# Regularizing threshold priors with sparse response patterns in Bayesian factor analysis with categorical indicators

ONLINE SUPPLEMENT

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# 1 Implementation of Bayesian Categorical SEM Model

## 1.1 Sampling Latent Response Distribution

The following Stan code is based on the implementation of Goodrich (2017).

```
//y = observed response vector
//mu = latent response mean vector (if marginal likelihood, then this is 0
//L = lower Cholesky decomposition of covariance matrix of latent responses
//b = matrix (nitems by nlevs) of thresholds
//u = nuisance (0,1) variable
//nlevs = vector designating number of levels/number of categories for each item
vector[] tmvn(int[] y, vector mu, matrix L, vector[] b, real[] u, int[] nlevs) {
  int K = rows(mu);
  vector[K] d;
  vector[K] z;
  vector[K] out[2];
  for (k in 1:K) {
    int km1 = k - 1;
    real nu;
    if (y[k] == 1) {
      real z_star = (b[k,1] - (mu[k] + ((k > 1) ? L[k,1:km1] * head(z, km1) : 0)
          )) /  L[k,k];
      real u_star = Phi(z_star); //normal CDF for implied density of TMVN
      nu = u_star * u[k];
      d[k] = u_star;
    } else if(y[k] == nlevs[k]) {
      real z_star = (b[k,(nlevs[k]-1)] - (mu[k] + ((k > 1) ? L[k,1:km1] * head(z
          , km1) : 0))) /  L[k,k];
      real u_star = Phi(z_star);
      d[k] = 1 - u_star;
      nu = u_star + d[k] * u[k];
    } else {
      int lb = y[k]-1;
      int ub = y[k];
      real z_starL = (b[k,lb] - (mu[k] + ((k > 1) ? L[k,1:km1] * head(z, km1) :
          0))) / L[k,k];
      real z_starU = (b[k,ub] - (mu[k] + ((k > 1) ? L[k,1:km1] * head(z, km1) :
          0))) / L[k,k];
      real u_starL = Phi(z_starL);
      real u_starU = Phi(z_starU);
      nu = u_starL + (u_starU - u_starL)*u[k];
      d[k] = u_starU - u_starL;
    }
    z[k] = inv_Phi(nu); //convert back to z-score from uniform variate
  }
  out[1] = z; //simulated ystar value
  out[2] = d; //density
  return(out);
}
```

## 1.2 Induced-Dirichlet Prior Implementation

The Dirichlet distribution is defined as

$$\text{Dirichlet}(\theta|\alpha) = \frac{\Gamma\left(\sum_{k=1}^{K} \alpha_k\right)}{\prod_{k=1}^{k} \Gamma\left(\alpha_k\right)} \prod_{k=1}^{K} \theta_k^{\alpha_k - 1}, \tag{1}$$

where $K \in \mathbb{N}, \alpha \in (\mathbb{R}^{+})^{K}$, for a $\theta \in K$-simplex. The hyper-parameter vector $\boldsymbol{\alpha}$ is strictly positive, with each element $\alpha > 0$. Sampling from the Dirichlet distribution produces a vector of probabilities that sum to unity. The probability density function for the Dirichlet distribution can then be used to compute the likelihood of a given vector of probabilities that define a simplex based on the given hyper-parameter for the Dirichlet distribution. In this work, the Dirichlet distribution is used to compute the probability/likelihood associated with a given simplex of probabilities induced by a set of thresholds for an item.

The density function of the Dirichlet distribution provides a convenient mechanism for using the probabilities induced by a set of thresholds for an item. We can randomly sample thresholds *indirectly* through the Dirichlet distribution. First, the simplex of probabilities are simulated, then thresholds are computed based on the simulated probabilities.

```
vector phi_logit_approx_vec(vector b) {
  int N = num_elements(b);
  vector[N] a;
  a = inv_logit( 1.702*b );
  return(a);
}
//c = vector of thresholds
//alpha = dirichlet hyper-prior parameters default=rep(1, num_elements(c)+1)
//phi = center of latent response distribution, default=0
real induced_dirichlet_lpdf(vector c, vector alpha, real phi) {
  int K = num_elements(c) + 1;
  vector[K - 1] anchoredcutoffs = c - phi; //which is first is important c vs.
      phi...
  vector[K] sigma;
  vector[K] p;
  matrix[K, K] J = rep_matrix(0, K, K);
  sigma[1:(K-1)] = phi_logit_approx_vec(anchoredcutoffs);
  sigma[K] = 1;
  p[1] =  sigma[1];
  for (k in 2:(K - 1))
    p[k] = sigma[k] - sigma[k - 1];
  p[K] = 1 - sigma[K - 1];
  for (k in 1:K) J[k, 1] = 1;
  for (k in 2:K) {
    real rho = 1.702 * sigma[k - 1] * (1 - sigma[k - 1]);
    J[k, k] =  - rho;
    J[k - 1, k] = + rho;
  }
  return dirichlet_lpdf(p | alpha)
          + log_determinant(J);
}
```

### 1.3 Locating the Complete .stan Files

We did not include the complete ".stan" files used for our analyses to conserve space and ease of reading. The files are available as part of the accompanying OSF repository (`https://osf.io/kpvwb/?view_only=fb9d9dcc743c4769aecade575284b9c0`). The files included in this repository are

- "ifa_idp.stan": This file contains the code needed to run the proposed prior distribution in a multigroup CFA model with categorical indicators. This implementation of the multigroup CFA model imposes the theta-parameterization within each group separately. An alternative implementation discussed in section 3 relaxes this assumption.
- "ifa_sen.stan": This file contains the code needed to run the sequential exponential of normal prior that is the default in Blavaan and Mplus.

- "applied_example_gwp.Rmd": The Rmarkdown file that was used to coordinate the running of the two scripts above for the GWP analyses.

## 2 Deriving informative prior for sequentially defined thresholds

Placing informative priors on individuals thresholds is not entirely obvious. For instance, consider the situation with an assessment where items are scored with four ordered categories, and previous results are available to create informative priors for the thresholds. The reported threshold point estimates for one of the items are $-2.00$, $-0.25$, and $1.75$. An informative prior for the first thresholds is straightforward,

$$\tau_1 \sim \text{Normal}(-2.00, 0.20).$$

An informative prior for the second threshold is unfortunately not simply $\tau_2 \sim \text{Normal}(-0.25, 0.25)$, but is instead specified *relative* to threshold 1. The informative prior for $\tau_2$ needs to be constructed using the solution to the following system.

1. We need to identify the distribution of $\tau_2^*$ such that $\tau_2 \sim \text{Normal}(-0.25, 0.25)$.
2. Rewrite $\tau_2 = \tau_1 + exp(\tau_2^*)$ in terms of the expected value and variances of each element, taking advantage of the fact that $exp(\tau_2^*) \sim \text{logNormal}(.)$.

$$E[\tau_2] = E[\tau_1] + exp\left(E[\tau_2^*] + \frac{Var[\tau_2^*]}{2}\right)$$
$$Var[\tau_2] = Var[\tau_1] + \left(exp\left(Var[\tau^*]\right) - 1\right) exp\left(2E[\tau_2^*] + Var[\tau_2^*]\right),\ \tau_1 \perp \tau_2^*,$$

3. The prior we need to specify to obtain the desired $E[\tau_2]$ and $Var[\tau_2]$ is then the solution system of equations above for values of $E[\tau_2^*]$ and $Var[\tau_2^*]$ under the conditions that $E[\tau_1] < E[\tau_2]$ and $Var[\tau_1] < Var[\tau_2]$. The solution is not immediately obvious but is analytically defined.
4. The solution (after some algebra) is

$$E[\tau_c] > E[\tau_{c-1}]$$
$$Var[\tau_c] > Var[\tau_{c-1}]$$
$$E[\tau_c^*] = log\left(E[\tau_c] - E[\tau_{c-1}]\right) - \frac{Var[\tau_c^*]}{2}$$
$$Var[\tau_c^*] = log\left(\frac{Var[\tau_c] - Var[\tau_{c-1}] + exp\left(2log(E[\tau_c] - E[\tau_{c-1}])\right)}{exp\left(2log(E[\tau_c] - E[\tau_{c-1}])\right)}\right).$$

5. The resulting prior for $\tau_2^*$ one needs to use to obtain the prior we want to place on $\tau_2$ is

$$\tau_2^* \sim \text{Normal}(0.55, 0.02).$$

A major caveat is that the resulting prior on $\tau_2$ is only approximately normally distributed (the larger the variance of $\tau_2^*$ is, the more asymmetric the prior on $\tau_2$ will be), though we will have the desired expected value and variance.

# 3 Monte Carlo Simulation Results Expanded

This section expands on the results presented in text of the Monte Carlo simulation study on the estimation performance between different prior distributions for thresholds. First, I provide a richer description of the simulation conditions. Then, the results are provided in deeper detail for individual cells of the design along with estimates of effect sizes.

## 3.1 Simulation Conditions

A Monte Carlo simulation study was conducted to evaluate the estimation performance of the proposed prior structure compared to the existing specifications. The proposed design of the Monte Carlo simulation systematically varies the distribution of the indicators (i.e., symmetric of category endorsement), the number of response categories (i.e., 3, 4, 5, 6), and sample size (i.e., 150, 500). The proposed set of response distribution conditions is shown in Figure 1. For the sparse category condition, three additional conditions were added: 1) all items contain a category without endorsement, 2) half of all items are simulated with a category without endorsement, and 3) each factor contains one item with a category without endorsement. The non-sparse items in these conditions were simulated using the asymmetric distribution condition. The number of response categories and sample size conditions were determined based on a recent literature review by Padgett (2022) which indicated that these values are commonly observed in applied research in psychological assessment. The results of the review by Padgett (2022) are described in more detail below.

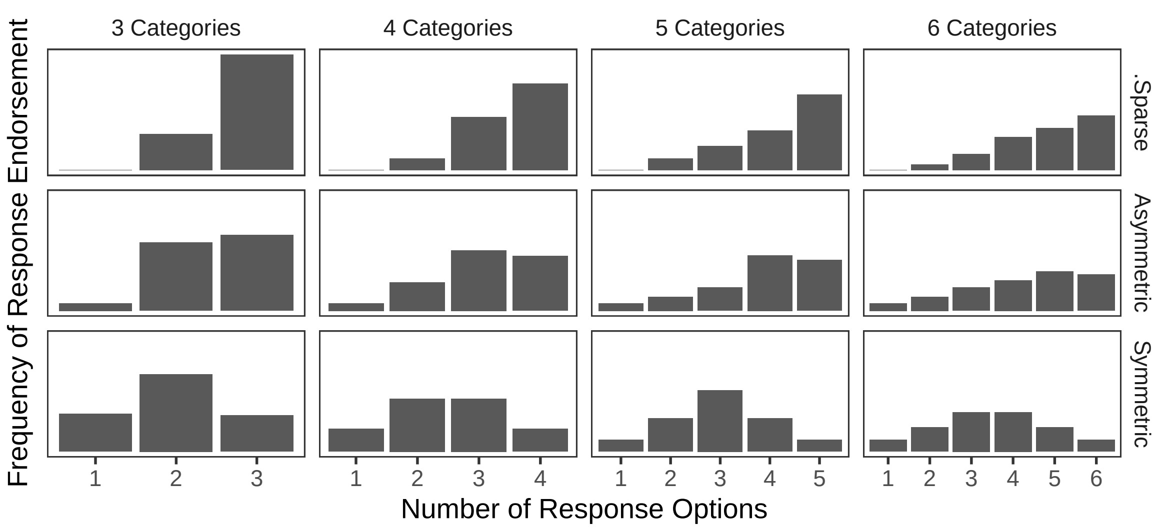



Figure 1: Category response distributions simulated in Monte Carlo study

A small subset of education and psychology journals were used in this review where researchers commonly use latent variable models to help answer their research questions. The journals selected for this review are: *Assessment*, *Journal of Psychoeducational Assessment*, *Psychological Assessment*, and *Individual Differences*. The applied literature review was kept focused to allow for an in-depth search of targeted journals. The search was be limited to a five-year range from January 2016 to December 2020. Because of the limited scope of the review, all articles were reviewed that were published within this range that match the key phrase of “("item factor analysis" OR "confirmatory factor analysis" OR "item response theory" OR "structural equation modeling" OR "covariance structure" OR "Rasch" OR "factor structure" OR "psychometric properties).”

