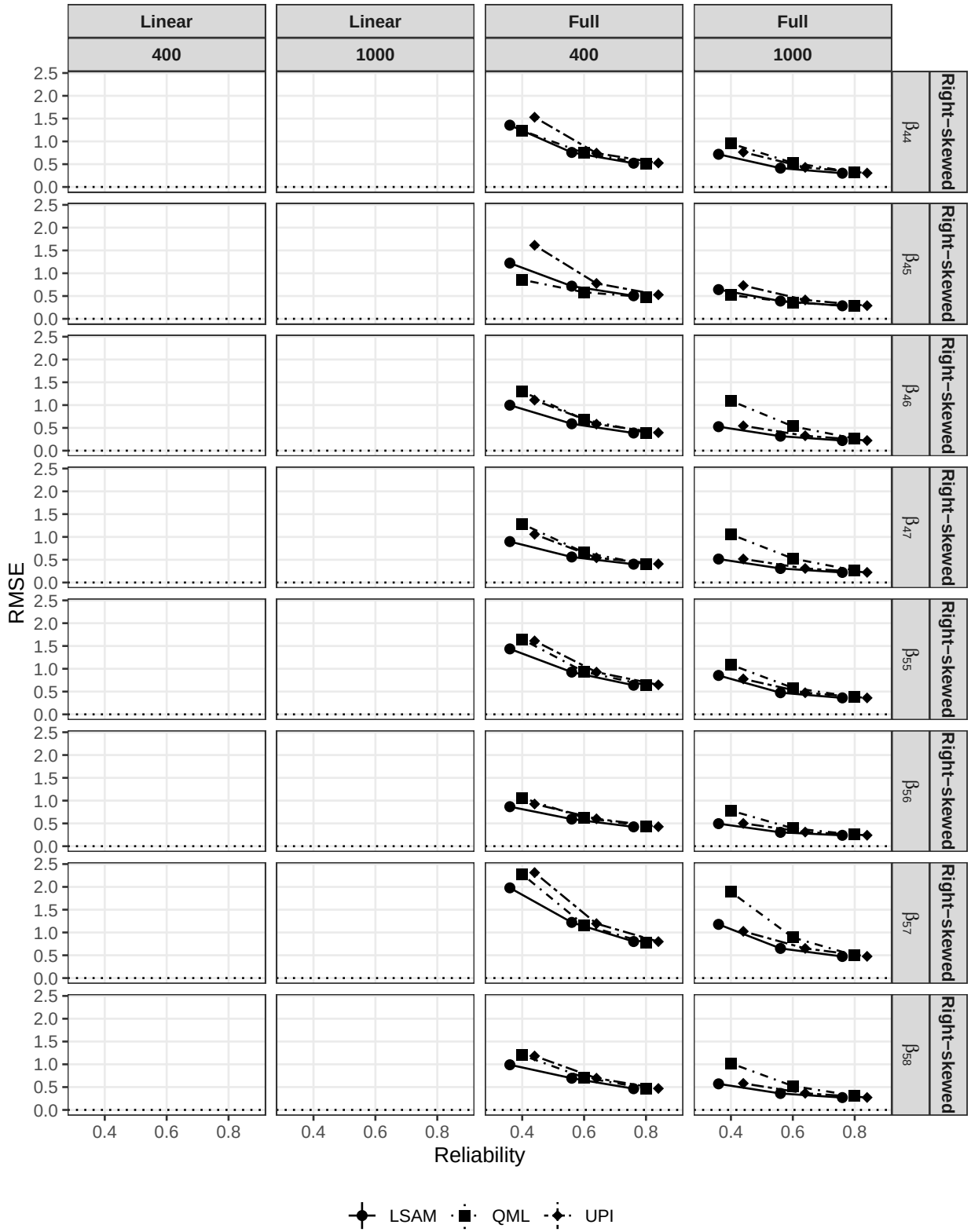


Figure S21

SE/SD Ratio for Simulation 2 (Right-skewed)

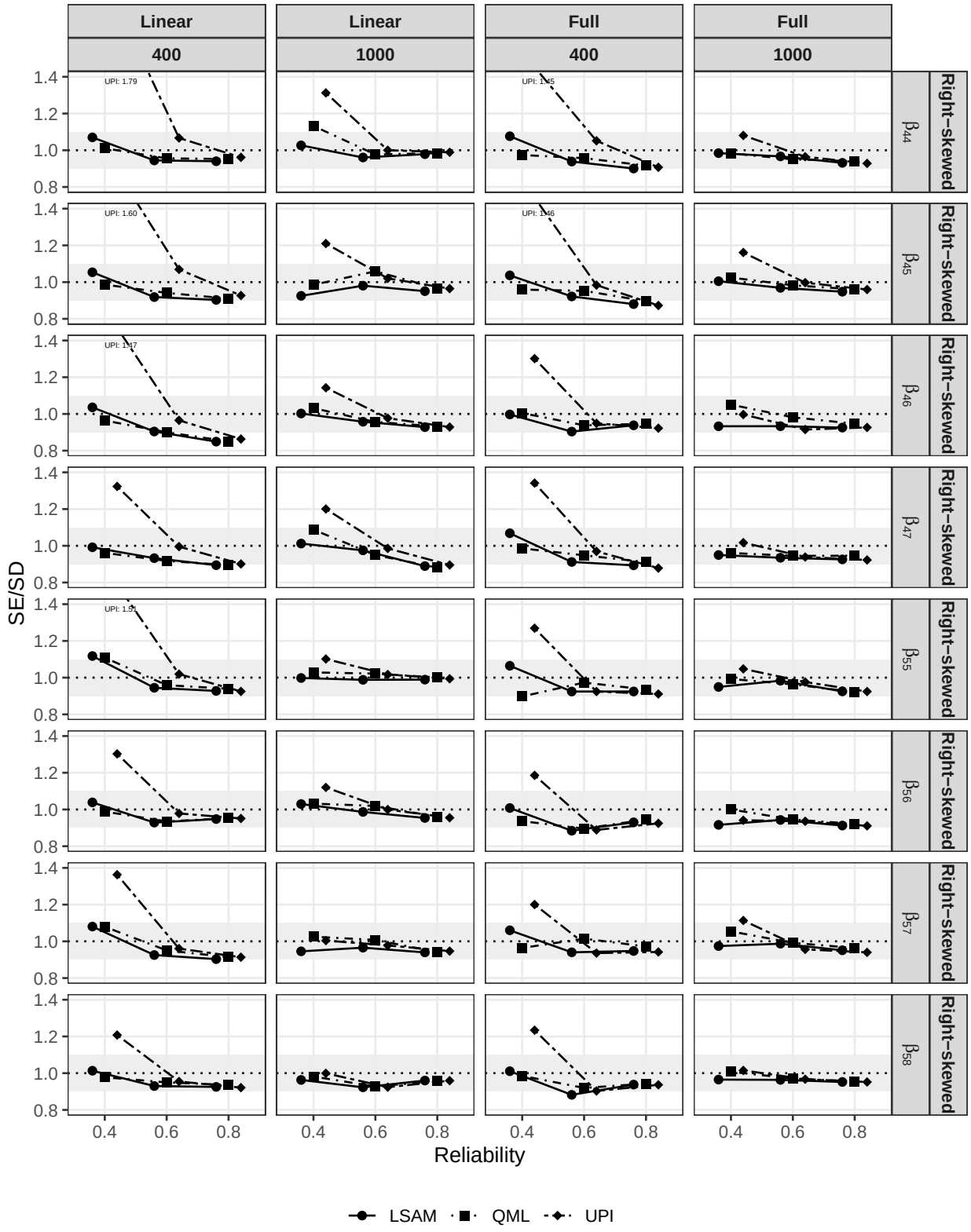


Figure S22

Coverage for Simulation 2 (Right-skewed)

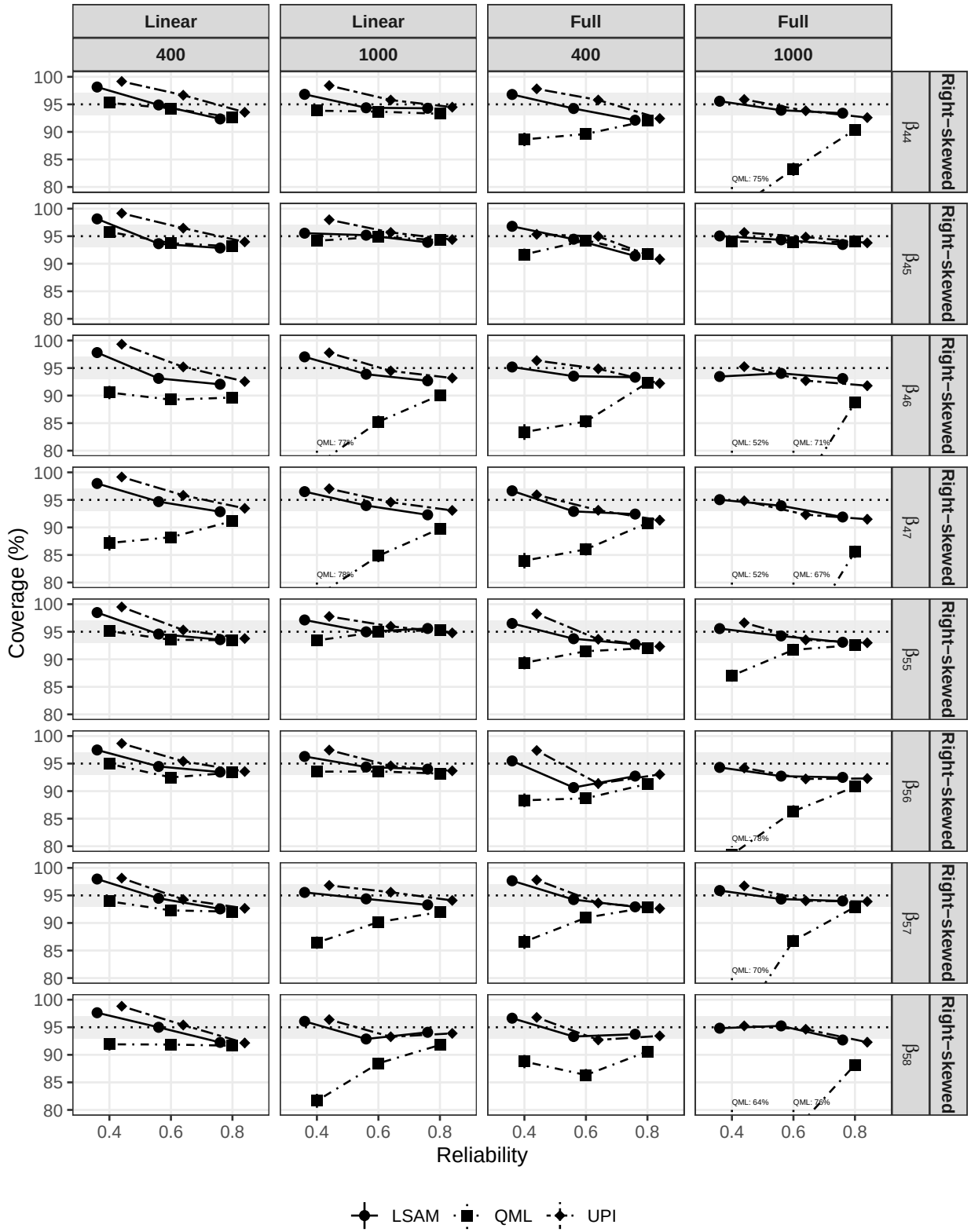


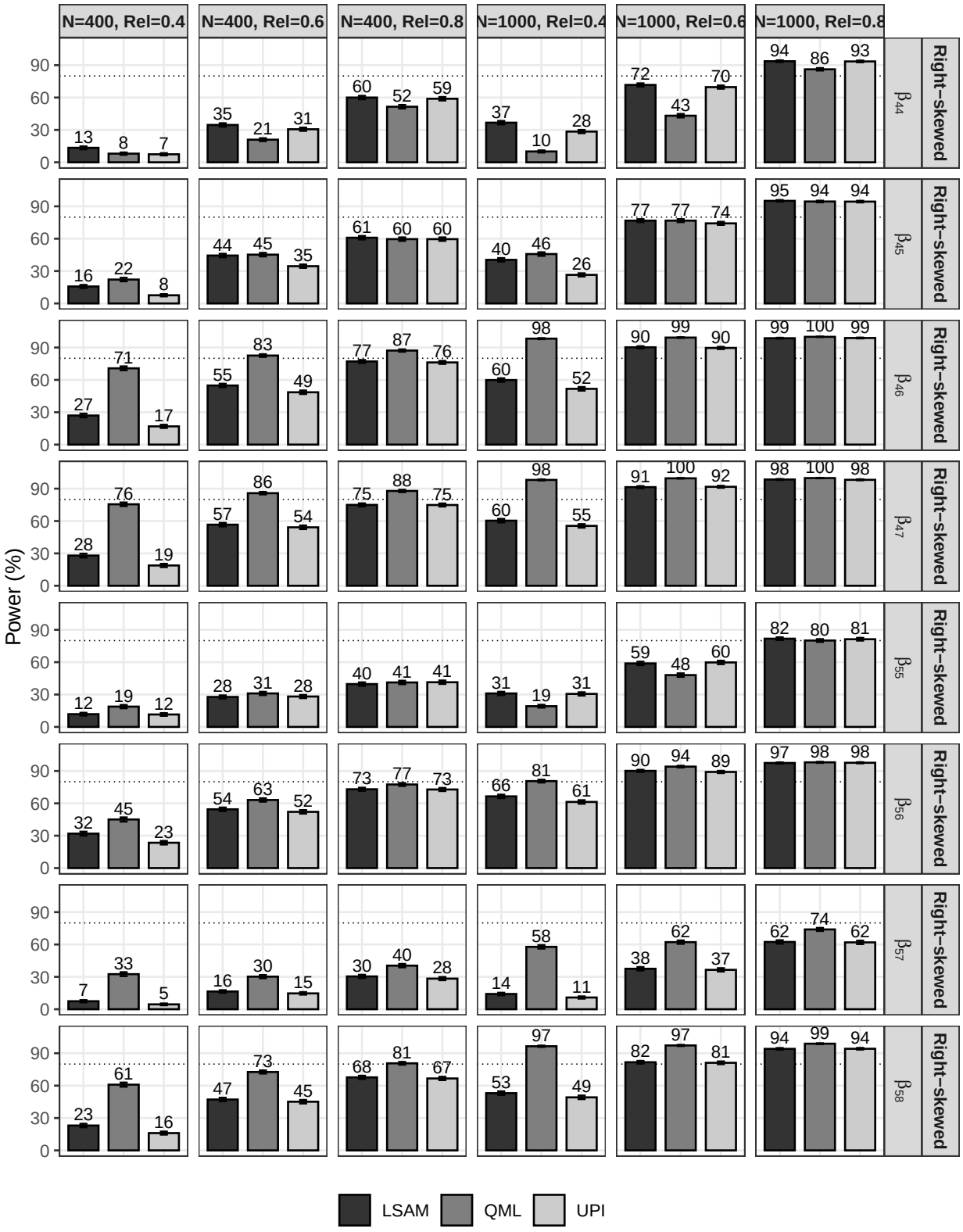
Figure S23

Type I Error Rate for Simulation 2 (Right-skewed)



Figure S24

Empirical Power for Simulation 2 (Right-skewed)



Uniform

Figure S25

Absolute Bias for Simulation 2 (Uniform)

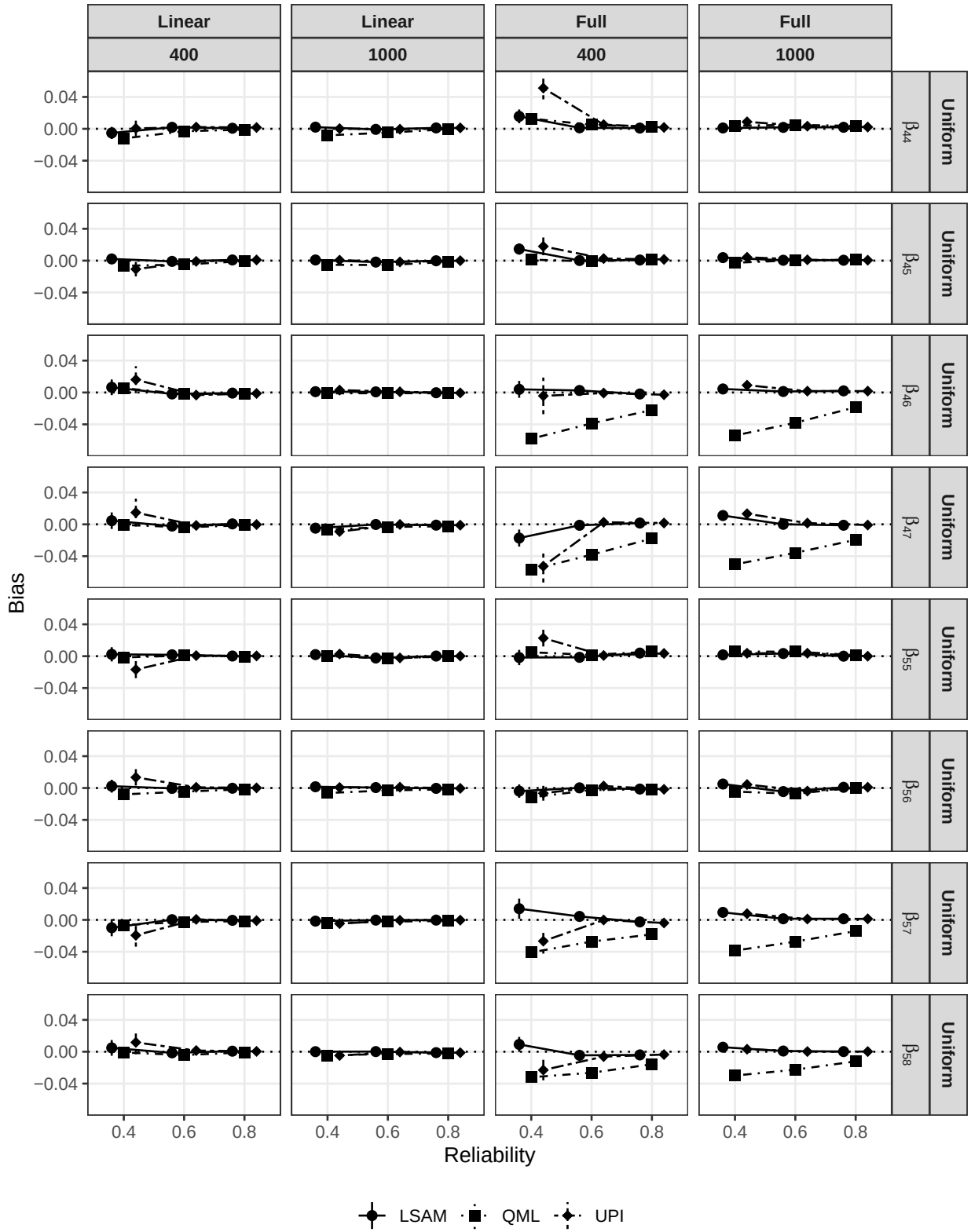


Figure S26

Relative Bias for Simulation 2 (Uniform)

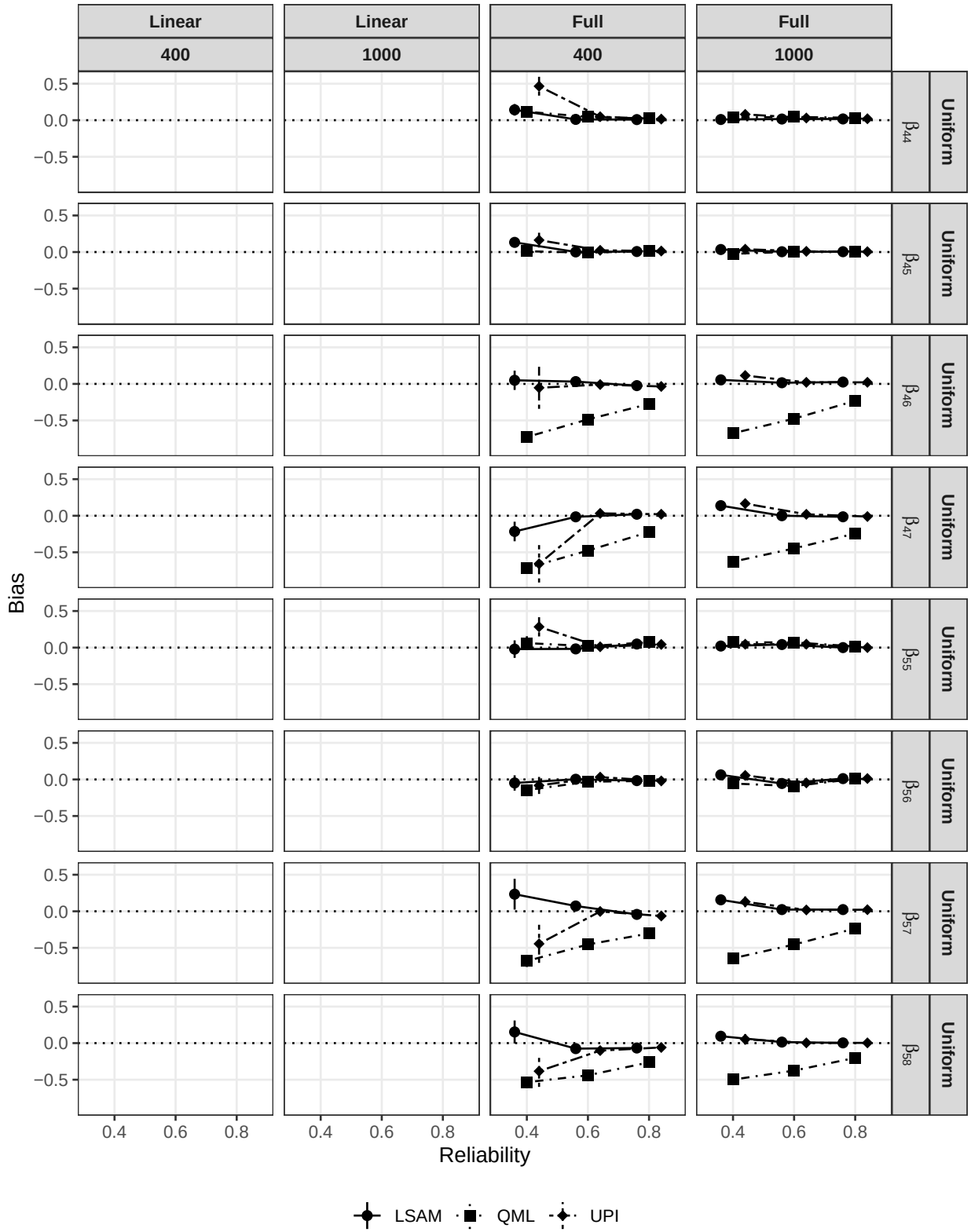


Figure S27

RMSE for Simulation 2 (Uniform)

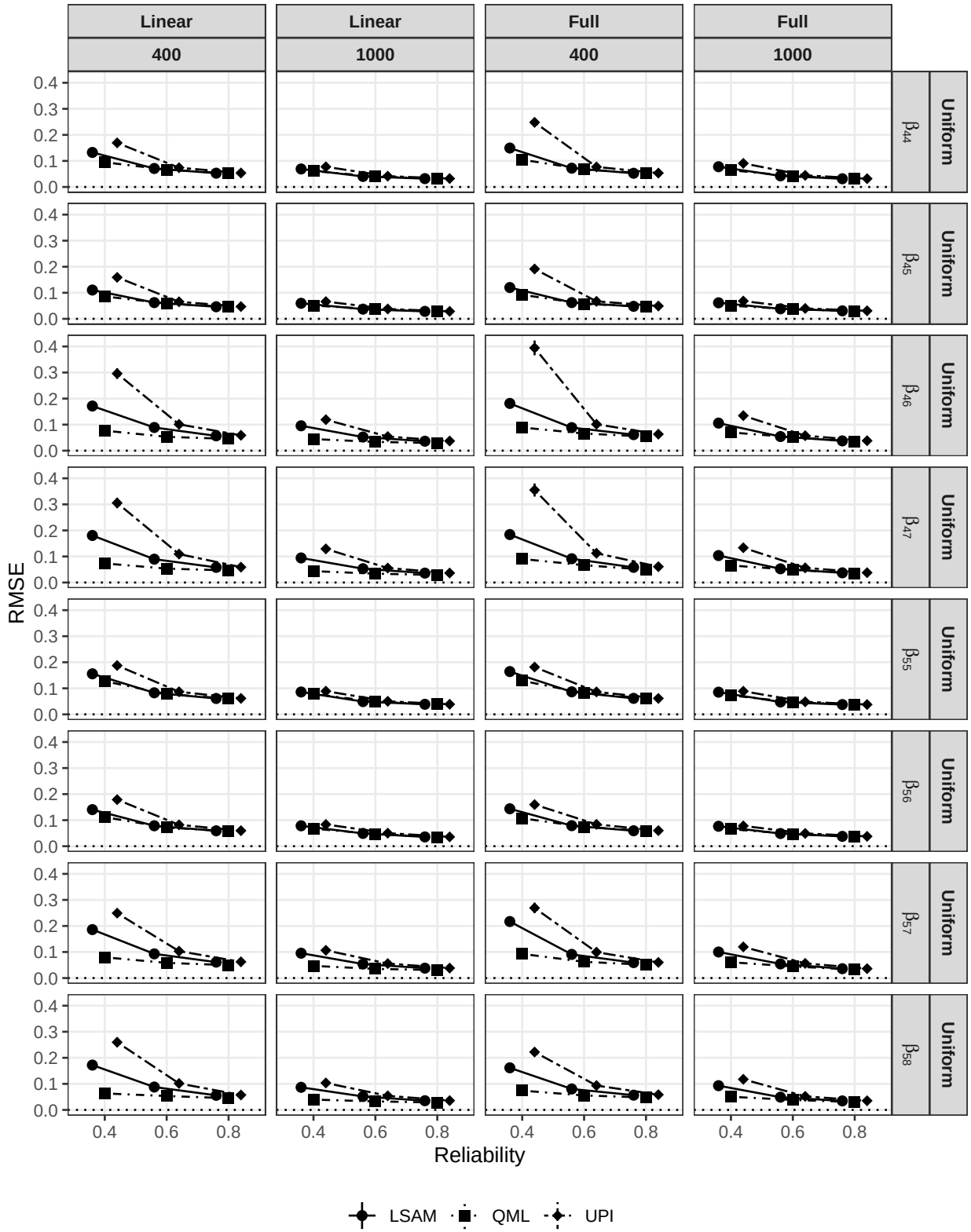


Figure S28

Relative RMSE for Simulation 2 (Uniform)

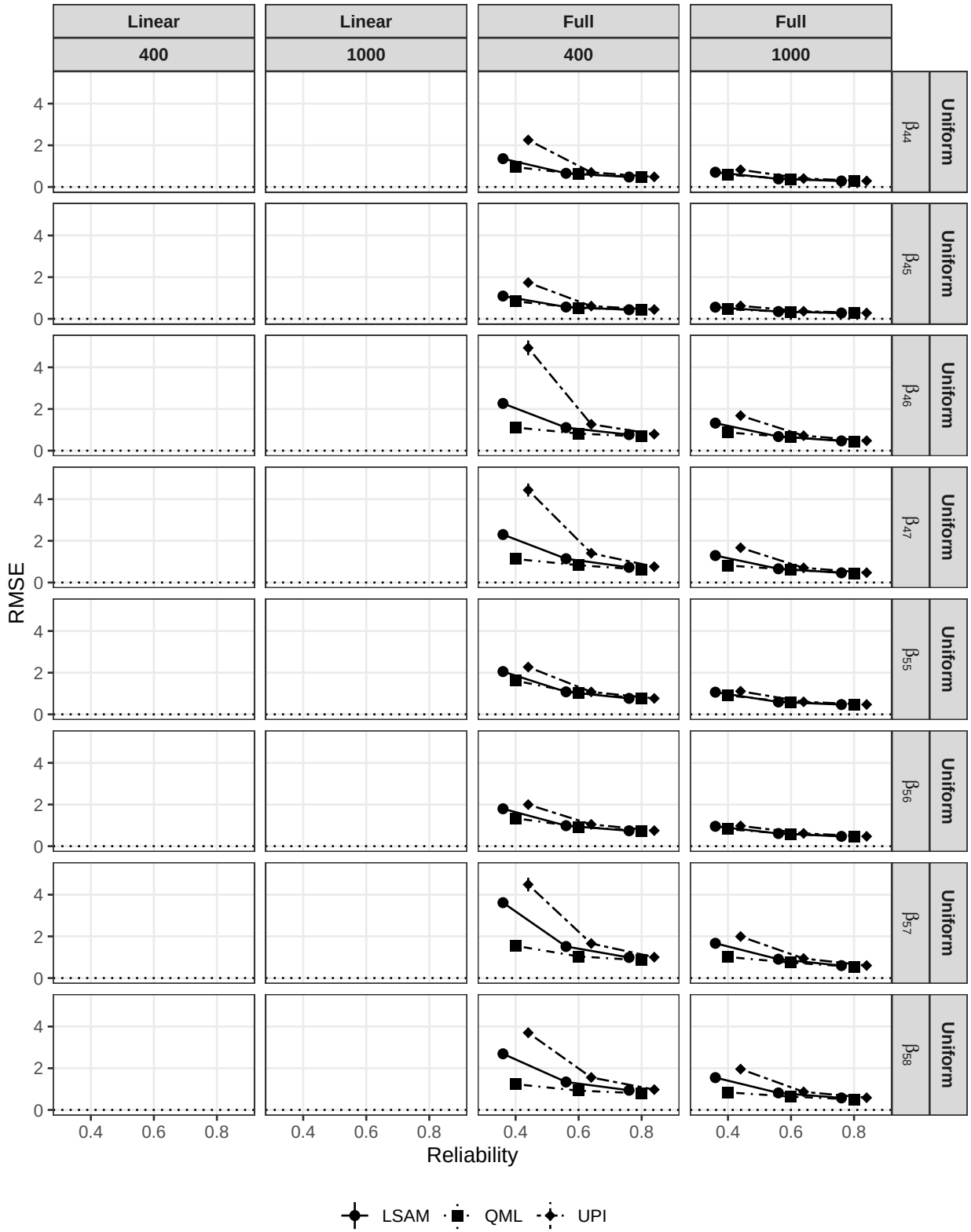


Figure S29

SE/SD Ratio for Simulation 2 (Uniform)

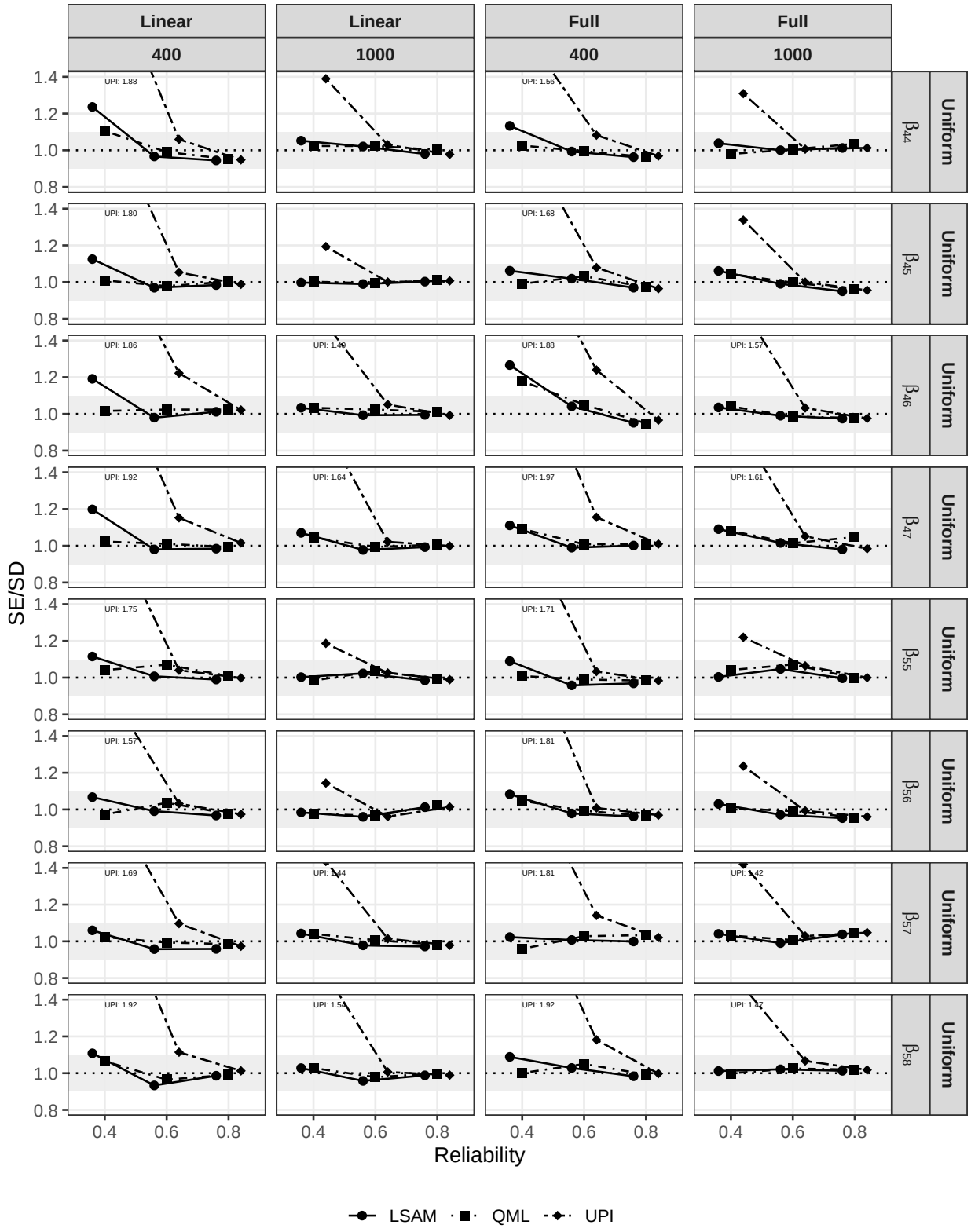


Figure S30

Coverage for Simulation 2 (Uniform)

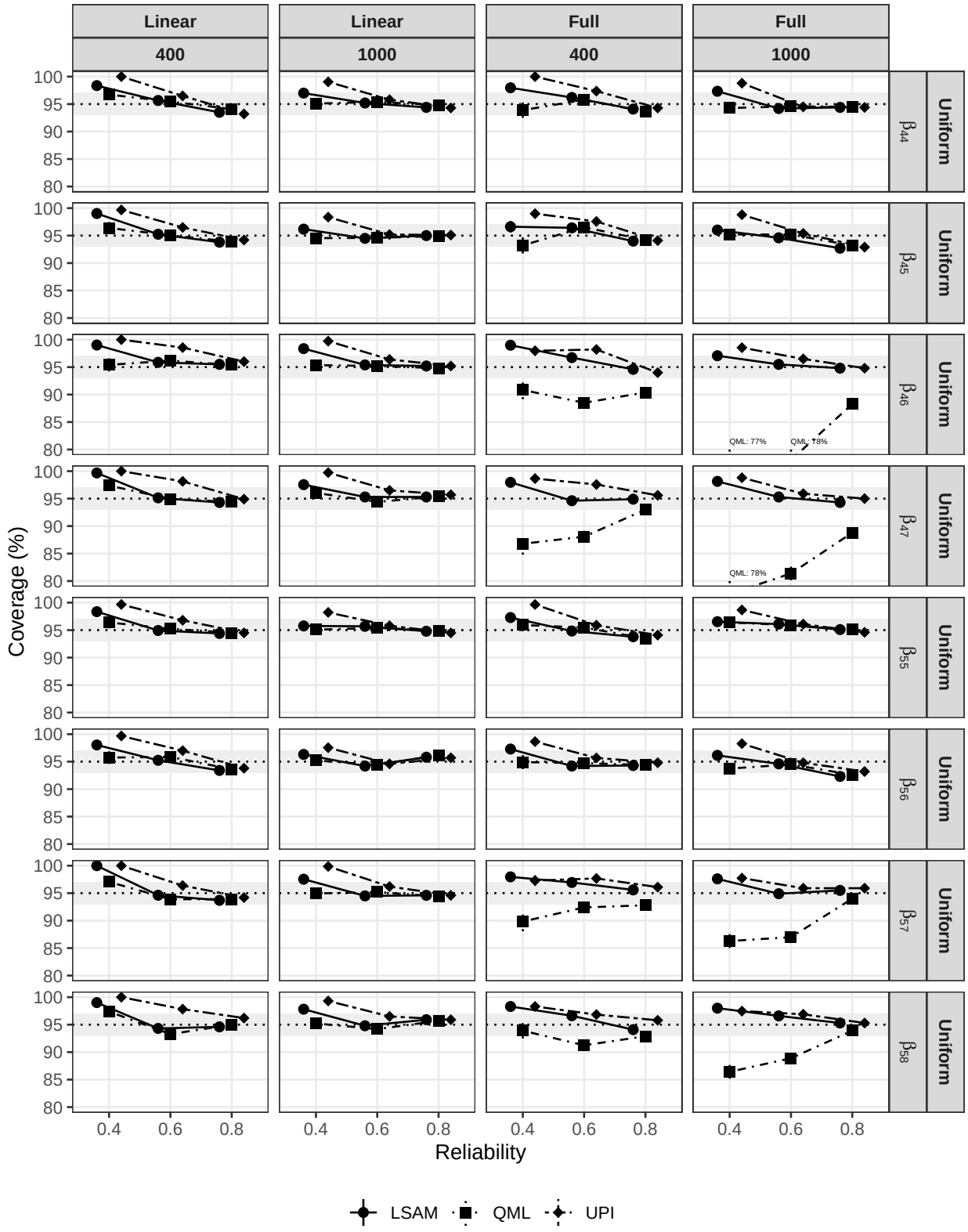


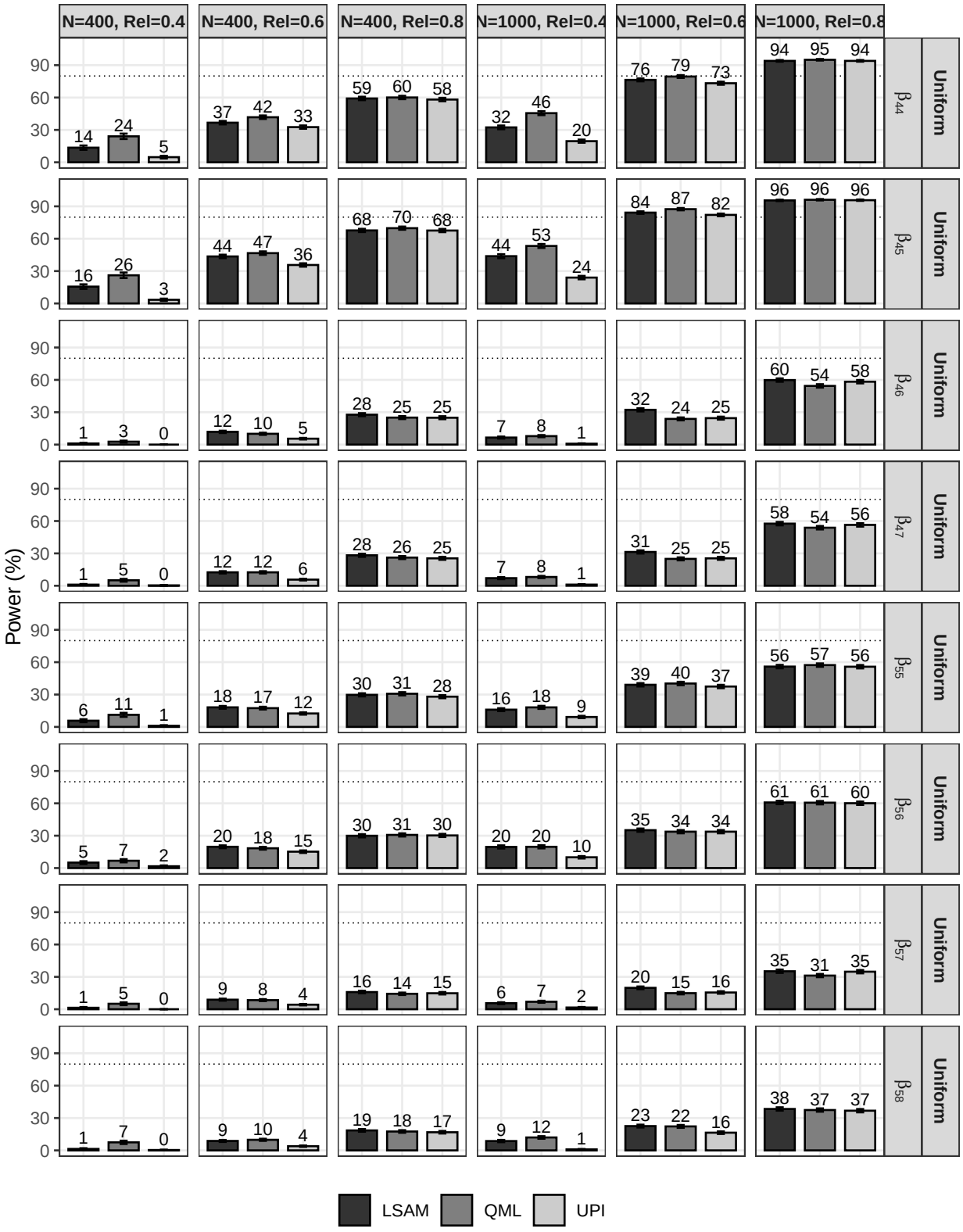
Figure S31

Type I Error Rate for Simulation 2 (Uniform)



Figure S32

Empirical Power for Simulation 2 (Uniform)



Additional Distributional Conditions Plots — Simulation 1

Figure S33

Additional Distributional Conditions: Relative Bias for Simulation 1 (Epsilon)

