

**Additional Materials to Support the paper “Dynamic Factor Analysis with
Multivariate Time Series of Multiple Individuals: An Error-Corrected
Estimation Method”**

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The document includes (1) the R code of the error-corrected (EC) method and a Naive method for estimating dynamic factor analysis with multiple individuals, (2) the R code of simulating data to assess different methods of estimating the model, and (3) tables for summarizing results of the simulation study.

Appendix A contains the R code of the EC method.

Appendix B contains the R code of generating data in the simulation studies.

Appendix C includes 13 tables that summarize results of the simulation study. More details are given next.

Table C1 reports the point estimates and standard error estimates of six between-individual parameters (intercepts, slopes, and variances of the two within-individual AR weights a_{11} and a_{12}) in the condition of normal variables and $n = 50$ individuals.

Table C2 reports the point estimates and standard error estimates of six between-individual parameters (intercepts, slopes, and variances of the two within-individual AR weights a_{11} and a_{12}) in the condition of t distributions and $n = 50$ individuals.

Table C3 reports the point estimates and standard error estimates of six between-individual parameters (intercepts, slopes, and variances of the two within-individual AR weights a_{11} and a_{12}) in the condition of normal variables and $n = 100$ individuals.

Table C4 reports the point estimates and standard error estimates of six between-individual parameters (intercepts, slopes, and variances of the two within-individual AR weights a_{11} and a_{12}) in the condition of t distributions and $n = 100$ individuals.

Table C5 reports the point estimates and standard error estimates of six between-individual parameters (intercepts, slopes, and variances of the two within-individual AR weights a_{21} and a_{22}) in the condition of t distributions and $n = 100$ individuals.

Table C6 reports the point estimates and standard error estimates of six between-individual parameters (intercepts, slopes, and variances of the two within-individual AR weights a_{11} and a_{12}) in the condition of normal variables and $n = 200$ individuals.

Table C7 reports the point estimates and standard error estimates of six between-individual parameters (intercepts, slopes, and variances of the two within-individual AR weights a_{21} and a_{22}) in the condition of normal variables and $n = 200$ individuals.

Table C8 reports the point estimates and standard error estimates of six between-individual parameters (intercepts, slopes, and variances of the two within-individual AR weights a_{11} and a_{12}) in the condition of t distributions and $n = 200$ individuals.

Table C9 reports the point estimates and standard error estimates of six between-individual parameters (intercepts, slopes, and variances of the two within-individual AR weights a_{21} and a_{22}) in the condition of t distributions and $n = 200$ individuals.

Table C10 reports the point estimates and standard error estimates of six between-individual parameters (intercepts, slopes, and variances of the two within-individual AR weights a_{11} and a_{12}) in the condition of normal variables and $n = 500$ individuals.

Table C11 reports the point estimates and standard error estimates of six between-individual parameters (intercepts, slopes, and variances of the two within-individual AR weights a_{21} and a_{22}) in the condition of normal variables and $n = 500$

individuals.

Table C12 reports the point estimates and standard error estimates of six between-individual parameters (intercepts, slopes, and variances of the two within-individual AR weights a_{11} and a_{12}) in the condition of t distributions and $n = 500$ individuals.

Table C13 reports the point estimates and standard error estimates of six between-individual parameters (intercepts, slopes, and variances of the two within-individual AR weights a_{21} and a_{22}) in the condition of t distributions and $n = 500$ individuals.

Appendix D includes six figures that summarize results of the simulation study. More details are given next.

Figure D1 depicts empirical coverage rates of 95% confidence intervals of the intercepts, slopes, and the variances of a_{11} and a_{12} , normal variables.

Figure D2 depicts empirical coverage rates of 95% confidence intervals of the intercepts, slopes, and the variances of a_{11} and a_{12} , T variables.

Figure D3 depicts empirical coverage rates of 95% confidence intervals of the intercepts, slopes, and the variances of a_{21} and a_{22} , T variables.

Figure D4 depicts empirical rejection rates of the intercepts, slopes, and the variances of a_{11} and a_{12} , normal variables.

Figure D5 depicts empirical rejection rates of the intercepts, slopes, and the variances of a_{11} and a_{12} , T variables.

Figure D6 depicts empirical rejection rates of the intercepts, slopes, and the variances of a_{21} and a_{22} , T variables.

Appendix A

R code of implementing the EC Method and the Naive Method

The first step is to download all the relevant files at <https://drive.google.com/file/d/1BVP1830mN9S82K0P60wZBWL2choHcXox/view?usp=sharing>.

```
# Set up the working fold that include the data files,
# executable file, the batch file, and R functions
setwd("C:/TempFiles/")

# The EC method
# Conduct the bootstrap analysis
system("PANAS81b.bat")

# Select AR weights and their estimation error

source('RFunctions20240502.txt')
TestA = Select.A (filename = "PANAS81b")

# Read in the time invariant trait 'depression'
Depression = t(array( scan("Depression.txt"),dim=c(2,81)))

n = 81
p = 4

depression_mean = mean (Depression[1:n,2])

Total.A.M.c = array(rep(NA,p*p*2*n),dim=c(p,p*2,n))
# the design matrix with centered predictor

for (i in 1:n) {
Total.A.M.c[1:p,1:p,i] = diag(p)
Total.A.M.c[1:p,(p+1):(2*p),i] =
```

```

diag( rep((Depression[i,2] - depression_mean) ,p))
}

MBB.c.EM = SynMP.0 (BetaStar=TestA$Point,TotalDesign=Total.A.M.c,
TotalCMatrix=TestA$VError, Iter.EM = 80, estimation='EM')

round(MBB.c.EM$beta,3) # point estimates of the fixed effects
round(MBB.c.EM$SE.beta,3) # SE of the fixed effects
round(MBB.c.EM$Sigma,3) # point estimates of the random effects
round(MBB.c.EM$SE.Sigma,3) # SE of the random effects

# The Naive method

# Make the data set appropriate for lavaan
Point = cbind(TestA$Point, (Depression[1:n,2] - depression_mean))
colnames(Point) <- c("AR11","AR12","AR21","AR22","cesdT_1")
D.Point <- as.data.frame(Point)

library(lavaan)

# Specify the Naive method as a SEM model
model.mreg <- '
AR11 ~ c11 * cesdT_1 + i11 * 1
AR12 ~ c12 *cesdT_1 + i12 * 1
AR21 ~ c21 * cesdT_1 + i21 * 1
AR22 ~ c22 * cesdT_1 + i22 * 1 '

# Estimate the model
mreg.res<-sem(model.mreg, data=D.Point)
summary(mreg.res)

```

Appendix B

R code of generating simulation data

```
library(GPARotation)
library(EFAutilities)

# load a workspace file of parameters
load('parameters.RData')

high.loading = rep(rep(NA, 7), 82)
low.loading = rep(rep(NA, 7), 82)

for (i in 1:82){
  high.loading[(1+7*(i-1)):(7+7*(i-1))] =
    c(lambda82[1:4,1,i],lambda82[5:7,2,i])
  low.loading[(1+7*(i-1)):(7+7*(i-1))] =
    c(lambda82[1:4,2,i],lambda82[5:7,1,i])
}

# load a function to generate simulation data
scan('Gen_Samp.txt')

# Generate an individual time series of  $T = 1000$  under the condition of normal variabl

set.seed(1)
X = rnorm(1, 0, sd(X_d))

dat = Gen_Samp(seed = 1, X = X, TS = 1000)
```