The review focused on extracting the characteristics of the data. Specifically, the extracted characteristics were the journal and author information, sample size, the number of latent variables, the minimum factor loading magnitude, maximum factor loading magnitude, an average of reported factor loadings, reliability estimate(s), and how reliability was computed. In addition, characteristics of the observed indicators include the number of indicators per latent factor, the scale of the indicators (number of response options or continuous), and the distribution of the indicators.

The applied literature review provided a valuable source of information that provides evidence for the measurement models of interest in this research. The complete list of extracted information is compiled and freely available on the accompanying supplemental website. In this section, I describe the information extracted, which is summarized in Table 1.

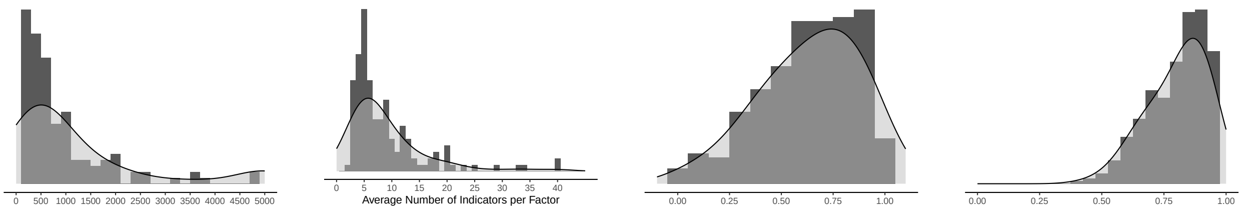
Table 1: Information extracted from studies reviewed

| Characteristic | Average | SD | Min | Median | Max |
|---|---|---|---|---|---|
| Sample Size | 4575.1 | 24451.4 | 36 | 603 | 263683 |
| Number of latent variables | 5.7 | 7.2 | 1.0 | 4.0 | 60.0 |
| Number of observed variables[a] | 52.42 | 83.5 | 5 | 24 | 442 |
| Number of indicators per factor | | | | | |
| Min | 7.1 | 6.6 | 1 | 5 | 40 |
| Avg | 9.3 | 7.2 | 2.7 | 6.7 | 40 |
| Max | 13.5 | 15.8 | 3 | 9 | 135[b] |
| Factor Loadings (Standardized) | | | | | |
| Min | 0.41 | 0.20 | 0.00 | 0.41 | 0.86 |
| Avg | 0.64 | 0.13 | 0.23 | 0.65 | 0.93 |
| Max | 0.85 | 0.10 | 0.49 | 0.87 | 1.08 |
| Reliability | | | | | |
| Min | .73 | .13 | .40 | .71 | .95 |
| Avg | .81 | .09 | .62 | .83 | .95 |
| Max | .88 | .08 | .67 | .90 | .98 |
| Proportion using McDonald’s $\omega$[c] | .16 | | | | |

*Note.* [a] Number of variables reportedly collected and analyzed in a measurement model. Many of the reported studies used the collected items to form sum-scores effectively reducing the number of “observed variables” to a small number of construct-specific scores to use in a CFA of those sum-scores. [b] The paper reporting the use of a latent variable with 135 indicators used the Coding subtest of Wechsler Adult Intelligence Scale (WAIS-IV). [c] Proportion using McDonald’s $\omega$ is conditional on whether reliability was reported.

The data conditions described in Table 1 focus on the measurement model of the latent trait. The measurement models for simulation studies 1 and 2 were restricted to a single construct to limit model complexity. However, applied studies commonly use four (median) to six (average of 5.7) latent variables in their studies. As a result, the sample size was highly variable, as shown in Figure 2(a). In addition, the number of indicators per factor varied greatly from 1-135 in the studies examined. The distribution of the number of indicators per factor is shown in Figure 2(b), demonstrating how the conditions simulated fall within the range many researchers encounter when using latent variable models. Similar conclusions about the factor loadings and reported reliability estimates could be drawn from Figure 2(c) and 2(d), respectively.

Additionally, the scale of the indicators was also extracted for each study. Twenty-two percent of reviewed studies reported using more than one survey where the observed indicators had a different




(a) Sample Size (b) Indicators (c) Factor Loadings (d) Reliability

Figure 2: Distribution of measurement model characteristics.

response scale (i.e., dichotomous and five response options). Studies reporting collecting data using dichotomous indicators (15%), three-(6%), four-(28%), five-(47%), six-(5%), and seven-point (17%) Likert-type response scales. Researchers reported using the total scores in 10% of studies instead of the item-level responses as part of their unit of analysis. The frequency of use of the response scales observed in applied studies shows that focusing the simulation study on dichotomous and five-point response scales aligns with over half of the applied studies reviewed.

A note about the estimation... The posterior distributions were sampled using Stan with four chains and 2000 iterations per chain. The first 1000 iterations were discarded as warm-up. The results of the simulation will focus on the coverage rates and interval widths of 95% credible intervals for all parameters and posterior convergence measured by the effective sample size and $\hat{R}$. Coverage rates were computed by summarizing the frequency with which the true value lay within the central 95% quantiles of the posterior distributions. The interval widths were computed using the difference between the 97.5-and 2.5 percentiles of the posterior distributions. Evaluating coverage rates and interval widths informed how well the posterior distributions captured the population parameters. Preferred are credible intervals with high coverage rates (≈95%) with shorter average interval widths. The results of the Monte Carlo simulation were summarized by averaging over conditions to obtain the average coverage rates and the average credible interval widths across conditions. The complete results describing convergence, coverage rates, and credible interval width are reported our supplemental material.

## 3.2 Simulation Results

### 3.2.1 Posterior Convergence

A summary of convergence details in all conditions of this study is available in the OSF repository in the file "Simulation Convergence Summary.csv".

| prior | rhat_avg | rhat_prop_leq1.1 | rhat_prop_leq1.05 | ess_bulk_avg | ess_tail_avg |
|---|---|---|---|---|---|
| Joint | 1.01 | 0.99 | 0.98 | 2479.90 | 2049.71 |
| Small Variance | 1.00 | 0.99 | 0.99 | 2470.09 | 2057.42 |
| Large Variance | 1.15 | 0.64 | 0.54 | 864.30 | 836.23 |

Table 2: Design main effects on posterior convergence (is $\hat{R} < 1.1$)

| | Estimate | Std. Error | z value | Pr(>\|z\|) | OR | ll | ul |
|---|---|---|---|---|---|---|---|
| (Intercept) | 4.29 | 0.07 | 63.41 | 0.00 | 73.16 | 64.19 | 83.71 |
| dist_spar | 0.48 | 0.08 | 6.01 | 0.00 | 1.62 | 1.38 | 1.89 |
| prior_joint | 0.10 | 0.09 | 1.17 | 0.24 | 1.11 | 0.93 | 1.31 |
| prior_large | -0.05 | 0.08 | -0.63 | 0.53 | 0.95 | 0.81 | 1.12 |
| sample_size500 | -0.04 | 0.01 | -3.38 | 0.00 | 0.96 | 0.93 | 0.98 |
| n_cat4 | 0.38 | 0.02 | 18.09 | 0.00 | 1.46 | 1.40 | 1.52 |
| n_cat5 | 0.73 | 0.02 | 36.55 | 0.00 | 2.08 | 2.00 | 2.17 |
| n_cat6 | 0.98 | 0.02 | 49.98 | 0.00 | 2.67 | 2.57 | 2.77 |
| dist_asym | 0.74 | 0.07 | 10.03 | 0.00 | 2.11 | 1.82 | 2.44 |
| sparse_ref | -0.04 | 0.02 | -2.85 | 0.00 | 0.96 | 0.93 | 0.99 |
| dist_spar:prior_joint | -1.29 | 0.10 | -12.61 | 0.00 | 0.27 | 0.22 | 0.34 |
| dist_spar:prior_large | -5.33 | 0.10 | -55.26 | 0.00 | 0.00 | 0.00 | 0.01 |

Table 3: Design main effects on threshold posterior convergence (is $\hat{R} < 1.1$)

| | Estimate | Std. Error | z value | Pr(>\|z\|) | OR | ll | ul |
|---|---|---|---|---|---|---|---|
| (Intercept) | 5.60 | 0.13 | 42.30 | 0.00 | 271.63 | 211.32 | 355.47 |
| dist_spar | 0.39 | 0.16 | 2.46 | 0.01 | 1.48 | 1.08 | 2.02 |
| prior_joint | -0.02 | 0.17 | -0.09 | 0.93 | 0.99 | 0.70 | 1.39 |
| prior_large | -0.62 | 0.15 | -4.09 | 0.00 | 0.54 | 0.39 | 0.72 |
| sample_size500 | -0.05 | 0.02 | -3.46 | 0.00 | 0.95 | 0.92 | 0.98 |
| n_cat4 | 0.14 | 0.03 | 5.40 | 0.00 | 1.15 | 1.10 | 1.22 |
| n_cat5 | 0.43 | 0.03 | 16.89 | 0.00 | 1.53 | 1.46 | 1.61 |
| n_cat6 | 0.60 | 0.02 | 24.33 | 0.00 | 1.82 | 1.73 | 1.91 |
| dist_asym | 0.87 | 0.14 | 6.34 | 0.00 | 2.38 | 1.83 | 3.13 |
| sparse_ref | -0.03 | 0.02 | -1.86 | 0.06 | 0.97 | 0.93 | 1.00 |
| dist_spar:prior_joint | -1.92 | 0.20 | -9.59 | 0.00 | 0.15 | 0.10 | 0.22 |
| dist_spar:prior_large | -5.38 | 0.18 | -30.04 | 0.00 | 0.00 | 0.00 | 0.01 |

### 3.2.2 Posterior Coverage and Bias

A summary of coverage and bias evaluation in all conditions of this study is available in the OSF repository in the file "Simulation Coverage and Relative Bias Summary.csv".