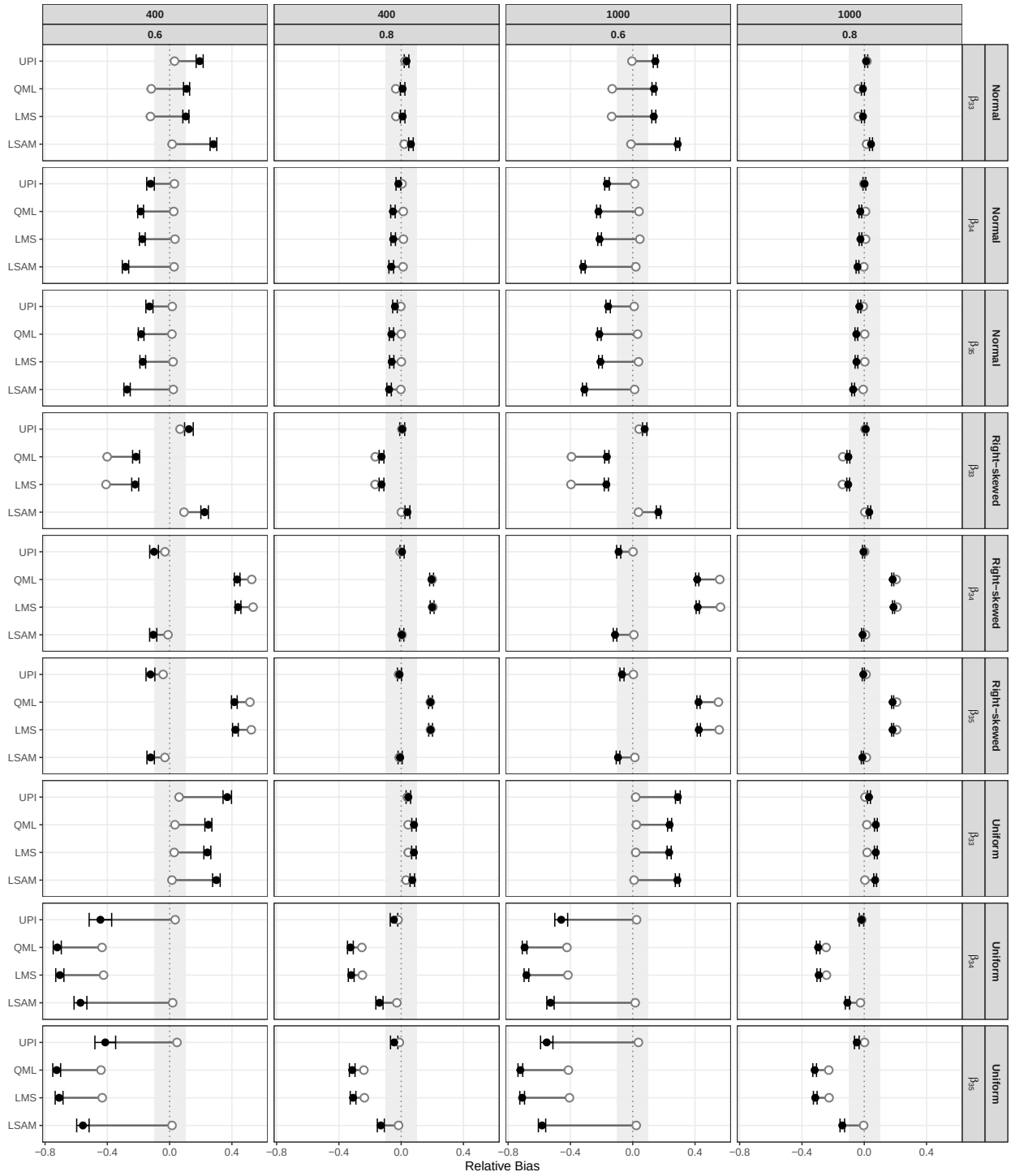


Figure S34

Additional Distributional Conditions: Relative Bias for Simulation 1 (Zeta)

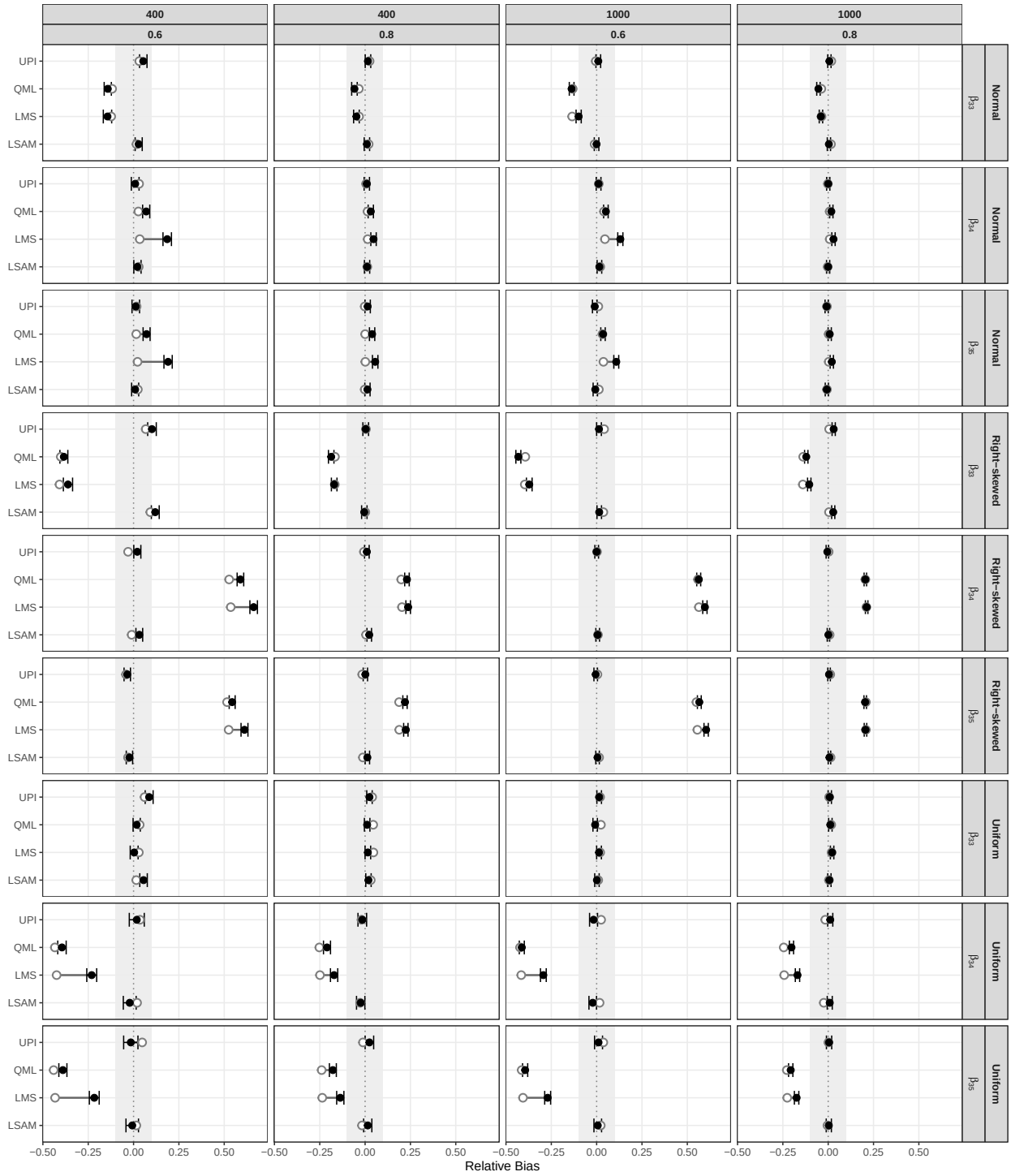


Figure S35

Additional Distributional Conditions: Relative Bias for Simulation 1 (Epsilon & Zeta)

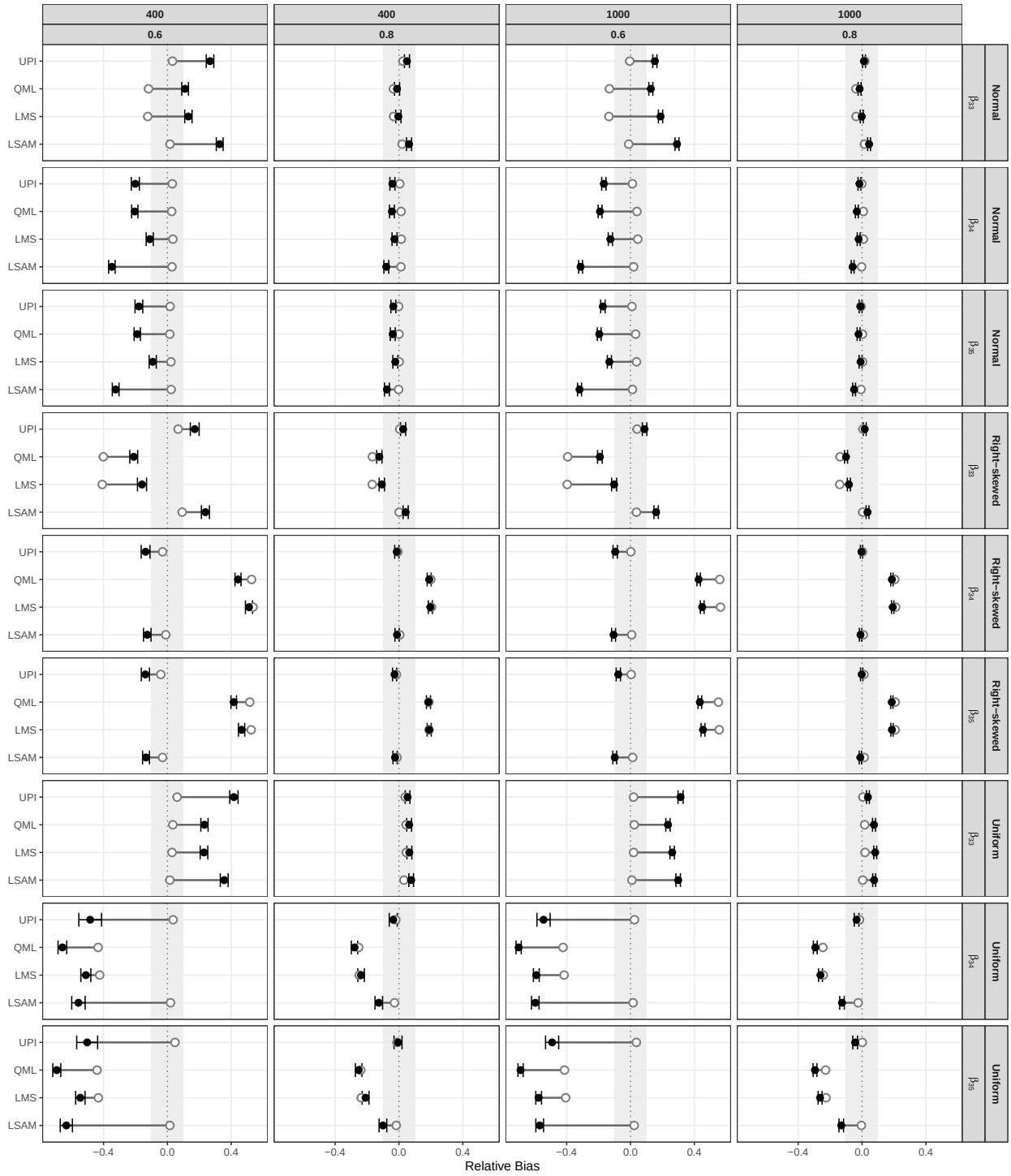


Figure S36

Additional Distributional Conditions: Relative RMSE for Simulation 1 (Epsilon)

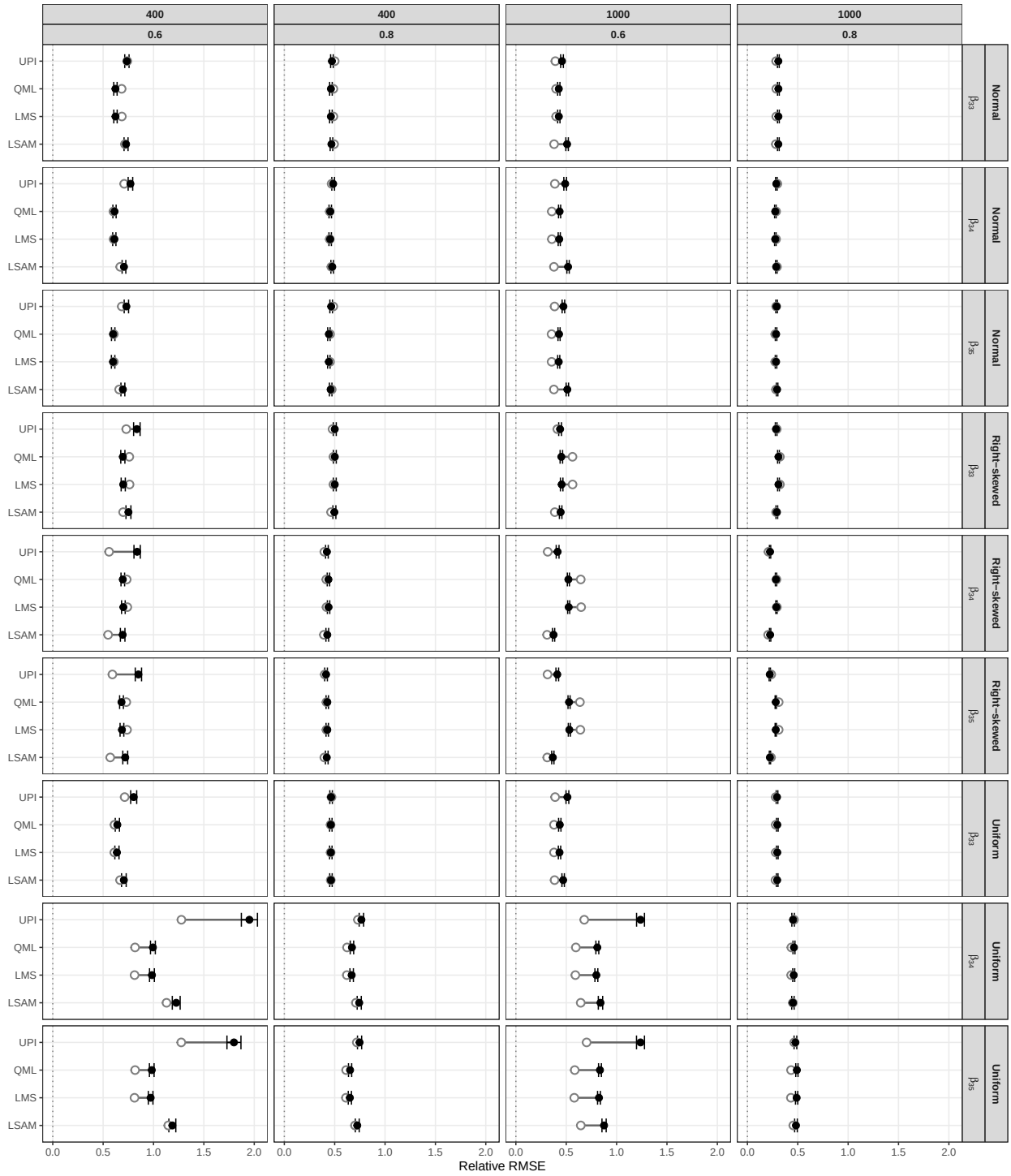


Figure S37

Additional Distributional Conditions: Relative RMSE for Simulation 1 (Zeta)

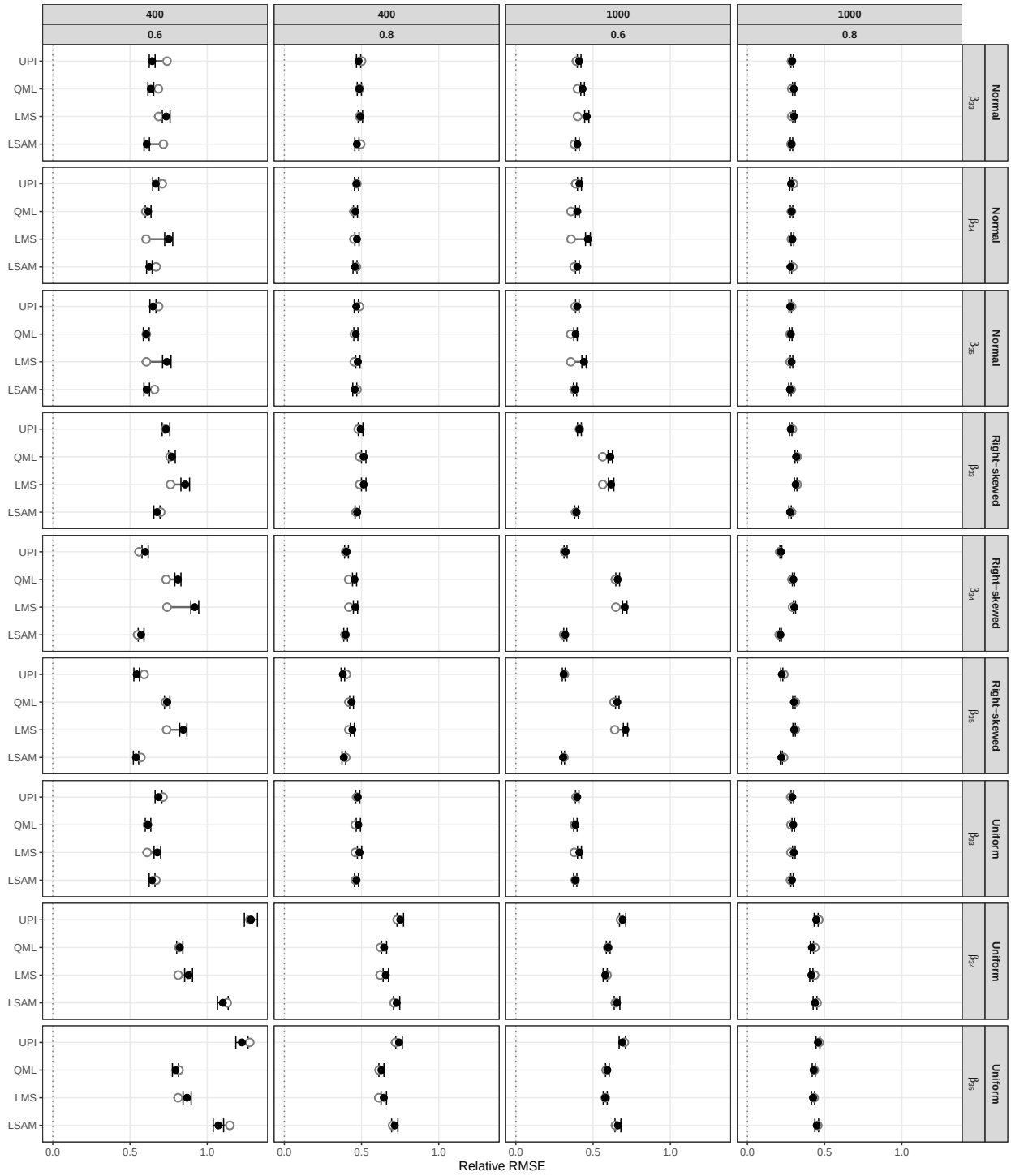


Figure S38

Additional Distributional Conditions: Relative RMSE for Simulation 1 (Epsilon & Zeta)

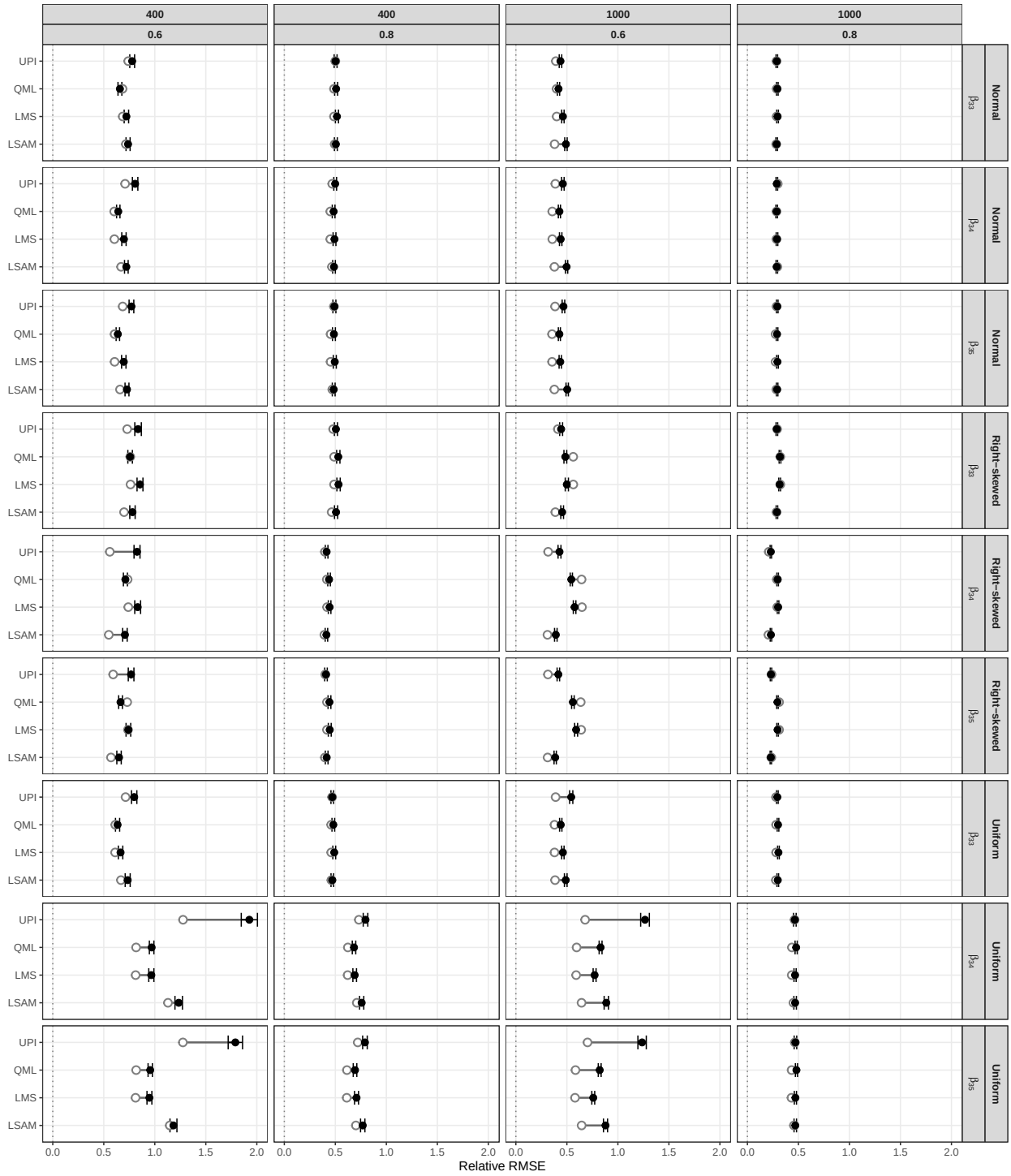


Figure S39

Additional Distributional Conditions: SE/SD Ratio for Simulation 1 (Epsilon)

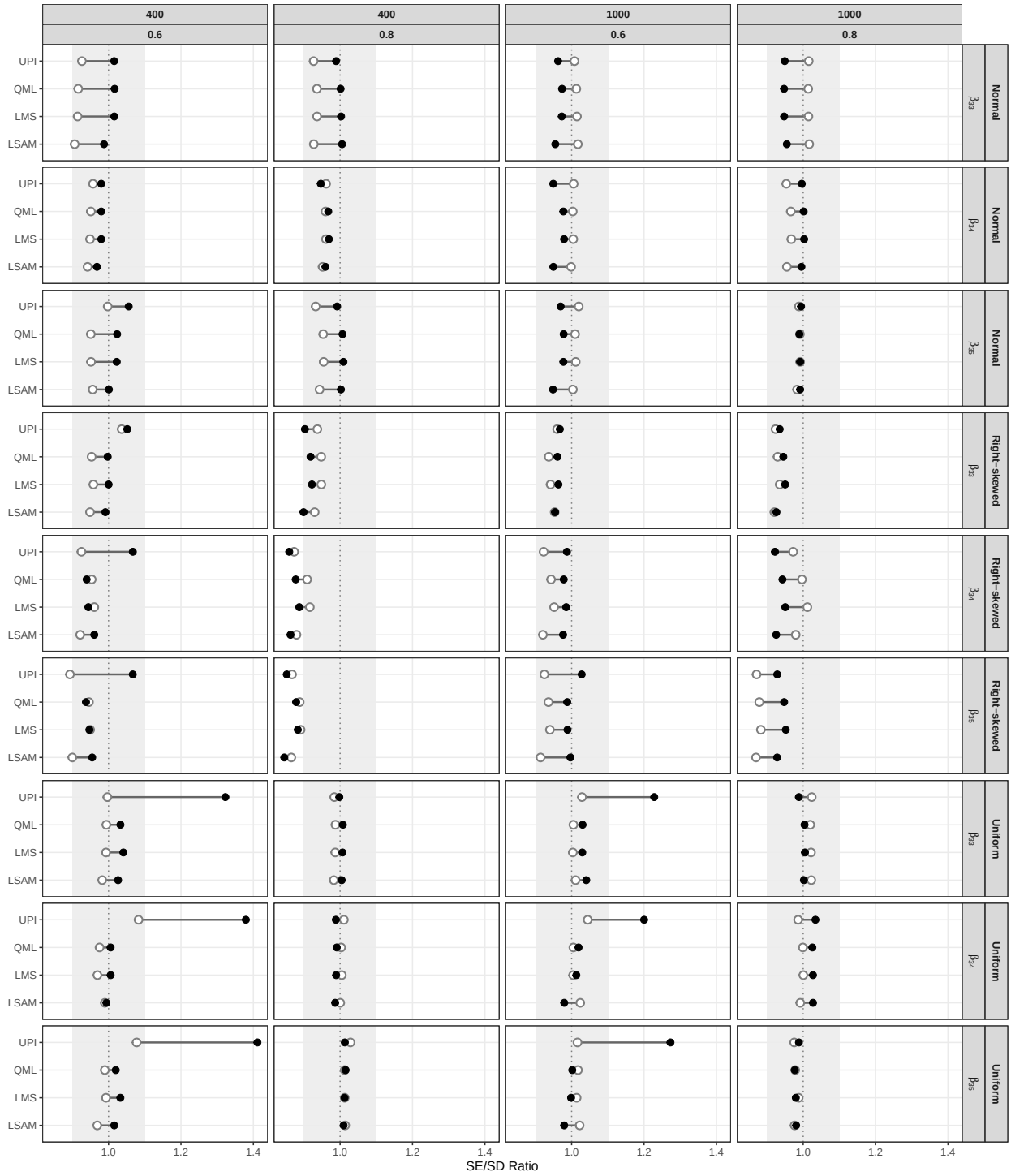


Figure S40

Additional Distributional Conditions: SE/SD Ratio for Simulation 1 (Zeta)

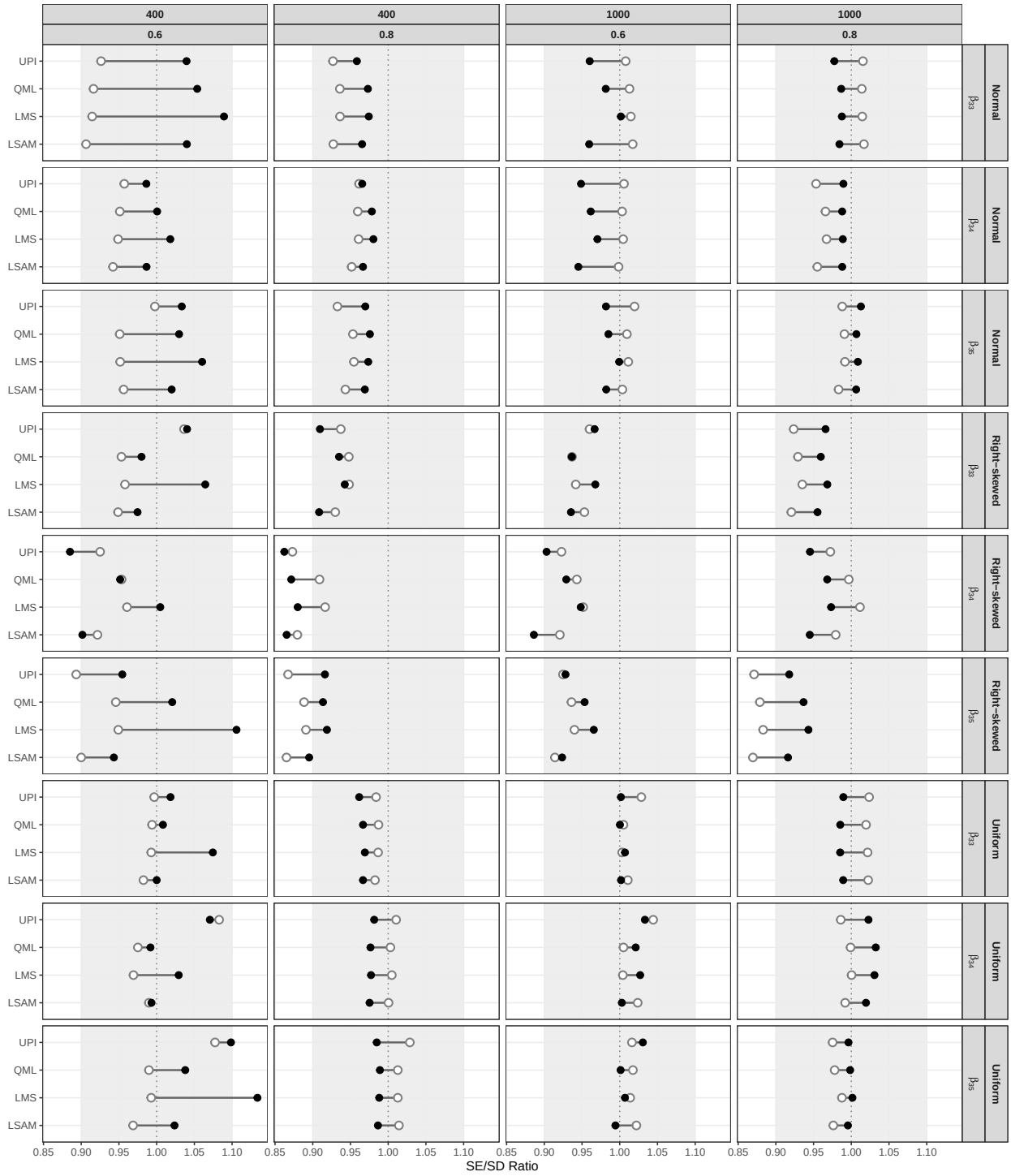


Figure S41

Additional Distributional Conditions: SE/SD Ratio for Simulation 1 (Epsilon & Zeta)

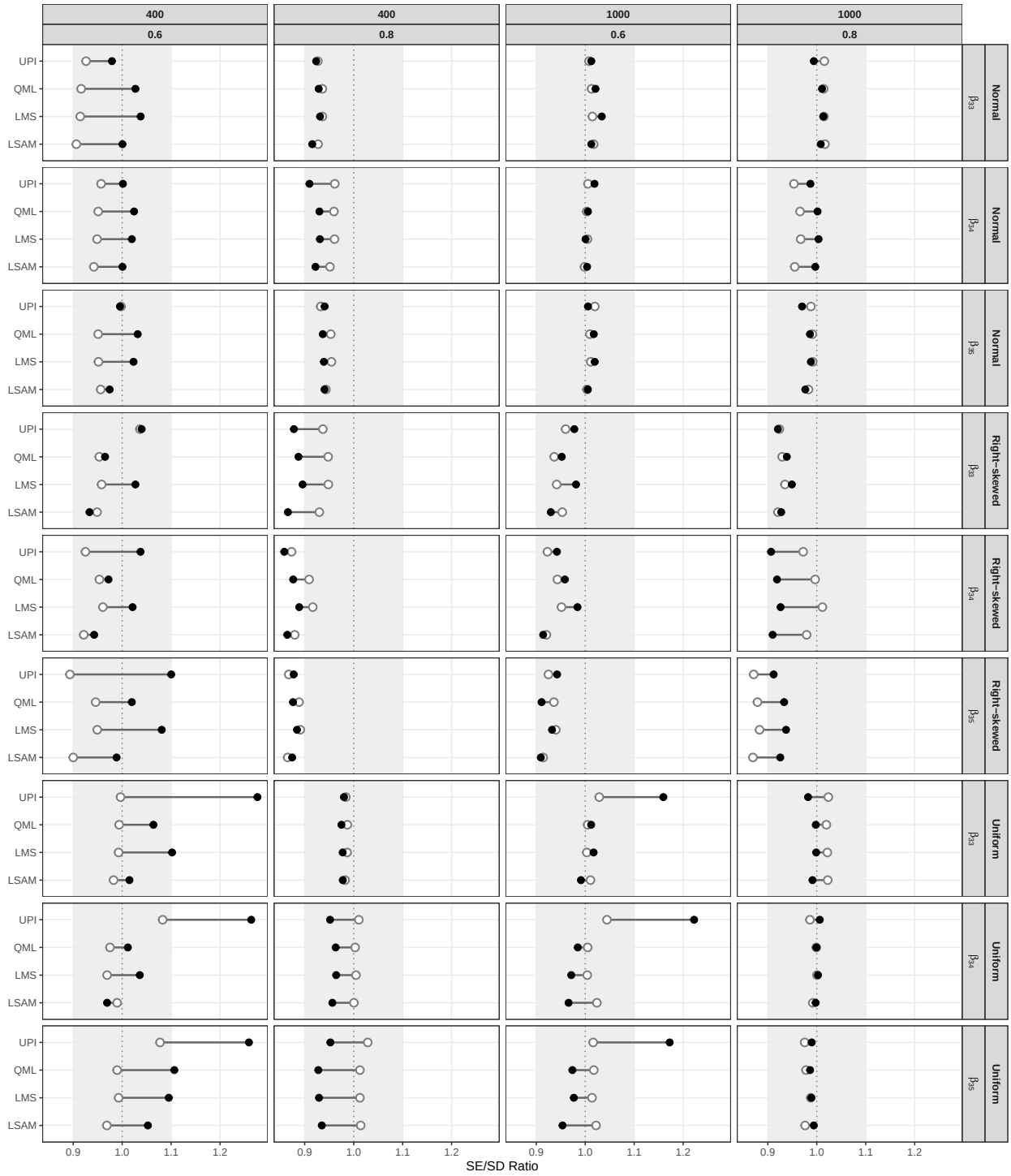


Figure S42

Additional Distributional Conditions: Coverage for Simulation 1 (Epsilon)

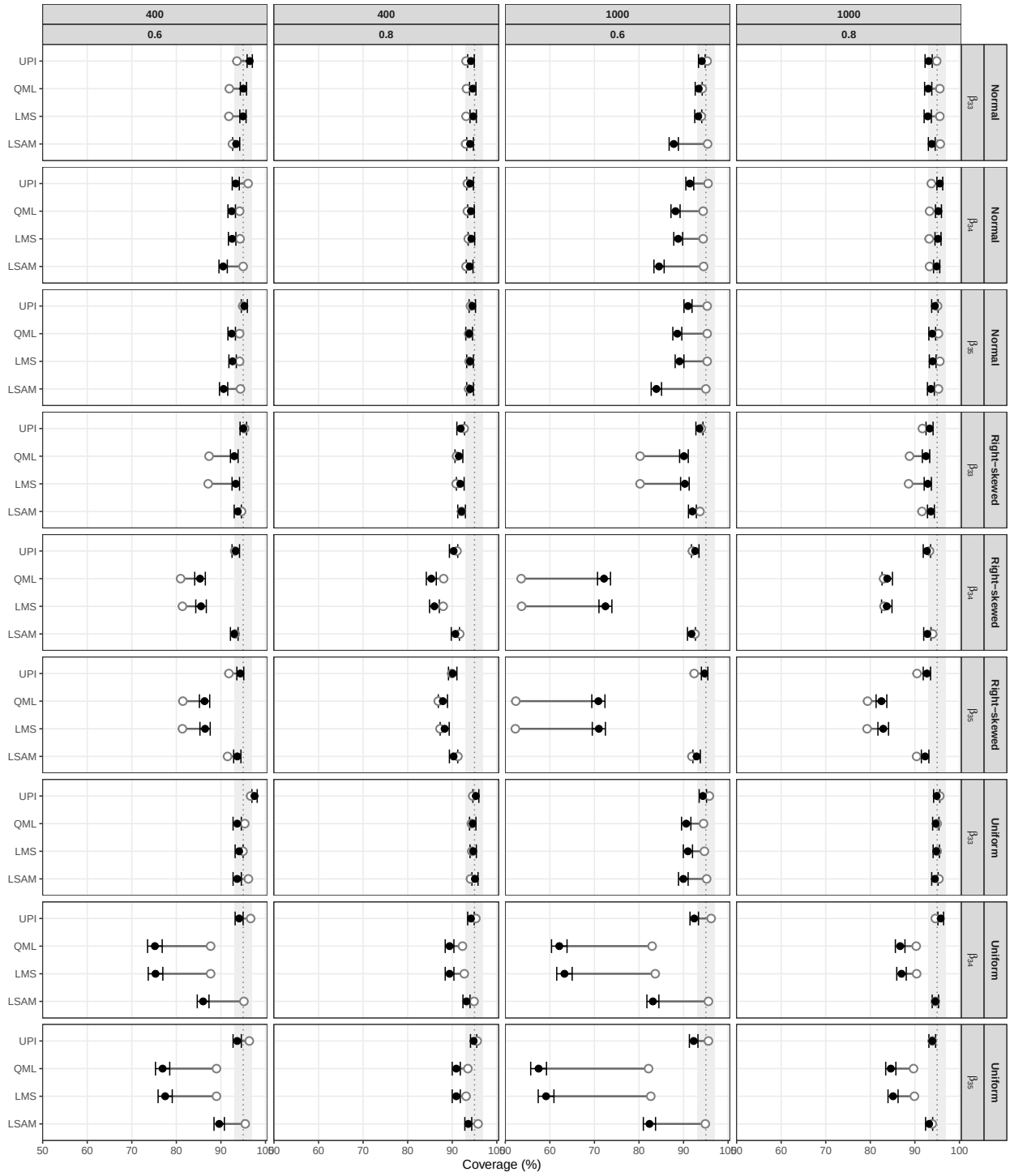


Figure S43

Additional Distributional Conditions: Coverage for Simulation 1 (Zeta)

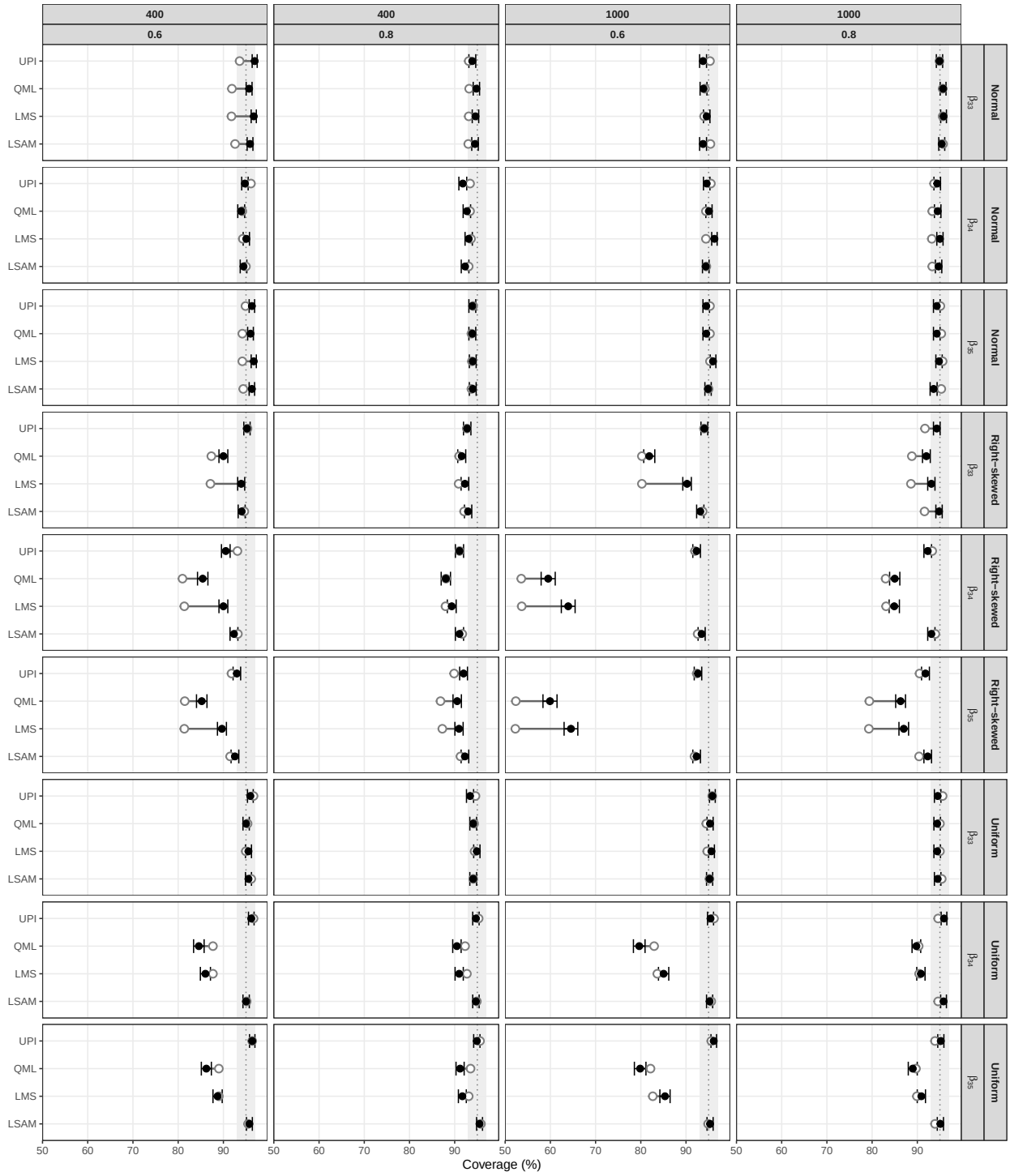


Figure S44

Additional Distributional Conditions: Coverage for Simulation 1 (Epsilon & Zeta)