Appendix C
Additional Tables

The EC method											
$T = 56$			$T = 100$			$T = 200$			$T = 1000$		
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE	$\bar{\sigma}$
a_{11} intercept	0.333	0.276	0.032	0.041	0.304	0.030	0.32	0.028	0.331	0.026	0.025
a_{12} intercept	0.000	-0.009	0.022	0.025	-0.006	0.018	-0.002	0.012	-0.001	0.005	0.006
a_{11} slope	0.040	0.031	0.066	0.069	0.039	0.057	0.038	0.058	0.038	0.052	0.05
a_{12} slope	0.000	0.003	0.049	0.056	-0.001	0.037	0.001	0.025	0.000	0.011	0.011
a_{11} variance	0.009	0.025	0.008	0.012	0.027	0.007	0.028	0.007	0.029	0.006	0.006
a_{12} variance	0.000	0.005	0.004	0.060	0.002	0.002	0.001	0.001	0.000	0.000	0.001
The Naive method											
$T = 56$			$T = 100$			$T = 200$			$T = 1000$		
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE	$\bar{\sigma}$
a_{11} intercept	0.333	0.027	0.032	0.031	0.297	0.030	0.315	0.028	0.329	0.026	0.025
a_{12} intercept	0.000	-0.009	0.023	0.024	-0.006	0.018	-0.003	0.013	-0.001	0.006	0.006
a_{11} slope	0.040	0.031	0.067	0.064	0.037	0.057	0.037	0.058	0.038	0.052	0.051
a_{12} slope	0.000	0.003	0.051	0.049	-0.001	0.039	0.001	0.027	0.000	0.012	0.012
a_{11} variance	0.009	0.049	0.010	0.010	0.040	0.008	0.035	0.007	0.031	0.006	0.006
a_{12} variance	0.000	0.029	0.006	0.006	0.016	0.004	0.008	0.002	0.002	0.000	0.000

Table C1

The point estimates and standard error estimates of six between-individual parameters, normal variables, and 50 individuals. “EC”: error corrected; T : the time series length; θ : the true value; $\bar{\theta}$: the mean of point estimates; “ESE”: empirical standard error; $\bar{\sigma}$: the mean of standard error estimates.

The EC method										
$T = 56$			$T = 100$			$T = 200$			$T = 1000$	
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE
a_{11} intercept	0.333	0.268	0.025	0.027	0.298	0.021	0.022	0.315	0.018	0.015
a_{12} intercept	0.000	-0.011	0.023	0.026	-0.008	0.017	0.019	-0.005	0.012	0.006
a_{11} slope	0.040	0.035	0.052	0.054	0.037	0.043	0.046	0.038	0.037	0.031
a_{12} slope	0.000	-0.002	0.049	0.051	0.000	0.037	0.038	-0.001	0.025	0.012
a_{11} variance	0.009	0.010	0.005	0.010	0.009	0.004	0.007	0.009	0.004	0.003
a_{12} variance	0.000	0.005	0.004	0.013	0.002	0.002	0.007	0.001	0.001	0.000

The Naive method										
$T = 56$			$T = 100$			$T = 200$			$T = 1000$	
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE
a_{11} intercept	0.333	0.260	0.026	0.026	0.290	0.022	0.021	0.310	0.018	0.014
a_{12} intercept	0.000	-0.011	0.023	0.024	-0.008	0.018	0.018	-0.005	0.013	0.006
a_{11} slope	0.040	0.034	0.053	0.052	0.036	0.044	0.044	0.038	0.036	0.029
a_{12} slope	0.000	-0.001	0.050	0.048	-0.001	0.038	0.037	-0.001	0.027	0.012
a_{11} variance	0.009	0.033	0.007	0.007	0.023	0.005	0.005	0.016	0.004	0.002
a_{12} variance	0.000	0.028	0.006	0.006	0.016	0.004	0.003	0.008	0.002	0.000

Table C2

The point estimates and standard error estimates of six between-individual parameters, t -distribution variables, and 50 individuals. “EC”: error corrected; T : the time series length; θ : the true value; $\bar{\theta}$: the mean of point estimates; “ESE”: empirical standard error; $\bar{\sigma}$: the mean of standard error estimates.

The EC method											
$T = 56$			$T = 100$			$T = 200$			$T = 1000$		
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE	$\bar{\sigma}$
a_{11} intercept	0.333	0.276	0.023	0.024	0.303	0.021	0.020	0.319	0.019	0.330	0.019
a_{12} intercept	0.000	-0.009	0.016	0.019	-0.005	0.013	0.012	-0.002	0.009	-0.001	0.004
a_{11} slope	0.040	0.034	0.046	0.043	0.040	0.042	0.040	0.038	0.039	0.035	0.035
a_{12} slope	0.000	0.000	0.034	0.033	0.000	0.026	0.024	0.001	0.018	0.000	0.008
a_{11} variance	0.009	0.026	0.006	0.007	0.027	0.005	0.006	0.029	0.005	0.030	0.004
a_{12} variance	0.000	0.004	0.003	0.005	0.002	0.002	0.003	0.001	0.001	0.000	0.000
The Naive method											
$T = 56$			$T = 100$			$T = 200$			$T = 1000$		
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE	$\bar{\sigma}$
a_{11} intercept	0.333	0.275	0.016	0.016	0.304	0.015	0.014	0.319	0.014	0.330	0.013
a_{12} intercept	0.000	-0.008	0.011	0.012	-0.004	0.009	0.009	-0.002	0.006	-0.001	0.003
a_{11} slope	0.040	0.035	0.032	0.031	0.039	0.029	0.029	0.038	0.027	0.040	0.025
a_{12} slope	0.000	-0.001	0.024	0.023	0.001	0.018	0.017	0.000	0.013	0.000	0.005
a_{11} variance	0.009	0.026	0.004	0.005	0.028	0.004	0.004	0.030	0.003	0.030	0.003
a_{12} variance	0.000	0.004	0.003	0.003	0.002	0.001	0.002	0.001	0.001	0.000	0.000

Table C3

The point estimates and standard error estimates of six between-individual parameters, normal variables, and 100 individuals. “EC”: error corrected; T : the time series length; θ : the true value; $\bar{\theta}$: the mean of point estimates; “ESE”: empirical standard error; $\bar{\sigma}$: the mean of standard error estimates.

The EC method										
$T = 56$			$T = 100$			$T = 200$			$T = 1000$	
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE
a_{11} intercept	0.333	0.268	0.018	0.018	0.298	0.015	0.316	0.013	0.329	0.011
a_{12} intercept	0.000	-0.011	0.017	0.017	-0.007	0.012	-0.004	0.009	-0.001	0.004
a_{11} slope	0.040	0.034	0.036	0.035	0.038	0.030	0.037	0.026	0.039	0.021
a_{12} slope	0.000	-0.003	0.034	0.032	-0.001	0.025	-0.002	0.017	0.000	0.008
a_{11} variance	0.009	0.009	0.004	0.004	0.009	0.003	0.009	0.003	0.009	0.002
a_{12} variance	0.000	0.005	0.003	0.004	0.002	0.002	0.001	0.001	0.000	0.000
The Naive method										
$T = 56$			$T = 100$			$T = 200$			$T = 1000$	
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE
a_{11} intercept	0.333	0.268	0.013	0.013	0.298	0.011	0.316	0.009	0.329	0.008
a_{12} intercept	0.000	-0.011	0.012	0.012	-0.007	0.009	-0.004	0.006	-0.001	0.003
a_{11} slope	0.040	0.036	0.026	0.025	0.037	0.021	0.038	0.018	0.039	0.014
a_{12} slope	0.000	0.000	0.023	0.023	0.000	0.018	0.000	0.012	0.000	0.005
a_{11} variance	0.009	0.010	0.003	0.003	0.009	0.002	0.009	0.002	0.009	0.002
a_{12} variance	0.000	0.004	0.002	0.003	0.002	0.001	0.001	0.001	0.000	0.000

Table C4

The point estimates and standard error estimates of six between-individual parameters, t -distribution variables, and 100 individuals. “EC”: error corrected; T : the time series length; θ : the true value; $\bar{\theta}$: the mean of point estimates; “ESE”: empirical standard error; $\bar{\sigma}$: the mean of standard error estimates.