Table 4: Posterior credible interval average coverage rates

| Parameter Grp | Joint | Small Variance | Large Variance |
|---|---|---|---|
| Loading | 0.935 | 0.935 | 0.893 |
| Factor Variance | 0.881 | 0.885 | 0.847 |
| Factor Covariance | 0.876 | 0.868 | 0.850 |
| Thresholds | 0.781 | 0.804 | 0.792 |
| Fixed Threshold | 0.011 | 0.000 | 0.897 |

Table 5: Posterior mean relative bias and sampling variability

| parameter_grp | prior | true | est_avg | sd_avg | avg_rb_avg | avg_rmse | avg_sq_bias | avg_samp_var |
|---|---|---|---|---|---|---|---|---|
| Loading | Joint | 1.00 | 1.13 | 0.25 | -27.82 | 1.02 | 0.89 | 0.89 |
| Loading | Small Variance | 1.00 | 1.13 | 0.25 | -28.56 | 1.01 | 0.88 | 0.88 |
| Loading | Large Variance | 1.00 | 1.12 | 0.23 | 4.32 | 0.35 | 0.24 | 0.24 |
| Factor Variance | Joint | 1.00 | 0.86 | 0.25 | -21.03 | 0.12 | 0.08 | 0.08 |
| Factor Variance | Small Variance | 1.00 | 0.86 | 0.25 | -22.75 | 0.13 | 0.08 | 0.08 |
| Factor Variance | Large Variance | 1.00 | 0.92 | 0.25 | -10.09 | 0.10 | 0.03 | 0.03 |
| Factor Covariance | Joint | 0.23 | 0.19 | 0.08 | -28.71 | 0.01 | 0.01 | 0.01 |
| Factor Covariance | Small Variance | 0.23 | 0.18 | 0.08 | -32.10 | 0.01 | 0.01 | 0.01 |
| Factor Covariance | Large Variance | 0.23 | 0.19 | 0.08 | -20.78 | 0.01 | 0.00 | 0.00 |
| Thresholds | Joint | -0.76 | -0.86 | 0.19 | 24.24 | 35.57 | 26.47 | 26.47 |
| Thresholds | Small Variance | -0.76 | -0.87 | 0.17 | 1.53 | 0.25 | 0.22 | 0.22 |
| Thresholds | Large Variance | -0.76 | -2.42 | 1.93 | 58.64 | 141.77 | 64.83 | 64.83 |
| Fixed Threshold | Joint | -15.00 | -4.74 | 1.29 | -48.39 | 459.57 | 312.38 | 312.38 |
| Fixed Threshold | Small Variance | -15.00 | -3.87 | 0.55 | -74.26 | 124.22 | 124.19 | 124.19 |
| Fixed Threshold | Large Variance | -15.00 | -61.06 | 63.77 | 152.26 | 3694.46 | 1595.78 | 1595.78 |

Table 6: Design main effects on posterior coverage (does 95% CI contain true value?)

| | Estimate | Std. Error | z value | Pr(>\|z\|) | OR | ll | ul |
|---|---|---|---|---|---|---|---|
| (Intercept) | 4.12 | 0.03 | 147.20 | 0.00 | 61.66 | 58.38 | 65.15 |
| dist_spar | -1.98 | 0.02 | -82.68 | 0.00 | 0.14 | 0.13 | 0.15 |
| prior_joint | -0.17 | 0.03 | -6.64 | 0.00 | 0.84 | 0.80 | 0.88 |
| prior_large | 0.03 | 0.03 | 1.22 | 0.22 | 1.03 | 0.98 | 1.09 |
| sample_size500 | 0.04 | 0.01 | 4.74 | 0.00 | 1.04 | 1.02 | 1.06 |
| n_cat4 | -0.02 | 0.01 | -1.36 | 0.17 | 0.98 | 0.96 | 1.01 |
| n_cat5 | 0.11 | 0.01 | 8.67 | 0.00 | 1.12 | 1.09 | 1.15 |
| n_cat6 | -0.05 | 0.01 | -4.34 | 0.00 | 0.95 | 0.93 | 0.97 |
| dist_asym | -0.17 | 0.02 | -7.61 | 0.00 | 0.85 | 0.81 | 0.88 |
| sparse_ref | -1.35 | 0.01 | -100.73 | 0.00 | 0.26 | 0.25 | 0.27 |
| dist_spar:prior_joint | 0.08 | 0.03 | 2.73 | 0.01 | 1.08 | 1.02 | 1.14 |
| dist_spar:prior_large | 0.05 | 0.03 | 1.54 | 0.12 | 1.05 | 0.99 | 1.11 |

Table 7: Design main effects on threshold posterior coverage (does 95% CI contain true value?)

| | Estimate | Std. Error | z value | Pr(>\|z\|) | OR | ll | ul |
|---|---|---|---|---|---|---|---|
| (Intercept) | 4.68 | 0.04 | 133.62 | 0.00 | 107.55 | 100.44 | 115.22 |
| dist_spar | -2.22 | 0.03 | -76.59 | 0.00 | 0.11 | 0.10 | 0.12 |
| prior_joint | -0.25 | 0.03 | -7.98 | 0.00 | 0.78 | 0.73 | 0.83 |
| prior_large | 0.08 | 0.03 | 2.28 | 0.02 | 1.08 | 1.01 | 1.15 |
| sample_size500 | -0.02 | 0.01 | -2.24 | 0.03 | 0.98 | 0.96 | 1.00 |
| n_cat4 | -0.03 | 0.02 | -1.57 | 0.12 | 0.97 | 0.94 | 1.01 |
| n_cat5 | 0.19 | 0.02 | 11.63 | 0.00 | 1.21 | 1.17 | 1.25 |
| n_cat6 | 0.03 | 0.02 | 2.10 | 0.04 | 1.03 | 1.00 | 1.06 |
| dist_asym | -0.25 | 0.03 | -9.40 | 0.00 | 0.78 | 0.74 | 0.82 |
| sparse_ref | -1.82 | 0.02 | -103.76 | 0.00 | 0.16 | 0.16 | 0.17 |
| dist_spar:prior_joint | 0.10 | 0.03 | 2.99 | 0.00 | 1.11 | 1.04 | 1.18 |
| dist_spar:prior_large | -0.19 | 0.04 | -5.24 | 0.00 | 0.83 | 0.77 | 0.89 |

### 3.2.3 Posterior Credible Interval Widths

The breakdown of credible interval widths across all conditions of this study is available in the OSF repository in the file "Simulation Coverage and Relative Bias Summary.csv".

Additionally, the *relative widths* of the 95% posterior credible interval across all parameters among the three prior specifications are provided in the file "Simulation Summary of Relative Efficiency of CI.csv". An evaluation of the relative efficiency, as measured by the relative widths of the resulting posterior distributions provided evidence of which prior specification results in narrow interval estimates.

Table 8: Design effects on posterior credible interval widths

| | $\omega^2$ | $\omega_p^2$ | $\eta$ |
|---:|---:|---:|---:|
| prior | 0.018 | 0.027 | 0.018 |
| parameter_grp | 0.109 | 0.141 | 0.109 |
| sample_size | 0.001 | 0.001 | 0.001 |
| n_cat | 0.002 | 0.003 | 0.002 |
| dist | 0.001 | 0.001 | 0.001 |
| nsparse | 0.004 | 0.005 | 0.004 |
| sparse_ref_ind | 0.000 | 0.000 | 0.000 |
| prior:parameter_grp | 0.201 | 0.233 | 0.201 |

Table 9: Average relative width of posterior CIs among prior specifications

| parameter_grp | Small Variance / Joint | Large Variance / Joint | Large Variance / Small Variance |
|---|---:|---:|---:|
| Factor Covariance | 1.00 | 1.02 | 1.02 |
| Factor Variance | 1.01 | 0.99 | 0.98 |
| Loading | 1.11 | 1.04 | 1.04 |
| Thresholds | 1.01 | 3.01 | 3.67 |
| Fixed Threshold | 0.74 | 72.44 | 96.95 |

# 4 Multigroup Extension and Identification

## 4.1 Identification strategy for thresholds

A multigroup CFA/IFA model with categorical indicators can be specified in several ways to identify parameter estimates (Wu and Estabrook, 2016). Thresholds are commonly identified by using the theta parameterization where the latent response intercepts ($\boldsymbol{\nu}$) are fixed to zero, and the latent response residual variances ($\boldsymbol{\theta}$) are fixed to one. In a multigroup model, these constraints are usually imposed separately within each group to achieve identified thresholds in each group. A difficulty is that the thresholds are then not comparable due to using arbitrary fixed values within each group. In a frequentist estimation approach using ML or DWLS, imposing these constraints separately within each group is necessary. However, in a Bayesian approach, imposing arbitrary fixed values are only necessary within a single "reference" group. In the other group(s), relatively informative priors can be specified that constrain the posterior to be in a logical range. Identification within a Bayesian model implies that the posterior distribution converges in distribution. The convergence can be assessed by inspecting the posterior (e.g., a histogram), the effective sample size, and the $\hat{R}$ of the parameter.

The multigroup model for $g = 1, ..., G$ groups with $n = 1, ..., N$ respondents can be specified as follows to estimate a categorical factor model with freely estimated thresholds in each group. The marginal (over latent variable $\boldsymbol{\eta} \sim \text{MVN}(\mathbf{0}, \boldsymbol{\Phi})$) factor model can be described as

$$\mathbf{y}^*_{gn} \sim \text{MVN}\left(\boldsymbol{\nu}_g, \boldsymbol{\Lambda}_g\boldsymbol{\Phi}_g\boldsymbol{\Lambda}'_g + \boldsymbol{\Theta}_g\right)$$

For identification, the following constraints were used for the latent response distributions for each item.

$$\begin{aligned}
&\text{Group 1}\\
&\qquad \nu = 0\\
&\qquad \theta = 1\\
&\text{Group 2}\\
&\qquad \nu \sim \text{Normal}(0, 0.25)\\
&\qquad \theta \sim \text{Cauchy}^+(1, 0.25)
\end{aligned}$$

Using the informative priors results in constrained posteriors for the latent response distributions that may update based on the data. The posteriors for the thresholds, latent response intercepts, and latent response variances provide an avenue for obtaining comparable thresholds between groups without relying on unnecessarily strict assumptions. Using relatively informative priors for the commonly unidentified parameters allows for uncertainty in these parameters to be incorporated into the analysis. The above modifications are incorporated into the model as shown below in an exert from our modified stan file: "mg_ifa_free_lr.stan".

```
parameters {
  ordered[nlevs-1] tau[Ng,D]; //threshold parameters
  vector[D] lambda_fr[Ng]; //factor loadings
  real<lower=0, upper=1> u[N, D]; // nuisance that absorbs inequality constraints
  cholesky_factor_corr[m] L_Psi[Ng];
  vector<lower=0>[m] L_Psi_sd[Ng];
  vector<lower=0>[D] theta_sd[Ng-1];
  vector<lower=-5,upper=5>[D] nu[Ng-1];
}
transformed parameters {
```

```
  matrix[D,m] Lambda[Ng];
  matrix[m,m] Psi[Ng];
  matrix[m,m] L_Psi_full[Ng];
  matrix[D,D] Sigma[Ng];
  matrix[D,D] L_Sigma[Ng];
  vector[D] Theta_sd[Ng];
  vector[D] Nu[Ng];
  for(g in 1:Ng){
    for(d in 1:D){
      for(mm in 1:m){
        Lambda[g,d,mm] = pos_mat[d,mm]*lambda_fr[g,d];
      }
      if(g == 1){
        Theta_sd[g,d] = 1;
        Nu[g,d] = 0;
      } else {
        Theta_sd[g,d] = theta_sd[g-1,d];
        Nu[g,d] = nu[g-1,d];
      }
    }
    L_Psi_full[g] = diag_pre_multiply(L_Psi_sd[g], L_Psi[g]);
    Psi[g] = multiply_lower_tri_self_transpose(L_Psi_full[g]);
    Sigma[g] = diag_matrix(to_vector(Theta_sd[g]));
    Sigma[g] = Sigma[g] + quad_form_sym(Psi[g], transpose(Lambda[g]));
    L_Sigma[g] = cholesky_decompose(Sigma[g]);
  }

}
model {
  for(n in 1:N)
    target += log(tmvn(y[n,1:D], to_vector(Nu[grpid[n]]), L_Sigma[grpid[n]], tau[
        grpid[n],], u[n,1:D], nlevs)[2]);

  for(g in 1:Ng){
    target += lkj_corr_cholesky_lpdf(L_Psi[g] | 4);
    target += cauchy_lpdf(L_Psi_sd[g] | 0, 2.5);
    for(d in 1:D){
      target += normal_lpdf(lambda_fr[g,d] | lambda_mu[d], lambda_sd[d]);
      target += induced_dirichlet_lpdf(tau[g,d,1:(nlevs-1)] | to_vector(idp_alpha[
          d]), 0);
    }
  }
  for(g in 1:(Ng-1)){
    target += cauchy_lpdf(theta_sd[g] | 1, 0.25);
    target += normal_lpdf(nu[g] | 0, 0.25);
  }
}
```

### 4.2 Alternative constrained thresholds approach

An alternative model is to constrain the thresholds to be equal across groups and then freely estimate the latent response intercepts and residual variances in all but the reference group. This is similar to a sequential model testing approach where the model fit of the threshold freely estimated model is compared to the constrained model with equal thresholds.