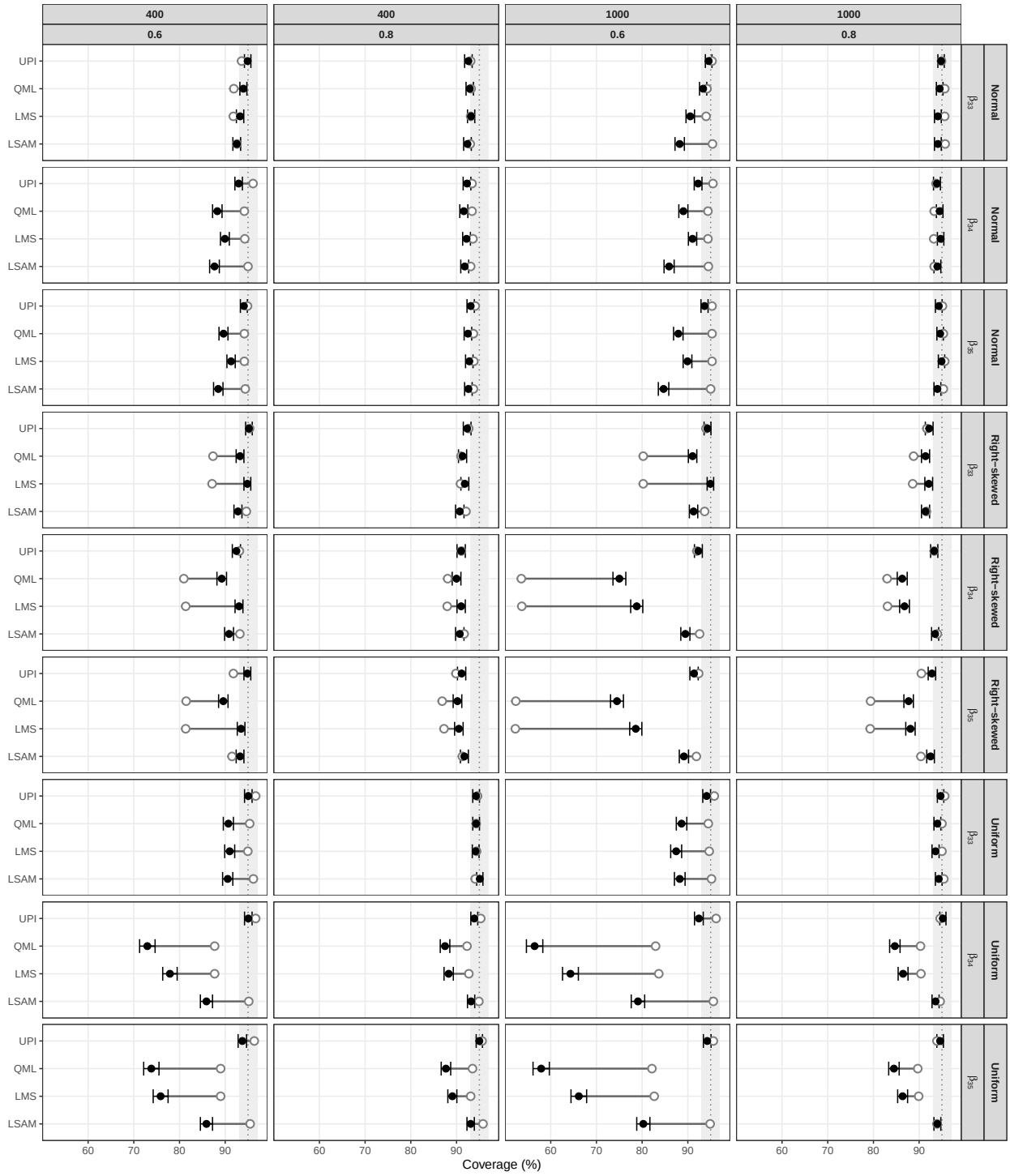


Figure S45

Additional Distributional Conditions: Type I Error for Simulation 1 (Epsilon)

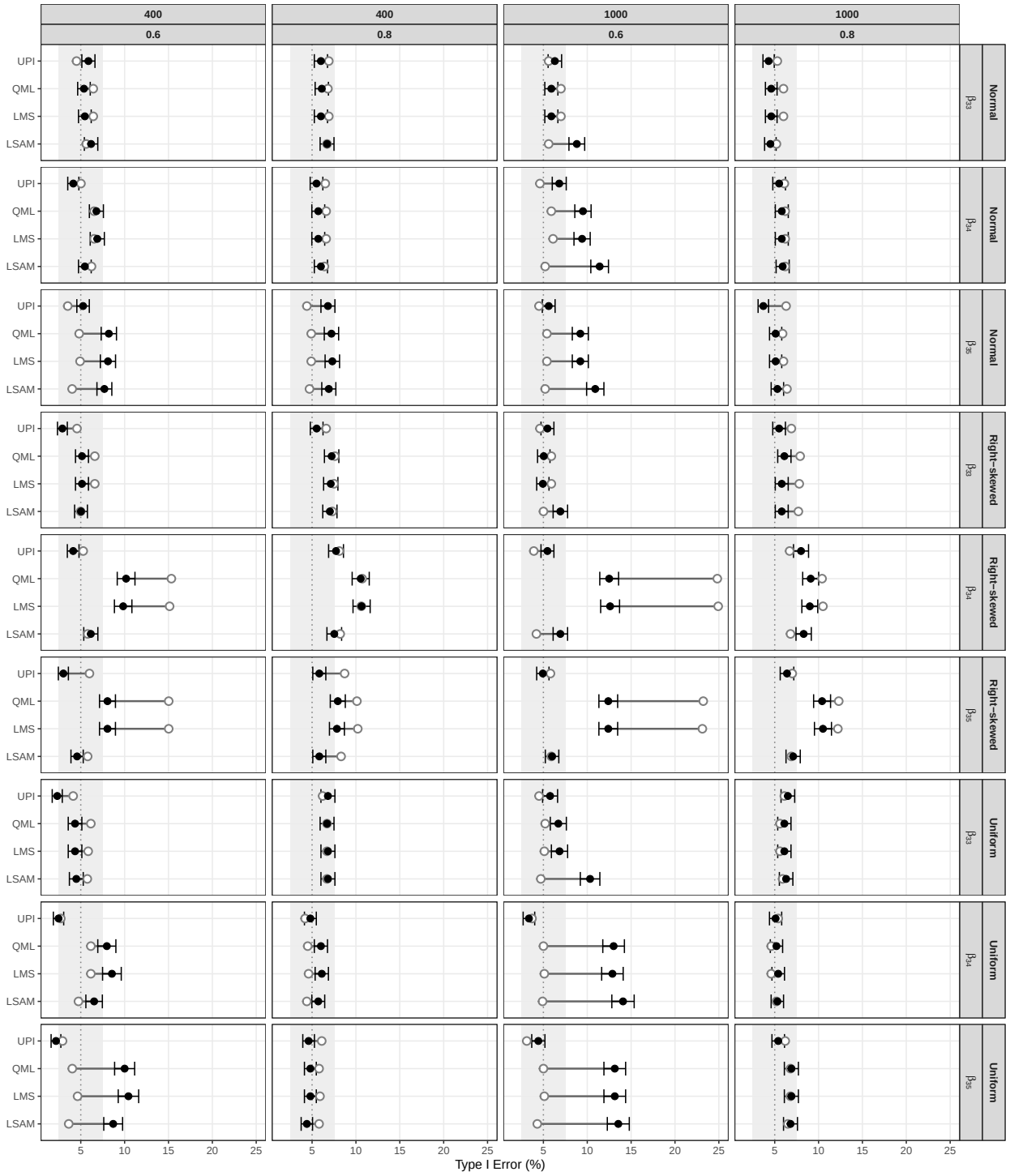


Figure S46

Additional Distributional Conditions: Type I Error for Simulation 1 (Zeta)

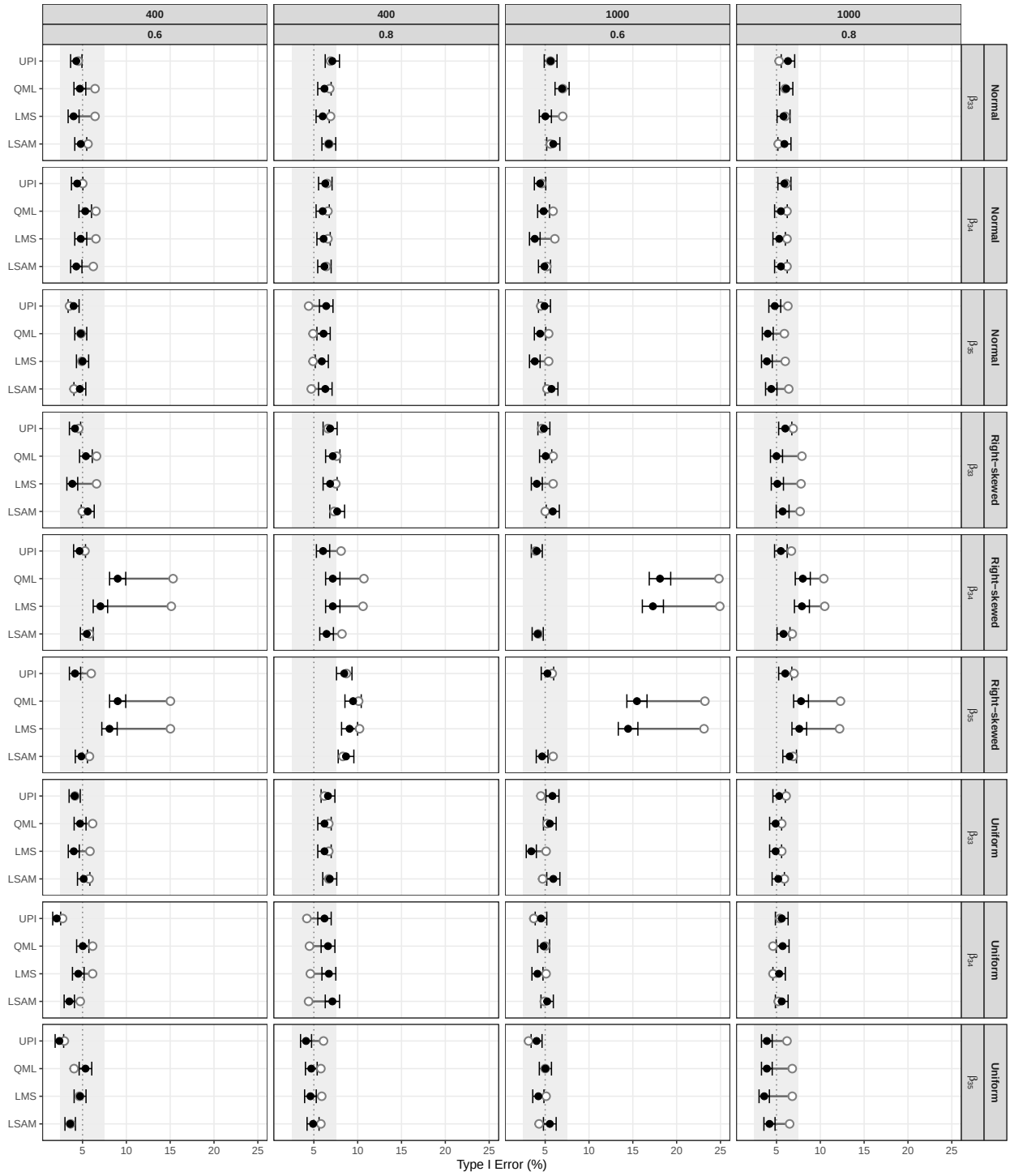


Figure S47

Additional Distributional Conditions: Type I Error for Simulation 1 (Epsilon & Zeta)

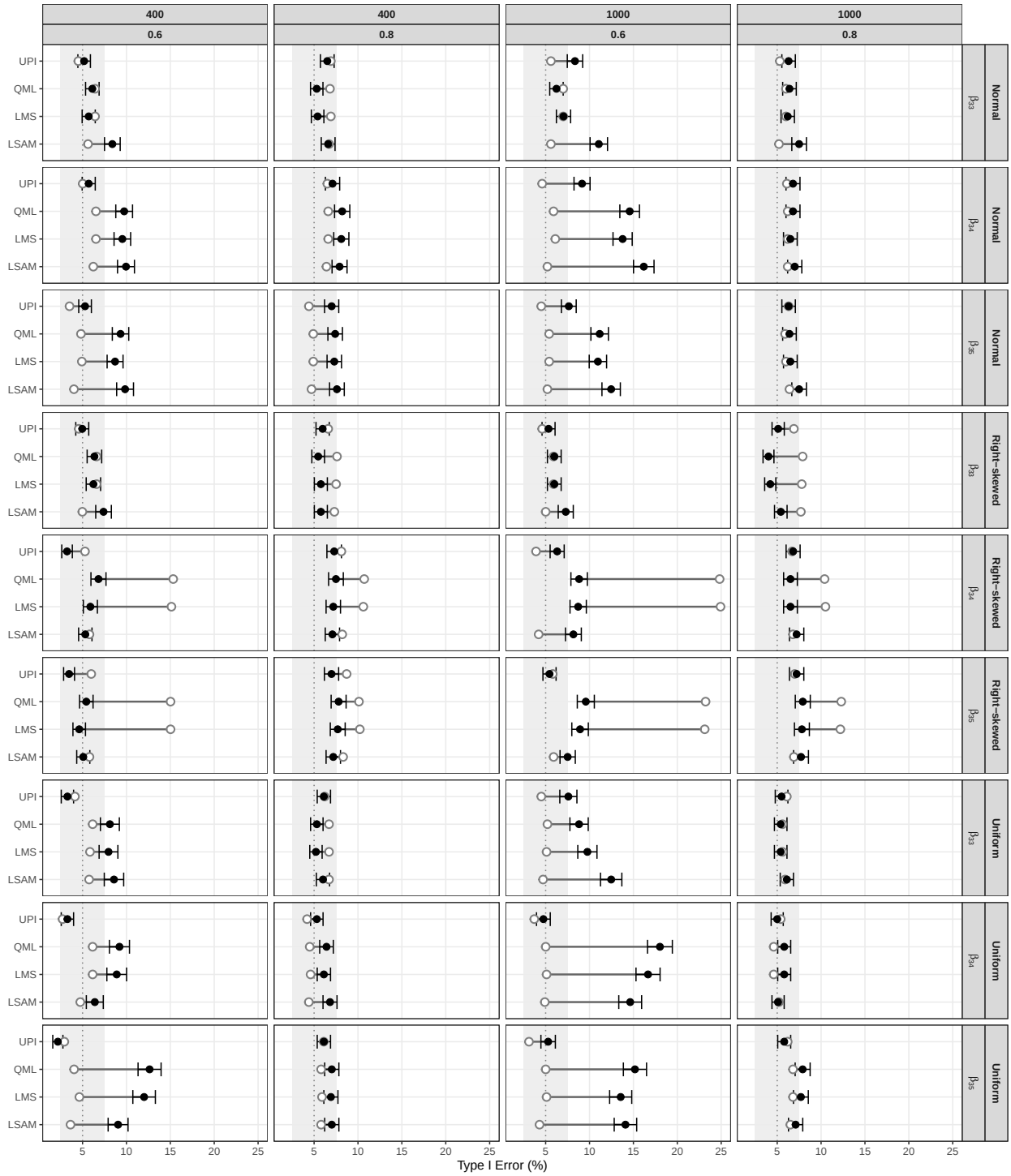


Figure S48

Additional Distributional Conditions: Power for Simulation 1 (Epsilon)

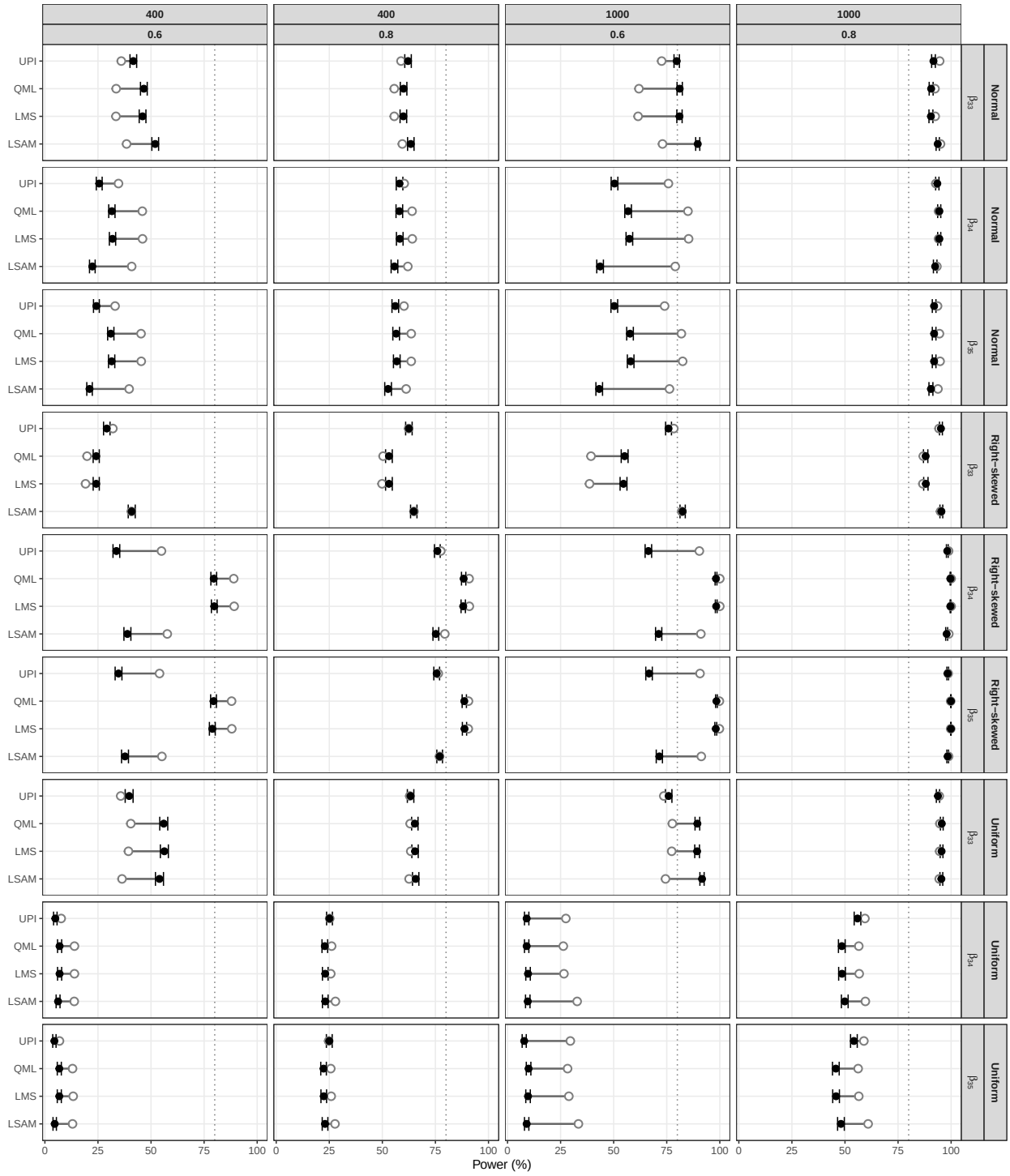


Figure S49

Additional Distributional Conditions: Power for Simulation 1 (Zeta)

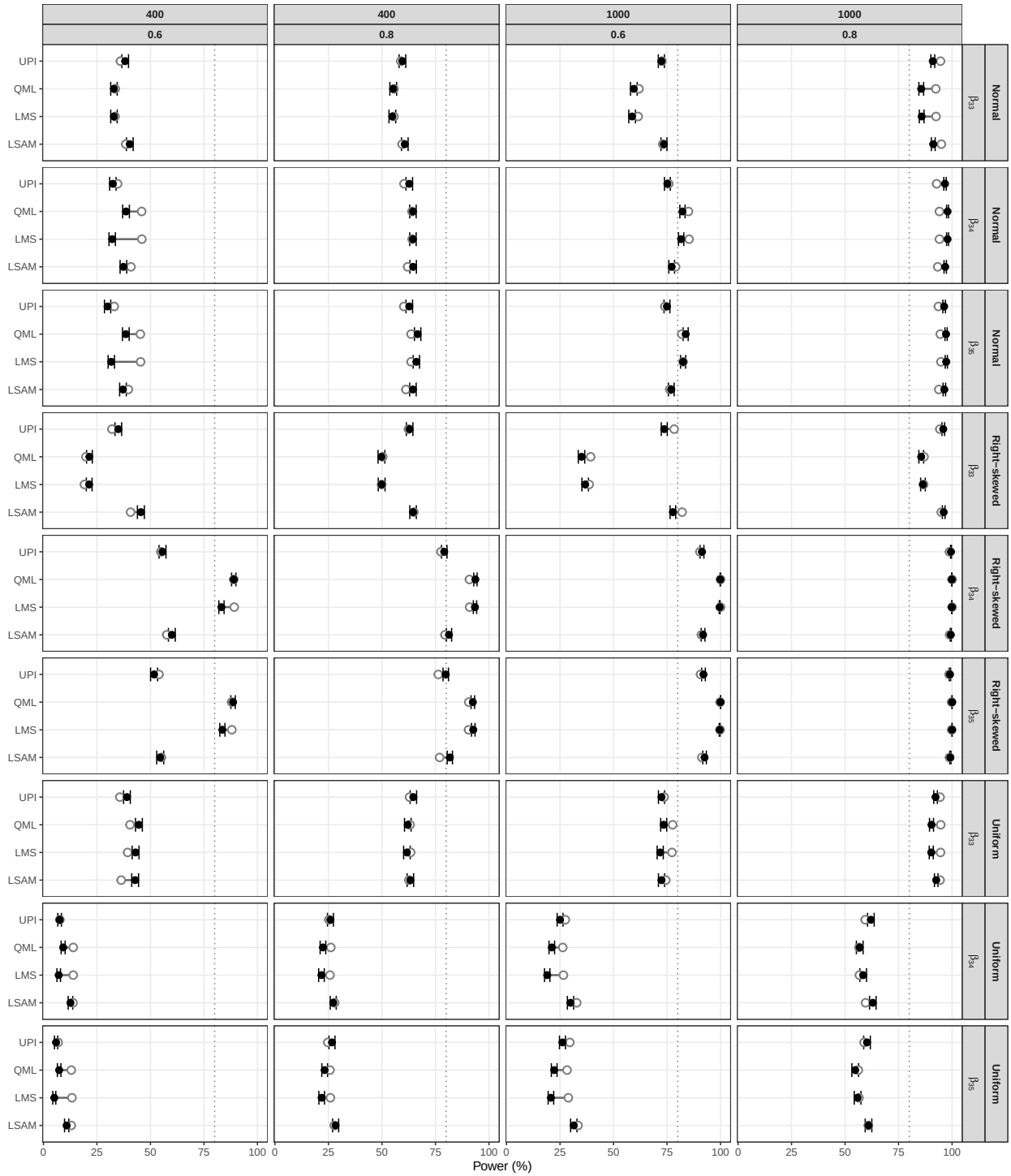
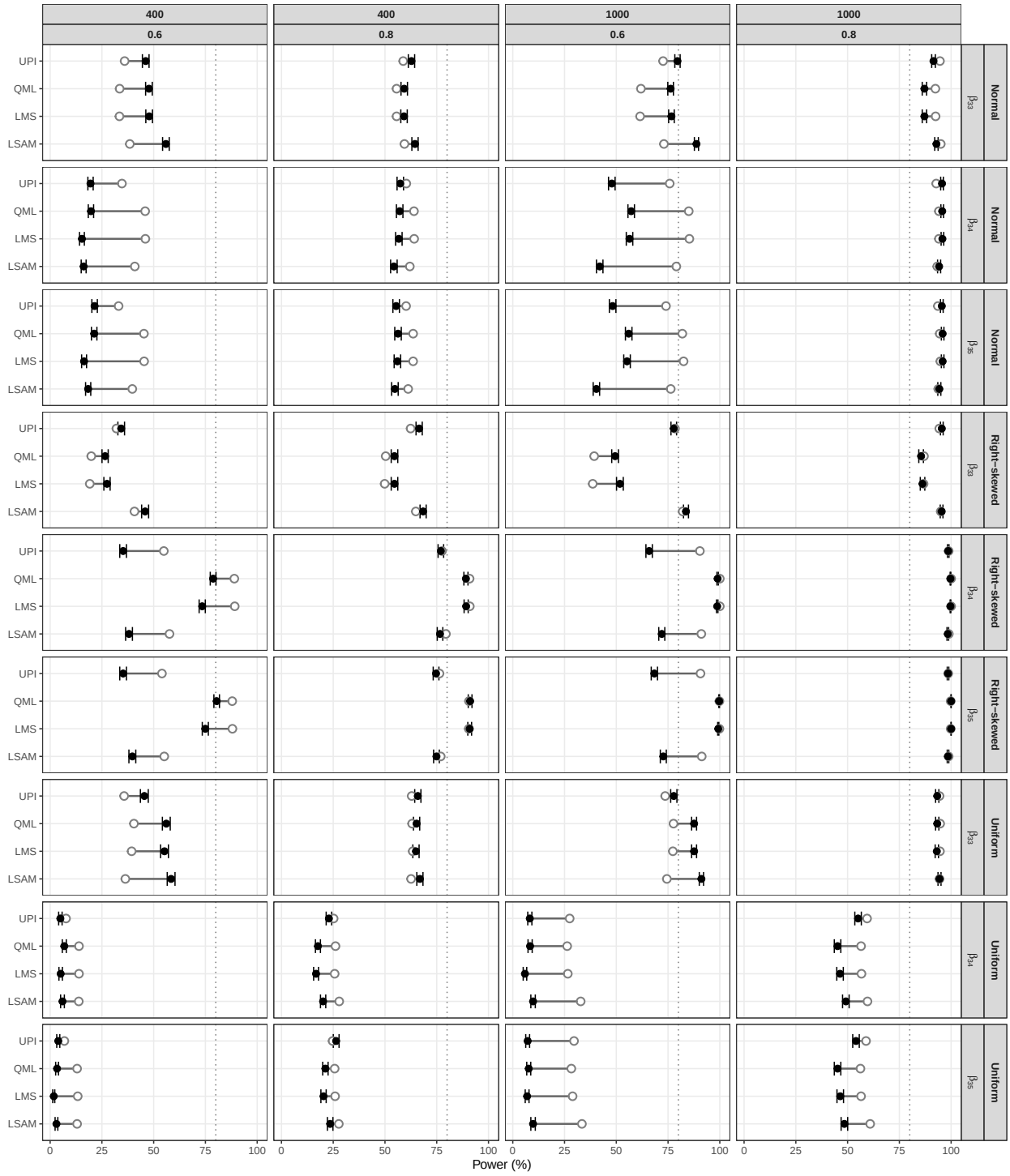


Figure S50

Additional Distributional Conditions: Power for Simulation 1 (Epsilon & Zeta)



Additional Distributional Conditions Plots — Simulation 2

Figure S51

Additional Distributional Conditions: Relative Bias for Simulation 2 — Normal (Epsilon)

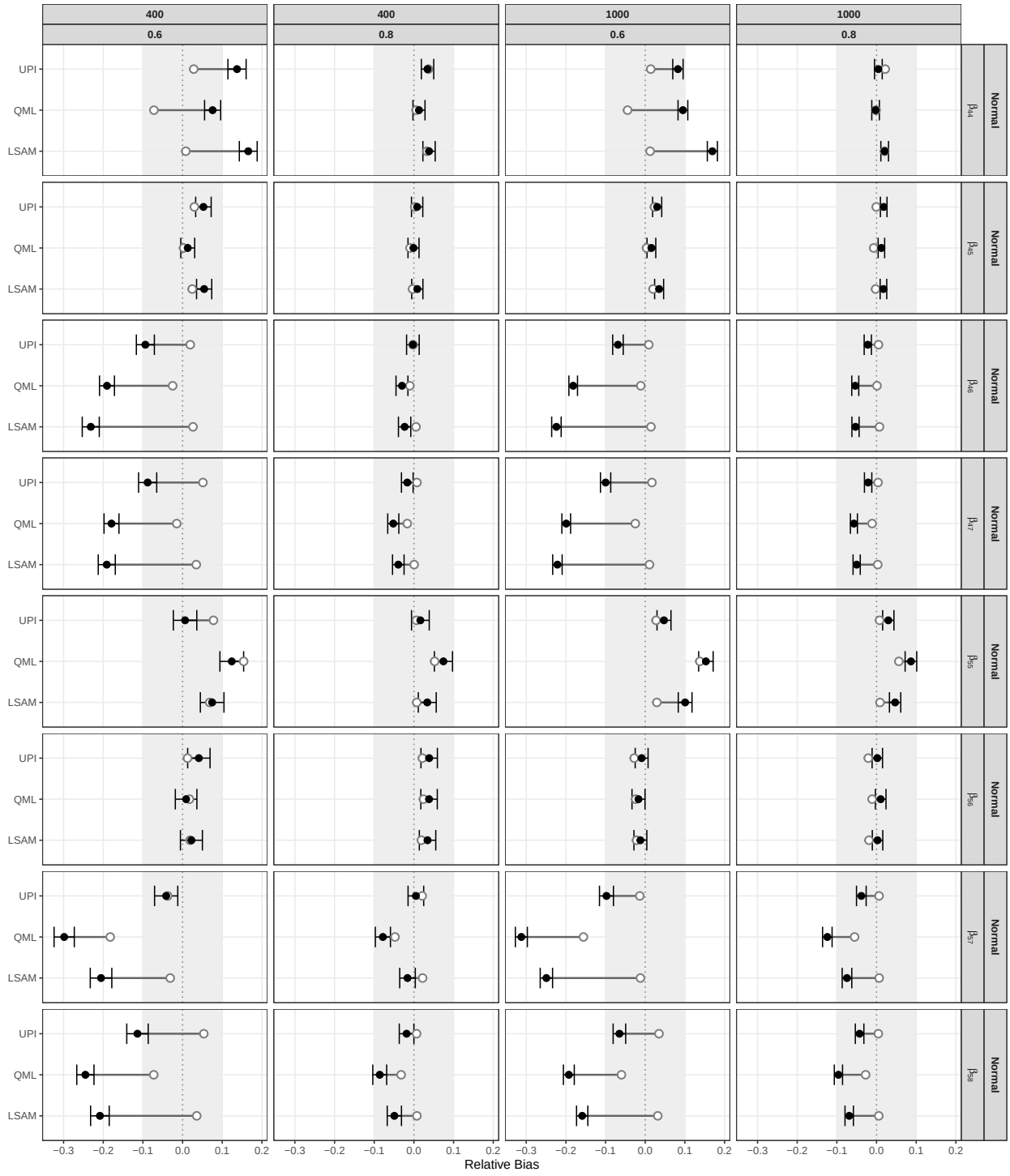


Figure S52

Additional Distributional Conditions: Relative Bias for Simulation 2 — Normal (Zeta)

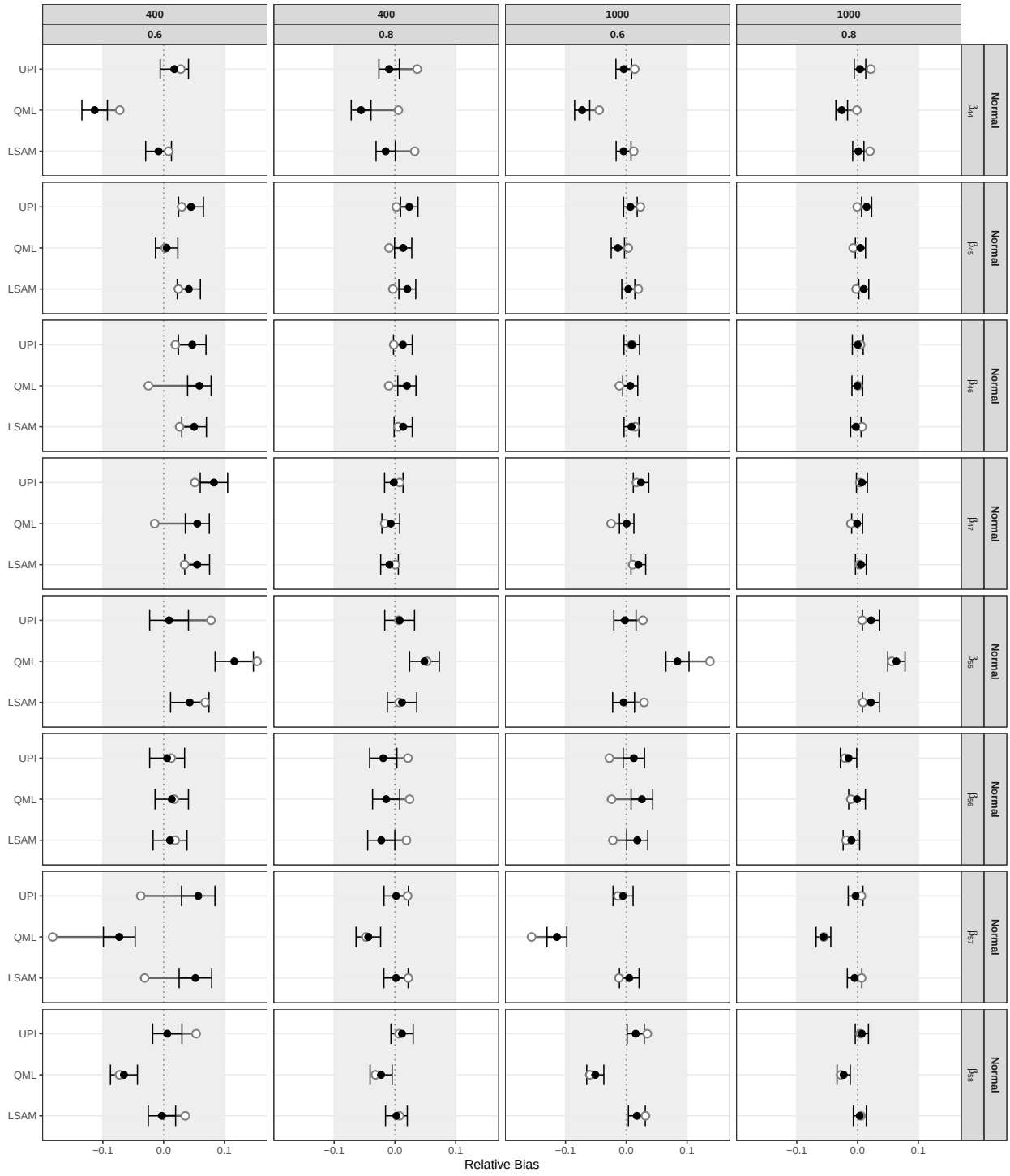


Figure S53

Additional Distributional Conditions: Relative Bias for Simulation 2 — Normal (Epsilon & Zeta)

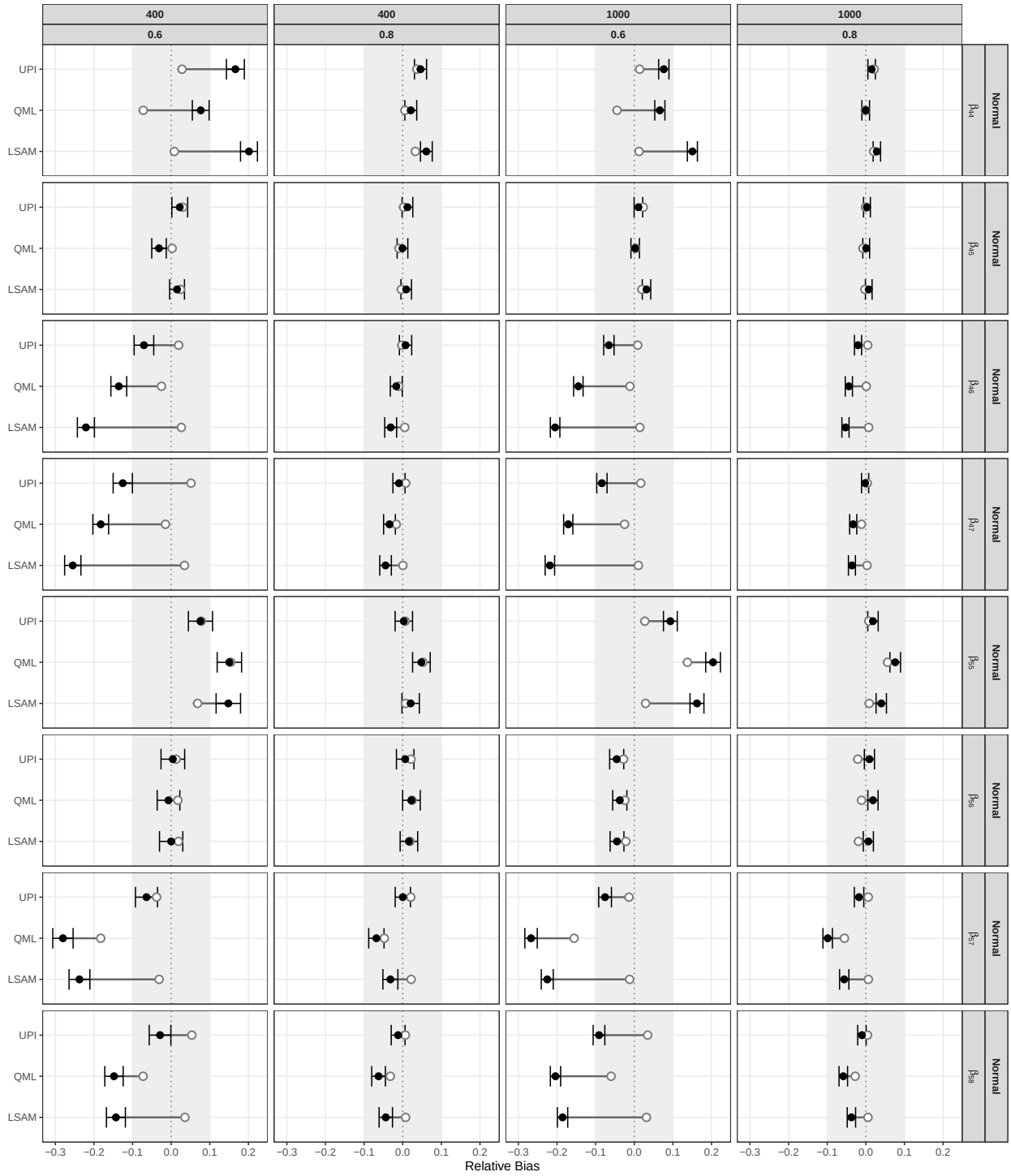


Figure S54

Additional Distributional Conditions: Relative RMSE for Simulation 2 — Normal (Epsilon)

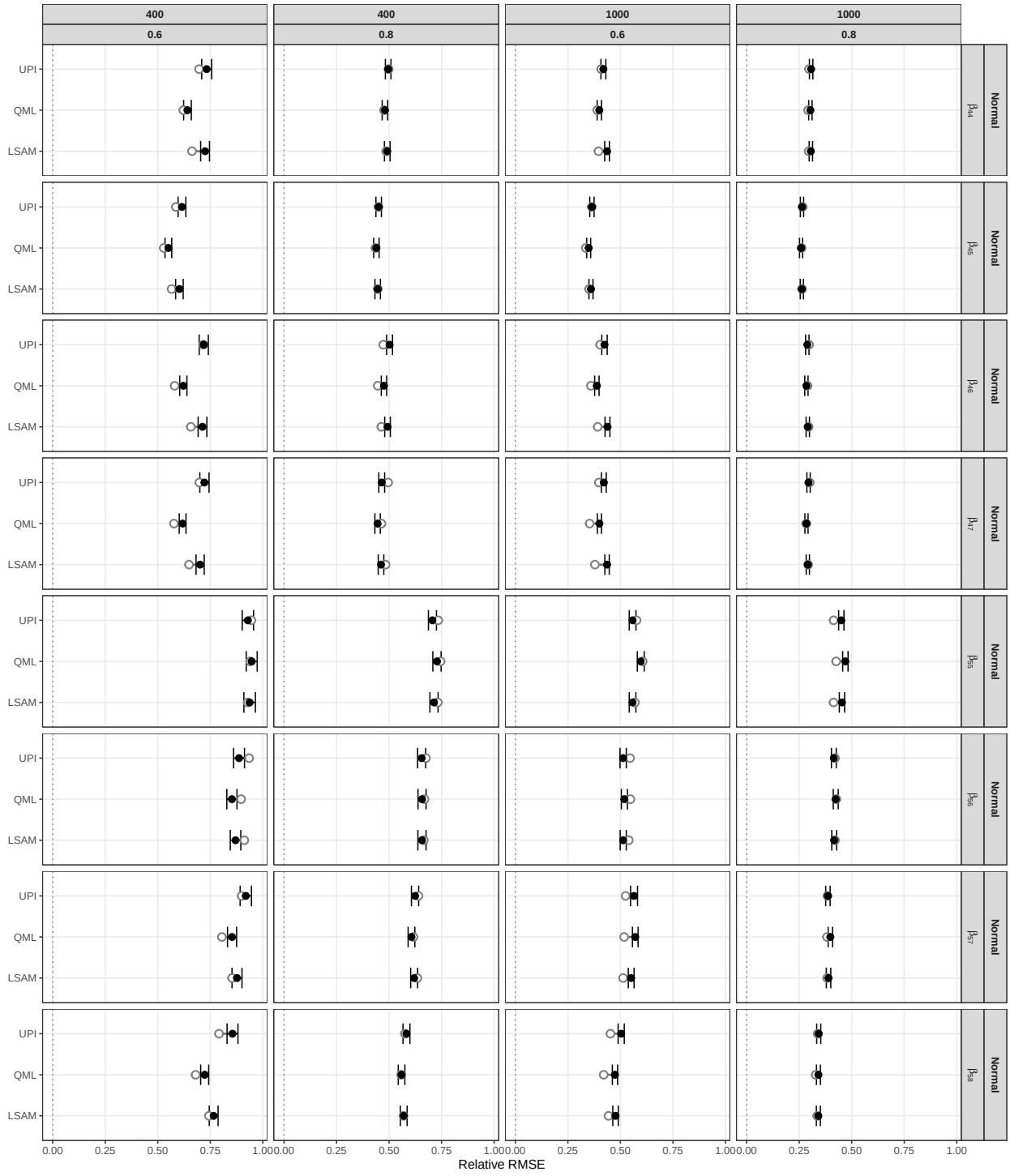


Figure S55

Additional Distributional Conditions: Relative RMSE for Simulation 2 — Normal (Zeta)