The EC method															
T = 56				T = 100				T = 200				T = 1000			
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$
a_{21} intercept	0.000	-0.012	0.017	0.017	-0.009	0.013	0.012	-0.005	0.009	0.009	-0.001	0.004	0.004	0.004	0.004
a_{22} intercept	0.334	0.252	0.019	0.018	0.287	0.015	0.015	0.310	0.013	0.012	0.328	0.011	0.009	0.011	0.009
a_{21} slope	0.000	-0.006	0.033	0.033	-0.003	0.025	0.024	-0.002	0.018	0.017	-0.001	0.008	0.008	0.008	0.008
a_{22} slope	0.087	0.072	0.036	0.035	0.080	0.029	0.029	0.082	0.025	0.024	0.086	0.019	0.018	0.019	0.018
a_{21} variance	0.000	0.005	0.003	0.004	0.002	0.002	0.002	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000
a_{22} variance	0.007	0.008	0.004	0.005	0.008	0.003	0.003	0.007	0.002	0.002	0.007	0.002	0.002	0.002	0.002

The Naive method															
T = 56				T = 100				T = 200				T = 1000			
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$
a_{21} intercept	0.000	-0.012	0.017	0.017	-0.010	0.013	0.013	-0.006	0.009	0.009	-0.002	0.004	0.004	0.004	0.004
a_{22} intercept	0.334	0.243	0.020	0.019	0.277	0.016	0.015	0.302	0.014	0.013	0.326	0.011	0.009	0.011	0.009
a_{21} slope	0.000	-0.007	0.034	0.034	-0.003	0.026	0.026	-0.003	0.019	0.019	-0.001	0.009	0.008	0.009	0.008
a_{22} slope	0.087	0.069	0.038	0.038	0.077	0.032	0.031	0.080	0.026	0.025	0.085	0.020	0.019	0.020	0.019
a_{21} variance	0.000	0.029	0.004	0.004	0.017	0.003	0.002	0.009	0.001	0.001	0.002	0.000	0.000	0.000	0.000
a_{22} variance	0.007	0.035	0.005	0.005	0.024	0.004	0.003	0.016	0.003	0.002	0.009	0.002	0.001	0.002	0.001

Table C5

The point estimates and standard error estimates of six between-individual parameters, t -distribution variables, and 100 individuals. “EC”: error corrected; T : the time series length; θ : the true value; $\bar{\theta}$: the mean of point estimates; “ESE”: empirical standard error; $\bar{\sigma}$: the mean of standard error estimates.

The EC method											
$T = 56$			$T = 100$			$T = 200$			$T = 1000$		
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE	$\bar{\sigma}$
a_{11} intercept	0.333	0.269	0.023	0.022	0.021	0.020	0.314	0.020	0.329	0.019	0.018
a_{12} intercept	0.000	-0.009	0.017	0.017	0.013	0.013	-0.003	0.009	-0.001	0.004	0.004
a_{11} slope	0.040	0.033	0.047	0.045	0.041	0.041	0.037	0.040	0.039	0.035	0.036
a_{12} slope	0.000	0.000	0.035	0.034	0.000	0.028	0.001	0.019	0.000	0.008	0.008
a_{11} variance	0.009	0.050	0.007	0.007	0.006	0.006	0.036	0.005	0.031	0.004	0.004
a_{12} variance	0.000	0.029	0.004	0.004	0.003	0.002	0.008	0.001	0.002	0.000	0.000
The Naive method											
$T = 56$			$T = 100$			$T = 200$			$T = 1000$		
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE	$\bar{\sigma}$
a_{11} intercept	0.333	0.269	0.016	0.016	0.015	0.014	0.314	0.014	0.329	0.013	0.013
a_{12} intercept	0.000	-0.008	0.012	0.012	0.009	0.009	-0.003	0.007	-0.001	0.003	0.003
a_{11} slope	0.040	0.035	0.033	0.032	0.030	0.029	0.037	0.027	0.039	0.026	0.025
a_{12} slope	0.000	-0.001	0.025	0.024	0.000	0.019	0.000	0.013	0.000	0.006	0.006
a_{11} variance	0.009	0.051	0.005	0.005	0.004	0.004	0.036	0.004	0.032	0.003	0.003
a_{12} variance	0.000	0.029	0.003	0.003	0.002	0.002	0.008	0.001	0.002	0.000	0.000

Table C6

The point estimates and standard error estimates of six between-individual parameters, normal variables, and 200 individuals. “EC”: error corrected; T : the time series length; θ : the true value; $\bar{\theta}$: the mean of point estimates; “ESE”: empirical standard error; $\bar{\sigma}$: the mean of standard error estimates.

The EC method															
T = 56				T = 100				T = 200				T = 1000			
θ	$\hat{\theta}$	ESE	$\hat{\sigma}$	$\hat{\theta}$	ESE	$\hat{\sigma}$	$\hat{\theta}$	ESE	$\hat{\sigma}$	$\hat{\theta}$	ESE	$\hat{\sigma}$	$\hat{\theta}$	ESE	$\hat{\sigma}$
a_{21} intercept	0.000	-0.010	0.012	0.012	-0.006	0.008	0.009	-0.003	0.006	-0.001	0.003	0.003	-0.001	0.003	0.003
a_{22} intercept	0.334	0.264	0.015	0.015	0.297	0.014	0.013	0.316	0.013	0.330	0.012	0.011	0.330	0.012	0.011
a_{21} slope	0.000	-0.003	0.023	0.023	-0.002	0.017	0.017	-0.001	0.012	0.000	0.005	0.005	0.000	0.005	0.005
a_{22} slope	0.087	0.075	0.030	0.029	0.082	0.027	0.026	0.084	0.024	0.087	0.022	0.022	0.087	0.022	0.022
a_{21} variance	0.000	0.004	0.002	0.003	0.002	0.001	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
a_{22} variance	0.024	0.019	0.004	0.004	0.021	0.003	0.003	0.022	0.003	0.023	0.002	0.002	0.023	0.002	0.002
The Naive method															
T = 56				T = 100				T = 200				T = 1000			
θ	$\hat{\theta}$	ESE	$\hat{\sigma}$	$\hat{\theta}$	ESE	$\hat{\sigma}$	$\hat{\theta}$	ESE	$\hat{\sigma}$	$\hat{\theta}$	ESE	$\hat{\sigma}$	$\hat{\theta}$	ESE	$\hat{\sigma}$
a_{21} intercept	0.000	-0.010	0.012	0.012	-0.006	0.009	0.009	-0.004	0.006	-0.001	0.003	0.003	-0.001	0.003	0.003
a_{22} intercept	0.334	0.256	0.016	0.015	0.289	0.014	0.014	0.310	0.013	0.328	0.012	0.011	0.328	0.012	0.011
a_{21} slope	0.000	-0.003	0.024	0.025	-0.002	0.018	0.019	-0.001	0.013	0.000	0.006	0.006	0.000	0.006	0.006
a_{22} slope	0.087	0.072	0.031	0.031	0.080	0.028	0.028	0.082	0.024	0.086	0.023	0.023	0.086	0.023	0.023
a_{21} variance	0.000	0.030	0.003	0.003	0.017	0.002	0.002	0.009	0.001	0.002	0.000	0.000	0.002	0.000	0.000
a_{22} variance	0.024	0.048	0.005	0.005	0.037	0.004	0.004	0.031	0.003	0.025	0.002	0.003	0.025	0.002	0.003

Table C7

The point estimates and standard error estimates of six between-individual parameters, normal variables, and 200 individuals. “EC”: error corrected; T : the time series length; θ : the true value; $\tilde{\theta}$: the mean of point estimates; “ESE”: empirical standard error; $\tilde{\sigma}$: the mean of standard error estimates.