This approach is implemented in the file: "mg_ifa_free_lr_equal_thresholds.stan"

Another alternative approach is to constrain the first two thresholds of each item to be equal across groups, then freely estimate the latent response intercept and residual variance (Wu and Estabrook, 2016). This approach was not implemented due to the focus of this study evaluating the induced-

Dirichlet prior. In this prior specification of thresholds, a prior distribution is placed on the set of thresholds for each item. It is not possible from our understanding to implement partial constraints on thresholds across items; only the full set of thresholds can be equal as the distribution is induced based on the hyperparameter vector of relative weights of each category using $\boldsymbol{\alpha}$. Future work might be able to find a workaround for this limitation.

## 5 Gallup World Poll Analysis Expanded Results

Here, insert an expanded set of results based on the GWP to provide better contextual information about the results and information. As a reminder, in-text we supplied the observed response categories proportions (see Table 10).

### 5.1 Data Availability

Data for these analyses are available from Gallup for research purposes, but we cannot directly share the files. Interested readers should request to gain access to these data from Gallup at `https://www.gallup.com/analytics/468179/global-wellbeing-initiative-dataset.aspx`.

### 5.2 Background on Data

Norway and the United States are relatively comparable countries in terms of developmental status, and one might not generally expect any differences in the operating characteristics of the assessment tool across these two countries. The assessment of balance and harmony could be revealing as to potential nuanced differences in the flourishing of these two countries. Indeed, it is intriguing that across the eight items, if we take the percentage of respondents in each country that report experiencing the various aspects of balance and harmony either "always" or "often," while Norway does better overall (with higher levels of flourishing than the USA on six items), the USA nevertheless fares better on two of them. These include: amount of things happening in life is "just right" (Norway ranks 33rd out of 141 countries, with 72.7% answering either "always" or "often," versus USA at 25th with 74.3%); life in balance (Norway at 13th with 81.8% versus USA at 51st with 70.8%); harmony with those around (Norway at 3rd with 96.3% versus USA at 29th with 91.1%); thoughts and feelings in harmony (Norway at 12th with 88.1% versus USA at 26th with 80.0%); feel stable and secure (Norward at 3rd with 93.1% versus USA at 18th with 89.0%), content (Norway 13th with 90.4% versus USA at 27th with 86.5%); mind at ease (Norway at 27th with 81.3% versus USA at 49th with 75.9%); and inner peace (Norway at 47th with 73.4% versus USA at 24th with 79.1%).

However, such nuances aside, the sum of responses (total score) by individuals is not significantly different on average between countries ($t(1878.2) = 0.91, p = .418$). Once we dive into these data, we see that the item "Harmony Around" (*You Are in Harmony With Those Around You*), we see that the Norway sample did not endorse *Never* (see Table 10). Leading to a sparse response distribution that would limit our ability to compare the invariance of the measurement properties more thoroughly. As mentioned in the introduction, we could collapse the categories of *Never* and *Rarely* into one category and then collapse *Often* and *Always* into another. However, collapsing categories and retesting the difference in total score changes our interpretation from "no difference" to a "significance difference" ($t(1935.6) = 2.9804, p = .002$). A dependence on interpretation based solely on how the responses are coded is not ideal, and this issue becomes exacerbated when one moves to analyze all the countries simultaneously.

Table 10: Response distribution of balance and harmony items in Norway and US samples

| | Norway | | | | United States | | | |
|---|---|---|---|---|---|---|---|---|
| Item | *Never* | *Rarely* | *Often* | *Always* | *Never* | *Rarely* | *Often* | *Always* |
| Balance | 11 | 166 | 703 | 96 | 44 | 241 | 554 | 137 |
| Amount Right | 28 | 237 | 601 | 110 | 58 | 194 | 561 | 163 |
| Harmony Around | 0* | 36 | 753 | 187 | 14 | 71 | 673 | 218 |
| Thoughts Harmony | 10 | 105 | 750 | 111 | 19 | 119 | 667 | 171 |
| Stable | 7 | 50 | 470 | 449 | 21 | 85 | 520 | 350 |
| Content | 8 | 85 | 754 | 129 | 31 | 101 | 592 | 252 |
| Mind Ease | 17 | 163 | 655 | 141 | 52 | 182 | 611 | 131 |
| Inner Peace | 27 | 234 | 610 | 105 | 29 | 172 | 584 | 191 |
| Total Score | 24.0 (3.10) | | | | 23.9 (3.78) | | | |

*Note.* *The full dataset had a single case endorsing *Never*, but we removed that single case for this study for demonstration purposes. *Never*=1, *Rarely*=2, *Often*=3, and *Always*=4; $t$-test comparing total (sum) score between countries was not significant ($t(1878.2) = 0.91, p = .42$).

## 5.3 Single-Group IFA Models

First, we provide the results of a categorical CFA model as all these data are from a single group. This is an oversimplification of the model to provide a baseline to demonstrate how the estimates change. The results are presented first within a frequentist estimation, using DWLS, in lavaan (Rosseel, 2012).

### 5.3.1 Single Group Analysis Model Fit Summary

The summary indices of the resulting model fit statistics for all models of the single group analyses are provided in Table 11.

Table 11: Summary of fit indices from single group analyses

| Country | Fit Indices | | | | |
|---|---|---|---|---|---|
| Frequentist (DWLS) | | | | | |
| | $\chi^2(20)$ | $p$ | RMSEA | CFI | TLI | SRMR |
| US | 182.9 | <.001 | .091 | .977 | .967 | .036 |
| Norway | 167.4 | <.001 | .087 | .981 | .973 | .040 |

### 5.3.2 Results of IFA (DWLS) for US Sample

The model, in lavaan syntax, is

```
mod1 <- "
bh =~ 1*balance + amount_right + harmony_around + thoughts_harmony + stable +
    content + mind_ease + inner_peace
"

fit1 <- lavaan::sem(
  mod1, data=data %>% filter(COUNTRY == "United States"),
  ordered=T, parameterization="theta"
)
summary(fit1, standardized=T, fit.measures=T)
```

The model had a significant $\chi^2$-test but fit moderately well according to the other fit indices. For the US sample, $\chi^2(20) = 182.9, p < .001, RMSEA = .091, CFI = .977, TLI = .967, SRMR = .036$. The complete model results, including the fully standardized solution, for the US sample are presented in Table 12.

Table 12: Results of estimated US sample

| | | | Unstandardized | | | | Standardized | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| lhs | op | rhs | est | se | Q.025 | Q.975 | est.std | se | Q.025 | Q.975 |
| bh | =~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 0.59 | 0.02 | 0.55 | 0.63 |
| bh | =~ | amo_ri | 0.78 | 0.06 | 0.66 | 0.89 | 0.49 | 0.03 | 0.44 | 0.55 |
| bh | =~ | har_ar | 0.93 | 0.07 | 0.80 | 1.07 | 0.57 | 0.02 | 0.52 | 0.61 |
| bh | =~ | tho_ha | 1.90 | 0.13 | 1.65 | 2.16 | 0.81 | 0.01 | 0.78 | 0.84 |
| bh | =~ | stable | 1.74 | 0.11 | 1.52 | 1.96 | 0.79 | 0.02 | 0.76 | 0.82 |
| bh | =~ | content | 1.91 | 0.12 | 1.67 | 2.14 | 0.81 | 0.01 | 0.79 | 0.84 |
| bh | =~ | min_ea | 1.98 | 0.13 | 1.72 | 2.23 | 0.82 | 0.01 | 0.80 | 0.85 |
| bh | =~ | inn_pe | 1.42 | 0.09 | 1.24 | 1.60 | 0.72 | 0.02 | 0.69 | 0.76 |
| balance | \| | t1 | -2.10 | 0.08 | -2.26 | -1.94 | -1.69 | 0.07 | -1.83 | -1.56 |
| balance | \| | t2 | -0.68 | 0.05 | -0.78 | -0.58 | -0.55 | 0.04 | -0.63 | -0.46 |
| balance | \| | t3 | 1.34 | 0.06 | 1.22 | 1.46 | 1.08 | 0.05 | 0.98 | 1.18 |
| amo_ri | \| | t1 | -1.79 | 0.07 | -1.94 | -1.65 | -1.56 | 0.06 | -1.69 | -1.43 |
| amo_ri | \| | t2 | -0.75 | 0.05 | -0.84 | -0.65 | -0.65 | 0.04 | -0.73 | -0.56 |
| amo_ri | \| | t3 | 1.11 | 0.06 | 1.00 | 1.22 | 0.97 | 0.05 | 0.87 | 1.06 |
| har_ar | \| | t1 | -2.65 | 0.13 | -2.90 | -2.41 | -2.19 | 0.10 | -2.39 | -1.98 |
| har_ar | \| | t2 | -1.65 | 0.07 | -1.78 | -1.51 | -1.36 | 0.06 | -1.47 | -1.25 |
| har_ar | \| | t3 | 0.92 | 0.05 | 0.82 | 1.03 | 0.76 | 0.04 | 0.67 | 0.85 |
| tho_ha | \| | t1 | -3.54 | 0.17 | -3.88 | -3.20 | -2.06 | 0.09 | -2.25 | -1.88 |
| tho_ha | \| | t2 | -1.84 | 0.09 | -2.02 | -1.66 | -1.07 | 0.05 | -1.17 | -0.98 |
| tho_ha | \| | t3 | 1.60 | 0.08 | 1.44 | 1.76 | 0.93 | 0.05 | 0.84 | 1.03 |
| stable | \| | t1 | -3.28 | 0.14 | -3.55 | -3.02 | -2.02 | 0.09 | -2.20 | -1.85 |
| stable | \| | t2 | -2.00 | 0.09 | -2.19 | -1.82 | -1.23 | 0.05 | -1.34 | -1.13 |
| stable | \| | t3 | 0.59 | 0.07 | 0.46 | 0.72 | 0.36 | 0.04 | 0.28 | 0.44 |
| content | \| | t1 | -3.19 | 0.13 | -3.44 | -2.93 | -1.86 | 0.08 | -2.01 | -1.70 |
| content | \| | t2 | -1.89 | 0.10 | -2.08 | -1.71 | -1.10 | 0.05 | -1.20 | -1.00 |
| content | \| | t3 | 1.12 | 0.07 | 0.97 | 1.26 | 0.65 | 0.04 | 0.56 | 0.73 |
| min_ea | \| | t1 | -2.84 | 0.12 | -3.08 | -2.61 | -1.61 | 0.07 | -1.74 | -1.48 |
| min_ea | \| | t2 | -1.24 | 0.08 | -1.40 | -1.09 | -0.71 | 0.04 | -0.79 | -0.62 |
| min_ea | \| | t3 | 1.95 | 0.09 | 1.77 | 2.12 | 1.11 | 0.05 | 1.01 | 1.21 |
| inn_pe | \| | t1 | -2.72 | 0.12 | -2.95 | -2.49 | -1.89 | 0.08 | -2.04 | -1.73 |
| inn_pe | \| | t2 | -1.18 | 0.07 | -1.32 | -1.05 | -0.82 | 0.05 | -0.91 | -0.73 |
| inn_pe | \| | t3 | 1.24 | 0.07 | 1.11 | 1.36 | 0.86 | 0.05 | 0.77 | 0.95 |
| balance | ~~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 0.65 | 0.03 | 0.60 | 0.70 |
| amo_ri | ~~ | amo_ri | 1.00 | 0.00 | 1.00 | 1.00 | 0.76 | 0.03 | 0.70 | 0.81 |
| har_ar | ~~ | har_ar | 1.00 | 0.00 | 1.00 | 1.00 | 0.68 | 0.03 | 0.63 | 0.73 |
| tho_ha | ~~ | tho_ha | 1.00 | 0.00 | 1.00 | 1.00 | 0.34 | 0.02 | 0.30 | 0.38 |
| stable | ~~ | stable | 1.00 | 0.00 | 1.00 | 1.00 | 0.38 | 0.02 | 0.33 | 0.43 |
| content | ~~ | content | 1.00 | 0.00 | 1.00 | 1.00 | 0.34 | 0.02 | 0.30 | 0.38 |
| min_ea | ~~ | min_ea | 1.00 | 0.00 | 1.00 | 1.00 | 0.32 | 0.02 | 0.28 | 0.36 |
| inn_pe | ~~ | inn_pe | 1.00 | 0.00 | 1.00 | 1.00 | 0.48 | 0.03 | 0.43 | 0.53 |
| bh | ~~ | bh | 0.54 | 0.06 | 0.42 | 0.65 | 1.00 | 0.00 | 1.00 | 1.00 |