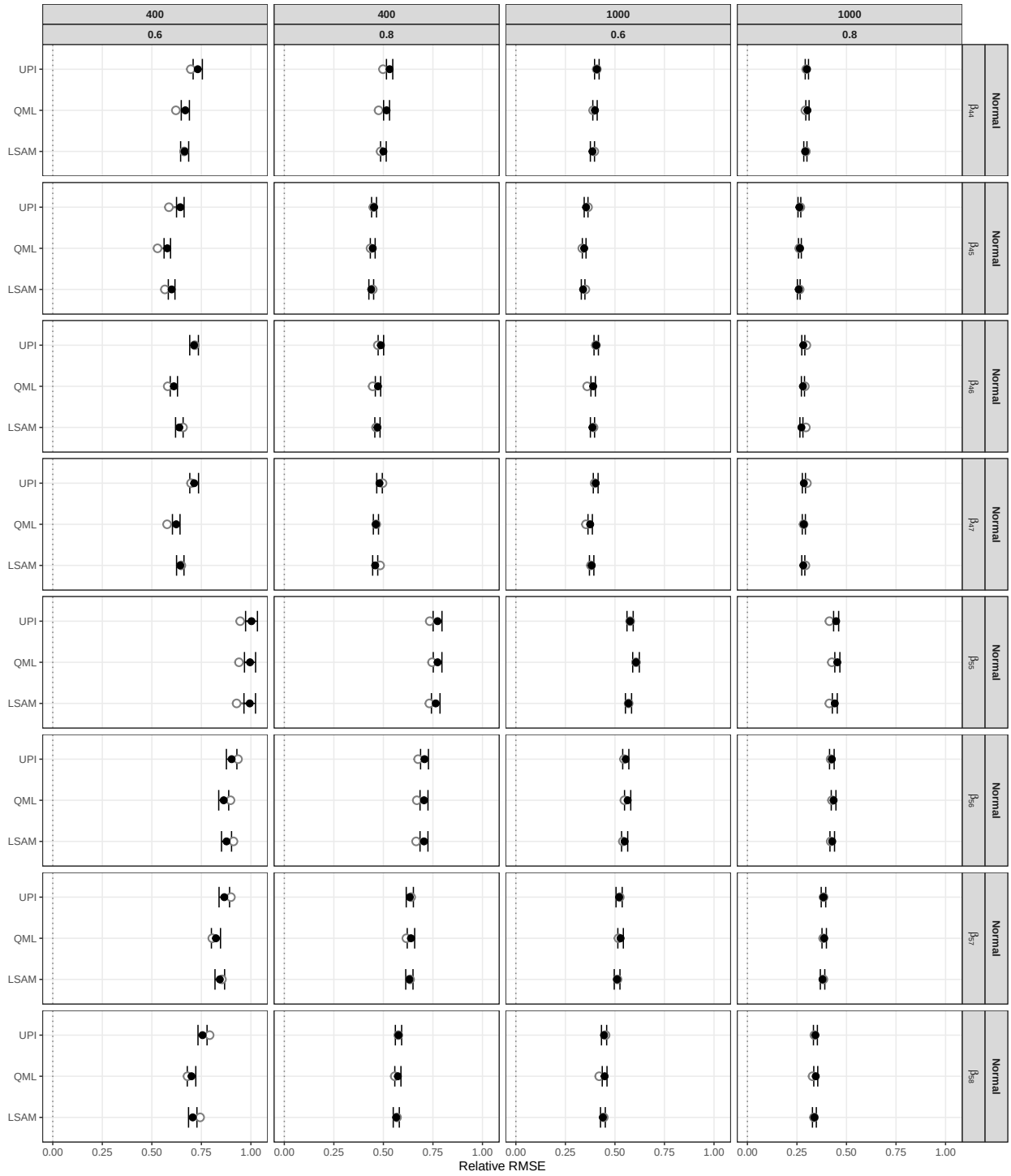


Figure S56

Additional Distributional Conditions: Relative RMSE for Simulation 2 — Normal (Epsilon & Zeta)

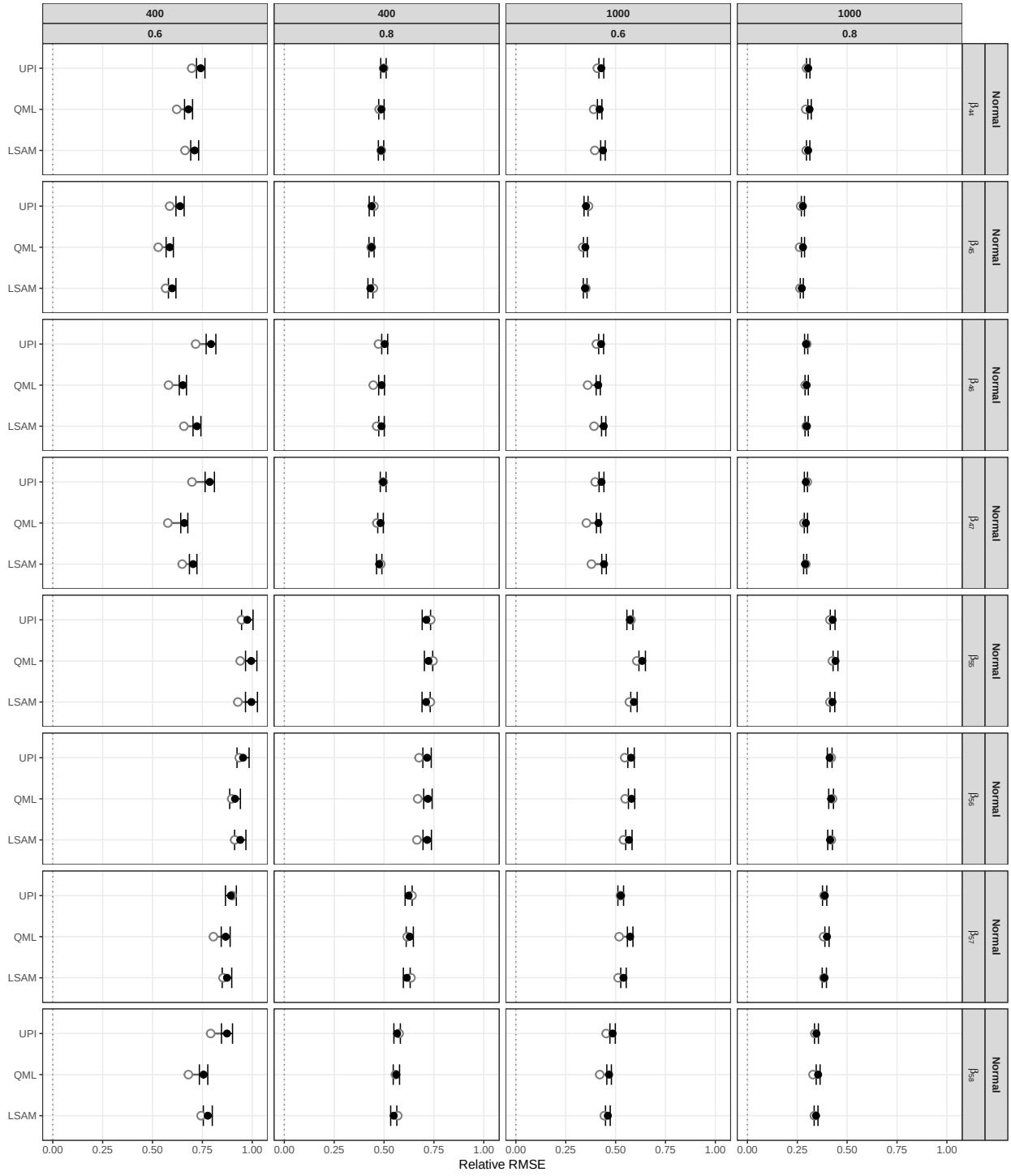


Figure S57

Additional Distributional Conditions: SE/SD Ratio for Simulation 2 — Normal (Epsilon)

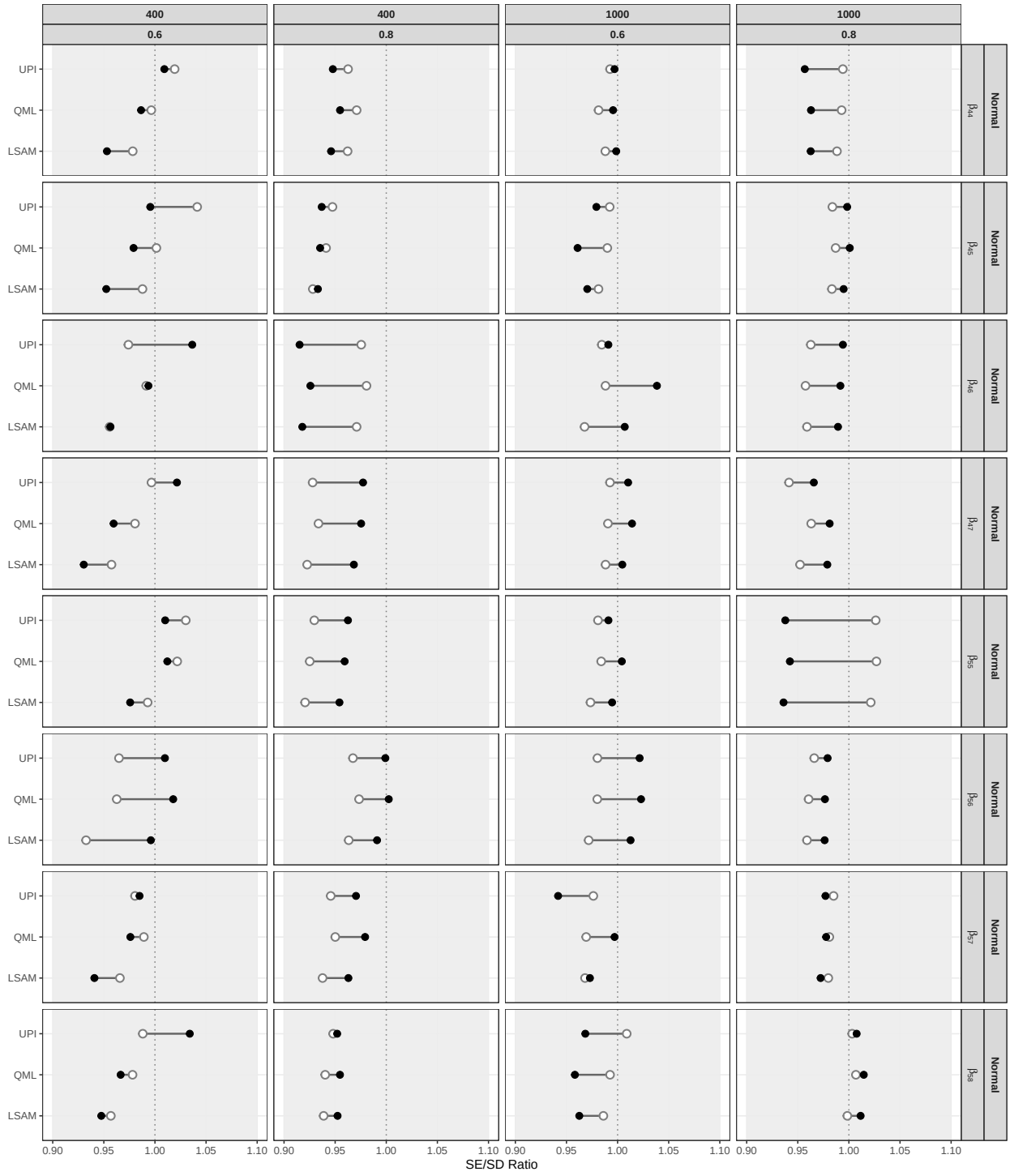


Figure S58

Additional Distributional Conditions: SE/SD Ratio for Simulation 2 — Normal (Zeta)

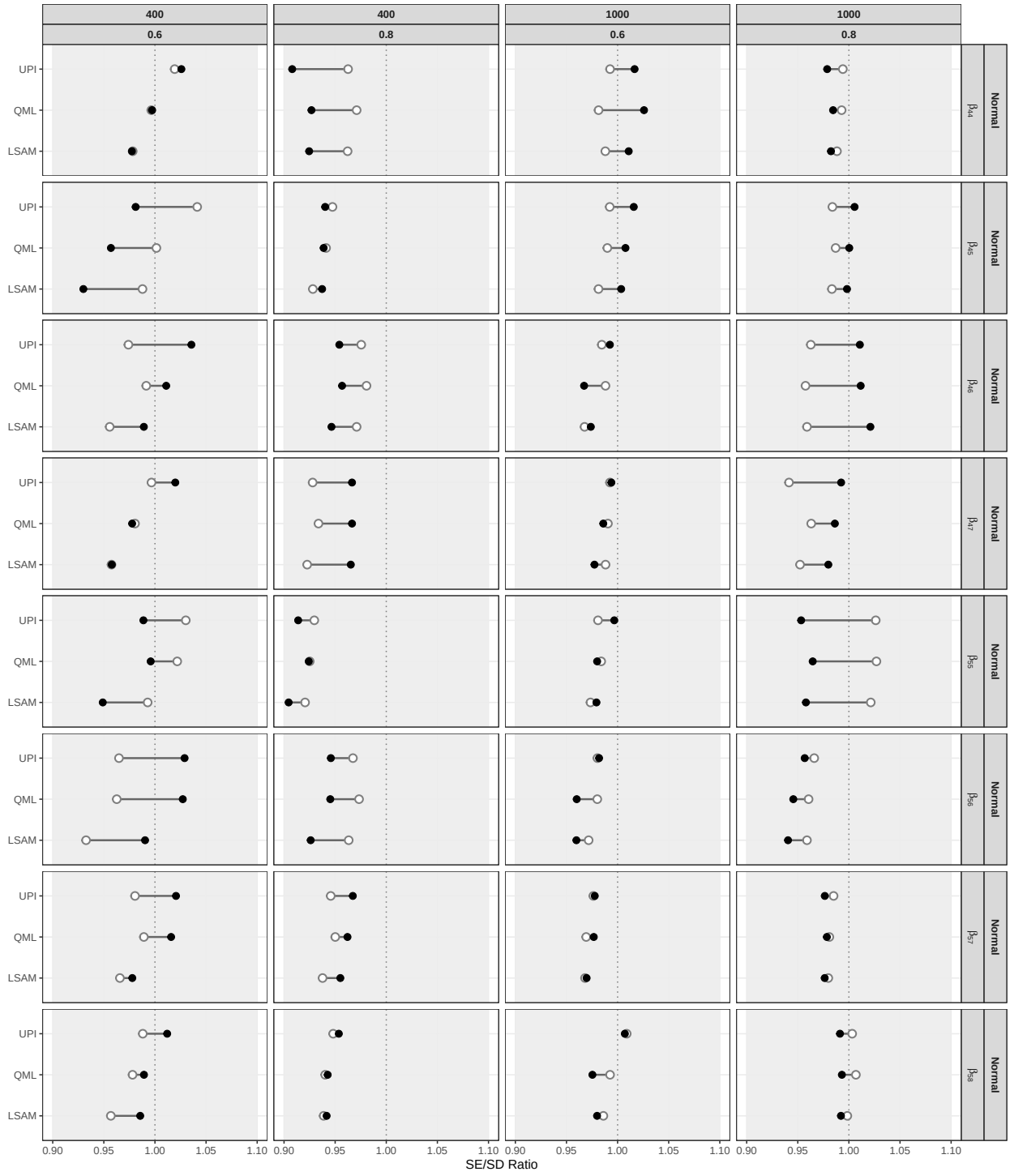


Figure S59

Additional Distributional Conditions: SE/SD Ratio for Simulation 2 — Normal (Epsilon & Zeta)

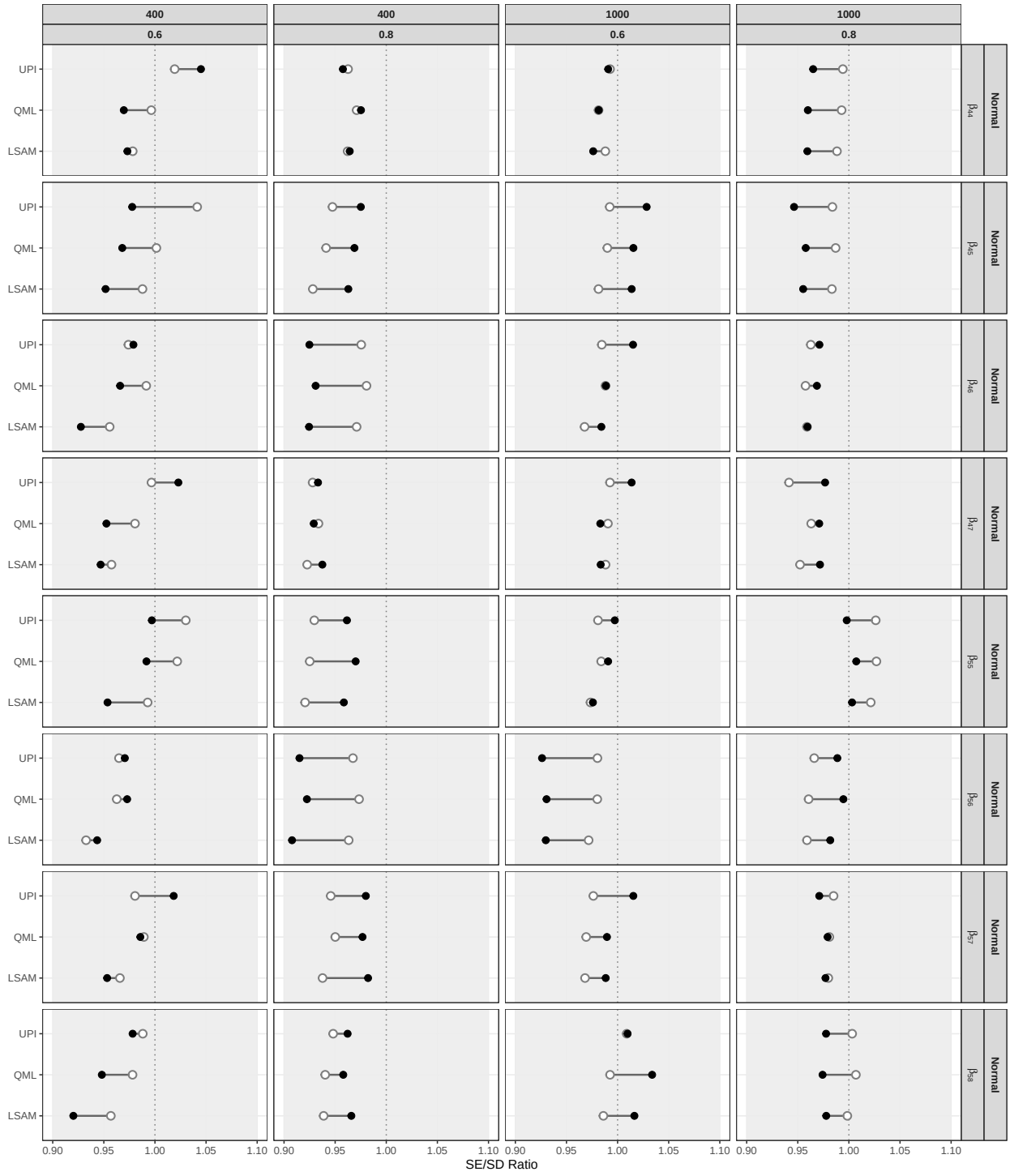


Figure S60

Additional Distributional Conditions: Coverage for Simulation 2 — Normal (Epsilon)

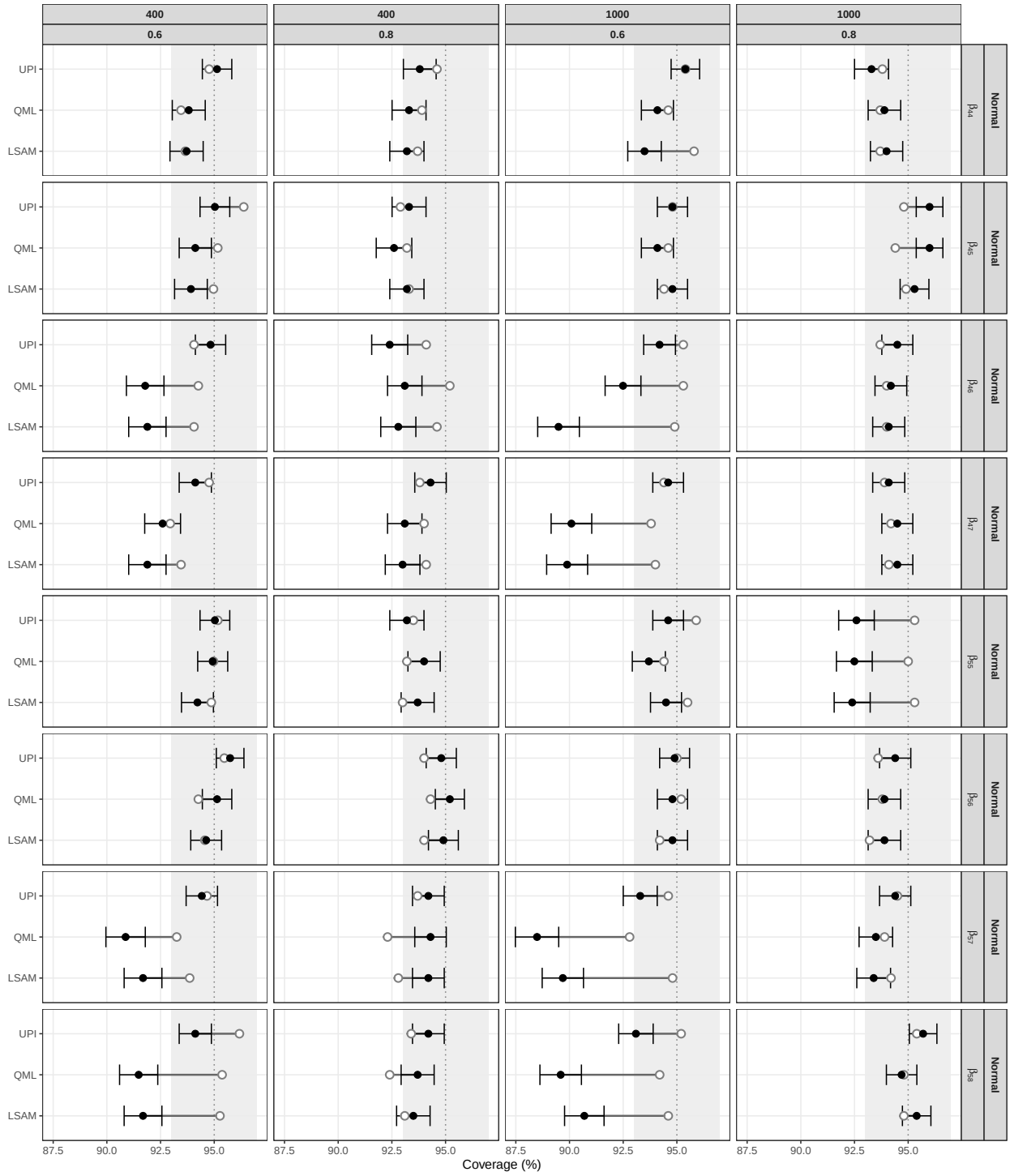


Figure S61

Additional Distributional Conditions: Coverage for Simulation 2 — Normal (Zeta)

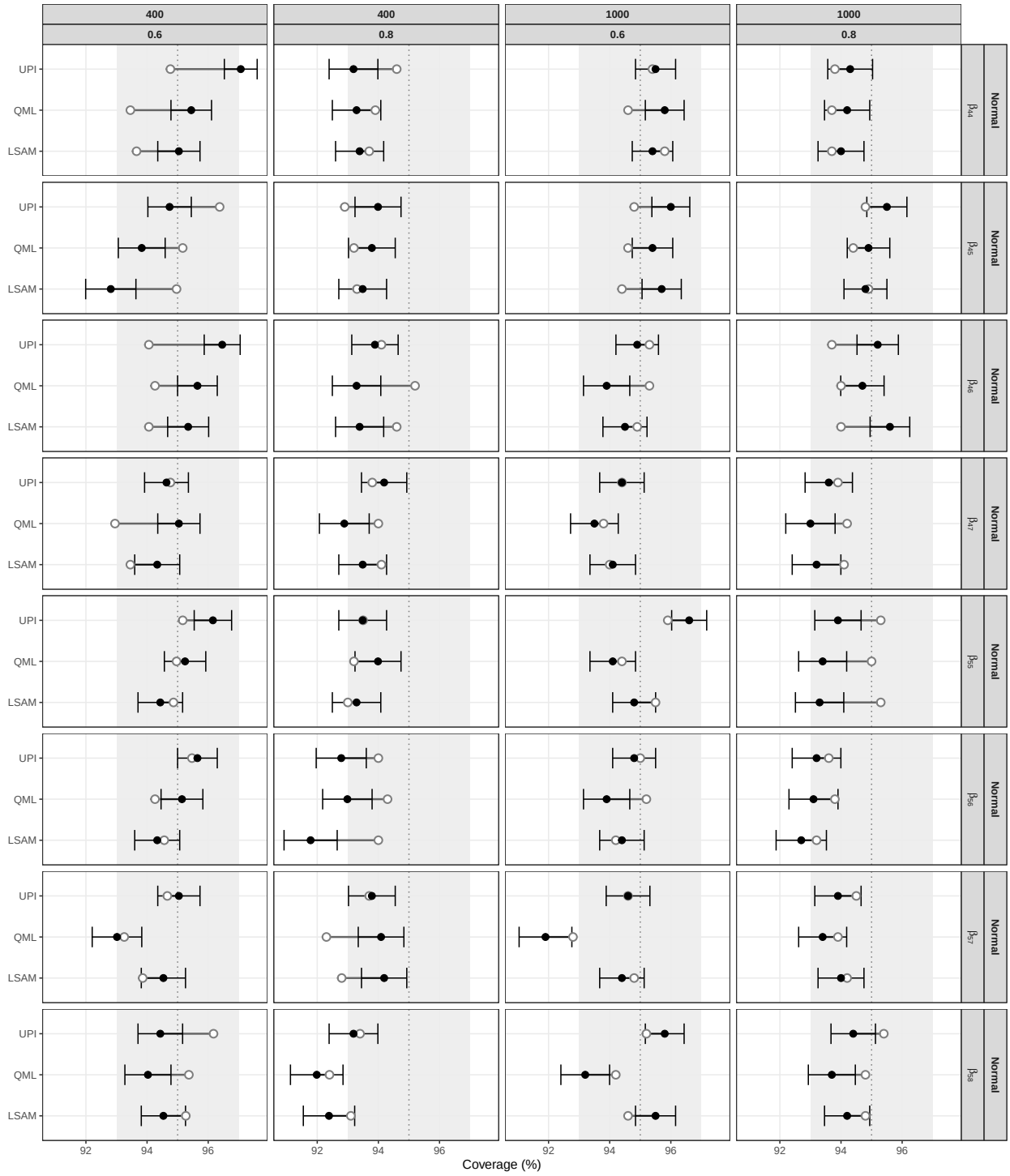


Figure S62

Additional Distributional Conditions: Coverage for Simulation 2 — Normal (Epsilon & Zeta)

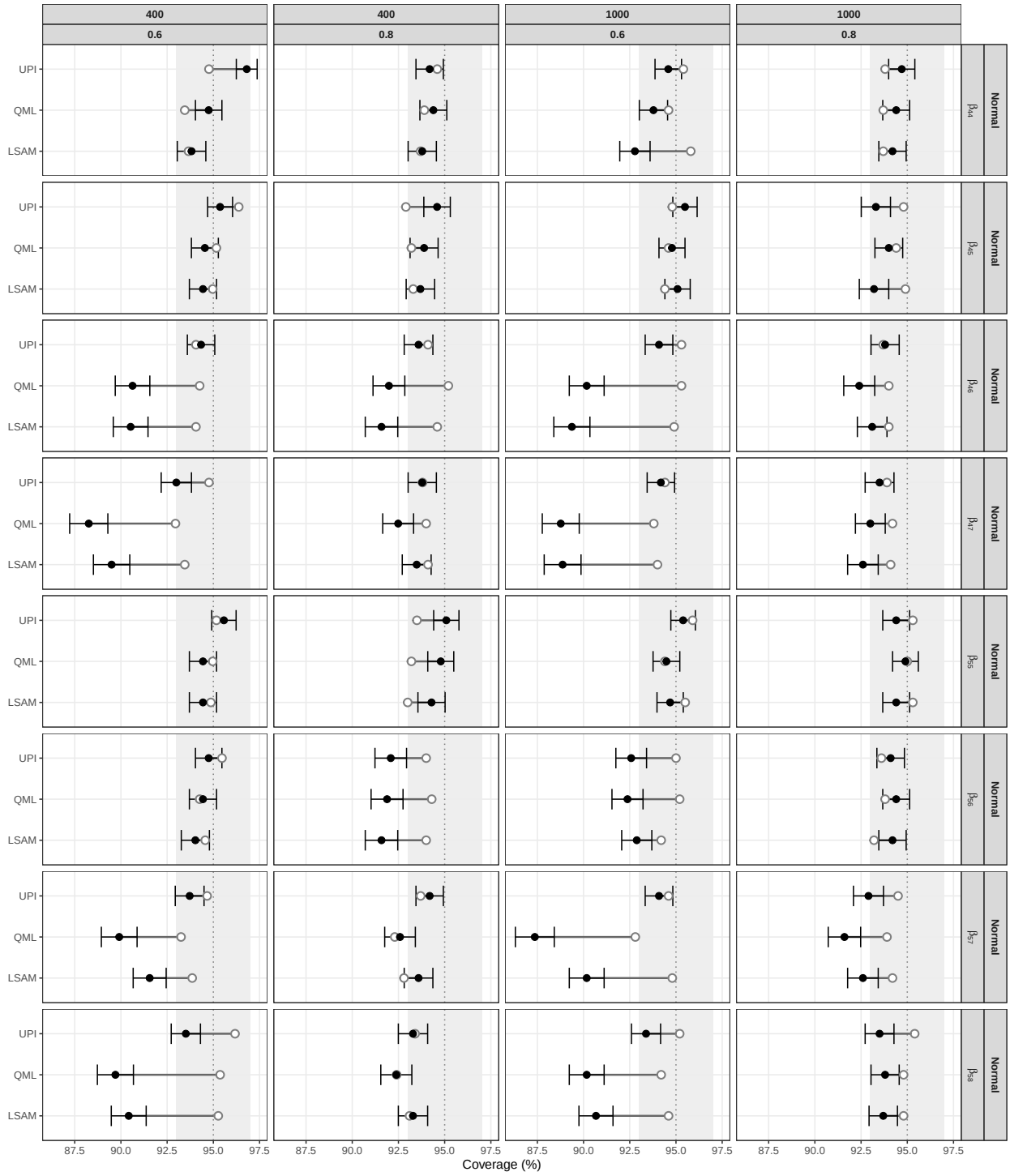


Figure S63

Additional Distributional Conditions: Type I Error for Simulation 2 — Normal (Epsilon)

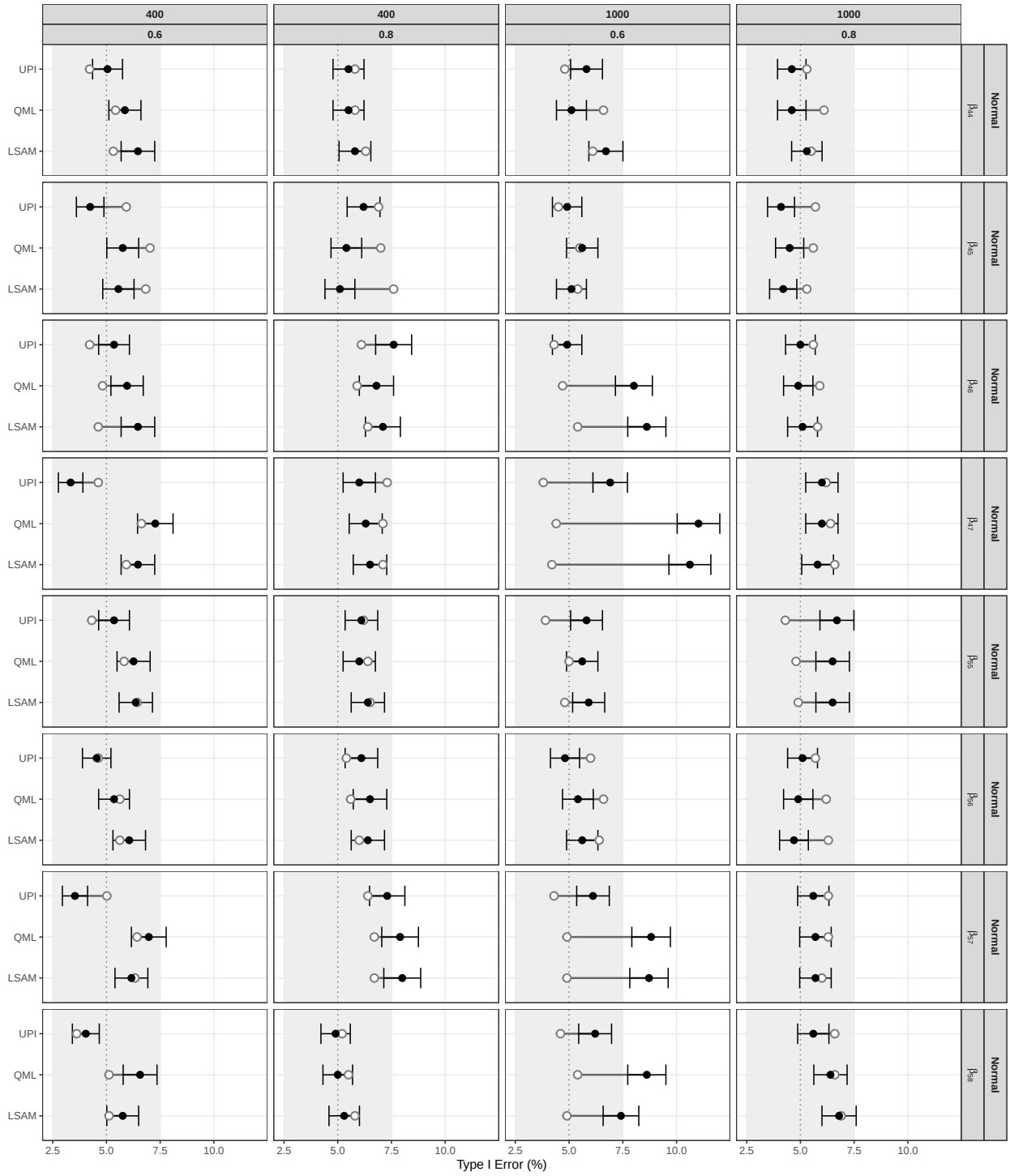


Figure S64

Additional Distributional Conditions: Type I Error for Simulation 2 — Normal (Zeta)

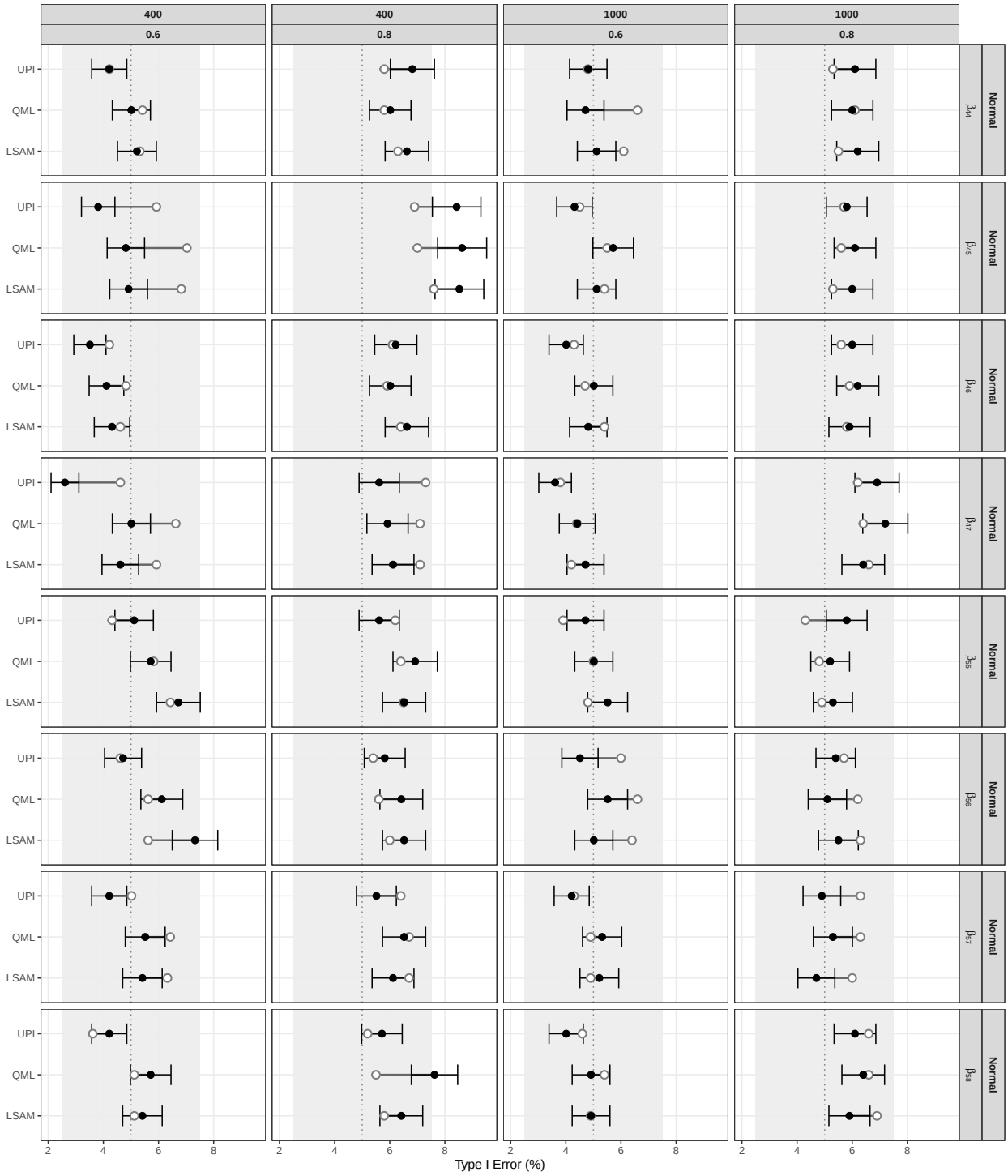


Figure S65

Additional Distributional Conditions: Type I Error for Simulation 2 — Normal (Epsilon & Zeta)

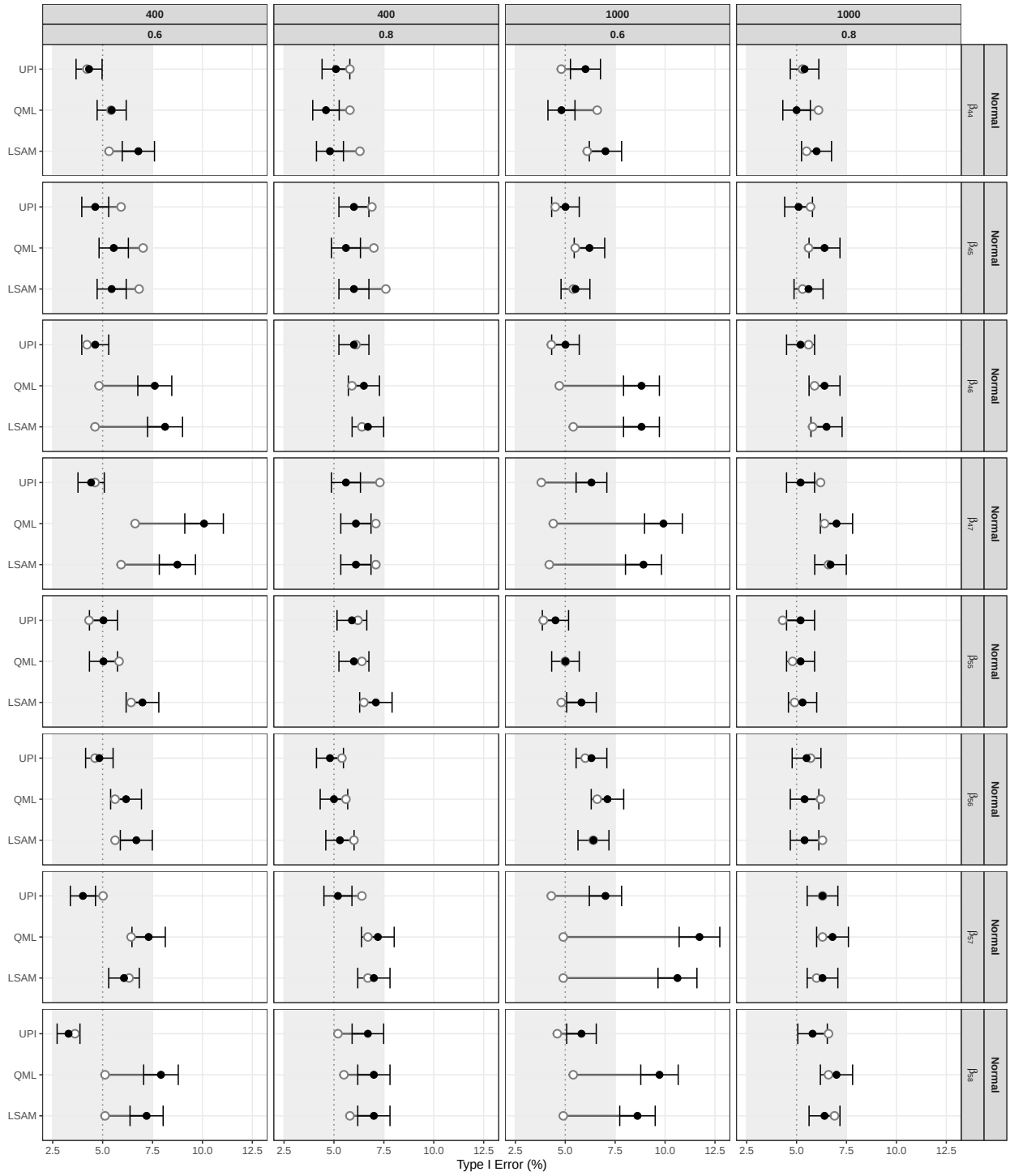


Figure S66

Additional Distributional Conditions: Power for Simulation 2 — Normal (Epsilon)