The EC method											
$T = 56$			$T = 100$			$T = 200$			$T = 1000$		
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE	$\bar{\sigma}$
a_{11} intercept	0.333	0.260	0.019	0.018	0.290	0.015	0.311	0.013	0.328	0.011	0.010
a_{12} intercept	0.000	-0.012	0.017	0.017	-0.008	0.013	-0.005	0.009	-0.001	0.004	0.004
a_{11} slope	0.040	0.034	0.037	0.037	0.038	0.031	0.036	0.027	0.039	0.021	0.021
a_{12} slope	0.000	-0.002	0.036	0.034	0.000	0.027	-0.001	0.018	0.000	0.008	0.008
a_{11} variance	0.009	0.034	0.005	0.005	0.023	0.004	0.016	0.003	0.010	0.002	0.001
a_{12} variance	0.000	0.029	0.004	0.004	0.016	0.002	0.008	0.001	0.002	0.000	0.000
The Naive method											
$T = 56$			$T = 100$			$T = 200$			$T = 1000$		
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE	$\bar{\sigma}$
a_{11} intercept	0.333	0.260	0.013	0.013	0.290	0.011	0.310	0.009	0.328	0.008	0.007
a_{12} intercept	0.000	-0.011	0.012	0.012	-0.007	0.009	-0.005	0.006	-0.001	0.003	0.003
a_{11} slope	0.040	0.035	0.026	0.026	0.036	0.022	0.037	0.019	0.039	0.014	0.015
a_{12} slope	0.000	0.000	0.024	0.024	-0.001	0.019	0.000	0.013	0.000	0.006	0.006
a_{11} variance	0.009	0.034	0.004	0.003	0.023	0.003	0.016	0.002	0.011	0.002	0.001
a_{12} variance	0.000	0.029	0.003	0.003	0.016	0.002	0.008	0.001	0.002	0.000	0.000

Table C8

The point estimates and standard error estimates of six between-individual parameters, t -distribution variables, and 200 individuals. “EC”: error corrected; T : the time series length; θ : the true value; $\bar{\theta}$: the mean of point estimates; “ESE”: empirical standard error; $\bar{\sigma}$: the mean of standard error estimates.

The EC method											
$T = 56$			$T = 100$			$T = 200$			$T = 1000$		
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE	$\bar{\sigma}$
a_{21} intercept	0.000	-0.013	0.012	0.012	-0.009	0.009	0.009	-0.005	0.006	-0.001	0.003
a_{22} intercept	0.334	0.253	0.013	0.013	0.287	0.011	0.010	0.309	0.008	0.328	0.007
a_{21} slope	0.000	-0.003	0.025	0.023	-0.004	0.018	0.017	-0.001	0.012	0.000	0.005
a_{22} slope	0.087	0.073	0.026	0.025	0.078	0.021	0.021	0.083	0.017	0.086	0.013
a_{21} variance	0.000	0.004	0.002	0.003	0.002	0.001	0.002	0.001	0.001	0.000	0.000
a_{22} variance	0.007	0.008	0.003	0.003	0.008	0.002	0.002	0.007	0.002	0.007	0.001

The Naive method											
$T = 56$			$T = 100$			$T = 200$			$T = 1000$		
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE	$\bar{\sigma}$
a_{21} intercept	0.000	-0.013	0.012	0.012	-0.010	0.009	0.009	-0.006	0.007	-0.002	0.003
a_{22} intercept	0.334	0.243	0.014	0.013	0.277	0.012	0.011	0.302	0.010	0.326	0.007
a_{21} slope	0.000	-0.004	0.026	0.024	-0.003	0.019	0.018	-0.002	0.014	-0.001	0.006
a_{22} slope	0.087	0.069	0.028	0.027	0.075	0.023	0.022	0.080	0.018	0.085	0.013
a_{21} variance	0.000	0.029	0.003	0.003	0.017	0.002	0.002	0.009	0.001	0.002	0.000
a_{22} variance	0.007	0.036	0.004	0.004	0.024	0.003	0.002	0.016	0.002	0.009	0.001

Table C9

The point estimates and standard error estimates of six between-individual parameters, t -distribution variables, and 200 individuals. “EC”: error corrected; T : the time series length; θ : the true value; $\bar{\theta}$: the mean of point estimates; “ESE”: empirical standard error; $\bar{\sigma}$: the mean of standard error estimates.

The EC method										
$T = 56$			$T = 100$			$T = 200$			$T = 1000$	
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE
a_{11} intercept	0.333	0.275	0.010	0.303	0.009	0.009	0.319	0.009	0.330	0.008
a_{12} intercept	0.000	-0.008	0.007	-0.004	0.006	0.006	-0.002	0.004	-0.001	0.002
a_{11} slope	0.040	0.034	0.020	0.037	0.018	0.018	0.038	0.017	0.040	0.016
a_{12} slope	0.000	-0.001	0.015	0.000	0.011	0.011	0.000	0.007	0.000	0.003
a_{11} variance	0.009	0.026	0.003	0.029	0.003	0.002	0.030	0.002	0.031	0.002
a_{12} variance	0.000	0.004	0.002	0.002	0.001	0.001	0.001	0.000	0.000	0.000
The Naive method										
$T = 56$			$T = 100$			$T = 200$			$T = 1000$	
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE
a_{11} intercept	0.333	0.268	0.010	0.296	0.009	0.009	0.314	0.009	0.329	0.008
a_{12} intercept	0.000	-0.008	0.008	-0.005	0.006	0.006	-0.003	0.004	-0.001	0.002
a_{11} slope	0.040	0.034	0.021	0.036	0.019	0.018	0.038	0.017	0.040	0.016
a_{12} slope	0.000	0.000	0.016	0.000	0.012	0.012	0.000	0.008	0.000	0.004
a_{11} variance	0.009	0.051	0.003	0.042	0.003	0.003	0.037	0.002	0.032	0.002
a_{12} variance	0.000	0.029	0.002	0.017	0.001	0.001	0.008	0.001	0.002	0.000

Table C10

The point estimates and standard error estimates of six between-individual parameters, normal variables, and 500 individuals. “EC”: error corrected; T : the time series length; θ : the true value; $\bar{\theta}$: the mean of point estimates; “ESE”: empirical standard error; $\bar{\sigma}$: the mean of standard error estimates.

The EC method															
T = 56				T = 100				T = 200				T = 1000			
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$
a_{21} intercept	0.000	-0.010	0.008	0.007	-0.006	0.006	0.006	-0.003	0.004	0.004	-0.001	0.002	0.002	0.002	0.002
a_{22} intercept	0.334	0.264	0.009	0.009	0.297	0.009	0.008	0.316	0.008	0.008	0.330	0.007	0.007	0.007	0.007
a_{21} slope	0.000	-0.003	0.014	0.015	-0.002	0.011	0.011	-0.001	0.008	0.008	0.000	0.003	0.003	0.003	0.003
a_{22} slope	0.087	0.074	0.019	0.019	0.081	0.017	0.017	0.084	0.015	0.015	0.087	0.014	0.014	0.014	0.014
a_{21} variance	0.000	0.004	0.002	0.002	0.002	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
a_{22} variance	0.024	0.020	0.003	0.003	0.022	0.002	0.002	0.023	0.002	0.002	0.024	0.002	0.002	0.002	0.002