Table 13: Residual correlations of U.S. group

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| balance | | 0.15 | -0.01 | -0.02 | -0.03 | -0.01 | -0.02 | -0.02 |
| amo_ri | 0.15 | | -0.03 | -0.02 | -0.05 | -0.04 | -0.00 | -0.03 |
| har_ar | -0.01 | -0.03 | | 0.08 | -0.02 | 0.00 | -0.04 | -0.04 |
| tho_ha | -0.02 | -0.02 | 0.08 | | -0.03 | -0.02 | -0.01 | 0.03 |
| stable | -0.03 | -0.05 | -0.02 | -0.03 | | 0.05 | 0.02 | -0.03 |
| content | -0.01 | -0.04 | 0.00 | -0.02 | 0.05 | | -0.01 | -0.01 |
| min_ea | -0.02 | -0.00 | -0.04 | -0.01 | 0.02 | -0.01 | | 0.02 |
| inn_pe | -0.02 | -0.03 | -0.04 | 0.03 | -0.03 | -0.01 | 0.02 | |

#### 5.3.3 Results of IFA (DWLS) for Norway Sample

The model, in lavaan syntax, is

```
mod1 <- "
bh =~ 1*balance + amount_right + harmony_around + thoughts_harmony + stable +
    content + mind_ease + inner_peace
"

fit1 <- lavaan::sem(
  mod1, data=data %>% filter(COUNTRY == "Norway"),
  ordered=T, parameterization="theta"
)
summary(fit1, standardized=T, fit.measures=T)
```

The model had a significant $\chi^2$-test but fit moderately well according to the other fit indices. For the Norway sample, $\chi^2(20) = 167.4, p < .001, RMSEA = .087, CFI = .981, TLI = .973, SRMR = .040$. The complete model results, including the fully standardized solution, for the Norway sample are presented in Table 14.

Table 14: Results of estimated Norway sample

| | | | Unstandardized | | | | Standardized | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| lhs | op | rhs | est | se | Q.025 | Q.975 | est.std | se | Q.025 | Q.975 |
| bh | =~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 0.78 | 0.02 | 0.75 | 0.81 |
| bh | =~ | amo_ri | 0.42 | 0.04 | 0.34 | 0.49 | 0.46 | 0.03 | 0.41 | 0.52 |
| bh | =~ | har_ar | 0.49 | 0.04 | 0.41 | 0.57 | 0.52 | 0.03 | 0.47 | 0.58 |
| bh | =~ | tho_ha | 1.40 | 0.11 | 1.18 | 1.63 | 0.87 | 0.01 | 0.84 | 0.90 |
| bh | =~ | stable | 0.74 | 0.05 | 0.64 | 0.84 | 0.68 | 0.02 | 0.64 | 0.72 |
| bh | =~ | content | 1.15 | 0.09 | 0.98 | 1.31 | 0.82 | 0.02 | 0.79 | 0.85 |
| bh | =~ | min_ea | 1.29 | 0.09 | 1.13 | 1.46 | 0.85 | 0.01 | 0.83 | 0.87 |
| bh | =~ | inn_pe | 0.85 | 0.06 | 0.74 | 0.97 | 0.73 | 0.02 | 0.70 | 0.77 |
| balance | \| | t1 | -3.66 | 0.17 | -4.00 | -3.32 | -2.28 | 0.11 | -2.51 | -2.06 |
| balance | \| | t2 | -1.46 | 0.08 | -1.62 | -1.30 | -0.91 | 0.05 | -1.00 | -0.82 |
| balance | \| | t3 | 2.07 | 0.10 | 1.87 | 2.27 | 1.29 | 0.06 | 1.18 | 1.40 |
| amo_ri | \| | t1 | -2.14 | 0.09 | -2.32 | -1.96 | -1.90 | 0.08 | -2.06 | -1.74 |
| amo_ri | \| | t2 | -0.69 | 0.05 | -0.78 | -0.59 | -0.61 | 0.04 | -0.69 | -0.52 |
| amo_ri | \| | t3 | 1.37 | 0.06 | 1.25 | 1.48 | 1.21 | 0.05 | 1.11 | 1.32 |
| har_ar | \| | t1 | -2.10 | 0.09 | -2.27 | -1.92 | -1.79 | 0.07 | -1.93 | -1.64 |
| har_ar | \| | t2 | 1.02 | 0.05 | 0.92 | 1.13 | 0.87 | 0.05 | 0.78 | 0.96 |
| tho_ha | \| | t1 | -4.69 | 0.29 | -5.26 | -4.11 | -2.32 | 0.12 | -2.55 | -2.09 |
| tho_ha | \| | t2 | -2.40 | 0.14 | -2.67 | -2.12 | -1.19 | 0.05 | -1.29 | -1.08 |
| tho_ha | \| | t3 | 2.44 | 0.14 | 2.17 | 2.71 | 1.21 | 0.05 | 1.10 | 1.31 |
| stable | \| | t1 | -3.34 | 0.18 | -3.70 | -2.98 | -2.45 | 0.14 | -2.71 | -2.18 |
| stable | \| | t2 | -2.14 | 0.09 | -2.32 | -1.95 | -1.57 | 0.06 | -1.69 | -1.44 |
| stable | \| | t3 | 0.14 | 0.05 | 0.03 | 0.24 | 0.10 | 0.04 | 0.02 | 0.18 |
| content | \| | t1 | -4.20 | 0.24 | -4.66 | -3.74 | -2.40 | 0.13 | -2.65 | -2.15 |
| content | \| | t2 | -2.29 | 0.12 | -2.53 | -2.05 | -1.31 | 0.06 | -1.42 | -1.20 |
| content | \| | t3 | 1.95 | 0.11 | 1.74 | 2.17 | 1.12 | 0.05 | 1.02 | 1.22 |
| min_ea | \| | t1 | -4.02 | 0.20 | -4.41 | -3.63 | -2.11 | 0.10 | -2.30 | -1.92 |
| min_ea | \| | t2 | -1.71 | 0.09 | -1.90 | -1.53 | -0.90 | 0.05 | -0.99 | -0.81 |
| min_ea | \| | t3 | 2.02 | 0.10 | 1.83 | 2.21 | 1.06 | 0.05 | 0.96 | 1.16 |
| inn_pe | \| | t1 | -2.81 | 0.13 | -3.06 | -2.56 | -1.92 | 0.08 | -2.08 | -1.75 |
| inn_pe | \| | t2 | -0.91 | 0.06 | -1.04 | -0.78 | -0.62 | 0.04 | -0.71 | -0.54 |
| inn_pe | \| | t3 | 1.82 | 0.08 | 1.66 | 1.97 | 1.24 | 0.05 | 1.13 | 1.34 |
| balance | ~~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 0.39 | 0.03 | 0.34 | 0.44 |
| amo_ri | ~~ | amo_ri | 1.00 | 0.00 | 1.00 | 1.00 | 0.79 | 0.03 | 0.74 | 0.84 |
| har_ar | ~~ | har_ar | 1.00 | 0.00 | 1.00 | 1.00 | 0.73 | 0.03 | 0.67 | 0.78 |
| tho_ha | ~~ | tho_ha | 1.00 | 0.00 | 1.00 | 1.00 | 0.24 | 0.02 | 0.20 | 0.29 |
| stable | ~~ | stable | 1.00 | 0.00 | 1.00 | 1.00 | 0.54 | 0.03 | 0.48 | 0.60 |
| content | ~~ | content | 1.00 | 0.00 | 1.00 | 1.00 | 0.33 | 0.03 | 0.27 | 0.38 |
| min_ea | ~~ | min_ea | 1.00 | 0.00 | 1.00 | 1.00 | 0.28 | 0.02 | 0.24 | 0.31 |
| inn_pe | ~~ | inn_pe | 1.00 | 0.00 | 1.00 | 1.00 | 0.47 | 0.03 | 0.41 | 0.52 |
| bh | ~~ | bh | 1.57 | 0.17 | 1.23 | 1.91 | 1.00 | 0.00 | 1.00 | 1.00 |

Table 15: Residual correlations of Norway group

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| balance (1) | | 0.09 | 0.02 | -0.01 | -0.01 | 0.04 | -0.04 | -0.07 |
| amo_ri (2) | 0.09 | | -0.04 | -0.06 | -0.07 | 0.02 | -0.00 | -0.04 |
| har_ar (3) | 0.02 | -0.04 | | 0.06 | 0.02 | -0.05 | -0.04 | -0.03 |
| tho_ha (4) | -0.01 | -0.06 | 0.06 | | 0.02 | -0.03 | 0.01 | 0.02 |
| stable (5) | -0.01 | -0.07 | 0.02 | 0.02 | | 0.08 | -0.07 | -0.02 |
| content (6) | 0.04 | 0.02 | -0.05 | -0.03 | 0.08 | | -0.00 | -0.05 |
| min_ea (7) | -0.04 | -0.00 | -0.04 | 0.01 | -0.07 | -0.00 | | 0.07 |
| inn_pe (8) | -0.07 | -0.04 | -0.03 | 0.02 | -0.02 | -0.05 | 0.07 | |

### 5.4 Multiple-group IFA Model

Comparing the measurement properties between these two countries using the original response scale (*Never* to *Always*) is preferred so as not to induce a difference between countries artificially. Using the methods developed earlier in this work, we estimated a multi-group item factor model to compare the thresholds between countries. The invariance of some of the thresholds was dependent upon how the model was estimated. In one extreme using pairwise maximum likelihood (PML), the invariance of threshold one for the "Harmony Around" item could not even be assessed because the estimator of the threshold does not exist within the Norway sample. The existence of the maximum likelihood estimator depends on at least one case endorsing "Never." To be able to fully compare the estimates for thresholds between groups without modifying the observed responses by collapsing categories, we used the Bayesian IFA approach described in the current paper.