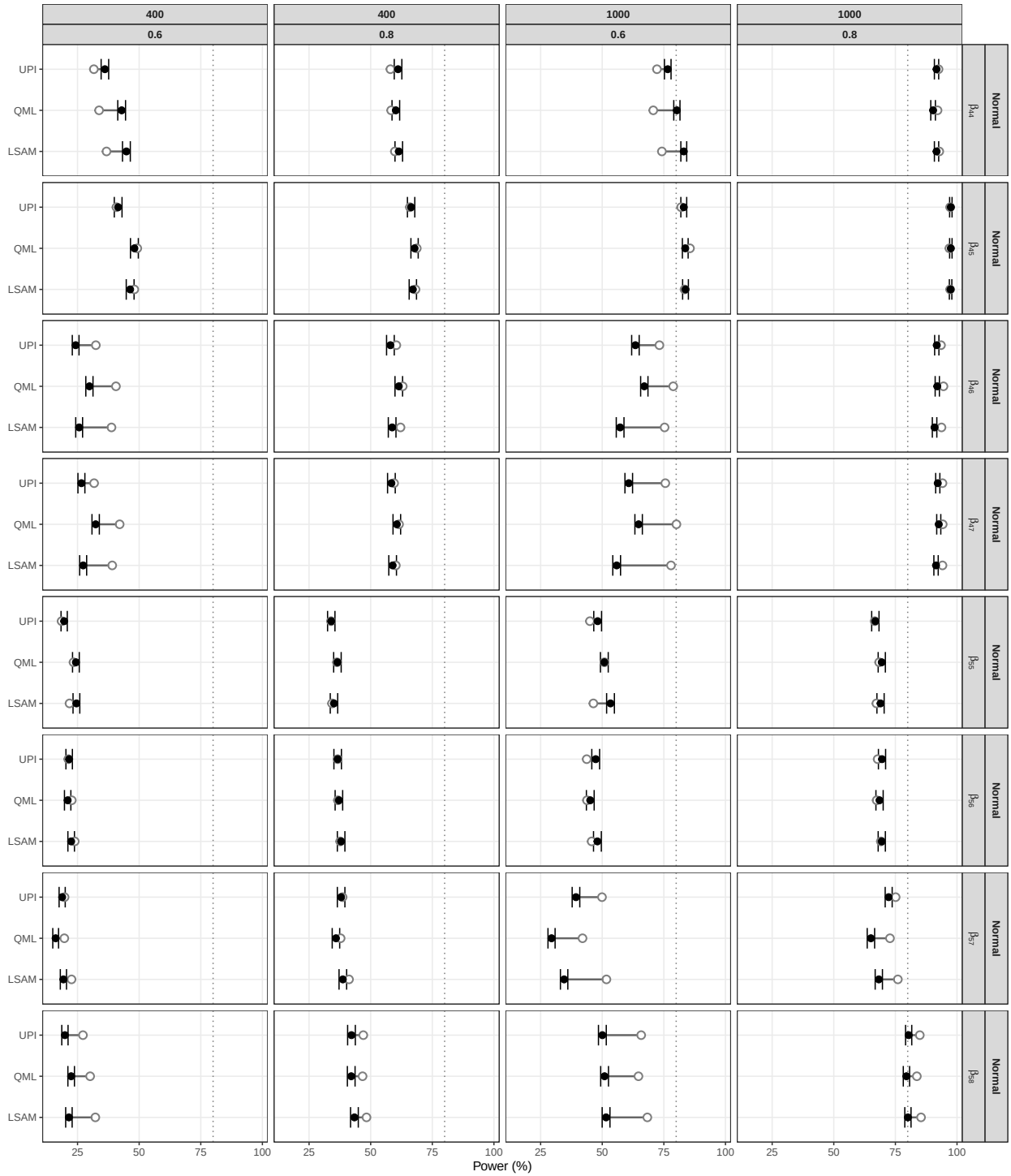


Figure S67

Additional Distributional Conditions: Power for Simulation 2 — Normal (Zeta)

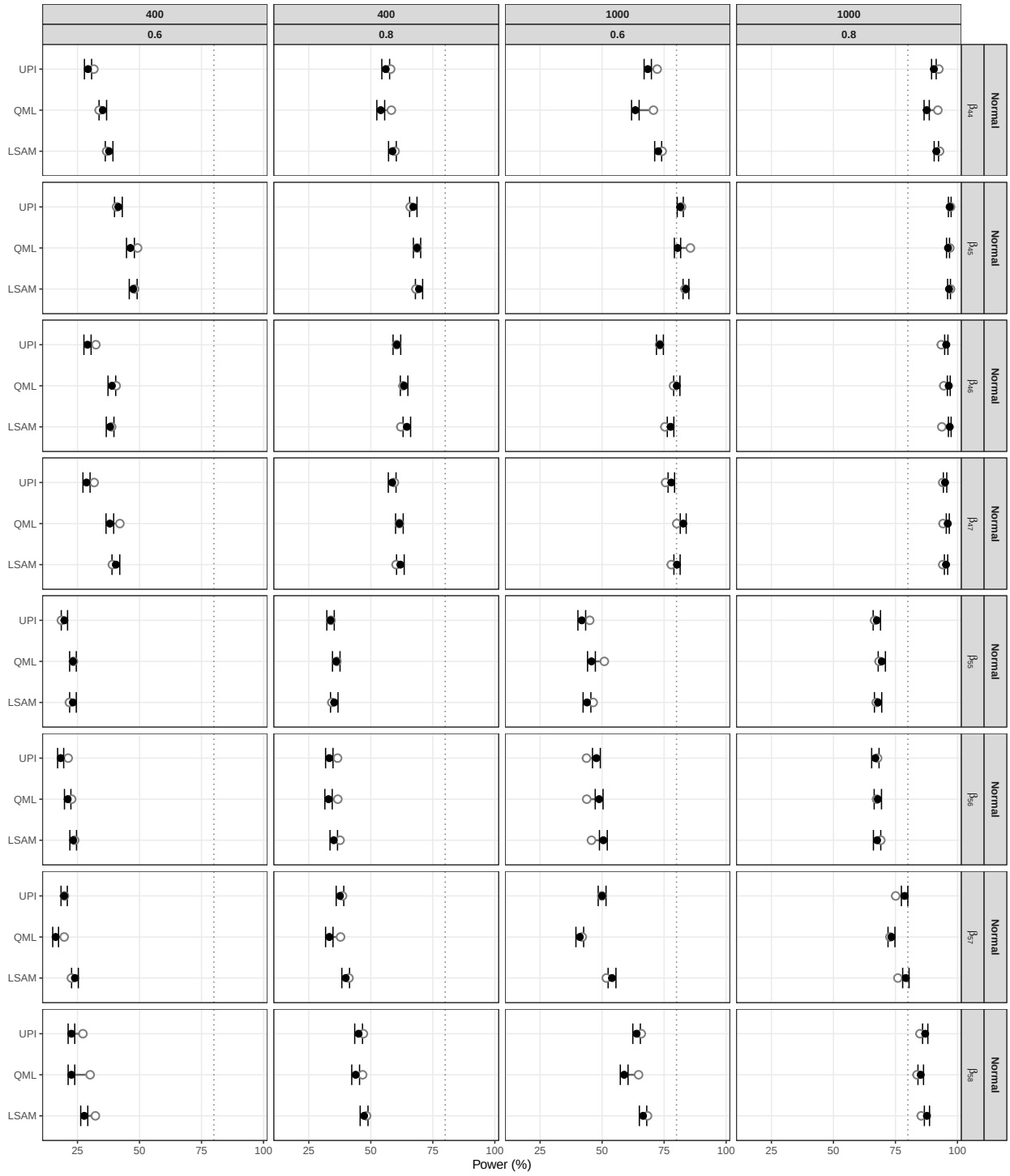


Figure S68

Additional Distributional Conditions: Power for Simulation 2 — Normal (Epsilon & Zeta)

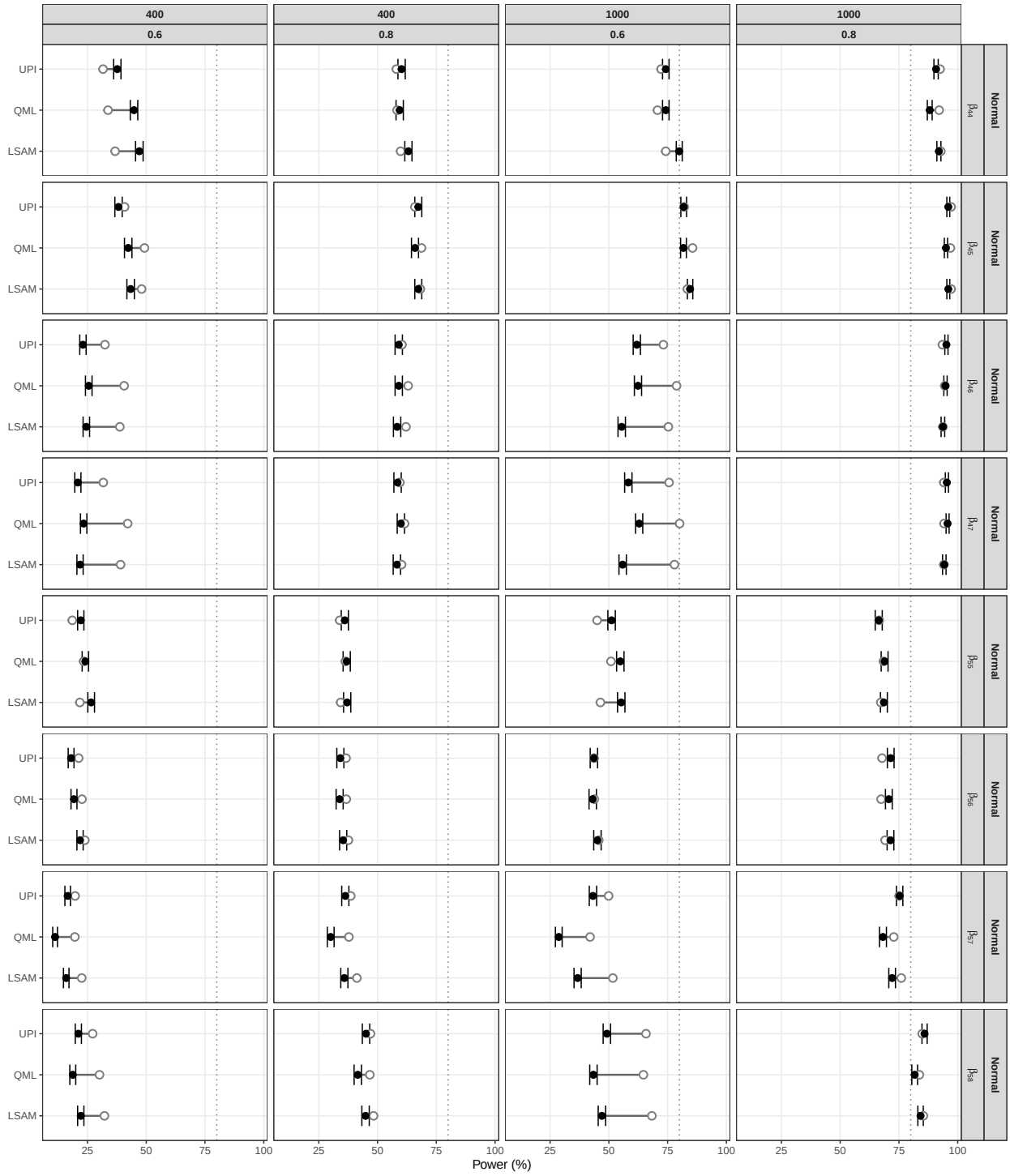


Figure S69

Additional Distributional Conditions: Relative Bias for Simulation 2 — Right-skewed (Epsilon)

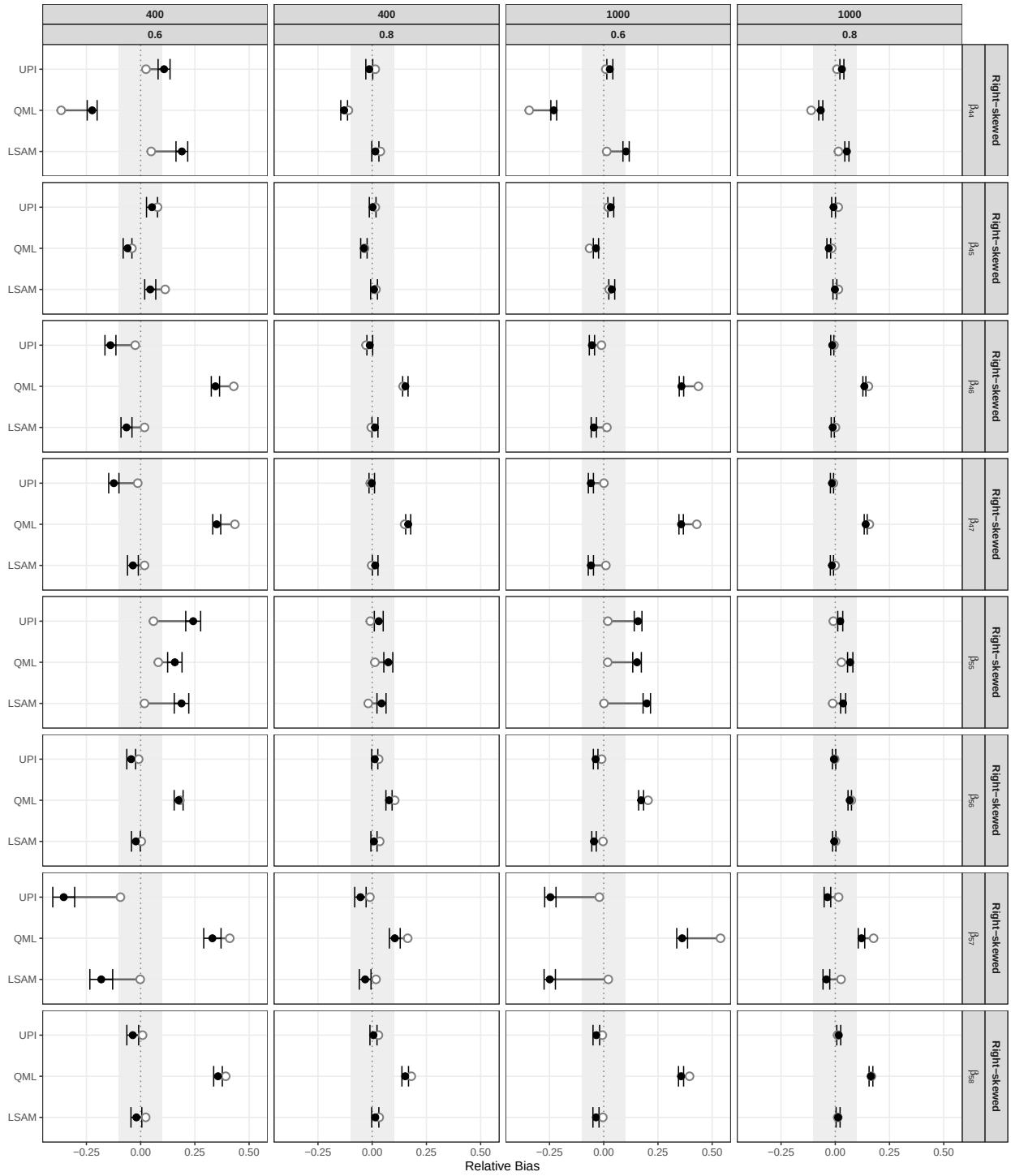


Figure S70

Additional Distributional Conditions: Relative Bias for Simulation 2 — Right-skewed (Zeta)

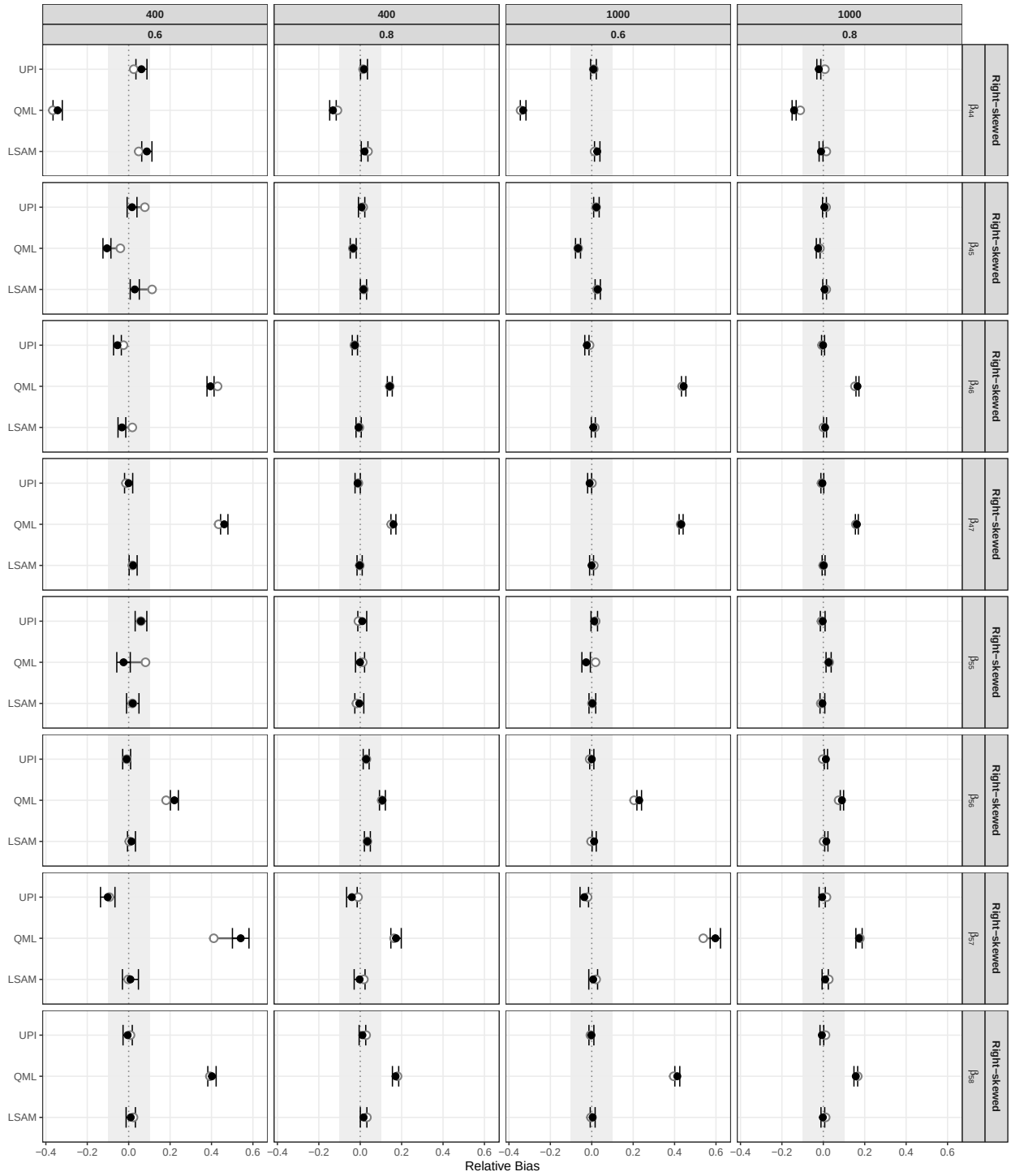


Figure S71

Additional Distributional Conditions: Relative Bias for Simulation 2 — Right-skewed (Epsilon & Zeta)

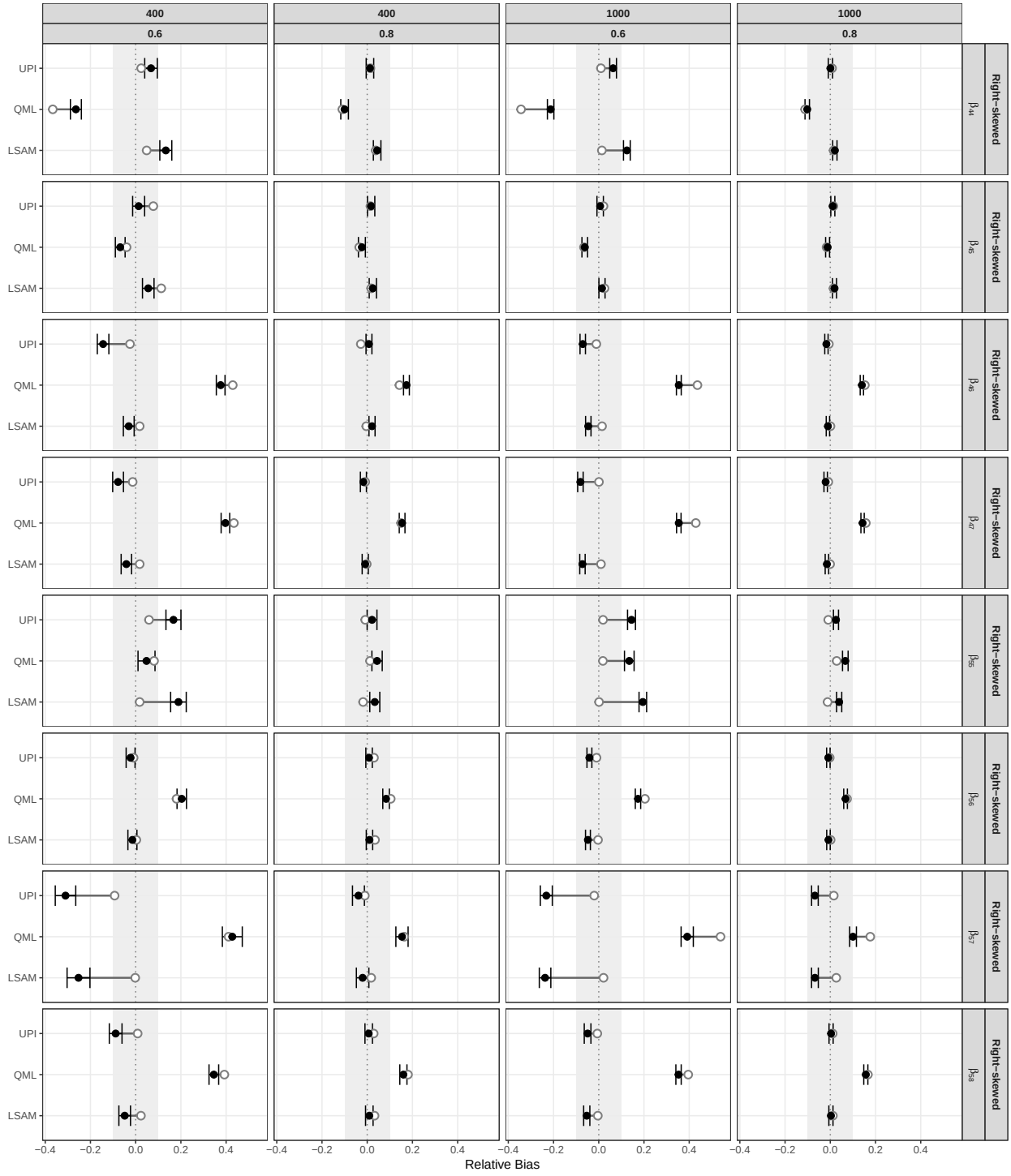


Figure S72

Additional Distributional Conditions: Relative RMSE for Simulation 2 — Right-skewed (Epsilon)

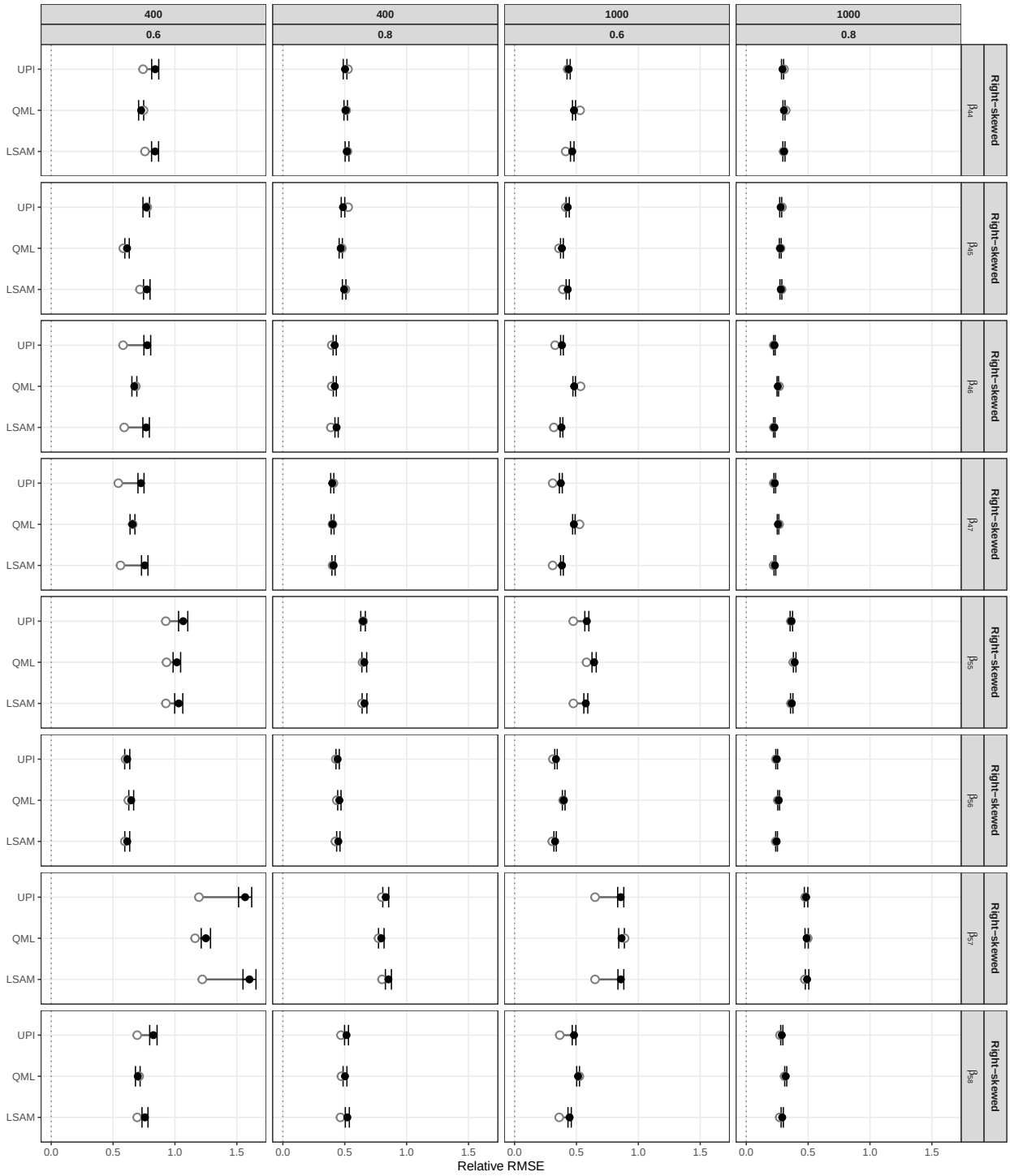


Figure S73

Additional Distributional Conditions: Relative RMSE for Simulation 2 — Right-skewed (Zeta)

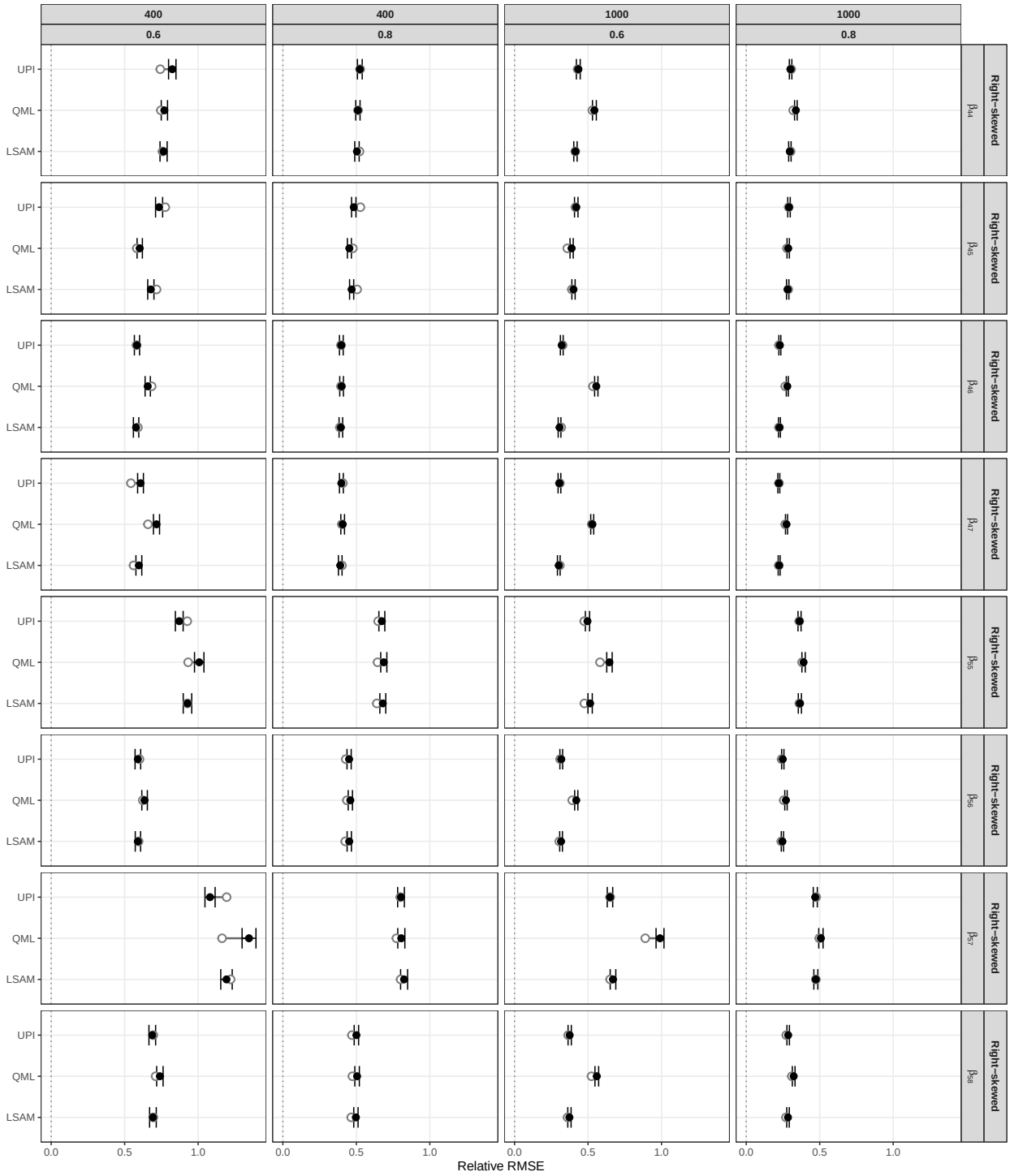


Figure S74

Additional Distributional Conditions: Relative RMSE for Simulation 2 — Right-skewed (Epsilon & Zeta)

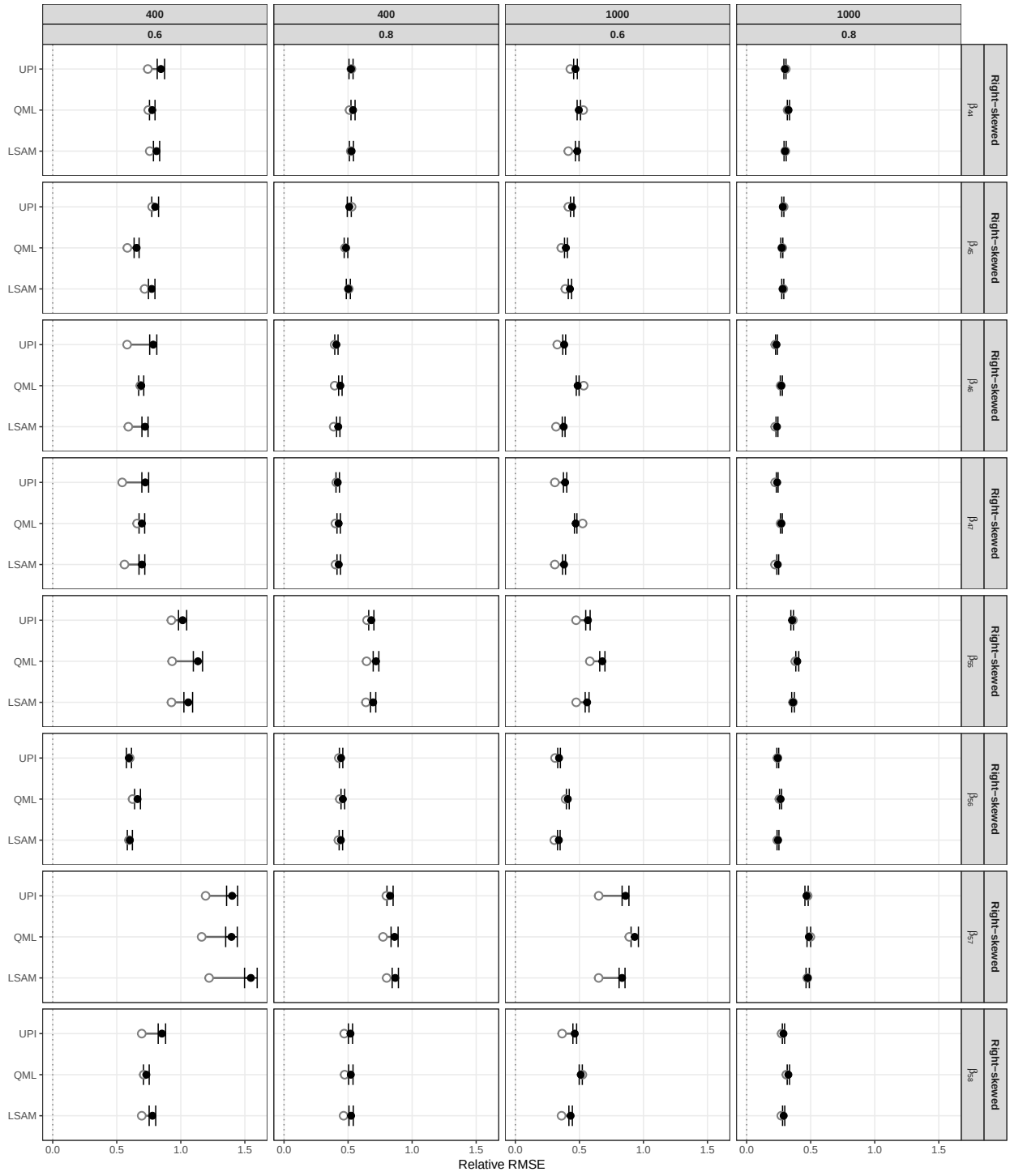


Figure S75

Additional Distributional Conditions: SE/SD Ratio for Simulation 2 — Right-skewed (Epsilon)

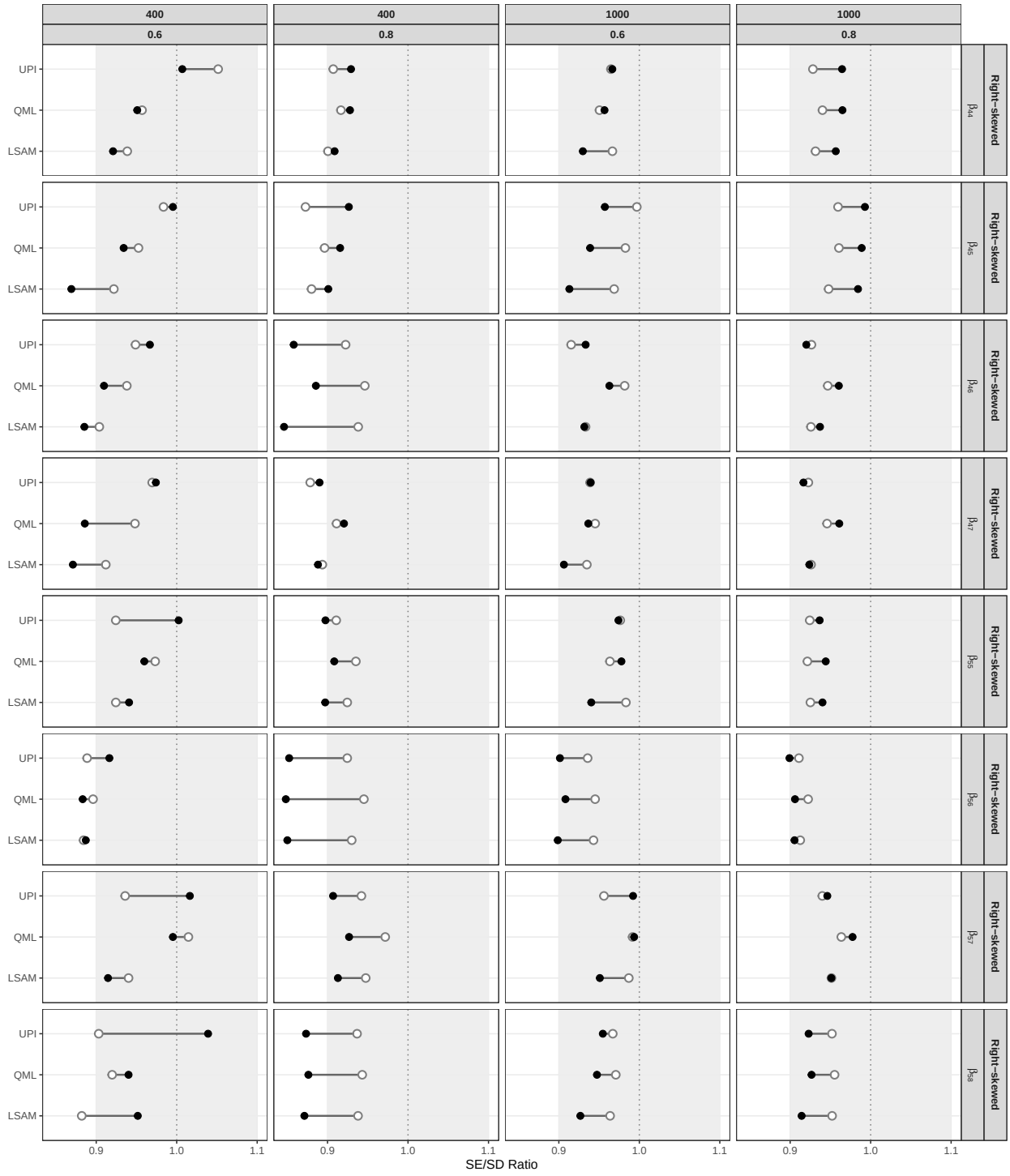


Figure S76

Additional Distributional Conditions: SE/SD Ratio for Simulation 2 — Right-skewed (Zeta)

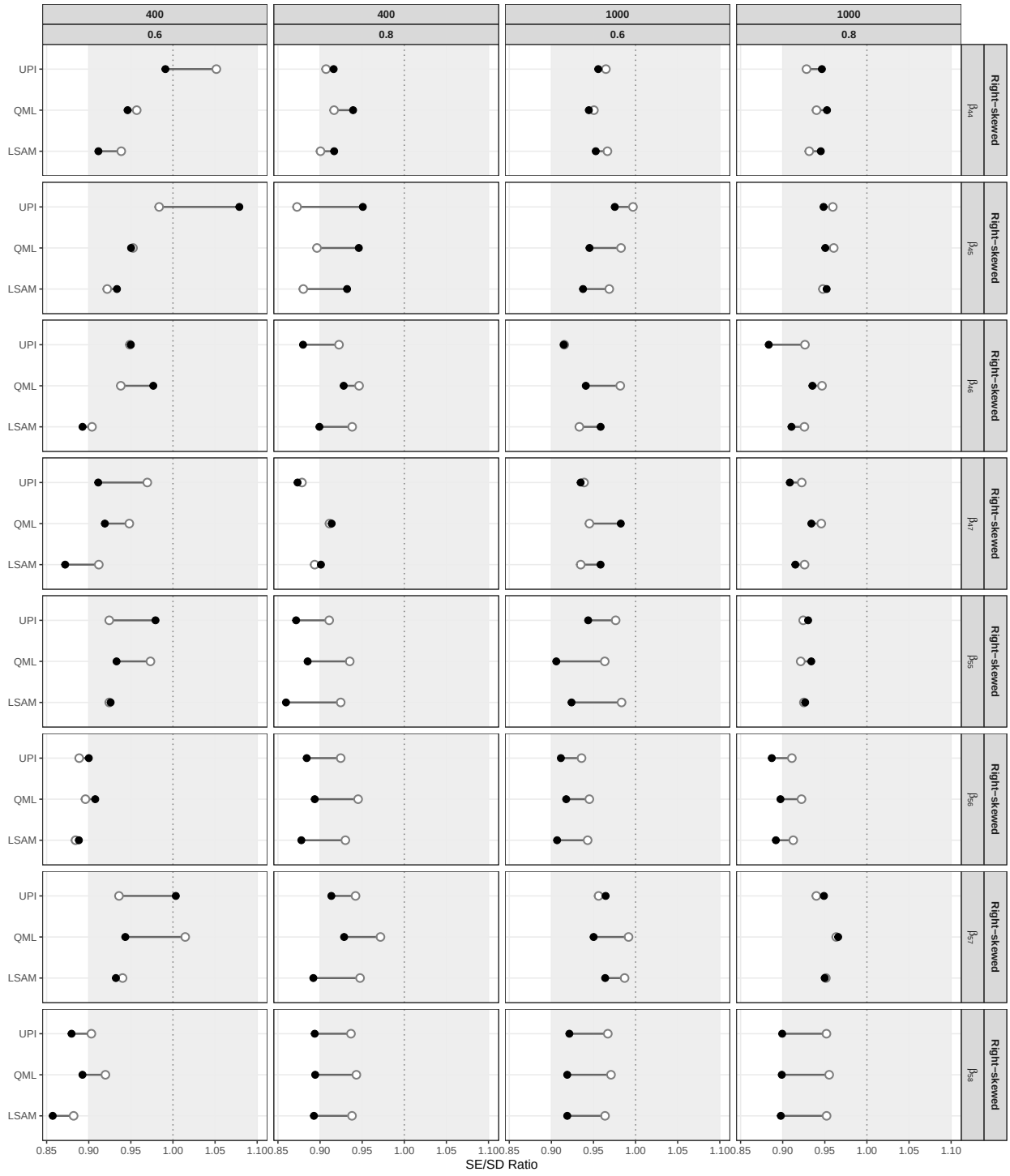


Figure S77

Additional Distributional Conditions: SE/SD Ratio for Simulation 2 — Right-skewed (Epsilon & Zeta)