The Naive method															
T = 56				T = 100				T = 200				T = 1000			
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$
a_{21} intercept	0.000	-0.010	0.008	0.008	-0.006	0.006	0.006	-0.003	0.004	0.004	-0.001	0.002	0.002	0.002	0.002
a_{22} intercept	0.334	0.257	0.010	0.010	0.289	0.009	0.009	0.310	0.008	0.008	0.328	0.007	0.007	0.007	0.007
a_{21} slope	0.000	-0.003	0.015	0.016	-0.002	0.012	0.012	-0.001	0.008	0.008	0.000	0.004	0.004	0.004	0.004
a_{22} slope	0.087	0.071	0.019	0.020	0.078	0.017	0.017	0.083	0.016	0.016	0.086	0.015	0.015	0.015	0.014
a_{21} variance	0.000	0.030	0.002	0.002	0.017	0.001	0.001	0.009	0.001	0.001	0.002	0.000	0.000	0.000	0.000
a_{22} variance	0.024	0.048	0.003	0.003	0.038	0.002	0.002	0.031	0.002	0.002	0.025	0.002	0.002	0.002	0.002

Table C11

The point estimates and standard error estimates of six between-individual parameters, normal variables, and 500 individuals. “EC”: error corrected; T : the time series length; θ : the true value; $\bar{\theta}$: the mean of point estimates; “ESE”: empirical standard error; $\bar{\sigma}$: the mean of standard error estimates.

The EC method											
$T = 56$			$T = 100$			$T = 200$			$T = 1000$		
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE	$\bar{\sigma}$
a_{11} intercept	0.333	0.268	0.008	0.008	0.007	0.007	0.316	0.006	0.329	0.005	0.005
a_{12} intercept	0.000	-0.010	0.007	0.007	-0.007	0.005	-0.004	0.004	-0.001	0.002	0.002
a_{11} slope	0.040	0.035	0.016	0.016	0.037	0.013	0.038	0.011	0.040	0.009	0.009
a_{12} slope	0.000	0.000	0.015	0.015	0.000	0.011	0.000	0.008	0.000	0.003	0.003
a_{11} variance	0.009	0.009	0.002	0.002	0.009	0.002	0.009	0.001	0.009	0.001	0.001
a_{12} variance	0.000	0.004	0.002	0.002	0.002	0.001	0.001	0.000	0.000	0.000	0.000
The Naive method											
$T = 56$			$T = 100$			$T = 200$			$T = 1000$		
θ	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\sigma}$	$\bar{\theta}$	ESE	$\bar{\theta}$	ESE	$\bar{\sigma}$
a_{11} intercept	0.333	0.260	0.008	0.008	0.290	0.007	0.310	0.006	0.328	0.005	0.005
a_{12} intercept	0.000	-0.010	0.008	0.008	-0.007	0.006	-0.004	0.004	-0.001	0.002	0.002
a_{11} slope	0.040	0.034	0.016	0.017	0.036	0.014	0.038	0.011	0.040	0.009	0.009
a_{12} slope	0.000	0.000	0.016	0.015	-0.001	0.012	0.000	0.008	0.000	0.004	0.004
a_{11} variance	0.009	0.034	0.002	0.002	0.024	0.002	0.016	0.001	0.011	0.001	0.001
a_{12} variance	0.000	0.029	0.002	0.002	0.017	0.001	0.008	0.001	0.002	0.000	0.000

Table C12

The point estimates and standard error estimates of six between-individual parameters, t -distribution variables, and 500 individuals. “EC”: error corrected; T : the time series length; θ : the true value; $\bar{\theta}$: the mean of point estimates; “ESE”: empirical standard error; $\bar{\sigma}$: the mean of standard error estimates.

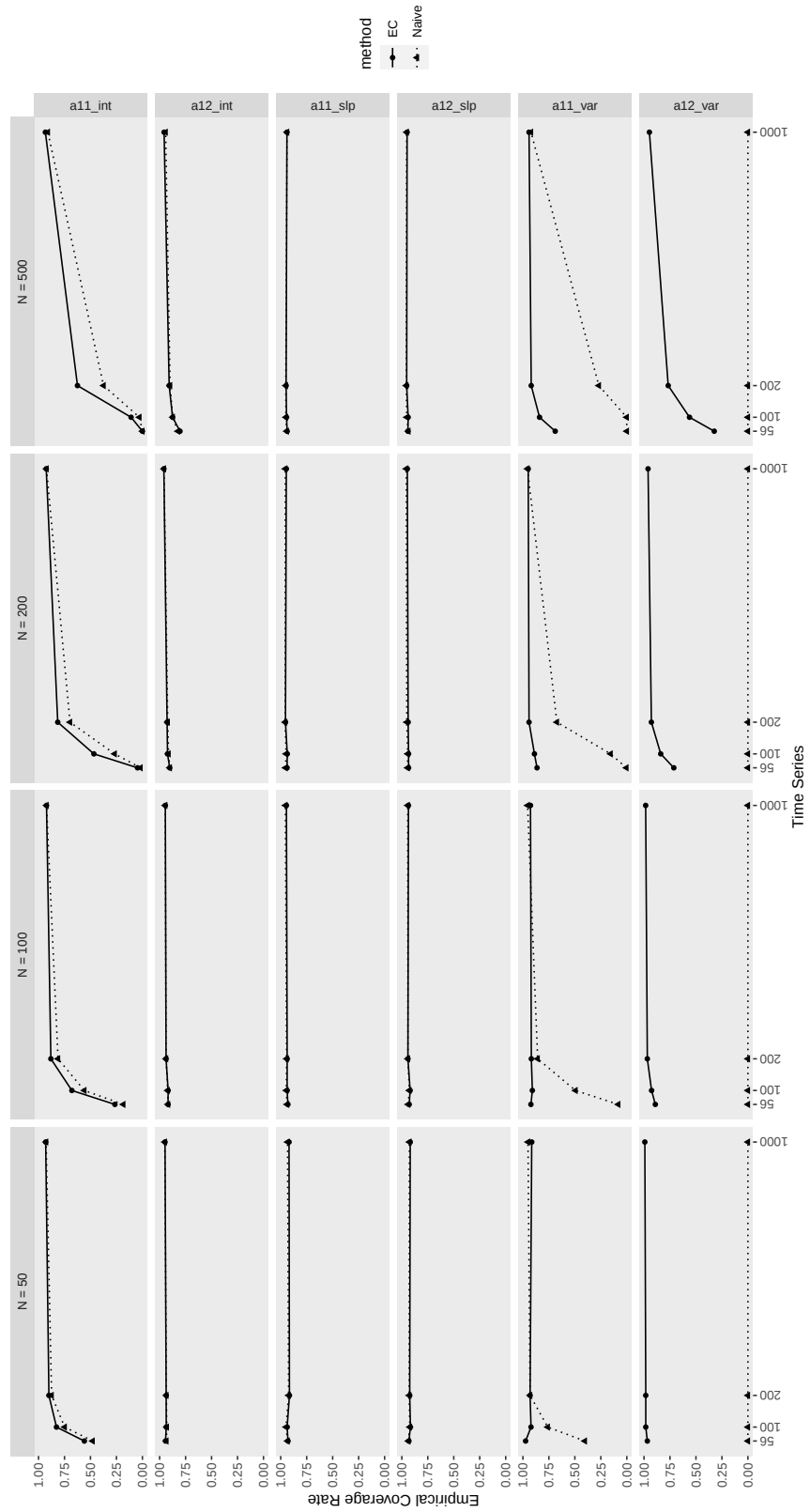
The EC method															
T = 56				T = 100				T = 200				T = 1000			
θ	$\hat{\theta}$	ESE	$\hat{\sigma}$	$\hat{\theta}$	ESE	$\hat{\sigma}$	$\hat{\theta}$	ESE	$\hat{\sigma}$	$\hat{\theta}$	ESE	$\hat{\sigma}$	$\hat{\theta}$	ESE	$\hat{\sigma}$
a_{21} intercept	0.000	-0.013	0.007	0.007	-0.009	0.006	0.006	-0.005	0.004	0.004	-0.001	0.002	0.002	0.002	0.002
a_{22} intercept	0.334	0.253	0.008	0.008	0.287	0.007	0.007	0.310	0.006	0.005	0.328	0.005	0.004	0.005	0.004
a_{21} slope	0.000	-0.004	0.015	0.015	-0.003	0.011	0.011	-0.001	0.008	0.008	0.000	0.003	0.003	0.003	0.003
a_{22} slope	0.087	0.072	0.017	0.016	0.078	0.013	0.013	0.083	0.011	0.011	0.086	0.008	0.008	0.008	0.008
a_{21} variance	0.000	0.004	0.002	0.002	0.002	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
a_{22} variance	0.007	0.008	0.002	0.002	0.008	0.001	0.001	0.007	0.001	0.001	0.007	0.001	0.001	0.001	0.001