To evaluate the effect of the threshold prior on the decision of the invariance of the thresholds, we used five separate approaches to specifying the prior. The first four priors are identical to the priors used above for the single-group analyses. That is, the Induced-Dirichlet prior, two separate specifications were also tried that is,

$$\begin{aligned}\boldsymbol{\tau} &\sim \text{Induced-Dirichlet}([1,1,1,1])\\ \boldsymbol{\tau} &\sim \text{Induced-Dirichlet}([1,3,8,4]).\end{aligned}$$

And the sequential exponential of normals with a standard deviation of 1.5 (blavaan) or $10^5$ (Mplus). That is,

$$\begin{aligned}\tau_c^* &\sim \text{Normal}(0, 1.5)\\ \tau_c^* &\sim \text{Normal}(0, 10^5).\end{aligned}$$

The last prior was chosen to act as a sensitivity analysis of the induced-Dirichlet prior. Below are the details of this final prior specification.

The induced-Dirichlet prior is used but with a modification of the hyper-prior. Instead of each set of thresholds for an item $i$ being sampled with the same prior,

$$\boldsymbol{\tau}_i \sim \text{Induced-Dirichlet}(\boldsymbol{\alpha} = [1,1,1,1]),$$

we added a hyper-prior to induce more or less informative priors. The modified prior can be denoted as

$$\begin{aligned}\boldsymbol{\tau}_i &\sim \text{Induced-Dirichlet}(\boldsymbol{\alpha}\delta_i)\\ \boldsymbol{\alpha} &= [1,1,1,1] \text{ OR} [1,3,8,4]\\ \delta &= [1,10,100].\end{aligned}$$

To help evaluate the effect of prior informativeness (i.e., $\delta$, on threshold posterior distributions, we used plots of the posterior thresholds as a function of $\delta$, leading to a 1x3 matrix of scatter plots for each item. The model was estimated three separate times for each value of delta. The posterior plots will include the relevant prior for comparisons.

### 5.4.1 Multi-Group Analysis Model Fit Summary

The summary indices of the resulting model fit statistics for all models of the multi-group analyses are provided in Table 16.

These models are shown by the identification method of the thresholds and then by prior specification. LR = Latent Response.

- Fixed LR: Model is identified by fixing the latent response distribution of each item in each group to equality where $\boldsymbol{\nu}_g = \mathbf{0}$, $\boldsymbol{\theta}_g = \mathbf{1}$.
- Fixed Thresholds: Model thresholds are identified by constraining thresholds to equality across groups, then freely estimating the latent response distribution intercept and residual variances in all but the reference group where $\boldsymbol{\nu}_1 = \mathbf{0}$, $\boldsymbol{\theta}_1 = \mathbf{1}$ and $\boldsymbol{\nu}_2 = \hat{\boldsymbol{\nu}}$, $\boldsymbol{\theta}_2 = \hat{\boldsymbol{\theta}}$
- Informative Prior on LR: Model thresholds are identified by using an informative prior on LR distribution of non-reference group to account for uncertainty in latent response intercepts and variances when comparing thresholds between groups.$\boldsymbol{\nu}_1 = \mathbf{0}$, $\boldsymbol{\theta}_1 = \mathbf{1}$ and $\nu_{2i} \sim N(0, 0.25)$, $\theta_{2i} \sim$ Cauchy$^{+}(1, 0.25)$

Table 16: Summary of fit indices from multi-group analyses

| Prior | DIC | Dbar | pV | WAIC | lppd | pWAIC |
|---|---|---|---|---|---|---|
| *Fixed Latent Response* $\boldsymbol{\nu}_{1,2} = \mathbf{0}$ *and* $\boldsymbol{\theta}_{1,2} = \mathbf{1}$ | | | | | | |
| ID([1,1,1,1]) | 32128.3 | 25396.6 | 6731.7 | 30311.0 | -11710.6 | 3444.9 |
| ID([1,3,8,4]) | 32382.6 | 25416.3 | 6966.3 | 30262.3 | -11731.8 | 3399.4 |
| SEN(0,1.5) | 32140.2 | 25398.4 | 6741.9 | 30346.0 | -11707.7 | 3465.3 |
| SEN(0,$10^5$) | 32611.7 | 25372.5 | 7239.2 | 30333.4 | -11694.2 | 3472.5 |
| *Fixed Thresholds* $\boldsymbol{\tau}_1 = \boldsymbol{\tau}_2$ | | | | | | |
| ID([1,1,1,1]) | 33010.3 | 25682.9 | 7327.4 | 30563.6 | -11858.9 | 3422.9 |
| ID([1,3,8,4]) | 33400.3 | 25676.1 | 7724.1 | 30604.3 | -11848.0 | 3454.1 |
| SEN(0,1.5) | 33531.7 | 25683.4 | 7848.3 | 30645.8 | -11848.1 | 3474.8 |
| SEN(0,$10^5$) | 33132.8 | 25670.3 | 7462.6 | 30655.0 | -11837.8 | 3489.7 |
| *Informative Latent Response Priors on* $\boldsymbol{\nu}_2$ *and* $\boldsymbol{\theta}_2$ | | | | | | |
| ID([1,1,1,1]) | 88807.3 | 25852.2 | 62955.1 | 31353.3 | -11732.7 | 3944.0 |
| ID([1,3,8,4]) | 61887.0 | 25827.1 | 36059.9 | 31107.8 | -11779.1 | 3774.8 |
| SEN(0,1.5) | 76936.3 | 25780.1 | 51156.2 | 31186.2 | -11720.9 | 3872.2 |
| SEN(0,$10^5$) | 37178.5 | 25568.8 | 11609.7 | 30667.0 | -11717.7 | 3615.8 |

### 5.4.2 Results with Fixed Latent Response in Both Groups

Table 17: Fixed LR with ID([1,1,1,1]) posterior summary

| | | | Norway | | | | United States | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| lhs | op | rhs | mean | sd | Q.025 | Q.975 | mean | sd | Q.025 | Q.975 |
| bh | =~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| bh | =~ | amount_right | 0.46 | 0.05 | 0.37 | 0.56 | 0.79 | 0.08 | 0.64 | 0.95 |
| bh | =~ | harmony_around | 0.55 | 0.06 | 0.44 | 0.67 | 0.97 | 0.10 | 0.80 | 1.18 |
| bh | =~ | thoughts_harmony | 1.45 | 0.15 | 1.18 | 1.76 | 1.94 | 0.18 | 1.62 | 2.33 |
| bh | =~ | stable | 0.80 | 0.08 | 0.66 | 0.96 | 1.83 | 0.17 | 1.52 | 2.19 |
| bh | =~ | content | 1.20 | 0.11 | 0.99 | 1.43 | 2.01 | 0.18 | 1.69 | 2.40 |
| bh | =~ | mind_ease | 1.43 | 0.14 | 1.17 | 1.72 | 2.09 | 0.19 | 1.75 | 2.48 |
| bh | =~ | inner_peace | 0.95 | 0.09 | 0.78 | 1.13 | 1.47 | 0.13 | 1.23 | 1.74 |
| balance | \| | t1 | -3.40 | 0.19 | -3.78 | -3.06 | -2.00 | 0.08 | -2.17 | -1.84 |
| balance | \| | t2 | -1.31 | 0.08 | -1.47 | -1.17 | -0.66 | 0.05 | -0.76 | -0.56 |
| balance | \| | t3 | 1.92 | 0.10 | 1.72 | 2.12 | 1.32 | 0.06 | 1.20 | 1.45 |
| amount_right | \| | t1 | -2.13 | 0.09 | -2.32 | -1.95 | -1.75 | 0.08 | -1.89 | -1.60 |
| amount_right | \| | t2 | -0.67 | 0.05 | -0.76 | -0.57 | -0.72 | 0.05 | -0.82 | -0.62 |
| amount_right | \| | t3 | 1.36 | 0.06 | 1.24 | 1.48 | 1.10 | 0.06 | 0.99 | 1.21 |
| harmony_around | \| | t1 | -4.28 | 0.72 | -6.02 | -3.32 | -2.62 | 0.13 | -2.90 | -2.37 |
| harmony_around | \| | t2 | -2.08 | 0.10 | -2.28 | -1.91 | -1.61 | 0.07 | -1.76 | -1.47 |
| harmony_around | \| | t3 | 1.01 | 0.06 | 0.90 | 1.12 | 0.91 | 0.05 | 0.81 | 1.02 |
| thoughts_harmony | \| | t1 | -4.54 | 0.31 | -5.19 | -3.96 | -3.51 | 0.19 | -3.90 | -3.15 |
| thoughts_harmony | \| | t2 | -2.17 | 0.14 | -2.46 | -1.91 | -1.73 | 0.10 | -1.92 | -1.54 |
| thoughts_harmony | \| | t3 | 2.22 | 0.14 | 1.96 | 2.51 | 1.53 | 0.09 | 1.35 | 1.70 |
| stable | \| | t1 | -3.31 | 0.19 | -3.68 | -2.96 | -3.23 | 0.16 | -3.55 | -2.92 |
| stable | \| | t2 | -2.10 | 0.10 | -2.30 | -1.91 | -1.95 | 0.10 | -2.16 | -1.76 |
| stable | \| | t3 | 0.15 | 0.05 | 0.05 | 0.26 | 0.57 | 0.07 | 0.44 | 0.71 |
| content | \| | t1 | -4.07 | 0.26 | -4.61 | -3.59 | -3.13 | 0.16 | -3.45 | -2.82 |
| content | \| | t2 | -2.13 | 0.13 | -2.39 | -1.90 | -1.83 | 0.10 | -2.04 | -1.63 |
| content | \| | t3 | 1.82 | 0.11 | 1.63 | 2.03 | 1.09 | 0.08 | 0.94 | 1.25 |
| mind_ease | \| | t1 | -4.06 | 0.27 | -4.64 | -3.57 | -2.78 | 0.15 | -3.08 | -2.51 |
| mind_ease | \| | t2 | -1.59 | 0.11 | -1.80 | -1.38 | -1.19 | 0.09 | -1.36 | -1.02 |
| mind_ease | \| | t3 | 1.93 | 0.12 | 1.71 | 2.16 | 1.93 | 0.11 | 1.72 | 2.16 |
| inner_peace | \| | t1 | -2.81 | 0.14 | -3.10 | -2.54 | -2.70 | 0.13 | -2.97 | -2.45 |
| inner_peace | \| | t2 | -0.85 | 0.07 | -0.98 | -0.72 | -1.12 | 0.07 | -1.26 | -0.99 |
| inner_peace | \| | t3 | 1.77 | 0.09 | 1.61 | 1.94 | 1.21 | 0.07 | 1.07 | 1.35 |
| balance | ~~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| amount_right | ~~ | amount_right | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| harmony_around | ~~ | harmony_around | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| thoughts_harmony | ~~ | thoughts_harmony | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| stable | ~~ | stable | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| content | ~~ | content | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| mind_ease | ~~ | mind_ease | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| inner_peace | ~~ | inner_peace | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| balance | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| amount_right | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| harmony_around | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| thoughts_harmony | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| stable | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| content | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| mind_ease | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| inner_peace | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| bh | ~~ | bh | 1.24 | 0.17 | 0.94 | 1.59 | 0.50 | 0.07 | 0.37 | 0.65 |