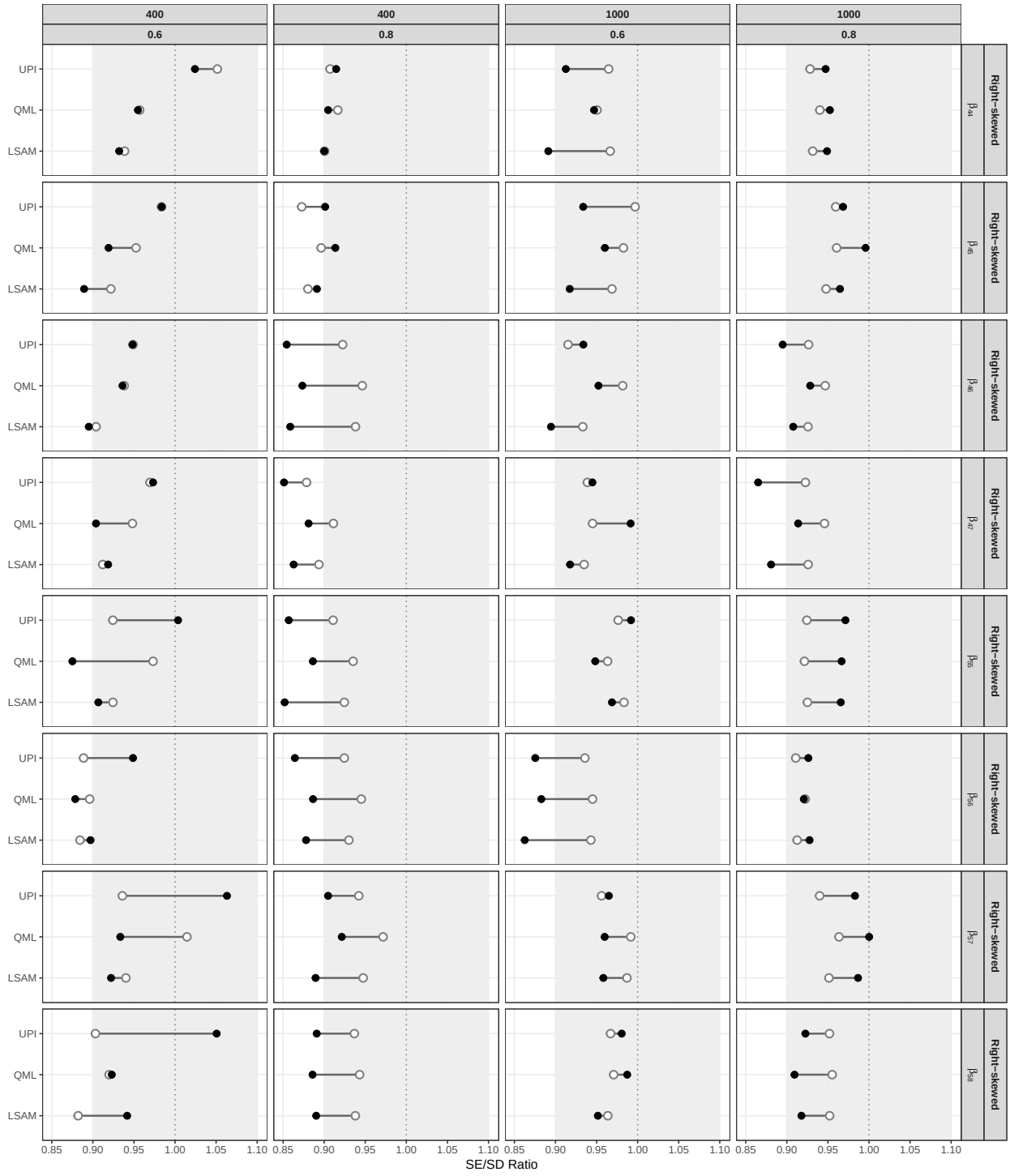


Figure S78

Additional Distributional Conditions: Coverage for Simulation 2 — Right-skewed (Epsilon)

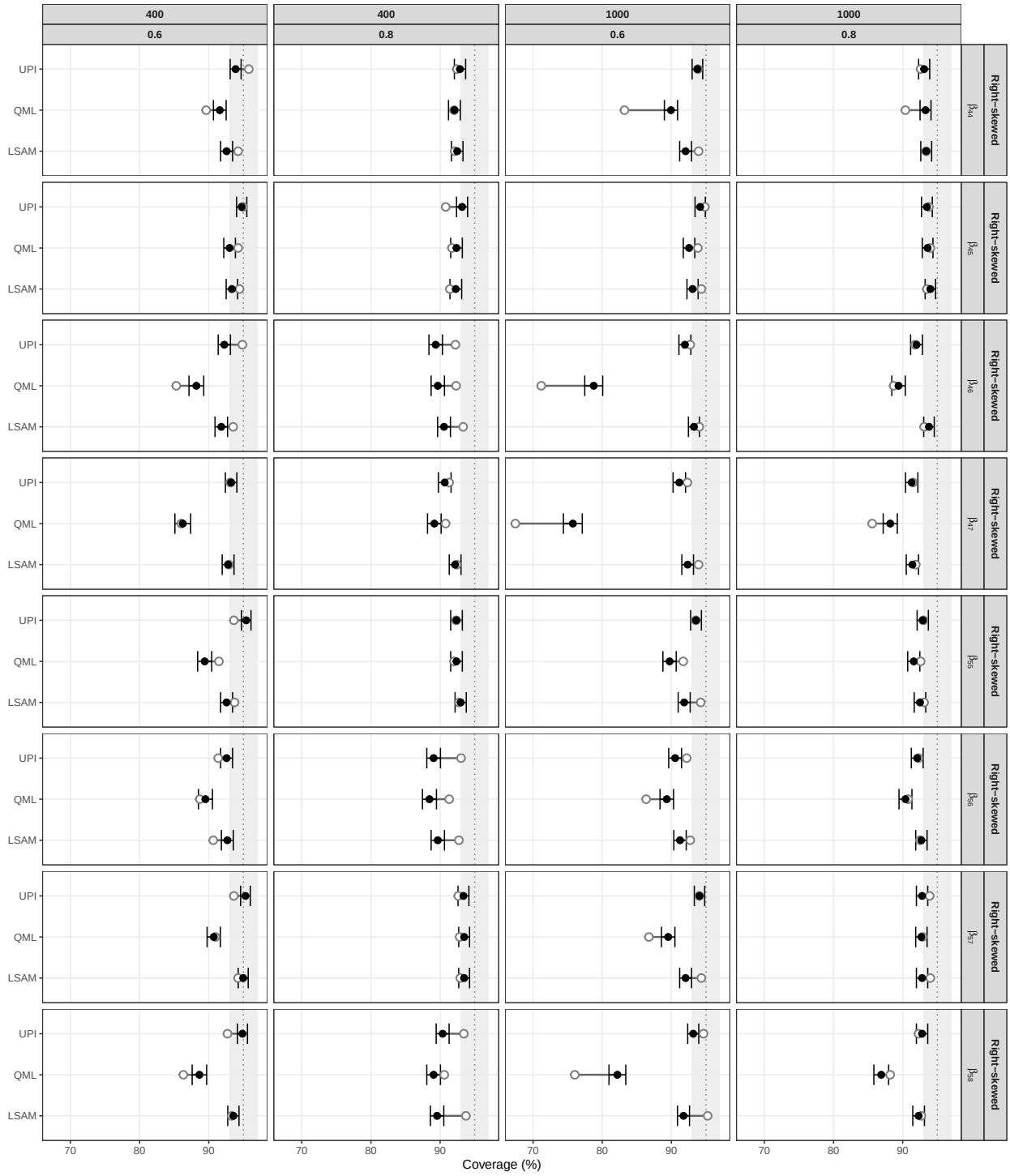


Figure S79

Additional Distributional Conditions: Coverage for Simulation 2 — Right-skewed (Zeta)

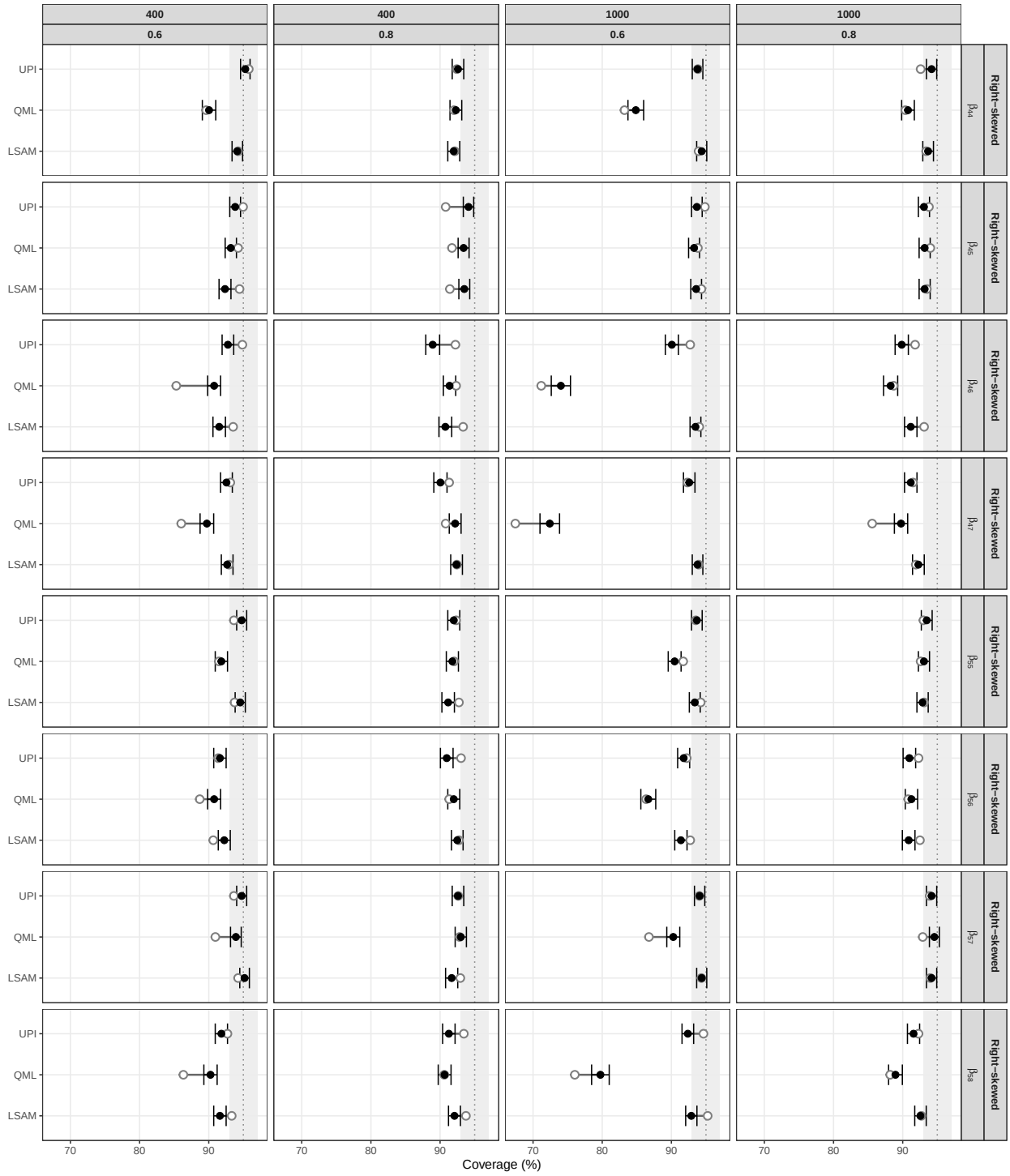


Figure S80

Additional Distributional Conditions: Coverage for Simulation 2 — Right-skewed (Epsilon & Zeta)

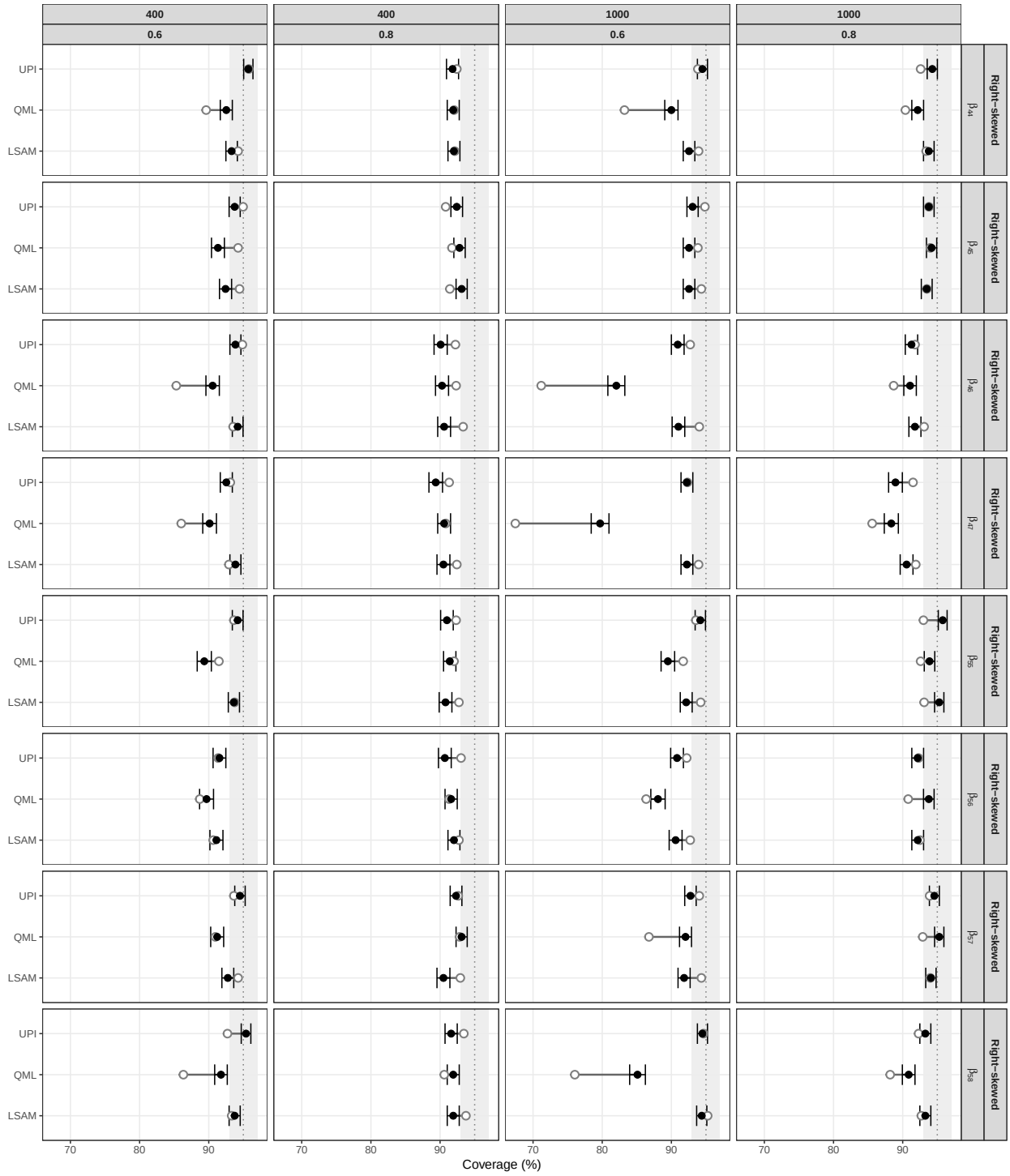


Figure S81

Additional Distributional Conditions: Type I Error for Simulation 2 — Right-skewed (Epsilon)

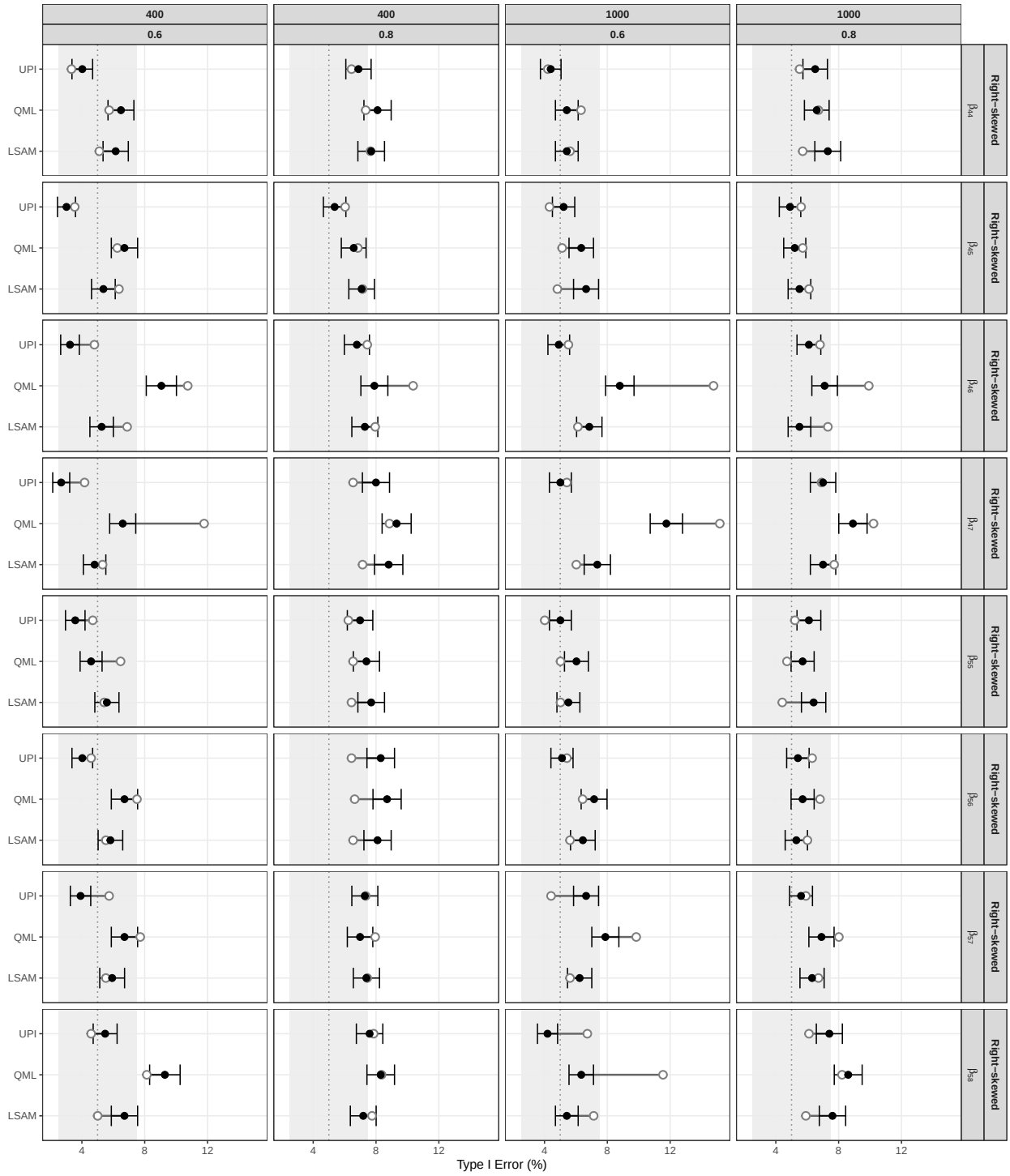


Figure S82

Additional Distributional Conditions: Type I Error for Simulation 2 — Right-skewed (Zeta)

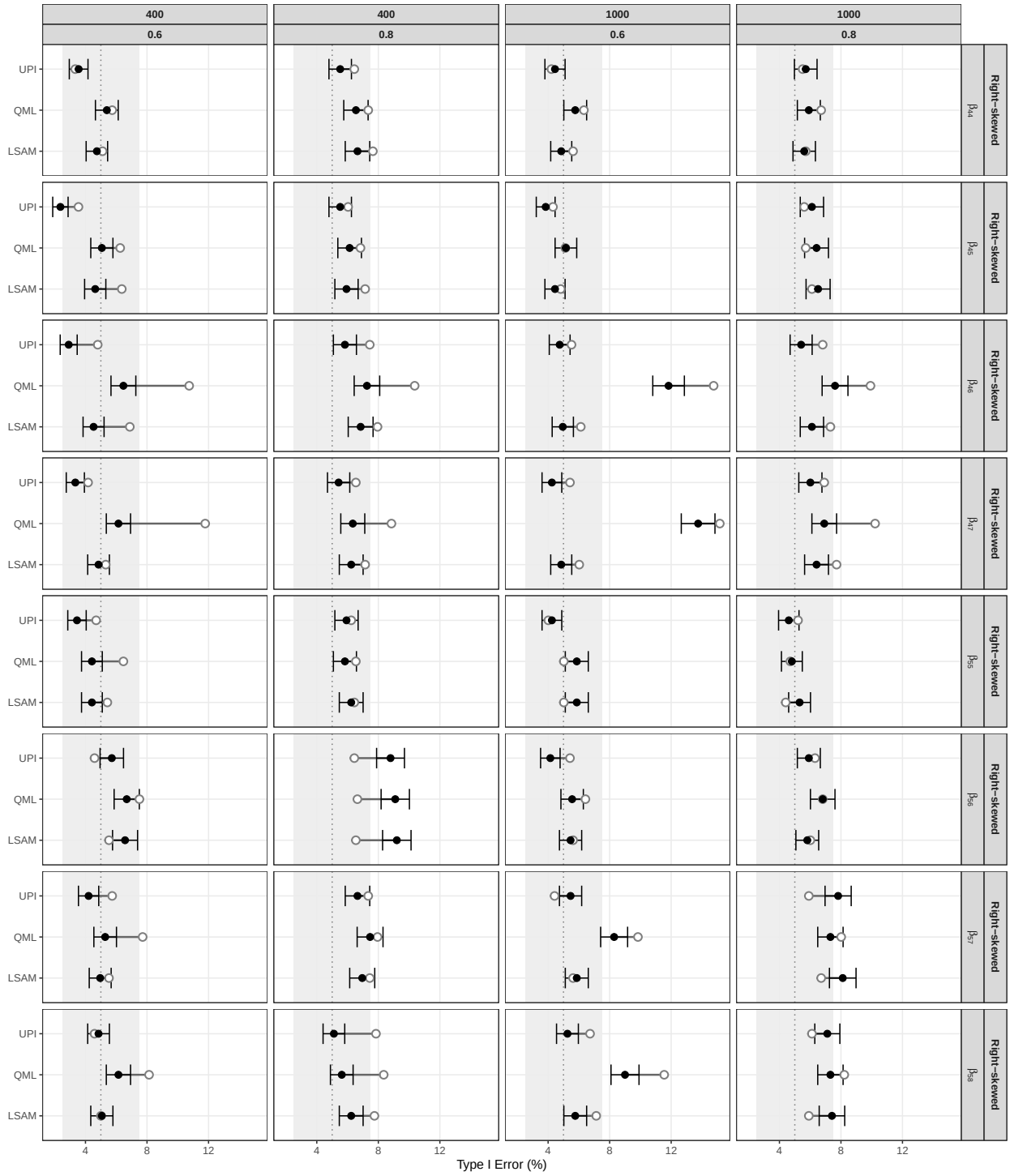


Figure S83

Additional Distributional Conditions: Type I Error for Simulation 2 — Right-skewed (Epsilon & Zeta)

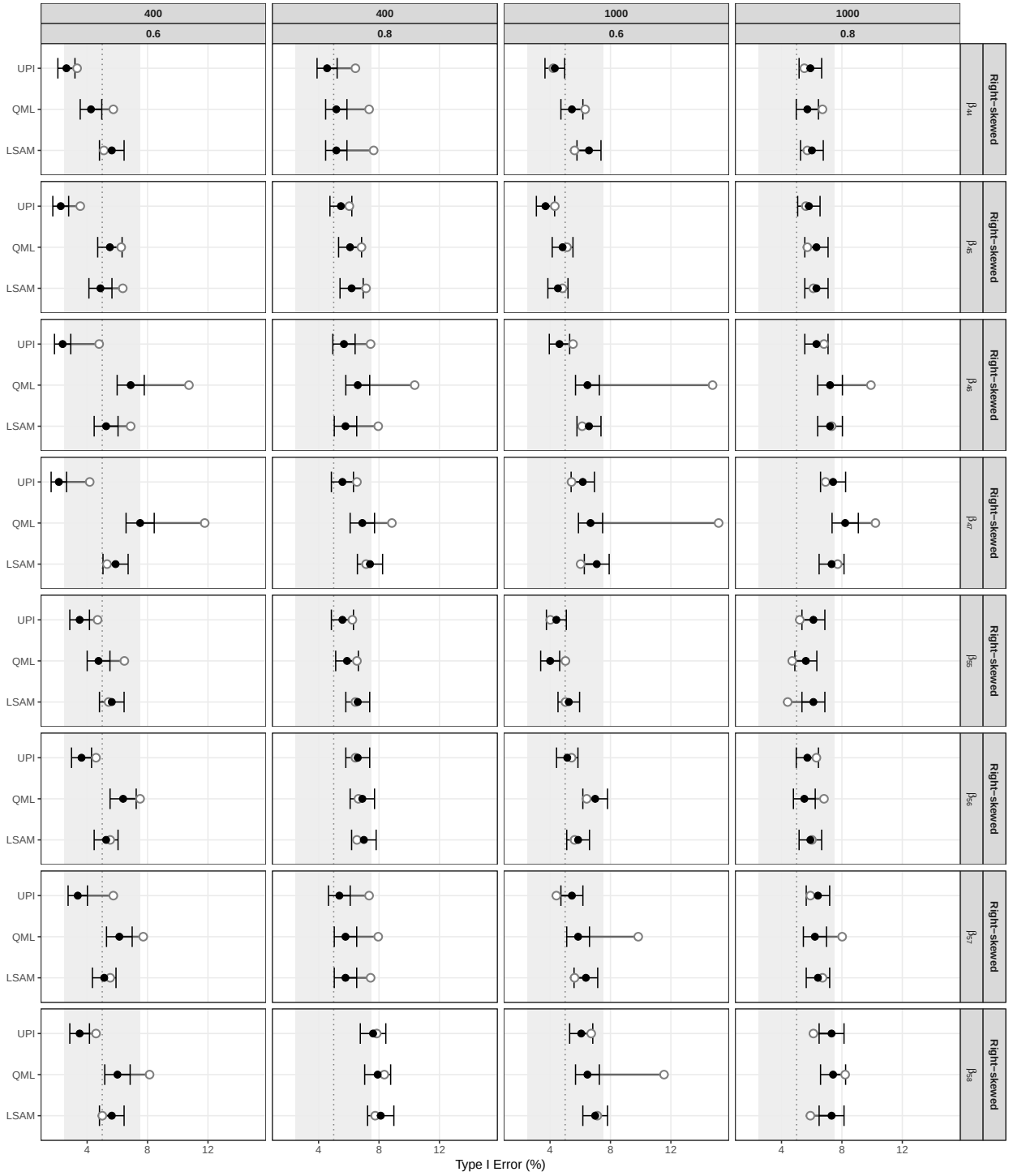


Figure S84

Additional Distributional Conditions: Power for Simulation 2 — Right-skewed (Epsilon)

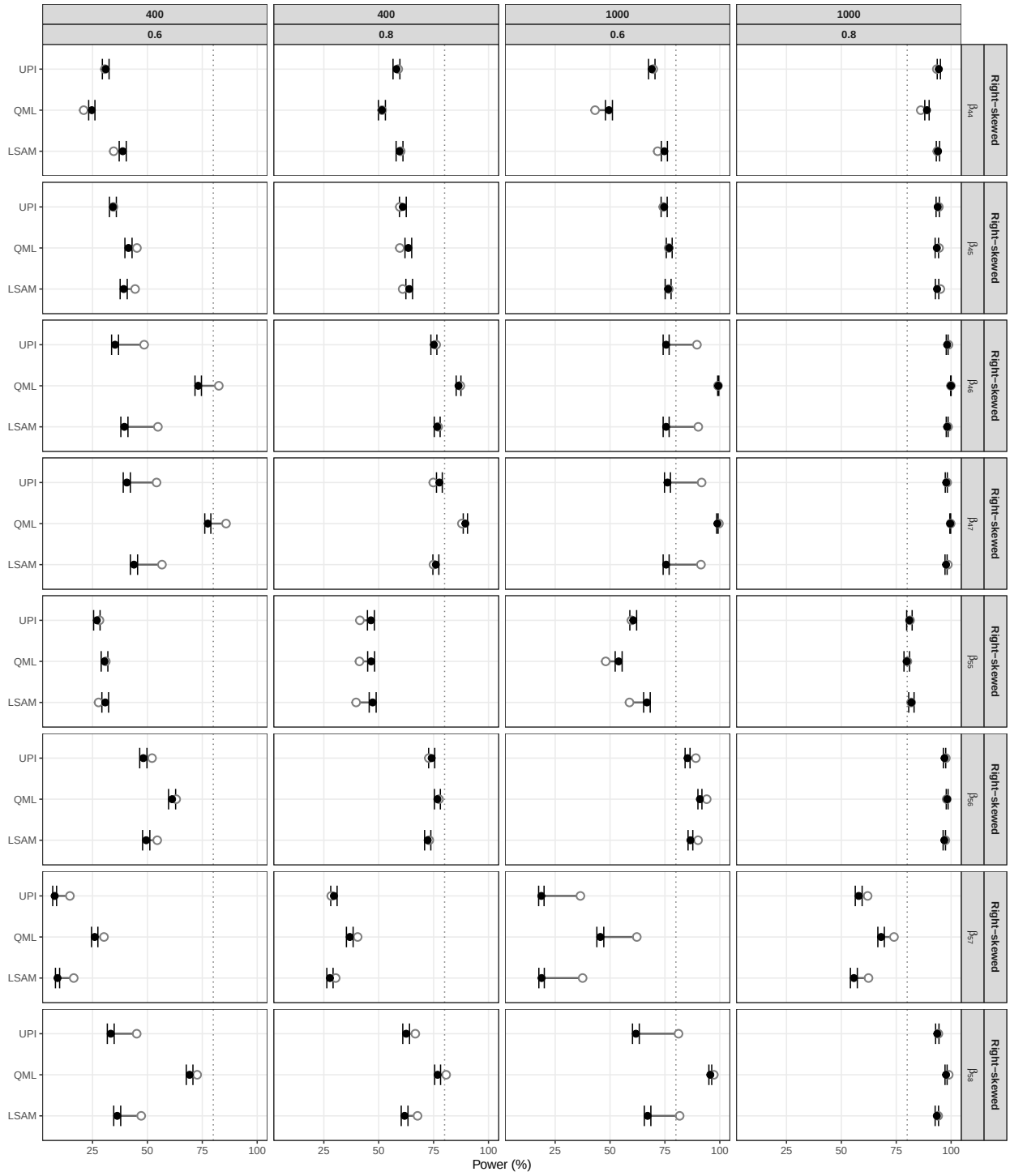


Figure S85

Additional Distributional Conditions: Power for Simulation 2 — Right-skewed (Zeta)

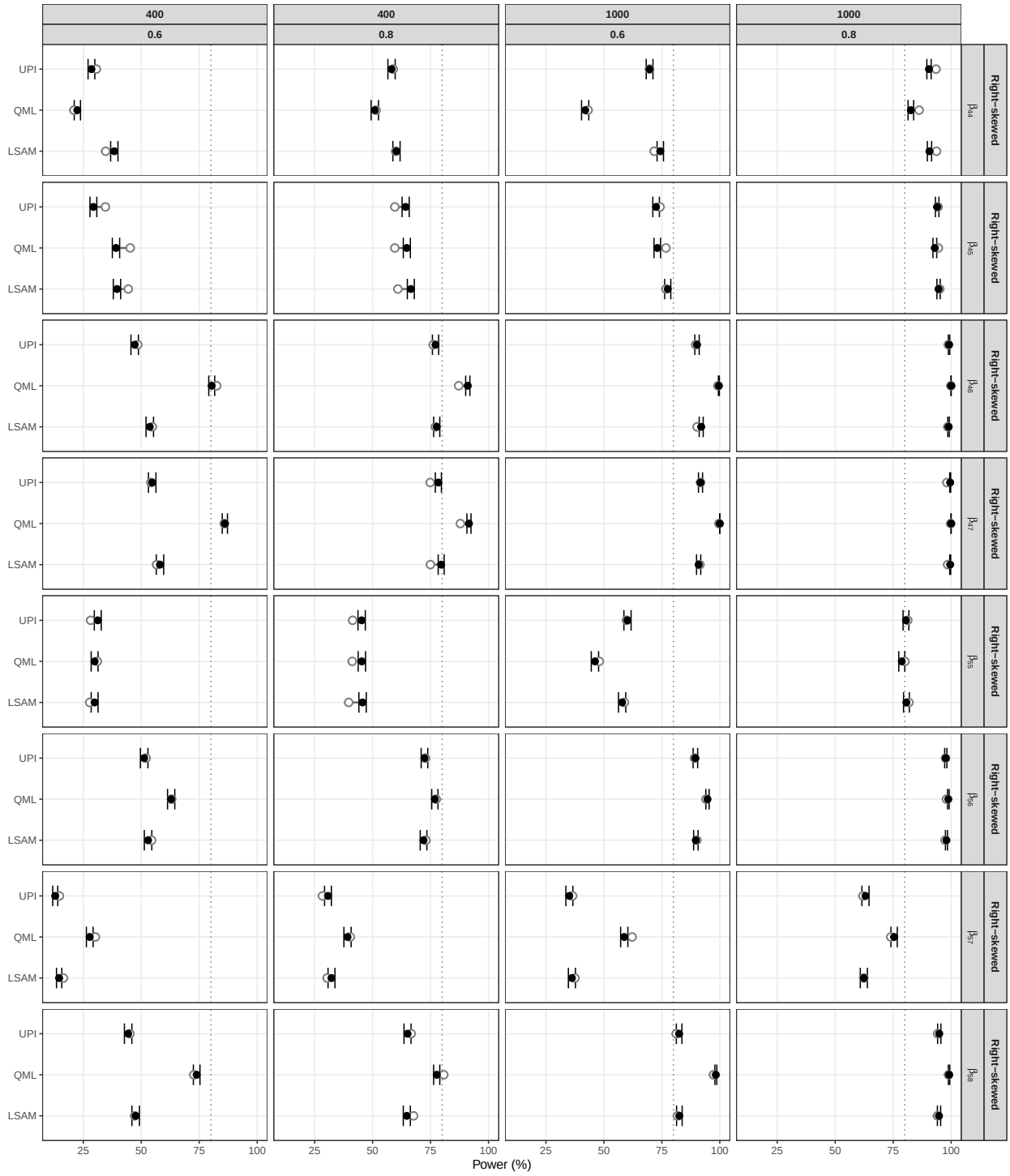


Figure S86

Additional Distributional Conditions: Power for Simulation 2 — Right-skewed (Epsilon & Zeta)

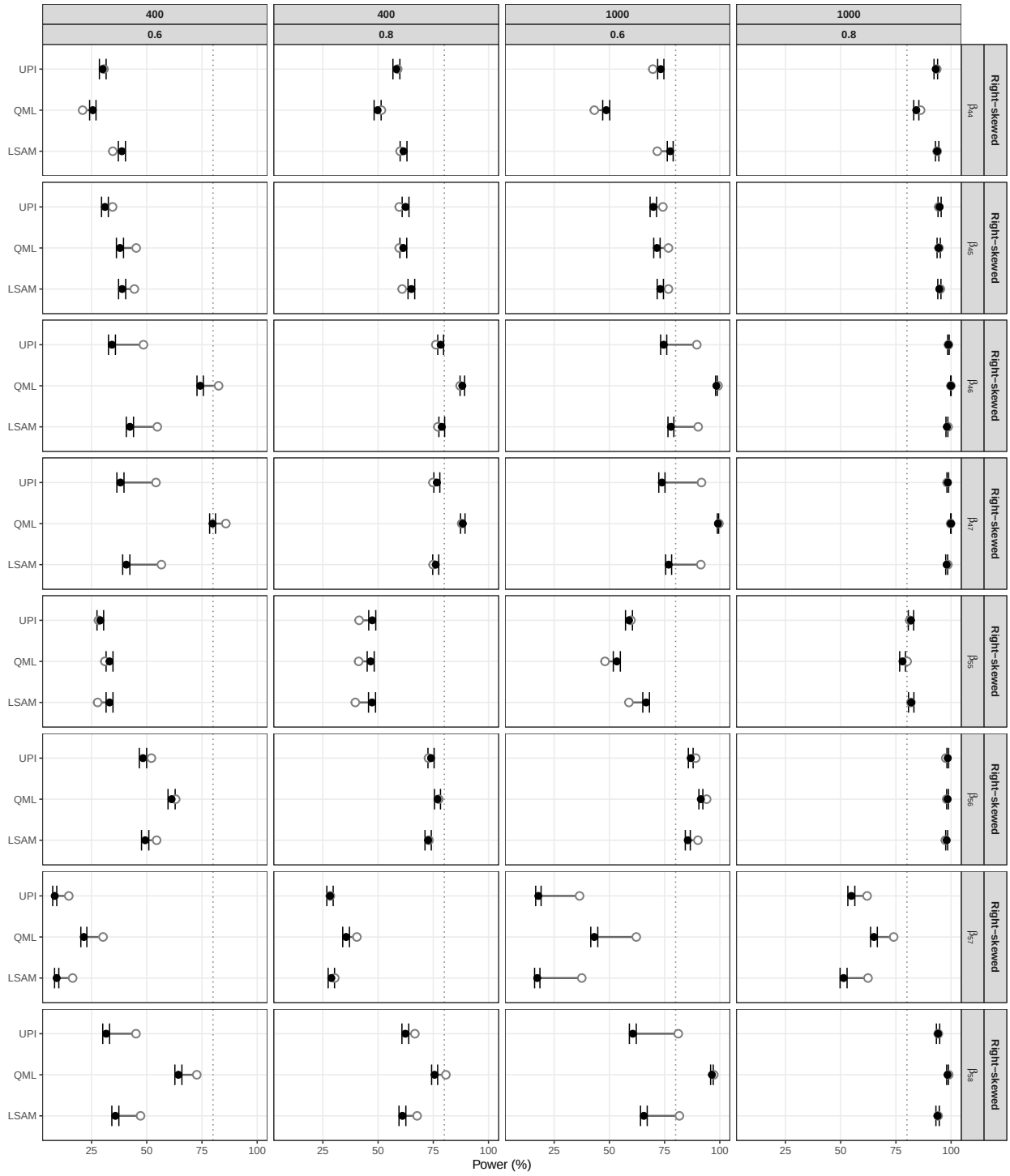


Figure S87

Additional Distributional Conditions: Relative Bias for Simulation 2 — Uniform (Epsilon)

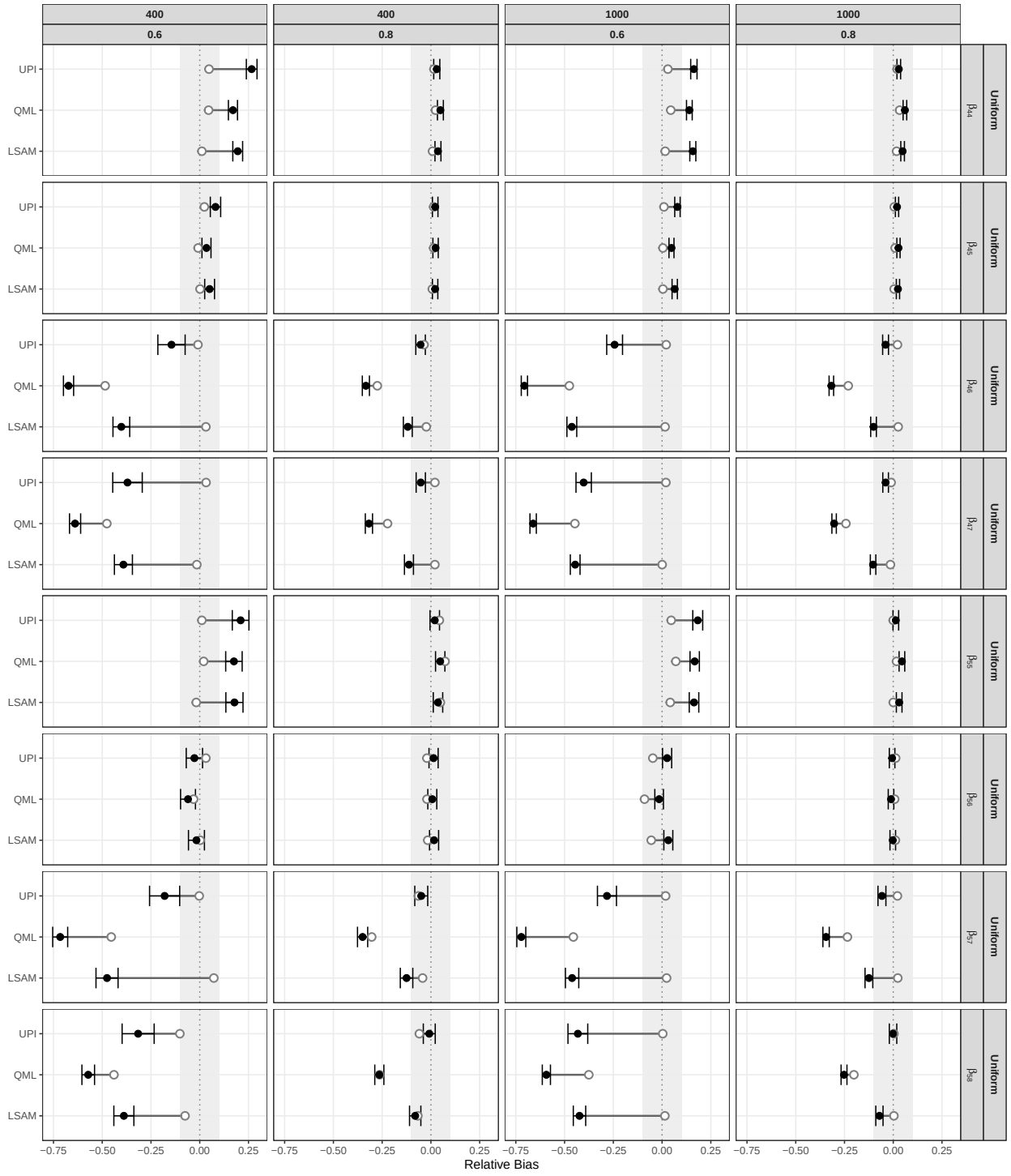


Figure S88

Additional Distributional Conditions: Relative Bias for Simulation 2 — Uniform (Zeta)