The Naive method															
T = 56				T = 100				T = 200				T = 1000			
θ	$\hat{\theta}$	ESE	$\hat{\sigma}$	$\hat{\theta}$	ESE	$\hat{\sigma}$	$\hat{\theta}$	ESE	$\hat{\sigma}$	$\hat{\theta}$	ESE	$\hat{\sigma}$	$\hat{\theta}$	ESE	$\hat{\sigma}$
a_{21} intercept	0.000	-0.013	0.008	0.008	-0.010	0.006	0.006	-0.006	0.004	0.004	-0.002	0.002	0.002	0.002	0.002
a_{22} intercept	0.334	0.243	0.008	0.008	0.277	0.007	0.007	0.303	0.006	0.006	0.326	0.005	0.004	0.005	0.004
a_{21} slope	0.000	-0.004	0.015	0.015	-0.003	0.012	0.012	-0.002	0.009	0.008	0.000	0.004	0.004	0.004	0.004
a_{22} slope	0.087	0.068	0.017	0.017	0.075	0.014	0.014	0.081	0.011	0.011	0.085	0.008	0.008	0.008	0.008
a_{21} variance	0.000	0.030	0.002	0.002	0.017	0.001	0.001	0.009	0.001	0.001	0.002	0.000	0.000	0.000	0.000
a_{22} variance	0.007	0.036	0.002	0.002	0.024	0.002	0.002	0.016	0.001	0.001	0.009	0.001	0.001	0.001	0.001

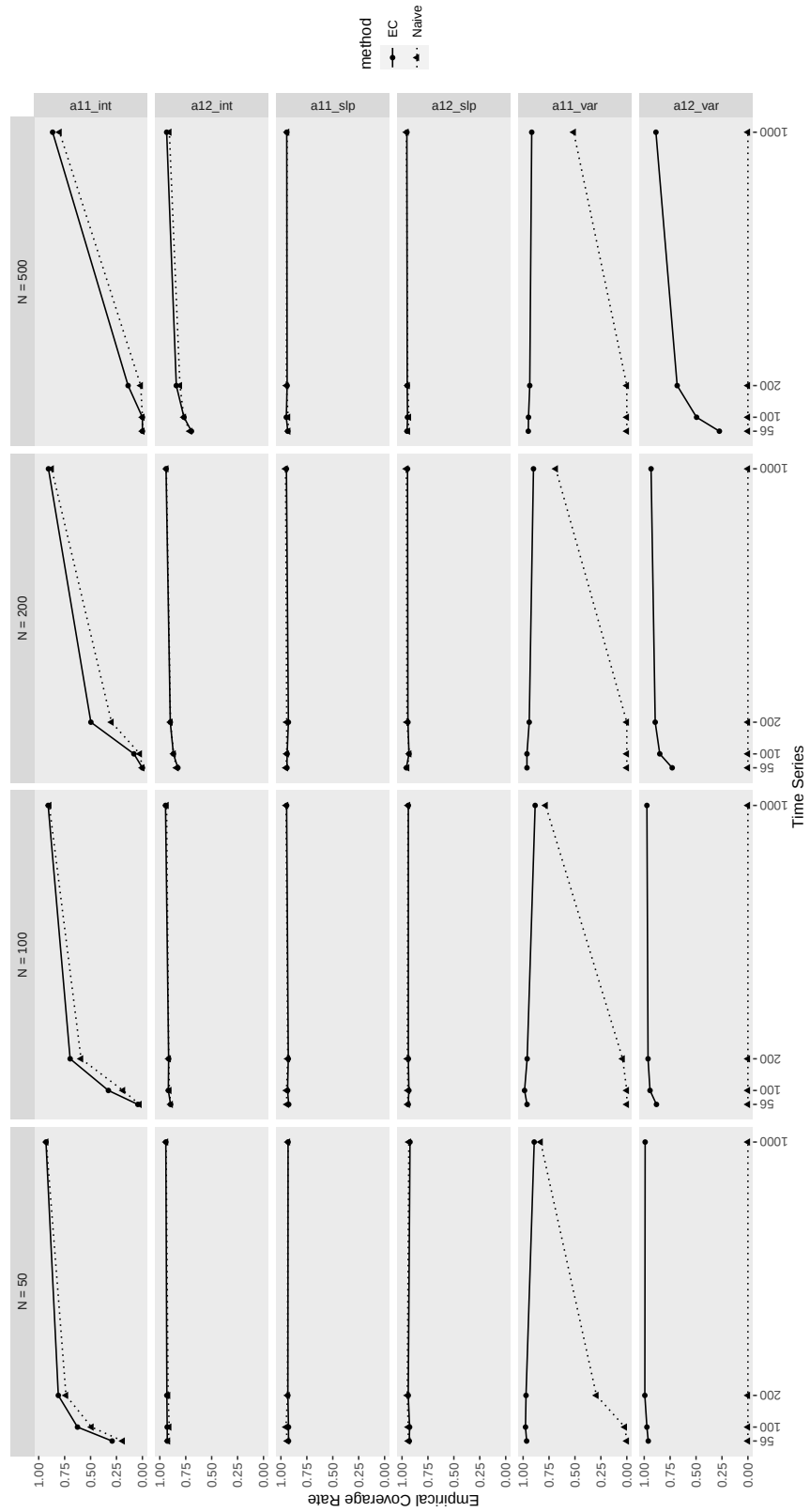
Table C13

The point estimates and standard error estimates of six between-individual parameters, t -distribution variables, and 500 individuals. “EC”: error corrected; T : the time series length; θ : the true value; $\bar{\theta}$: the mean of point estimates; “ESE”: empirical standard error; $\bar{\sigma}$: the mean of standard error estimates.