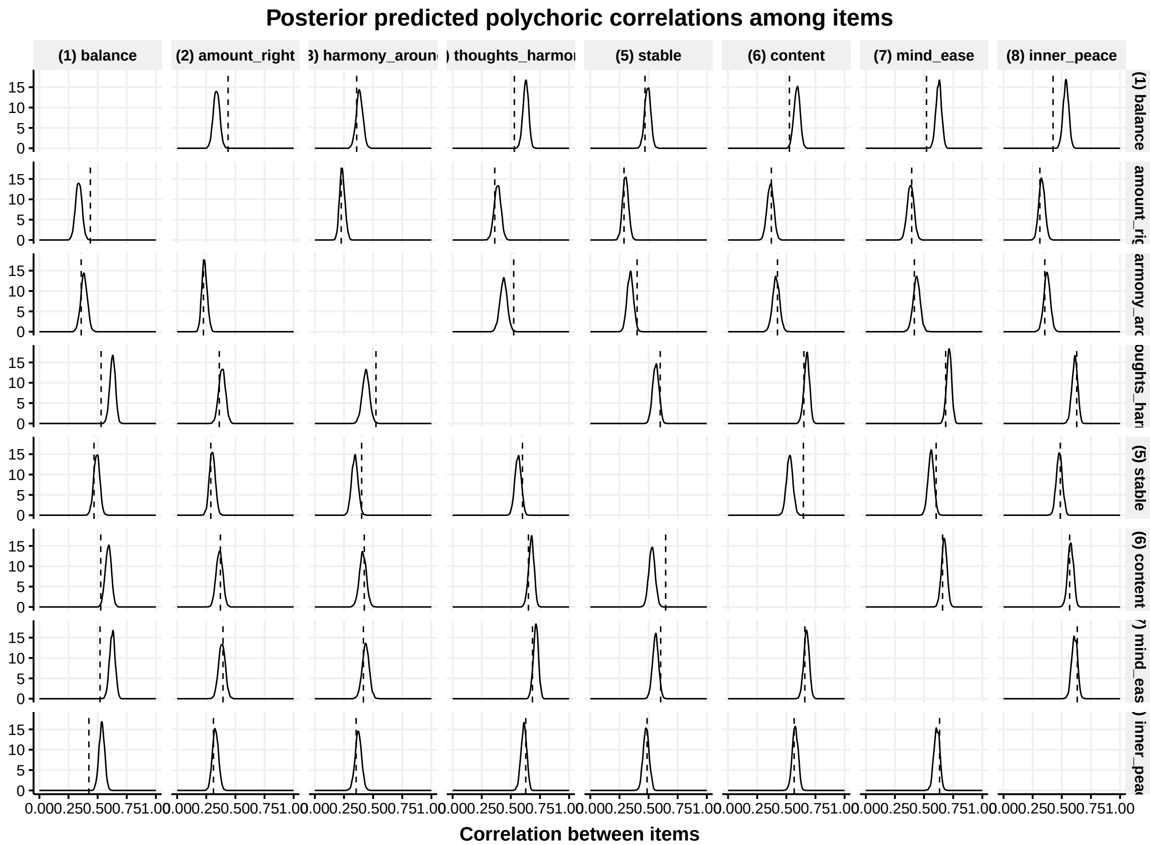



Figure 3: Fixed LR with ID([1,1,1,1]) posterior distribution of model implied polychoric correlation

*Note.* Vertical dashed lines represent the observed polychoric correlation among items. Polychoric correlations were estimated using the *psych* packages in R.

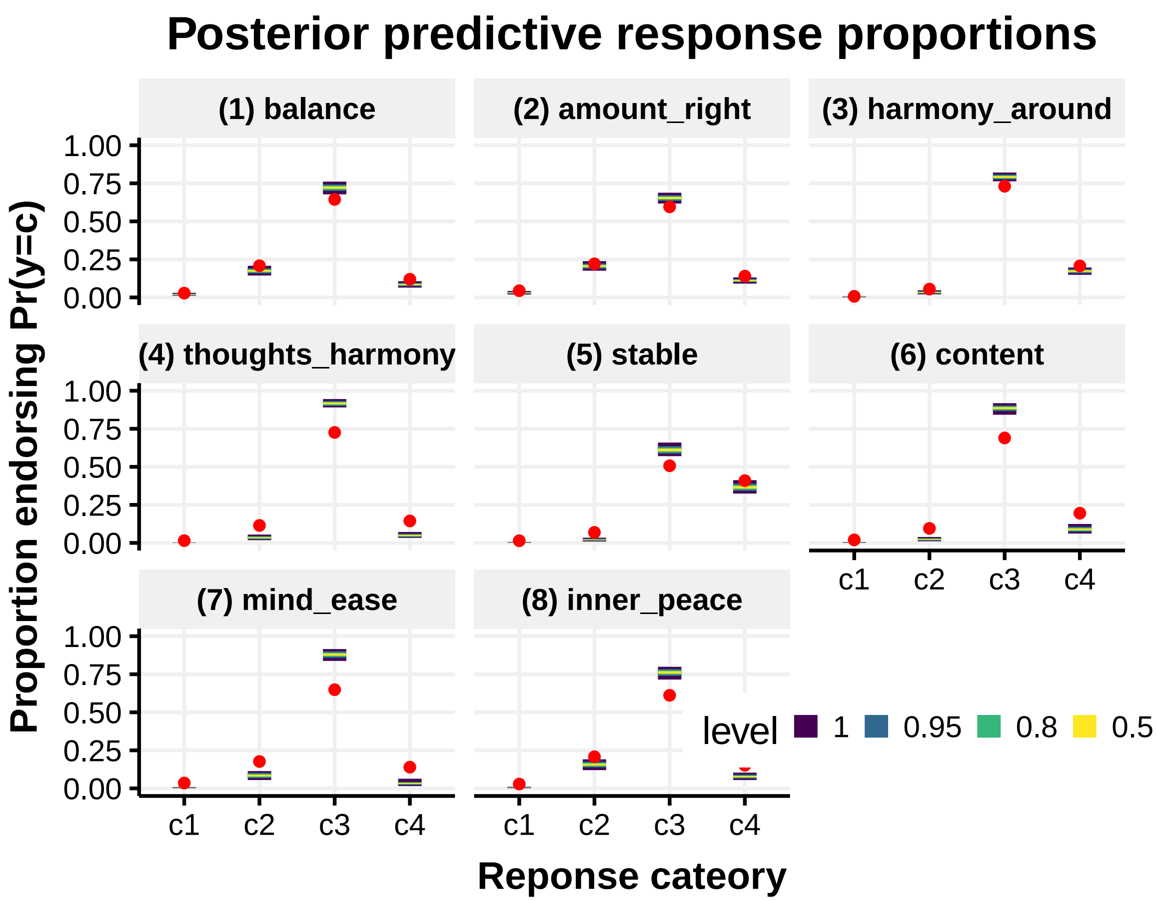



Figure 4: Fixed LR with ID([1,1,1,1]) PPD of item endorsement proportions across items

*Note.* Dots represent the observed proportion of the sample endorsing the category.

Table 18: Fixed LR with ID([1,3,8,4]) posterior summary

| | | | Norway | | | | United States | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| lhs | op | rhs | mean | sd | Q.025 | Q.975 | mean | sd | Q.025 | Q.975 |
| bh | =~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| bh | =~ | amount_right | 0.47 | 0.05 | 0.37 | 0.57 | 0.78 | 0.08 | 0.64 | 0.95 |
| bh | =~ | harmony_around | 0.55 | 0.06 | 0.44 | 0.67 | 0.96 | 0.10 | 0.78 | 1.17 |
| bh | =~ | thoughts_harmony | 1.40 | 0.14 | 1.14 | 1.70 | 1.89 | 0.17 | 1.58 | 2.26 |
| bh | =~ | stable | 0.81 | 0.08 | 0.67 | 0.98 | 1.80 | 0.17 | 1.49 | 2.17 |
| bh | =~ | content | 1.18 | 0.11 | 0.98 | 1.42 | 1.97 | 0.18 | 1.63 | 2.35 |
| bh | =~ | mind_ease | 1.41 | 0.14 | 1.16 | 1.70 | 2.04 | 0.19 | 1.70 | 2.45 |
| bh | =~ | inner_peace | 0.95 | 0.09 | 0.78 | 1.13 | 1.45 | 0.14 | 1.21 | 1.74 |
| balance | \| | t1 | -3.39 | 0.19 | -3.78 | -3.04 | -2.02 | 0.08 | -2.19 | -1.86 |
| balance | \| | t2 | -1.31 | 0.08 | -1.48 | -1.17 | -0.67 | 0.05 | -0.77 | -0.57 |
| balance | \| | t3 | 1.87 | 0.10 | 1.68 | 2.07 | 1.31 | 0.06 | 1.18 | 1.44 |
| amount_right | \| | t1 | -2.14 | 0.09 | -2.33 | -1.95 | -1.76 | 0.07 | -1.91 | -1.61 |
| amount_right | \| | t2 | -0.67 | 0.05 | -0.77 | -0.58 | -0.73 | 0.05 | -0.83 | -0.64 |
| amount_right | \| | t3 | 1.34 | 0.06 | 1.23 | 1.46 | 1.09 | 0.05 | 0.99 | 1.20 |
| harmony_around | \| | t1 | -4.26 | 0.69 | -5.95 | -3.33 | -2.63 | 0.13 | -2.90 | -2.39 |
| harmony_around | \| | t2 | -2.06 | 0.09 | -2.25 | -1.87 | -1.60 | 0.07 | -1.75 | -1.46 |
| harmony_around | \| | t3 | 1.00 | 0.06 | 0.89 | 1.11 | 0.90 | 0.06 | 0.79 | 1.02 |
| thoughts_harmony | \| | t1 | -4.41 | 0.29 | -5.01 | -3.88 | -3.48 | 0.19 | -3.87 | -3.13 |
| thoughts_harmony | \| | t2 | -2.10 | 0.13 | -2.37 | -1.86 | -1.71 | 0.10 | -1.91 | -1.53 |
| thoughts_harmony | \| | t3 | 2.11 | 0.13 | 1.88 | 2.38 | 1.49 | 0.09 | 1.31 | 1.67 |
| stable | \| | t1 | -3.32 | 0.19 | -3.71 | -2.98 | -3.22 | 0.17 | -3.56 | -2.91 |
| stable | \| | t2 | -2.09 | 0.10 | -2.29 | -1.89 | -1.94 | 0.10 | -2.14 | -1.75 |
| stable | \| | t3 | 0.14 | 0.05 | 0.04 | 0.25 | 0.56 | 0.07 | 0.43 | 0.69 |
| content | \| | t1 | -4.02 | 0.25 | -4.52 | -3.56 | -3.12 | 0.16 | -3.44 | -2.83 |
| content | \| | t2 | -2.10 | 0.12 | -2.33 | -1.88 | -1.81 | 0.10 | -2.03 | -1.62 |
| content | \| | t3 | 1.76 | 0.10 | 1.56 | 1.96 | 1.06 | 0.08 | 0.90 | 1.21 |
| mind_ease | \| | t1 | -4.01 | 0.25 | -4.54 | -3.55 | -2.77 | 0.14 | -3.06 | -2.51 |
| mind_ease | \| | t2 | -1.57 | 0.10 | -1.78 | -1.37 | -1.18 | 0.09 | -1.35 | -1.02 |
| mind_ease | \| | t3 | 1.86 | 0.12 | 1.63 | 2.09 | 1.87 | 0.11 | 1.68 | 2.09 |
| inner_peace | \| | t1 | -2.81 | 0.14 | -3.09 | -2.55 | -2.71 | 0.13 | -2.97 | -2.46 |
| inner_peace | \| | t2 | -0.86 | 0.06 | -0.98 | -0.74 | -1.13 | 0.07 | -1.26 | -1.00 |
| inner_peace | \| | t3 | 1.73 | 0.08 | 1.57 | 1.90 | 1.19 | 0.07 | 1.06 | 1.32 |
| balance | ~~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| amount_right | ~~ | amount_right | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| harmony_around | ~~ | harmony_around | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| thoughts_harmony | ~~ | thoughts_harmony | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| stable | ~~ | stable | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| content | ~~ | content | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| mind_ease | ~~ | mind_ease | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| inner_peace | ~~ | inner_peace | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| balance | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| amount_right | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| harmony_around | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| thoughts_harmony | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| stable | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| content | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| mind_ease | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| inner_peace | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| bh | ~~ | bh | 1.21 | 0.17 | 0.90 | 1.56 | 0.51 | 0.07 | 0.38 | 0.66 |