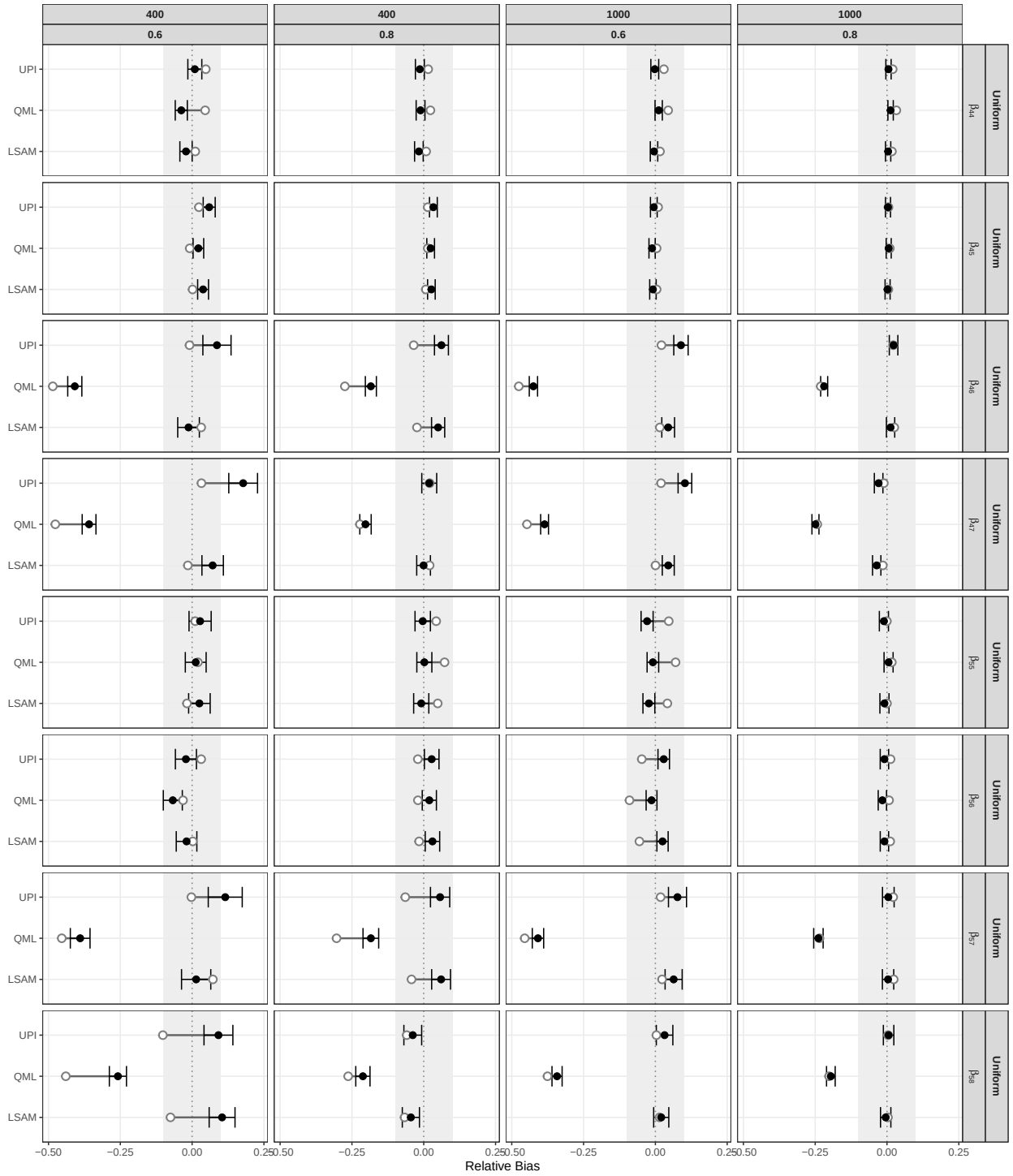


Figure S89

Additional Distributional Conditions: Relative Bias for Simulation 2 — Uniform (Epsilon & Zeta)

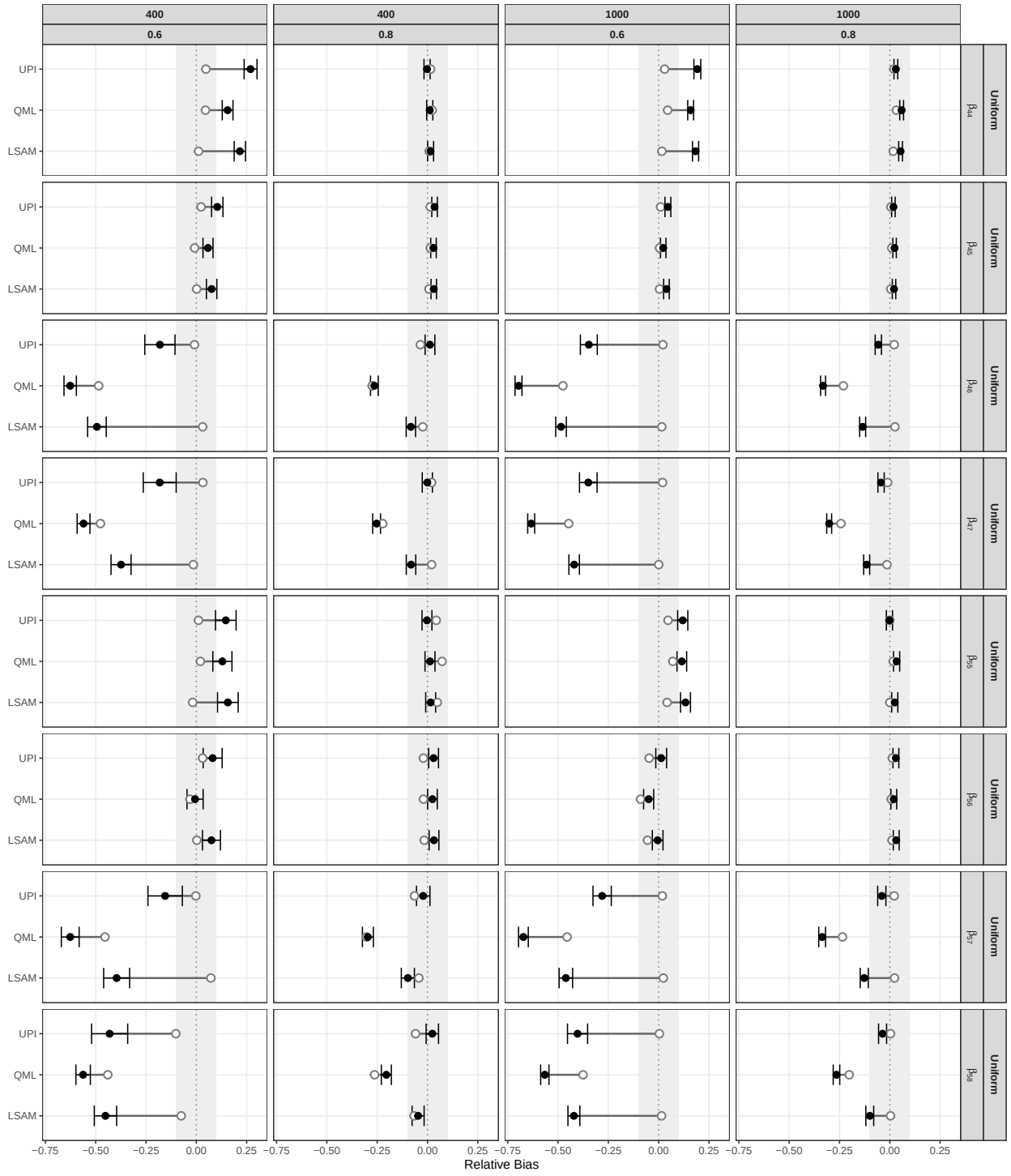


Figure S90

Additional Distributional Conditions: Relative RMSE for Simulation 2 — Uniform (Epsilon)

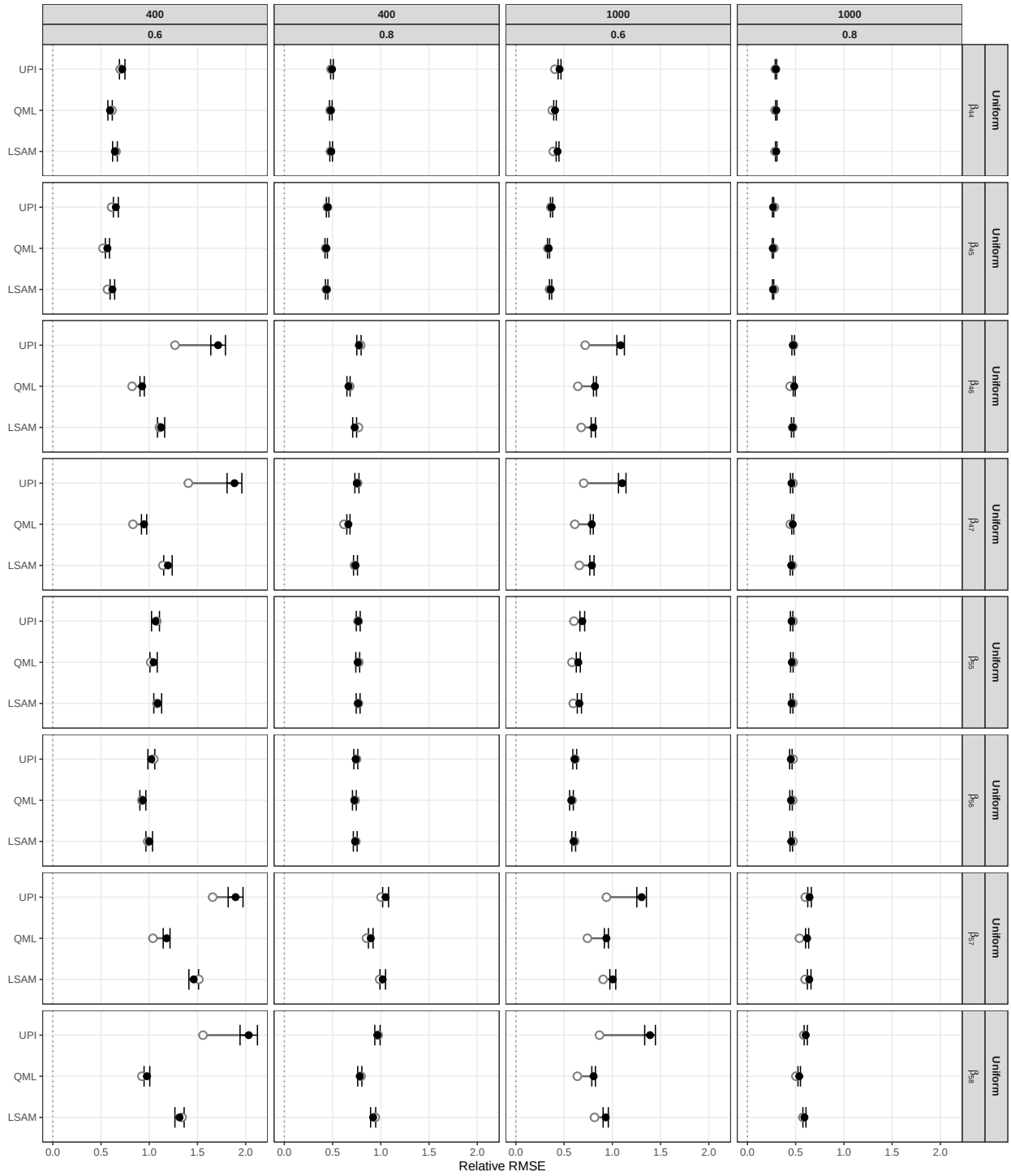


Figure S91

Additional Distributional Conditions: Relative RMSE for Simulation 2 — Uniform (Zeta)

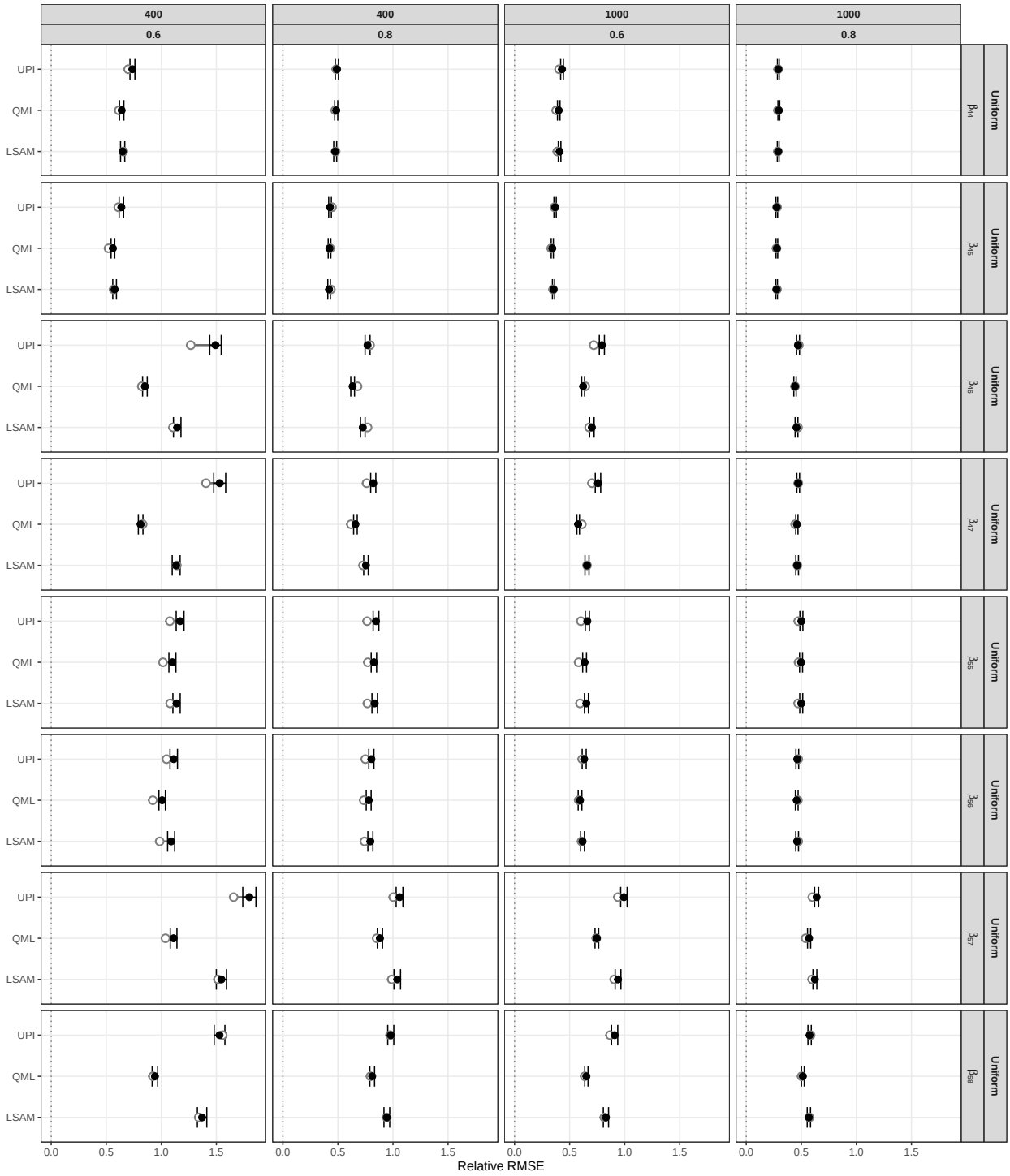


Figure S92

Additional Distributional Conditions: Relative RMSE for Simulation 2 — Uniform (Epsilon & Zeta)

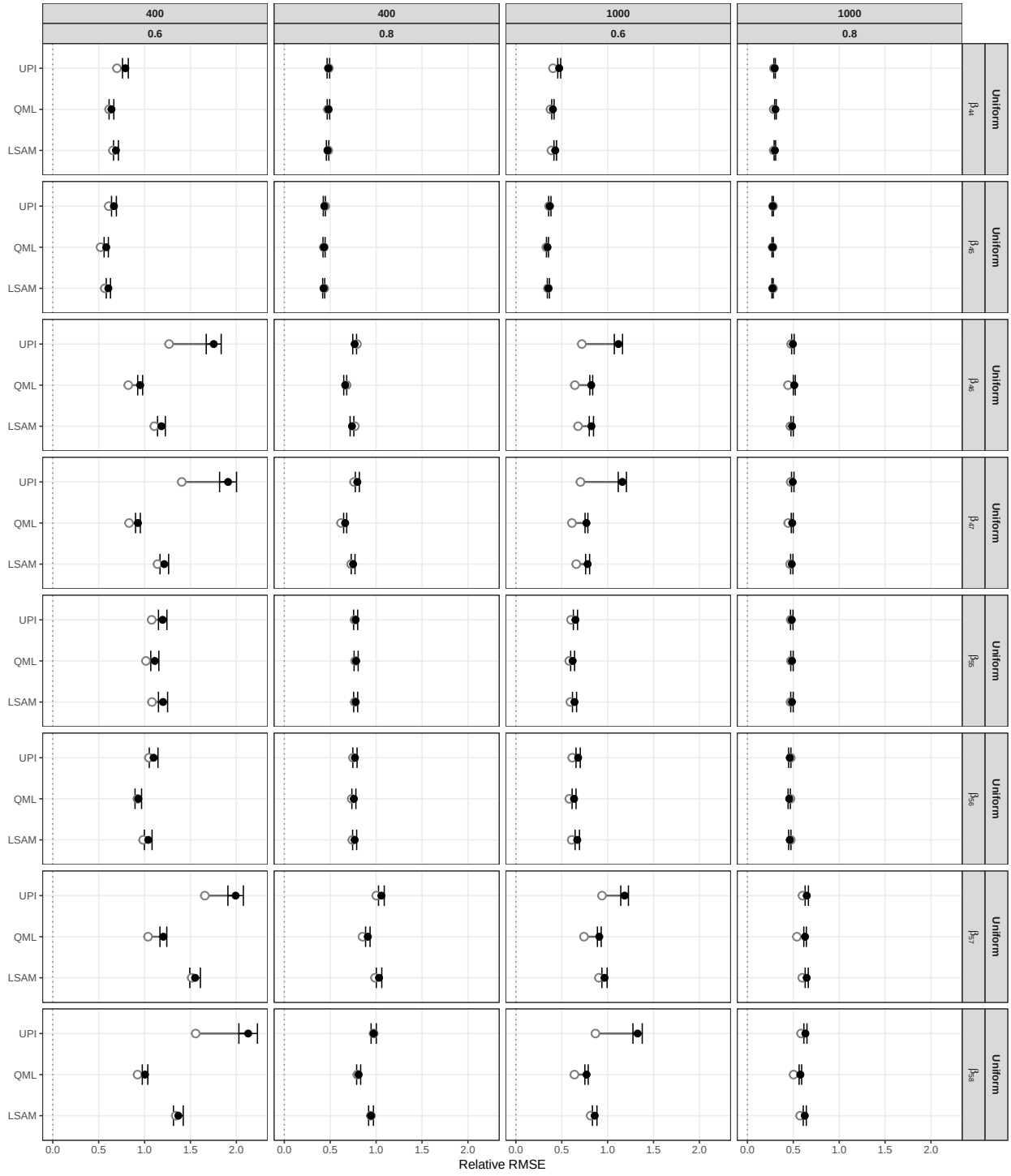


Figure S93

Additional Distributional Conditions: SE/SD Ratio for Simulation 2 — Uniform (Epsilon)

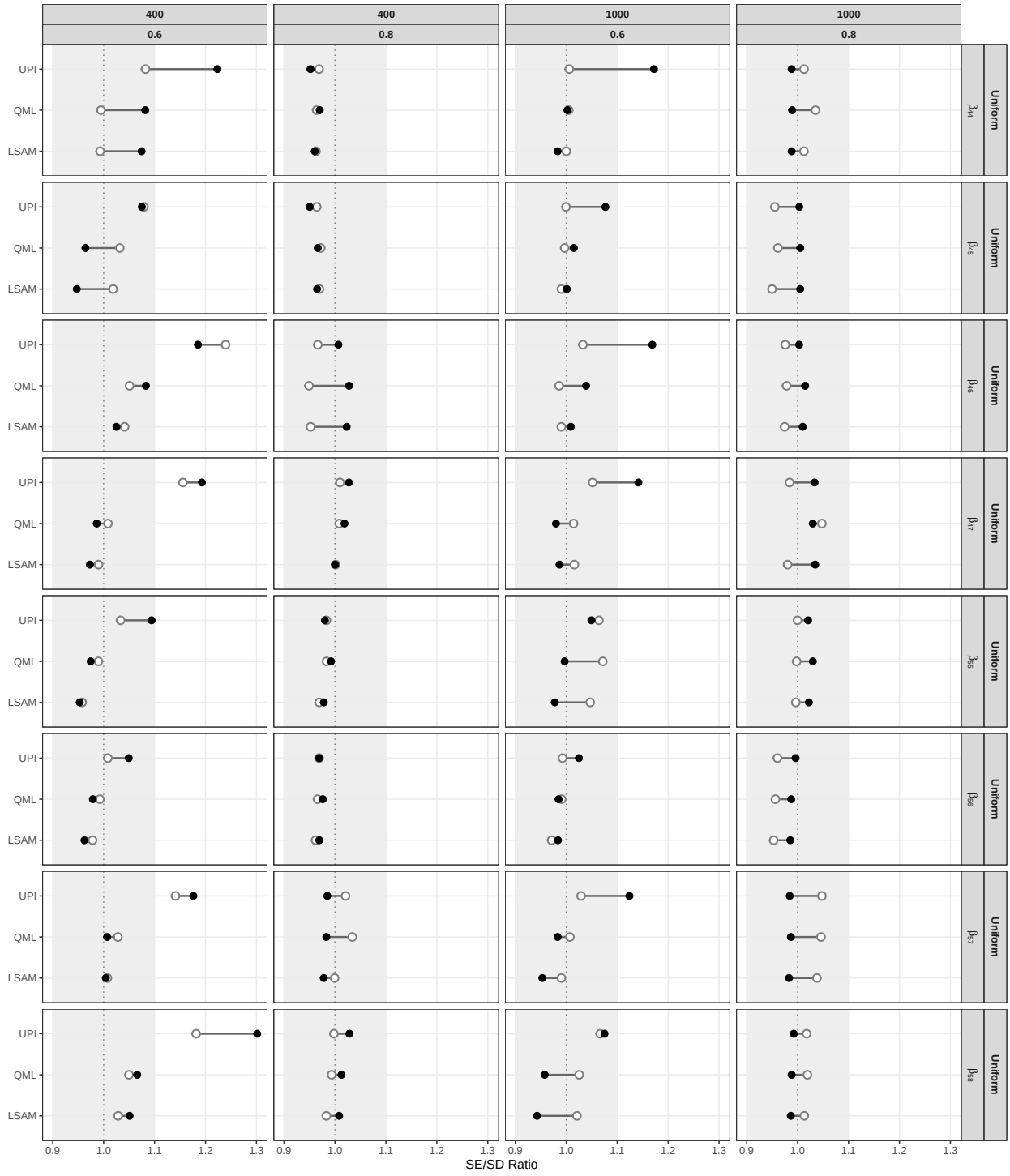


Figure S94

Additional Distributional Conditions: SE/SD Ratio for Simulation 2 — Uniform (Zeta)

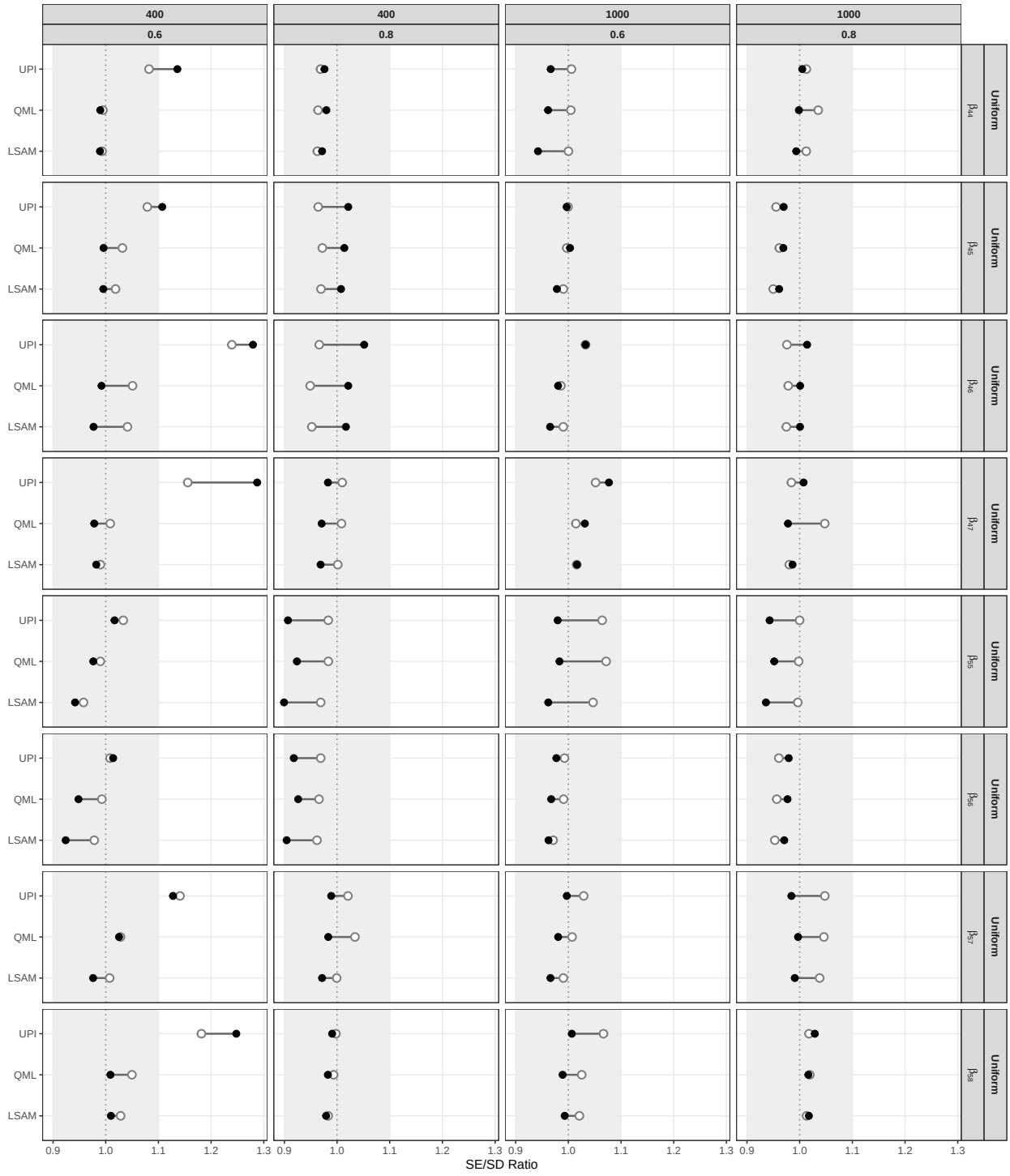


Figure S95

Additional Distributional Conditions: SE/SD Ratio for Simulation 2 — Uniform (Epsilon & Zeta)

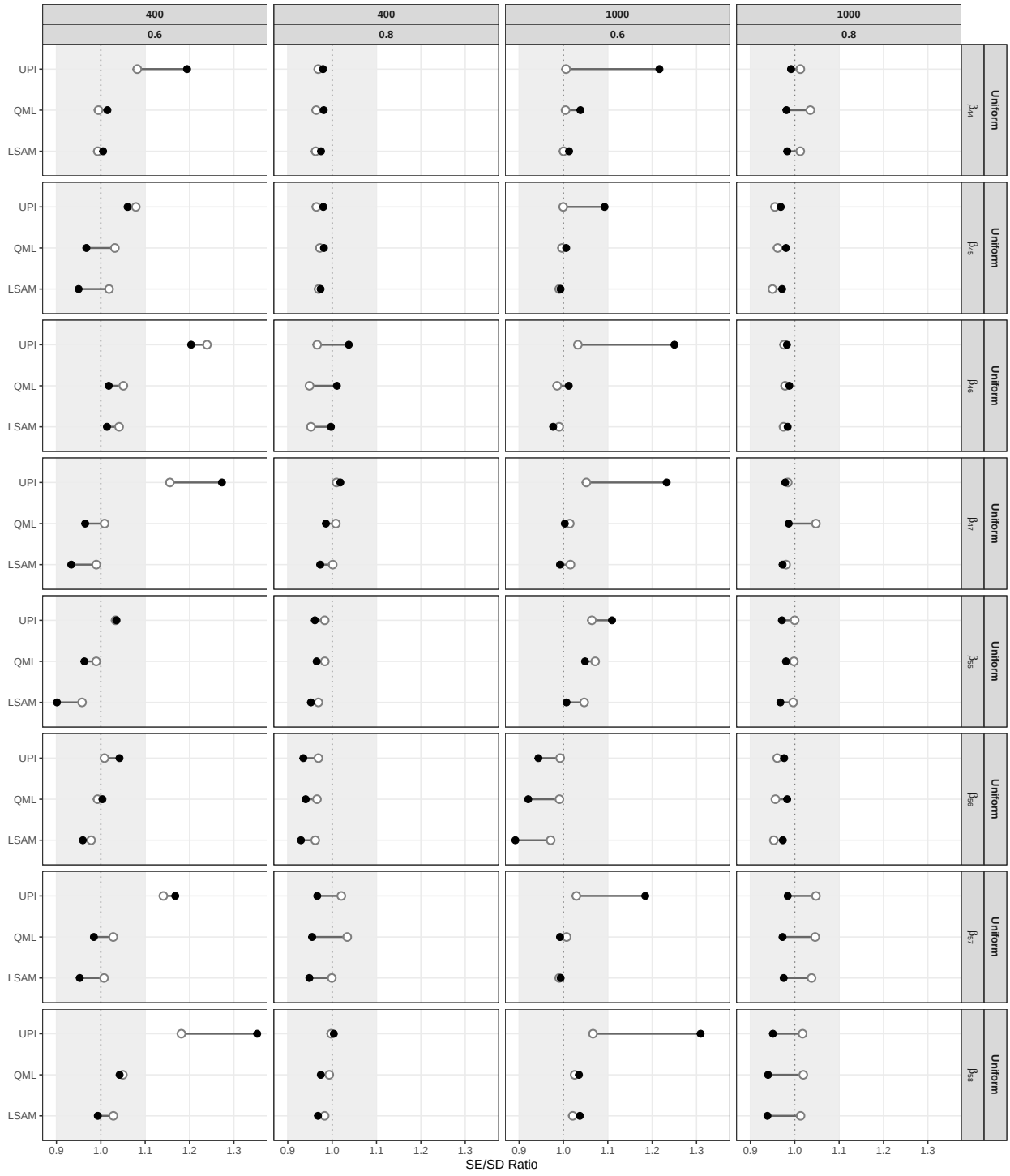


Figure S96

Additional Distributional Conditions: Coverage for Simulation 2 — Uniform (Epsilon)

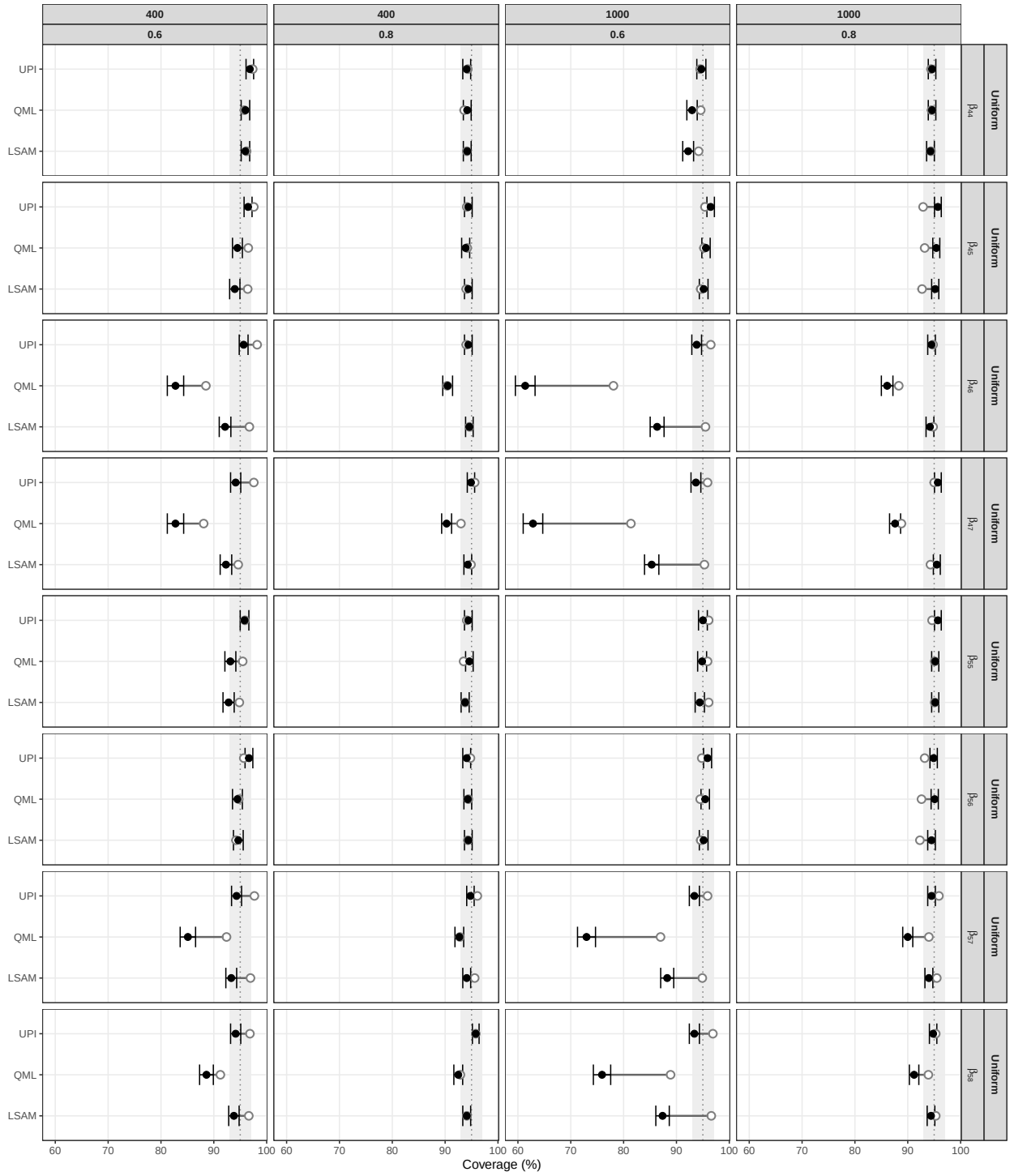


Figure S97

Additional Distributional Conditions: Coverage for Simulation 2 — Uniform (Zeta)

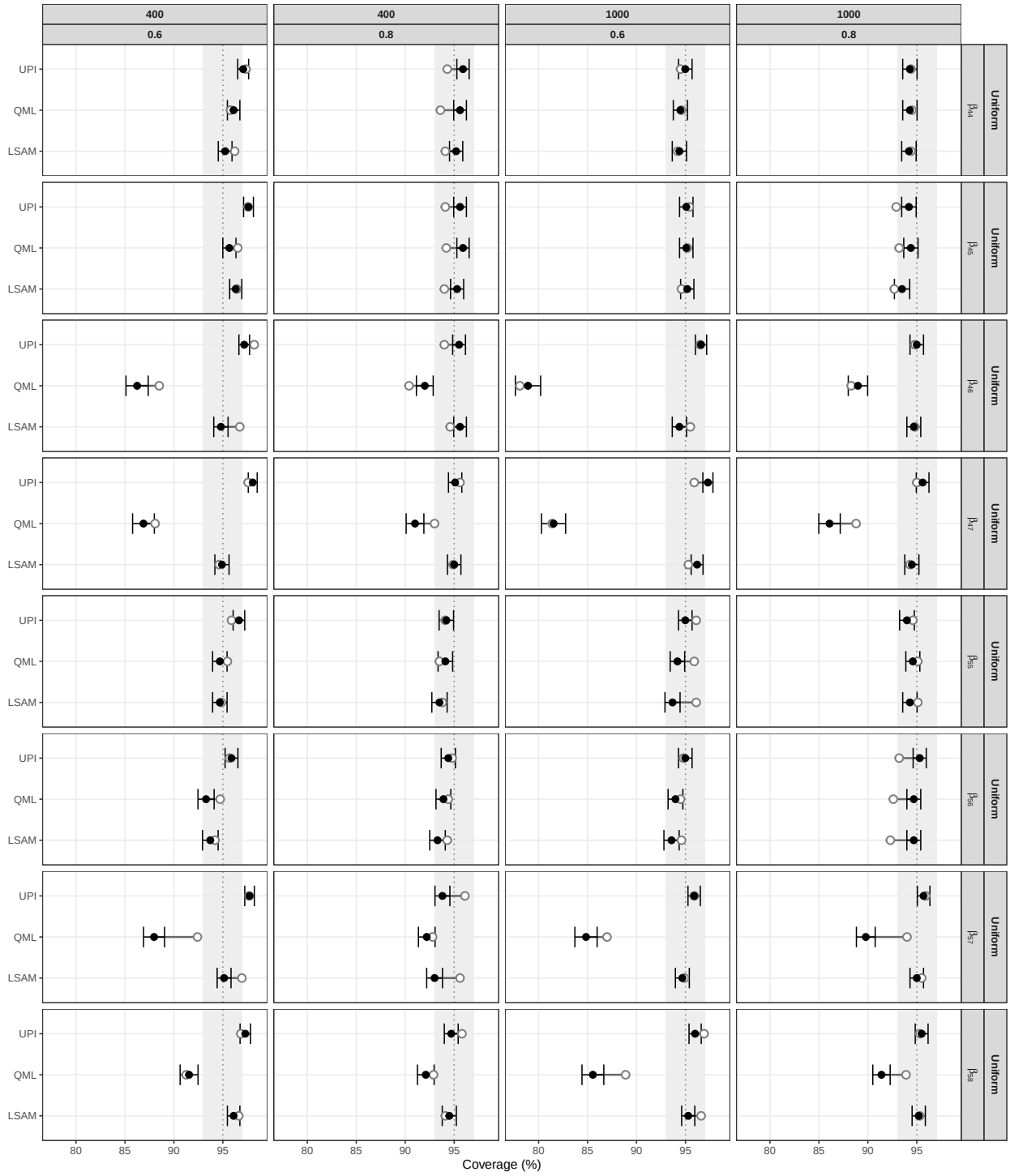


Figure S98

Additional Distributional Conditions: Coverage for Simulation 2 — Uniform (Epsilon & Zeta)