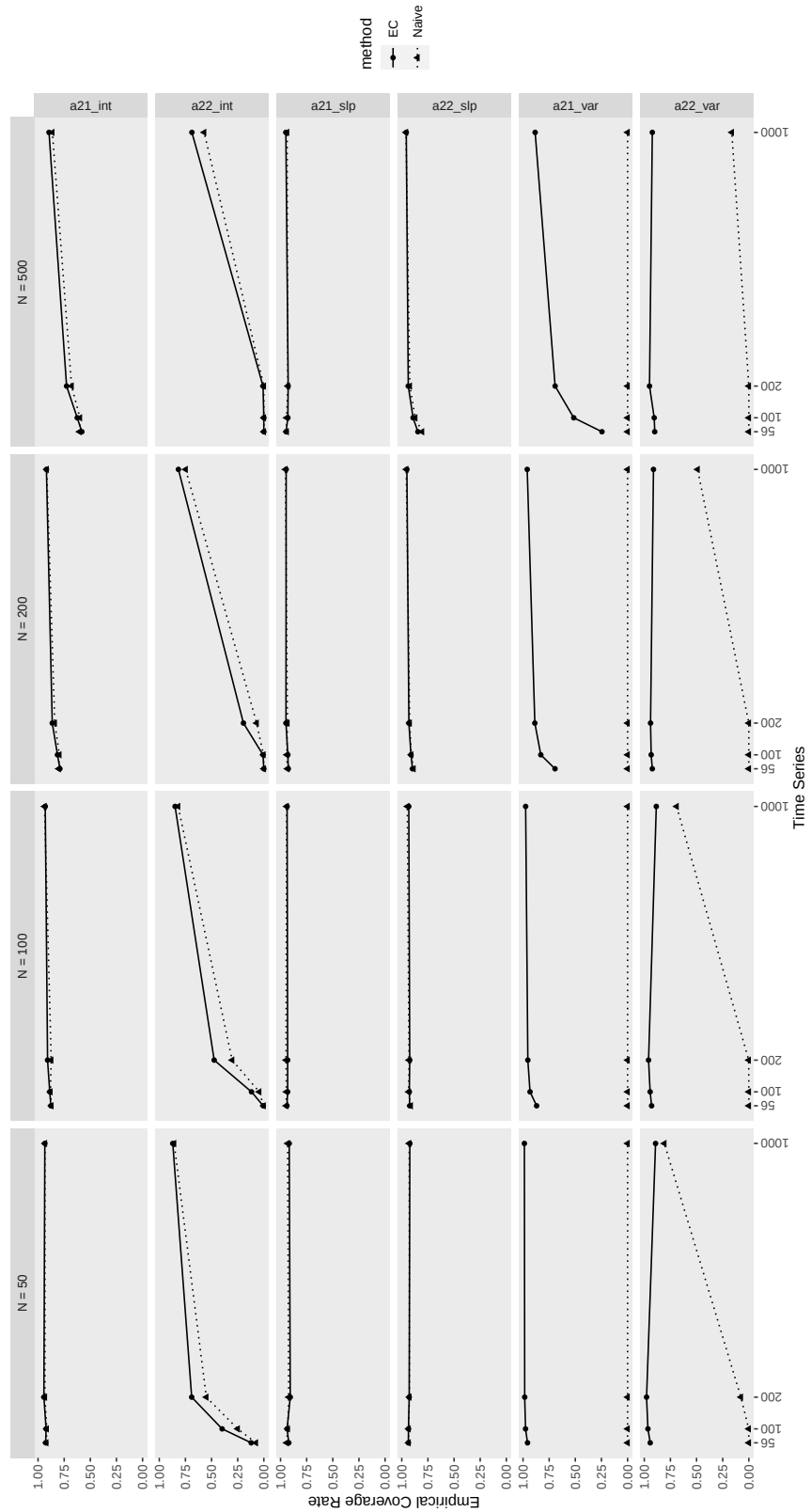
Appendix D
Additional Figures

**Figure D1**

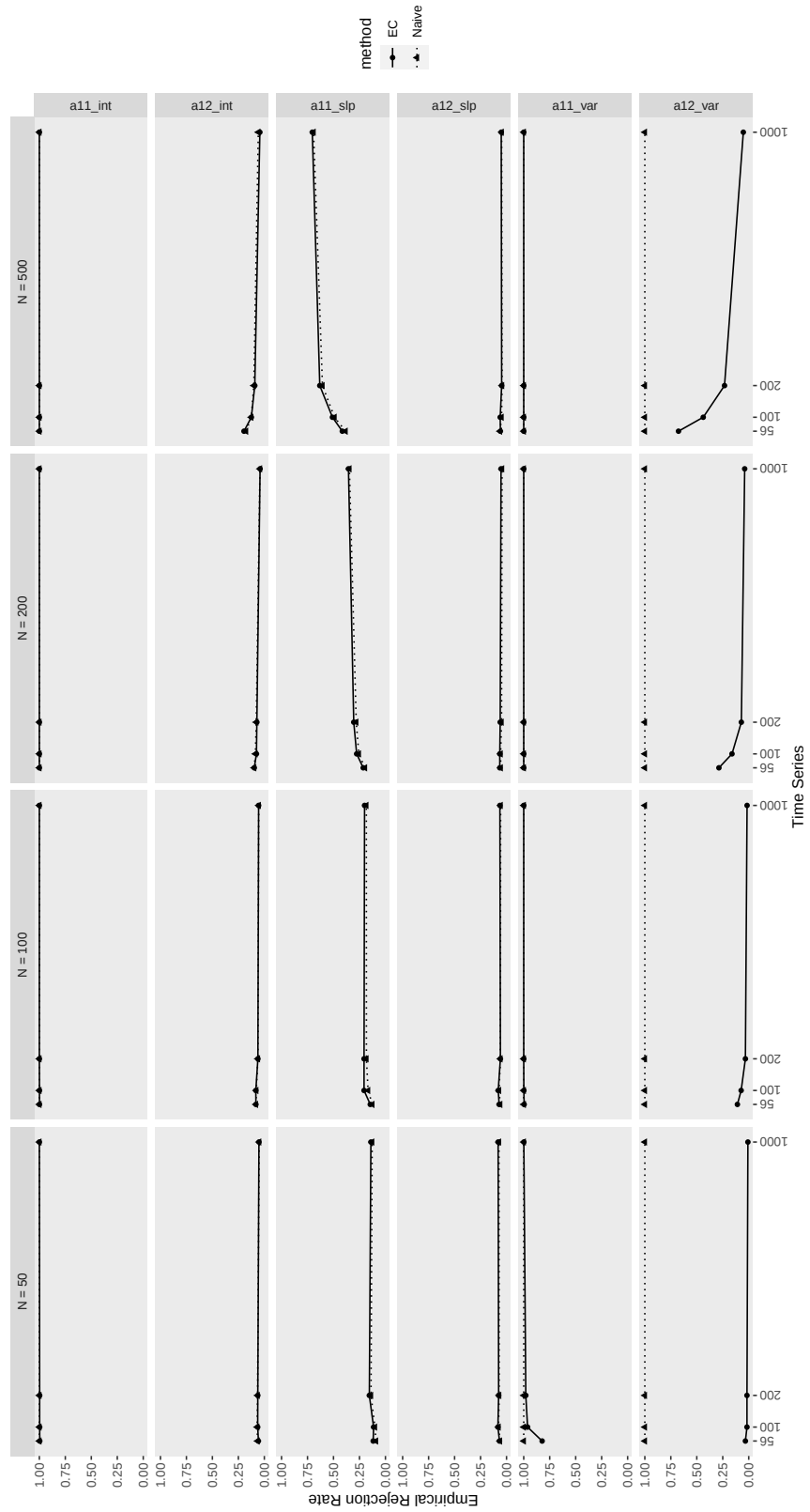
Empirical coverage rates of 95% confidence intervals of the intercepts, slopes, and the variances of a_{11} and a_{12} , normal variables.