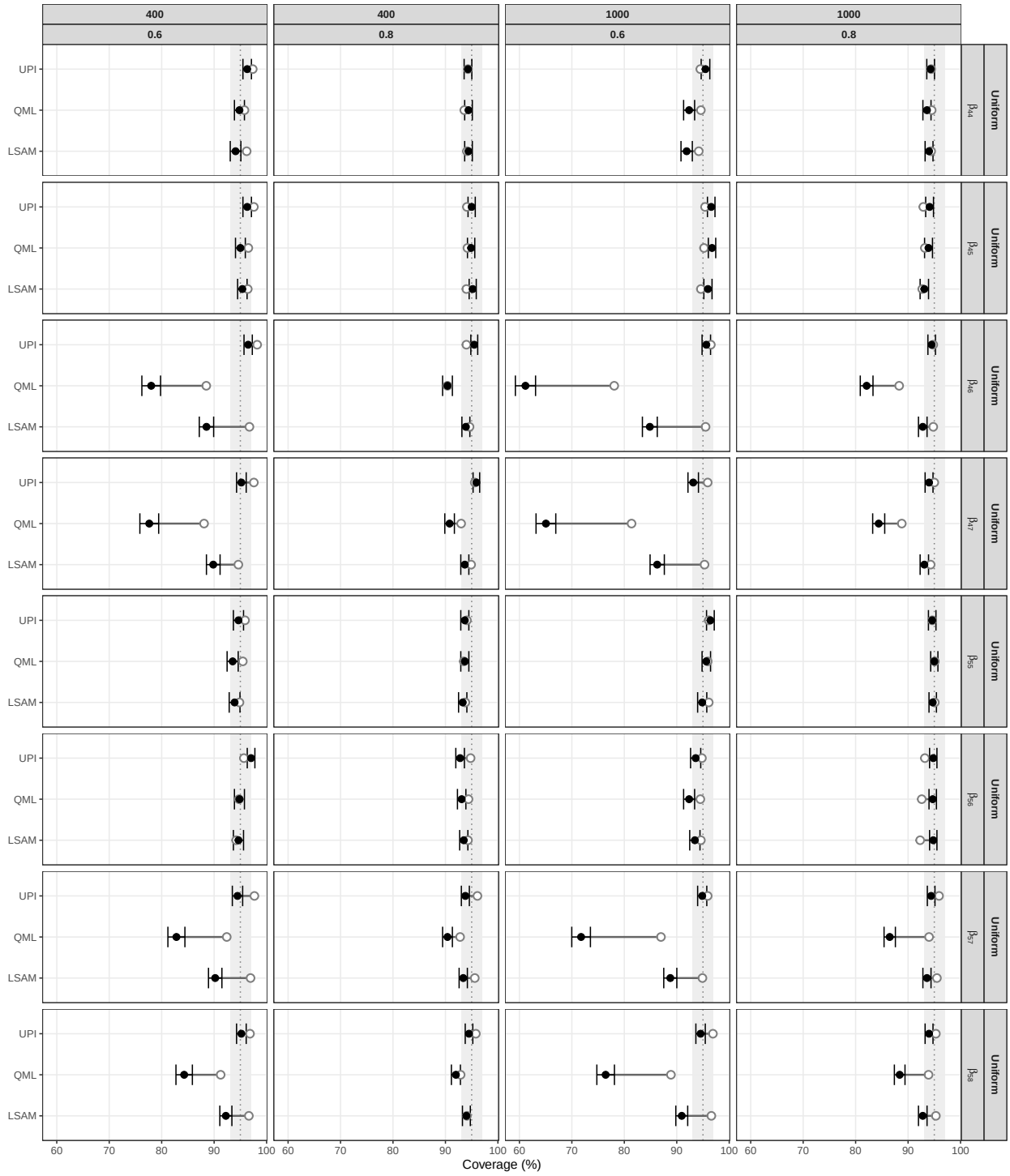


Figure S99

Additional Distributional Conditions: Type I Error for Simulation 2 — Uniform (Epsilon)

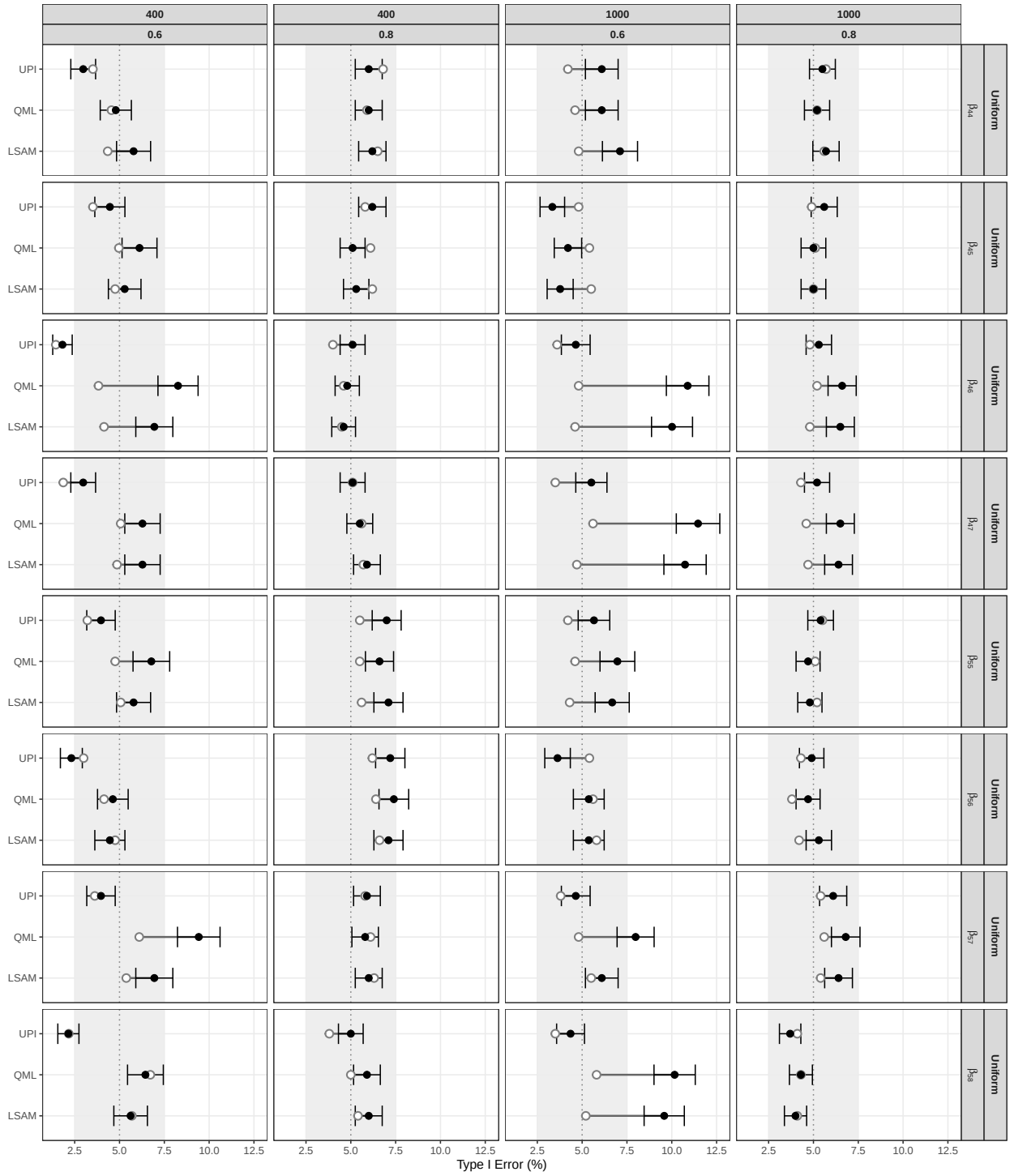


Figure S100

Additional Distributional Conditions: Type I Error for Simulation 2 — Uniform (Zeta)

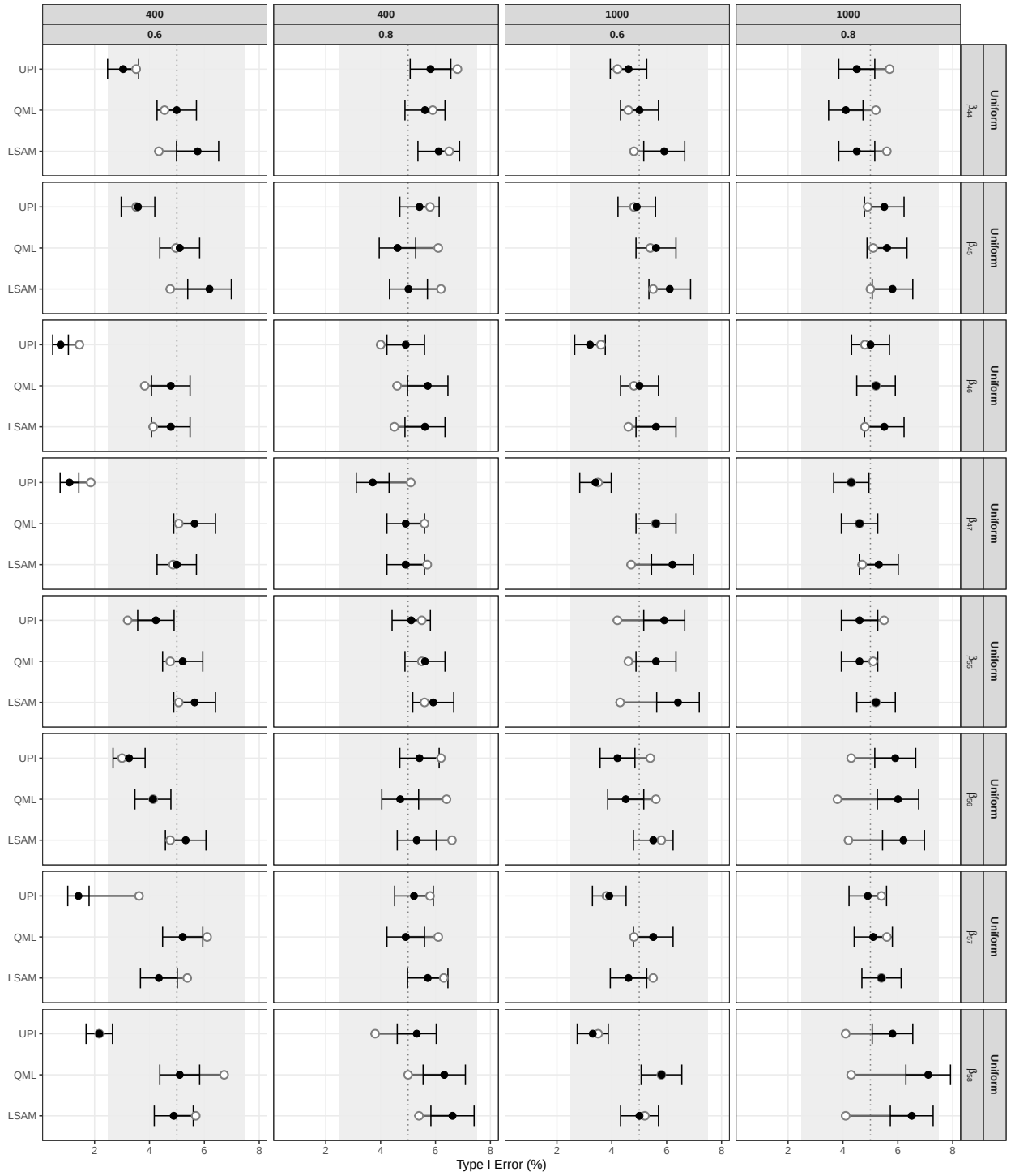


Figure S101

Additional Distributional Conditions: Type I Error for Simulation 2 — Uniform (Epsilon & Zeta)

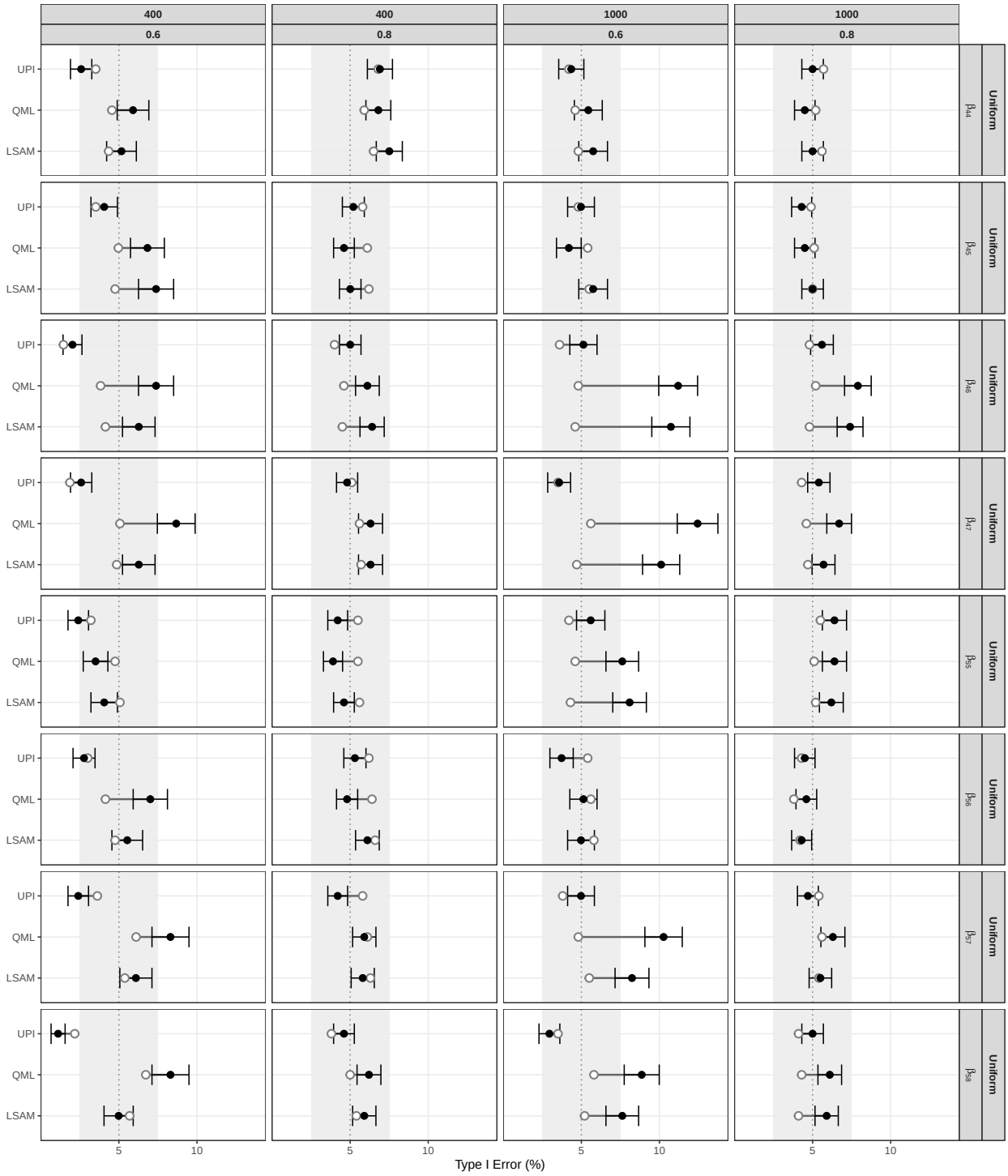


Figure S102

Additional Distributional Conditions: Power for Simulation 2 — Uniform (Epsilon)

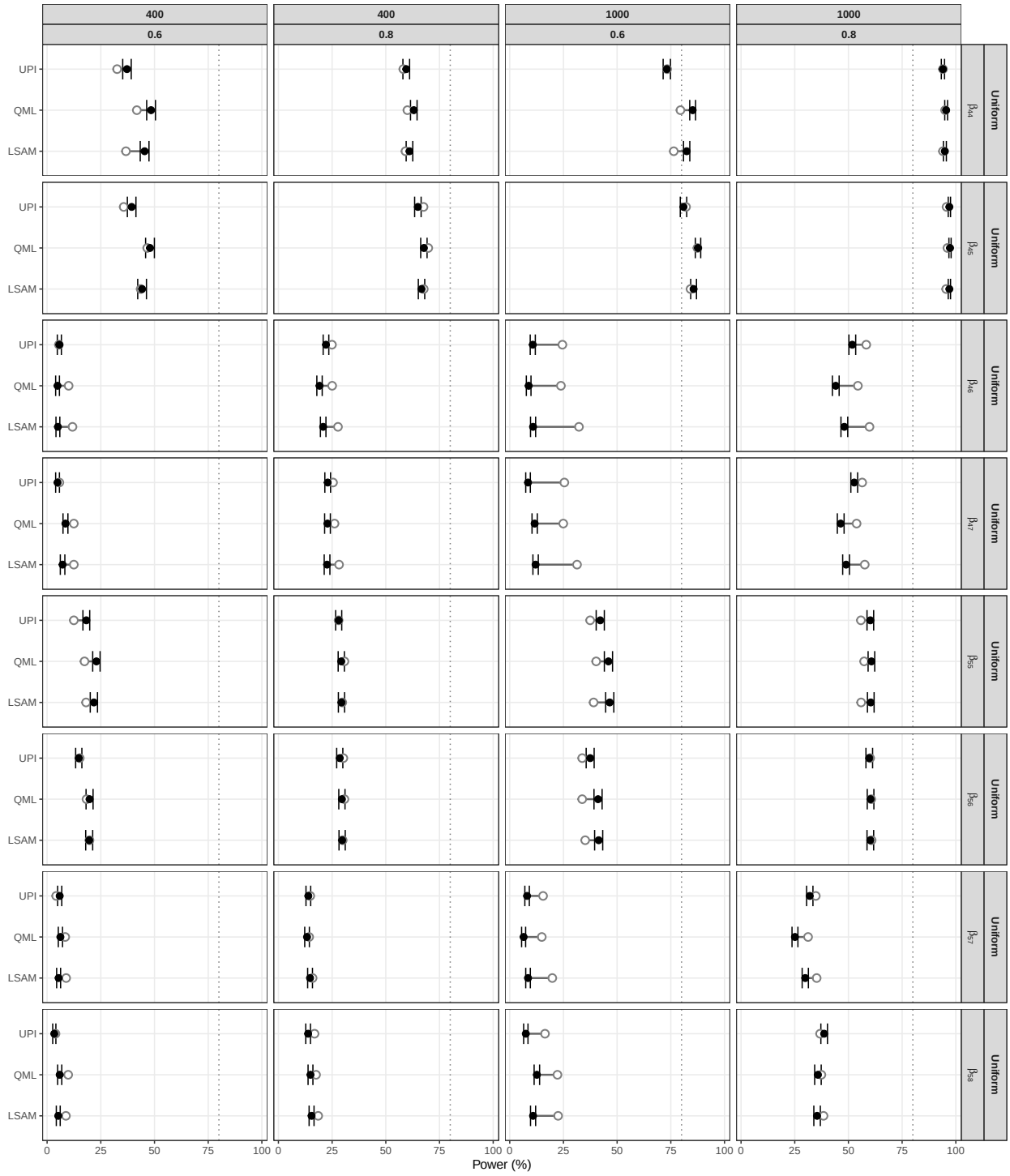


Figure S103

Additional Distributional Conditions: Power for Simulation 2 — Uniform (Zeta)

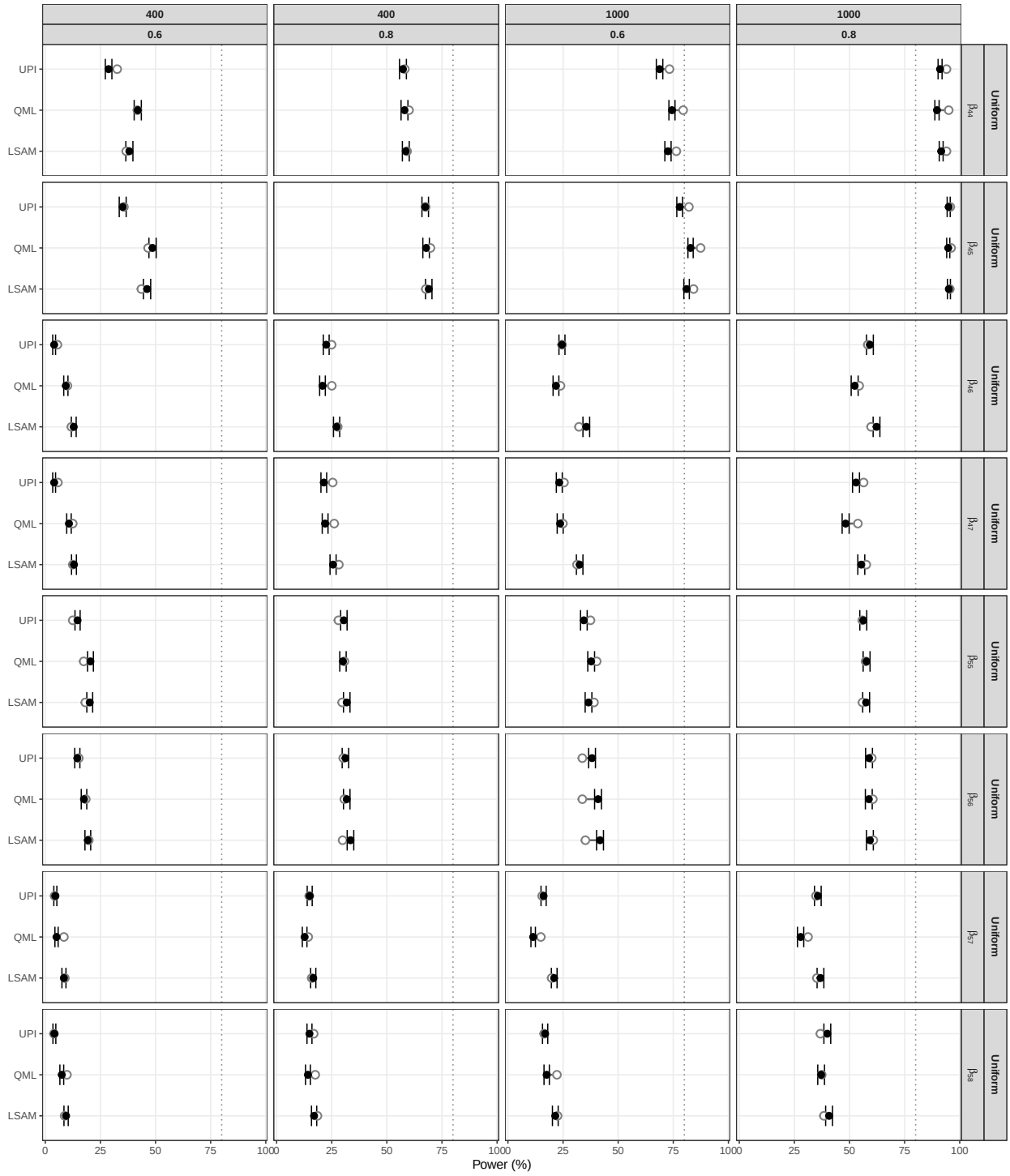
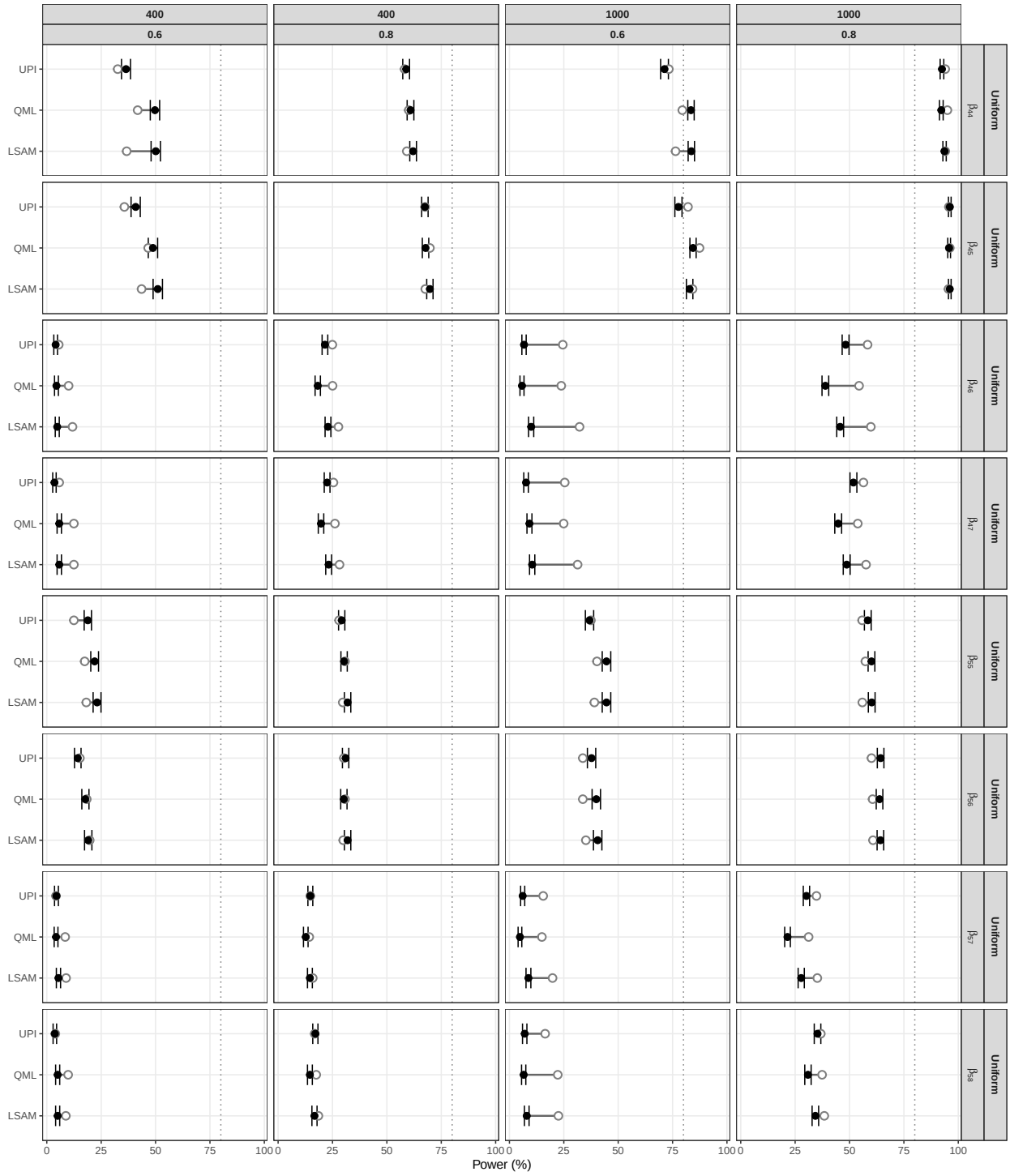


Figure S104

Additional Distributional Conditions: Power for Simulation 2 — Uniform (Epsilon & Zeta)



SEs Outlier Diagnostics for Condition 13 (Simulation 1)

Figure S105

SE Outliers for LSAM in Condition 13

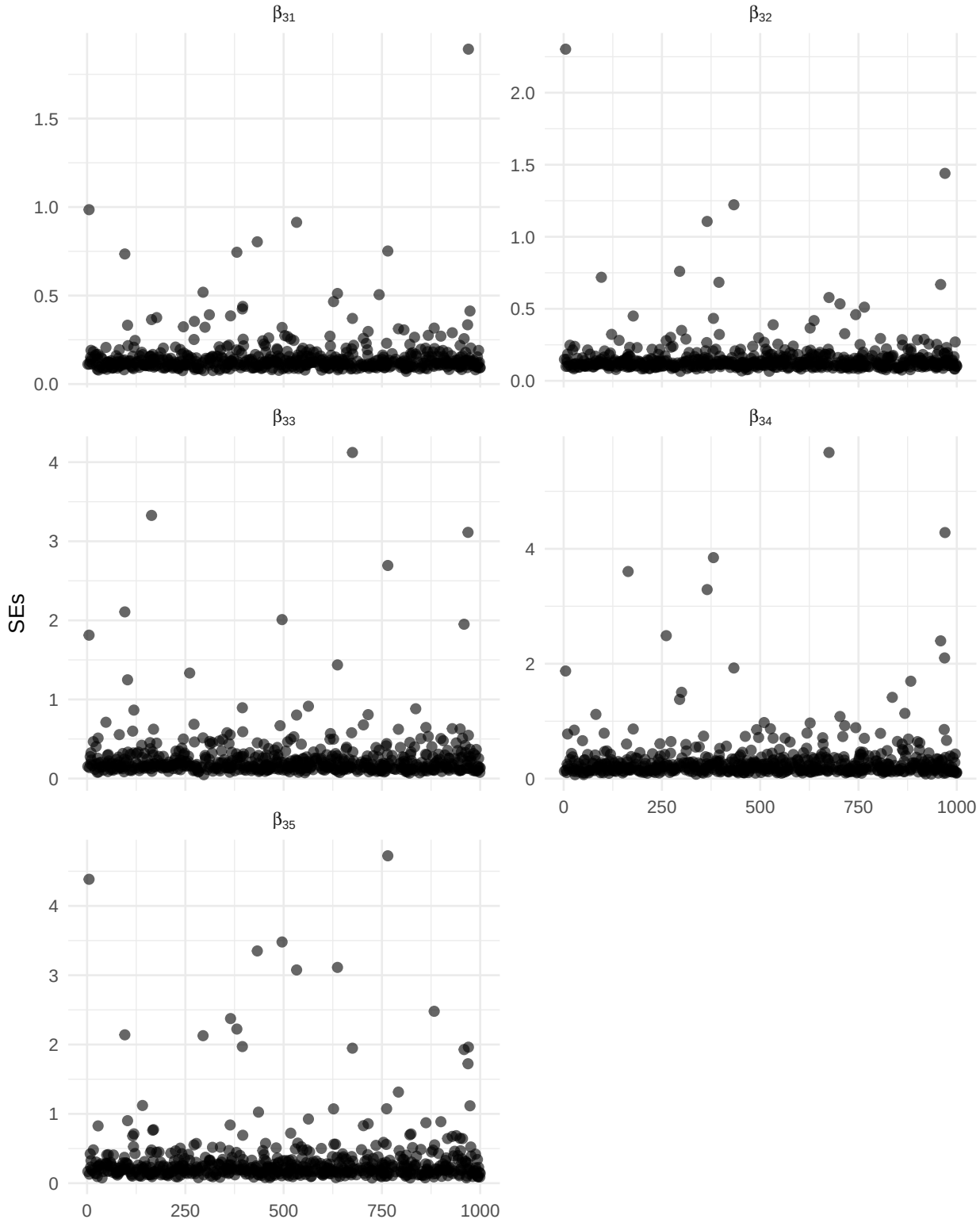


Figure S106

SE Outliers for LMS in Condition 13

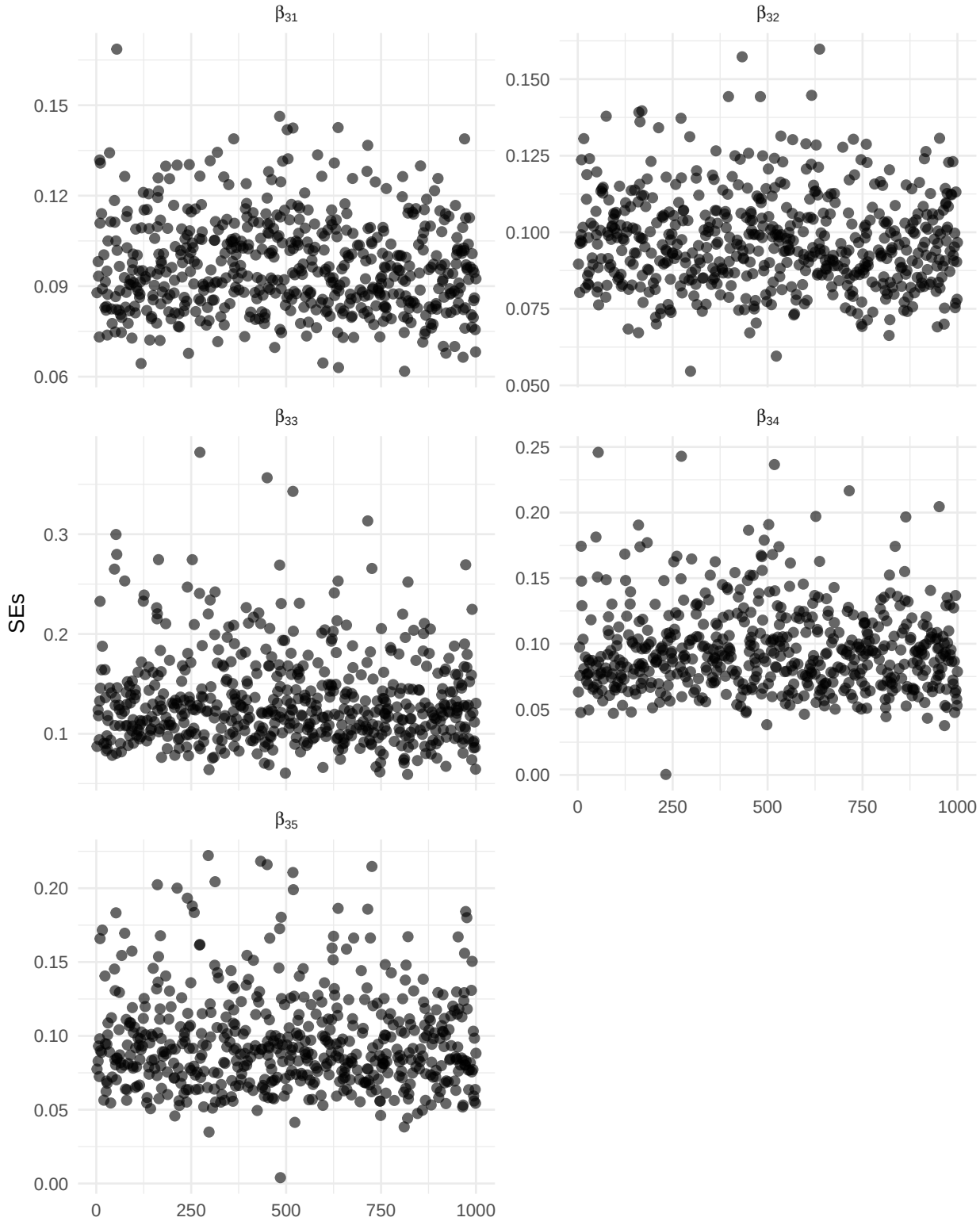


Figure S107

SE Outliers for QML in Condition 13

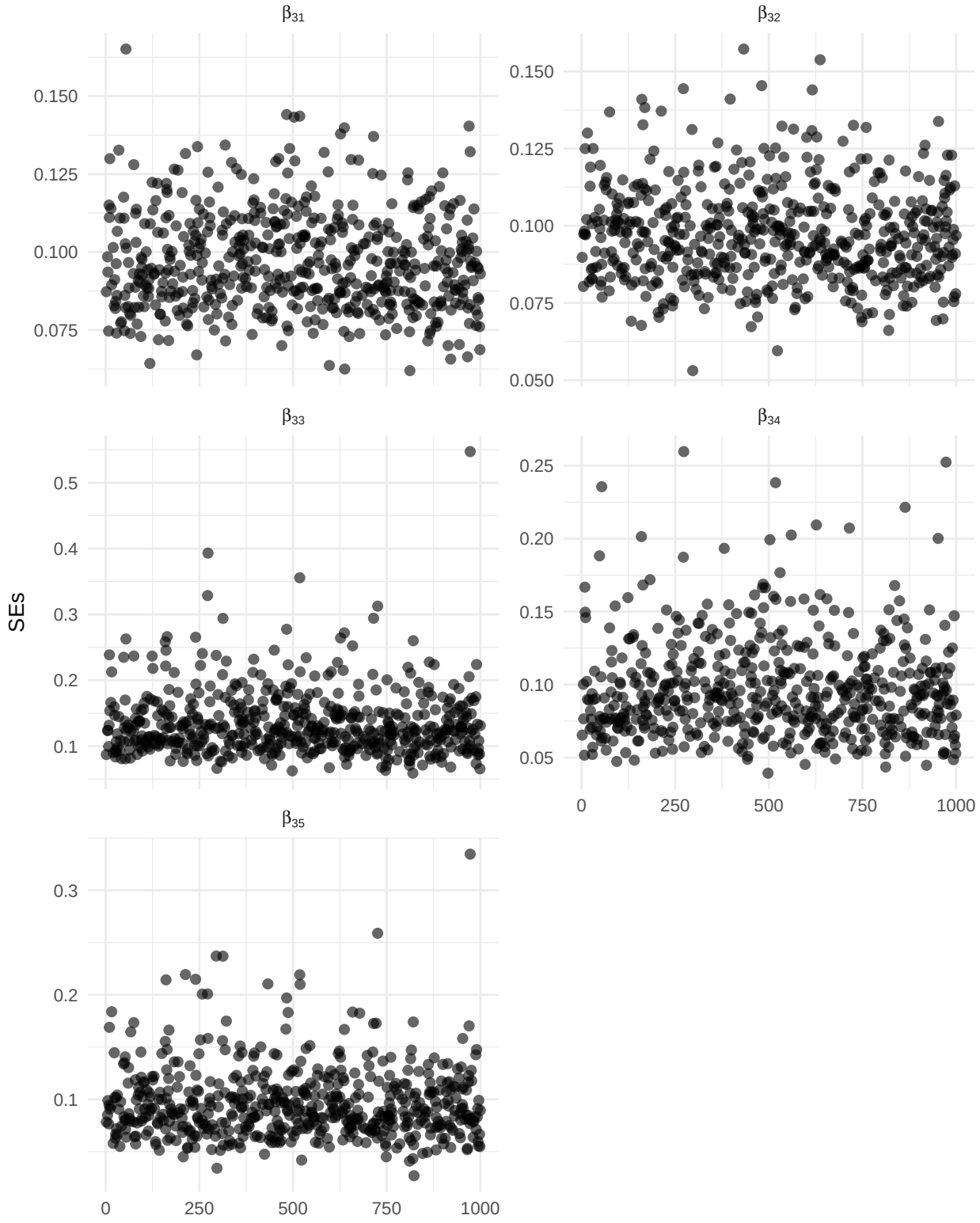
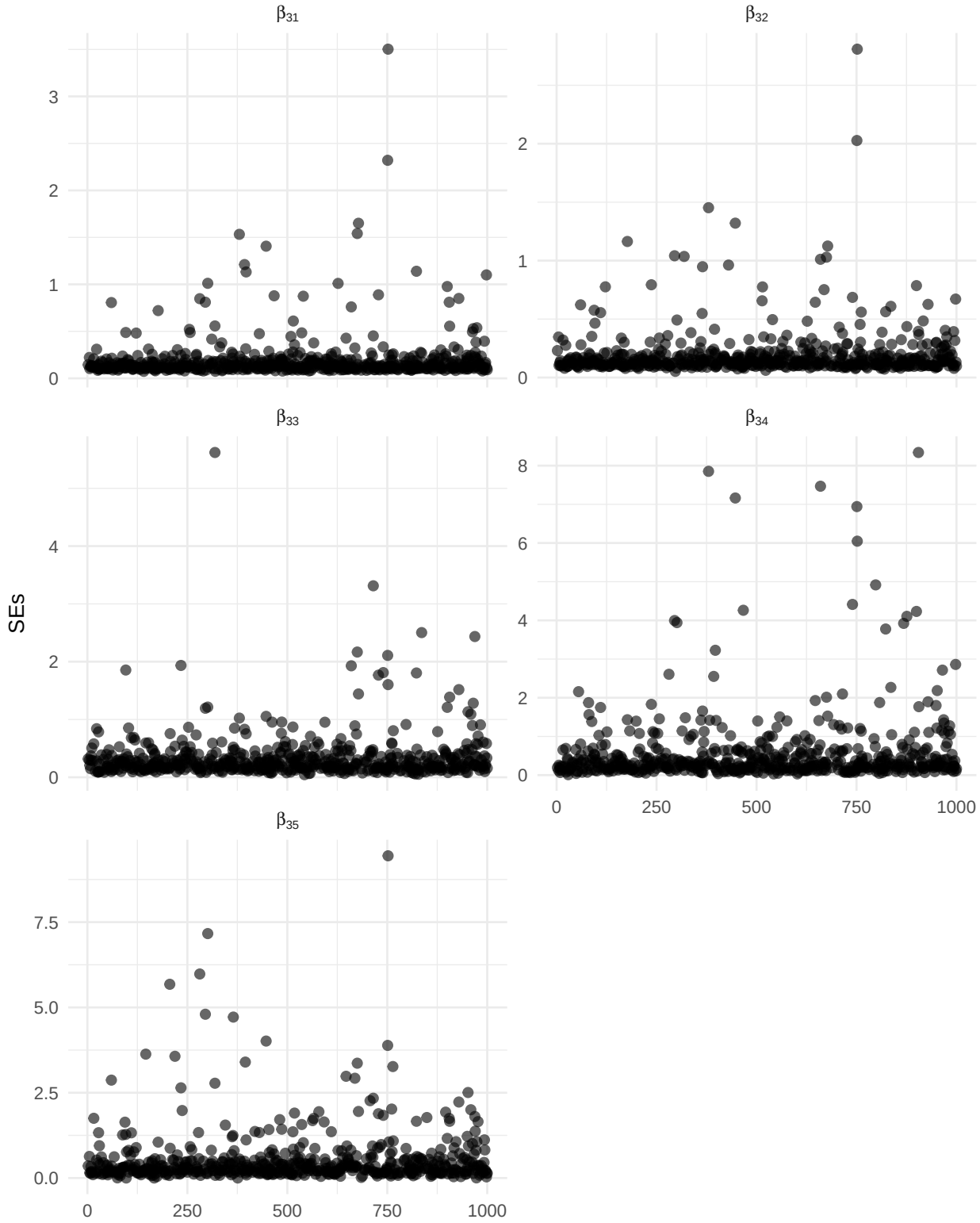


Figure S108

SE Outliers for UPI in Condition 13



Additional Distributional Conditions: Difference Uncertainty

The dumbbell plots in the manuscript and preceding sections show main results and additional distributional conditions values side by side. To formally quantify whether a performance metric changed meaningfully under nonnormal errors, we compute the difference

$$\Delta = \hat{\theta}_{\text{additional}} - \hat{\theta}_{\text{main}}$$

where $\hat{\theta}$ denotes a simulation performance metric (e.g., relative bias, coverage rate, etc). Because the main results and additional distributional conditions results were generated from independent sets of Monte Carlo replications, their estimators $\hat{\theta}_{\text{main}}$ and $\hat{\theta}_{\text{additional}}$ are statistically independent. Under independence, the variance of a difference equals the sum of the variances:

$$\text{Var}(\Delta) = \text{Var}\big(\hat{\theta}_{\text{additional}} - \hat{\theta}_{\text{main}}\big) = \text{Var}\big(\hat{\theta}_{\text{additional}}\big) + \text{Var}\big(\hat{\theta}_{\text{main}}\big)\,.$$

Because each MCSE estimates the standard deviation of $\hat{\theta}$ across replications, squaring it gives an estimate of the corresponding variance, $\text{MCSE}^2 \approx \text{Var}(\hat{\theta})$. Substituting into the expression above and taking the square root gives the MCSE of the difference:

$$\text{MCSE}_\Delta = \sqrt{\text{Var}\big(\hat{\theta}_{\text{main}}\big) + \text{Var}\big(\hat{\theta}_{\text{additional}}\big)} = \sqrt{\text{MCSE}^2_{\text{main}} + \text{MCSE}^2_{\text{additional}}}.$$

An approximate 95% confidence interval for the true difference is then

$$\Delta \pm 1.96 \times \text{MCSE}_\Delta.$$

The bar plots in the Supplemental Tables document (available on the GitHub repository) display Δ on the y-axis with 95% CI error bars and a dashed reference line at zero, making it straightforward to identify which method–metric–condition combinations were meaningfully affected.