**Figure D2**

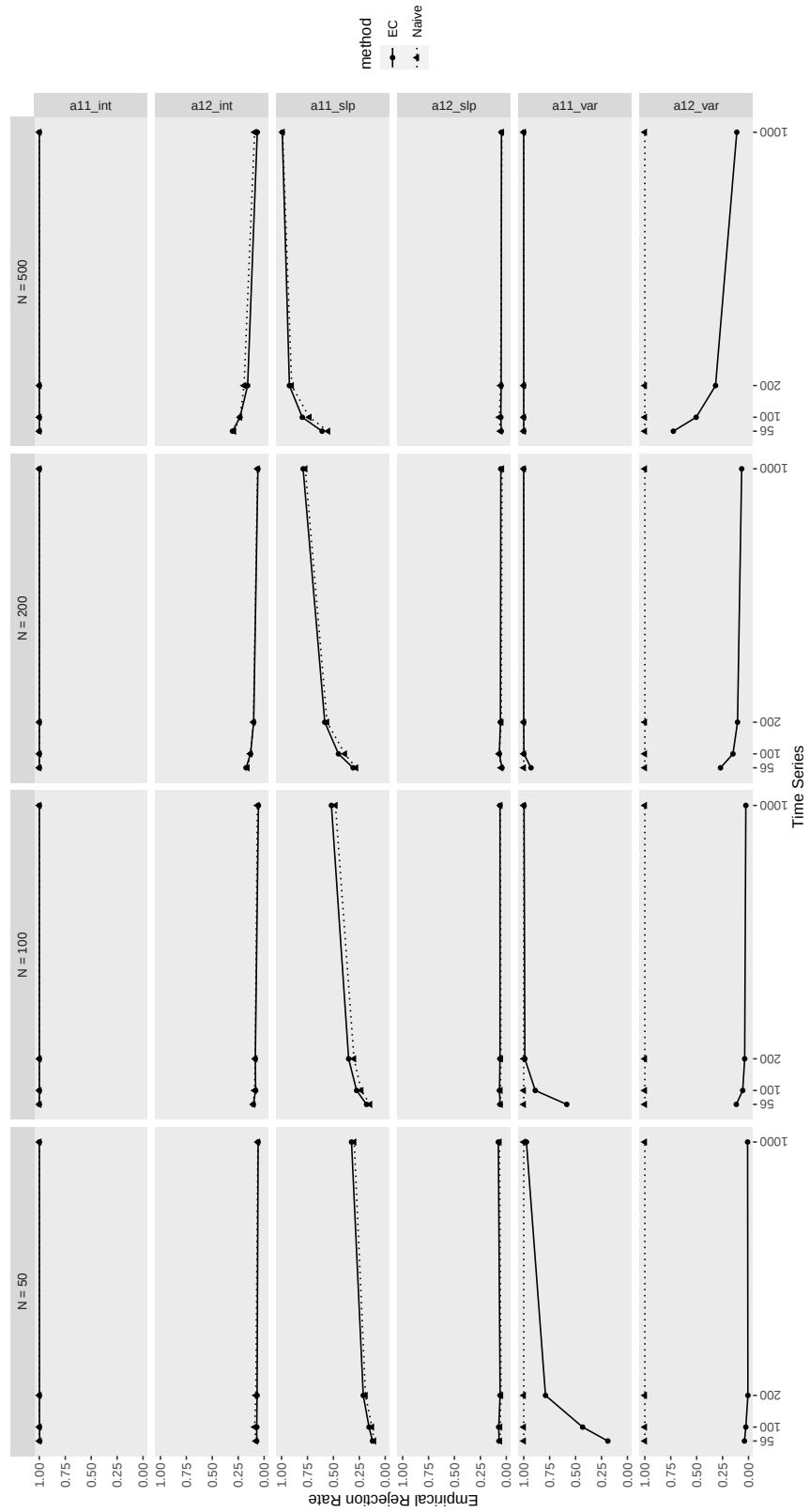
Empirical coverage rates of 95% confidence intervals of the intercepts, slopes, and the variances of a_{11} and a_{12} , T variables.

**Figure D3**

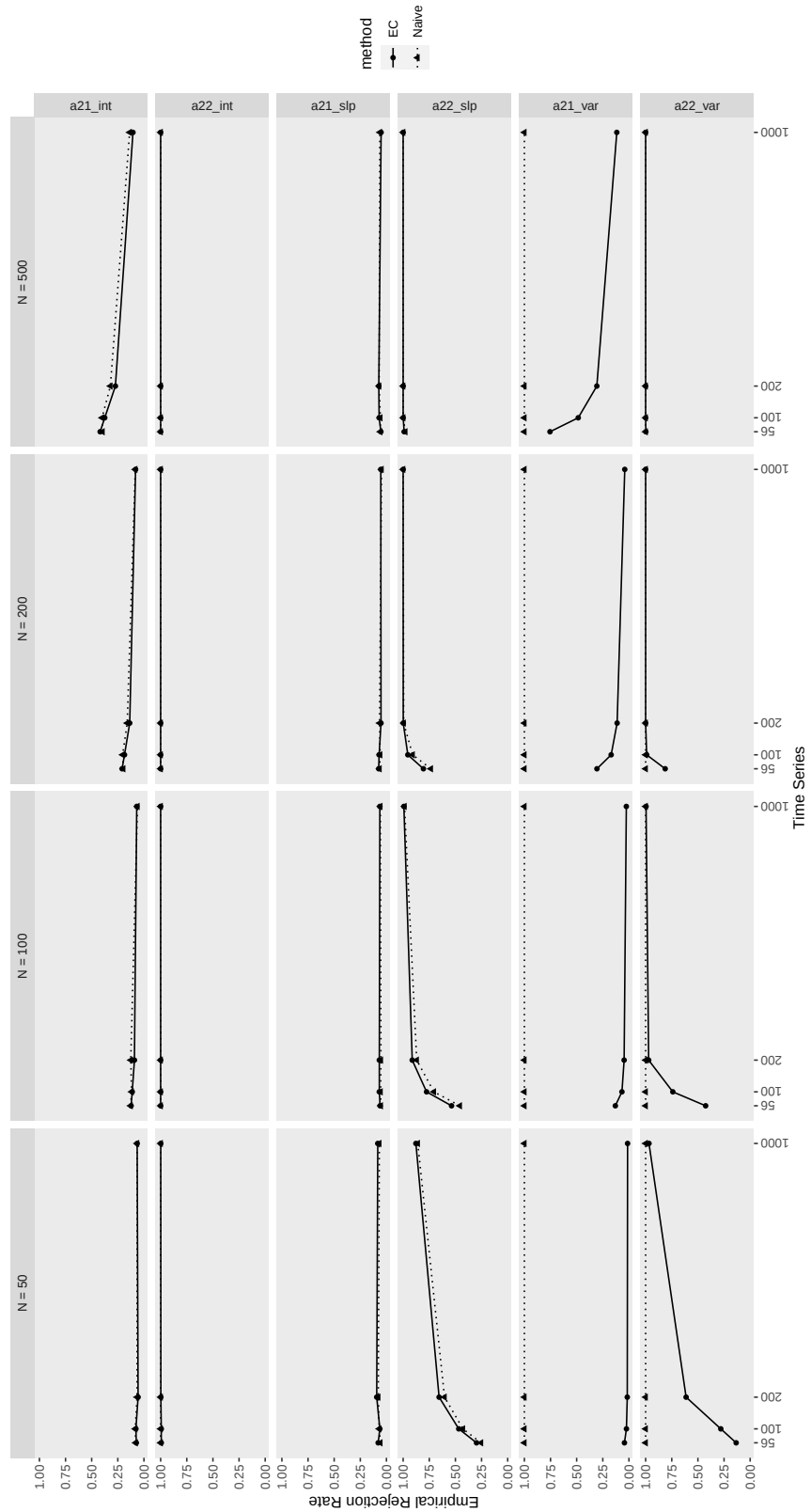
Empirical coverage rates of 95% confidence intervals of the intercepts, slopes, and the variances of a_{21} and a_{22} , normal variables.

**Figure D4**

Empirical rejection rates of the intercepts, slopes, and the variances of a_{11} and a_{12} , normal variables.

**Figure D5**

Empirical rejection rates of the intercepts, slopes, and the variances of a_{11} and a_{12} , T variables.

**Figure D6**

Empirical rejection rates of the intercepts, slopts, and the variances of a_{21} and a_{22} , normal variables.