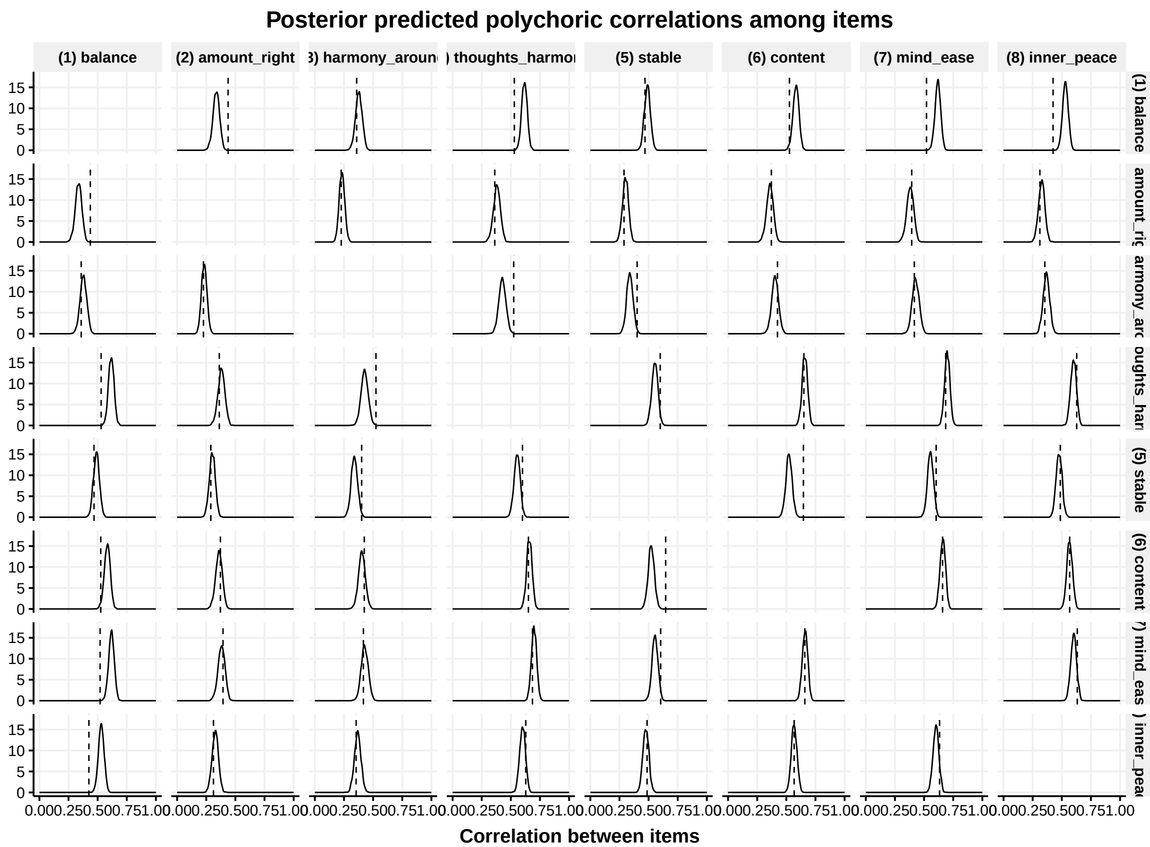



Figure 5: Fixed LR with ID([1,3,8,4]) posterior distribution of model implied polychoric correlation

*Note.* Vertical dashed lines represent the observed polychoric correlation among items. Polychoric correlations were estimated using the *psych* packages in R.

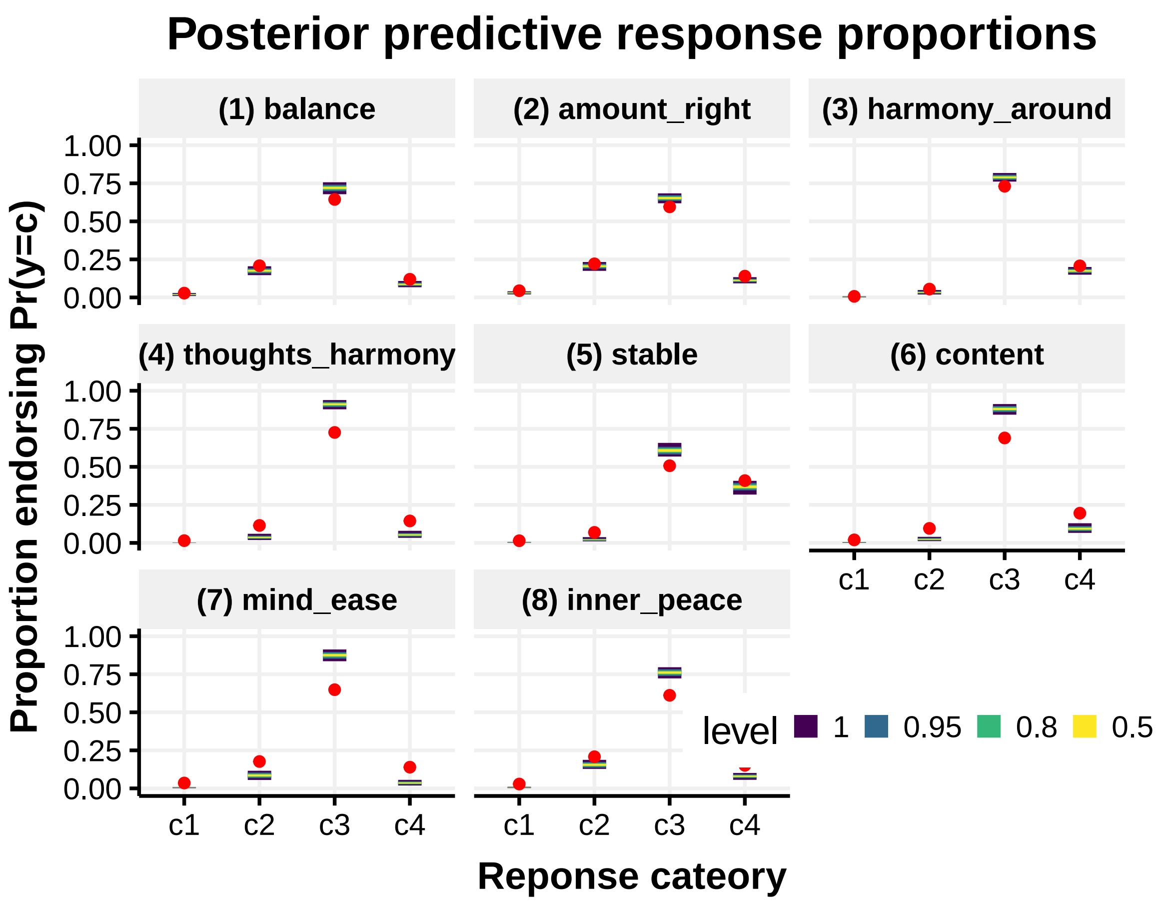



Figure 6: Fixed LR with ID([1,3,8,4]) PPD of item endorsement proportions across items

*Note.* Dots represent the observed proportion of the sample endorsing the category.

Table 19: Fixed LR with SEN(0,1.5) posterior summary

| | | | Norway | | | | United States | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| lhs | op | rhs | mean | sd | Q.025 | Q.975 | mean | sd | Q.025 | Q.975 |
| bh | =~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| bh | =~ | amount_right | 0.46 | 0.05 | 0.36 | 0.56 | 0.78 | 0.08 | 0.63 | 0.95 |
| bh | =~ | harmony_around | 0.54 | 0.06 | 0.43 | 0.67 | 0.97 | 0.10 | 0.79 | 1.17 |
| bh | =~ | thoughts_harmony | 1.45 | 0.15 | 1.18 | 1.76 | 1.93 | 0.18 | 1.61 | 2.31 |
| bh | =~ | stable | 0.80 | 0.08 | 0.66 | 0.96 | 1.83 | 0.17 | 1.53 | 2.18 |
| bh | =~ | content | 1.20 | 0.11 | 0.99 | 1.45 | 2.00 | 0.19 | 1.66 | 2.39 |
| bh | =~ | mind_ease | 1.42 | 0.15 | 1.15 | 1.73 | 2.08 | 0.19 | 1.74 | 2.48 |
| bh | =~ | inner_peace | 0.94 | 0.09 | 0.78 | 1.13 | 1.46 | 0.13 | 1.22 | 1.74 |
| balance | \| | t1 | -3.39 | 0.18 | -3.76 | -3.05 | -2.01 | 0.09 | -2.19 | -1.85 |
| balance | \| | t2 | -1.33 | 0.08 | -1.49 | -1.17 | -0.67 | 0.05 | -0.77 | -0.56 |
| balance | \| | t3 | 1.93 | 0.10 | 1.74 | 2.14 | 1.32 | 0.07 | 1.20 | 1.46 |
| amount_right | \| | t1 | -2.13 | 0.10 | -2.32 | -1.95 | -1.74 | 0.08 | -1.90 | -1.60 |
| amount_right | \| | t2 | -0.67 | 0.05 | -0.76 | -0.57 | -0.73 | 0.05 | -0.83 | -0.63 |
| amount_right | \| | t3 | 1.36 | 0.06 | 1.24 | 1.48 | 1.10 | 0.06 | 0.99 | 1.21 |
| harmony_around | \| | t1 | -3.98 | 0.51 | -5.16 | -3.23 | -2.63 | 0.13 | -2.90 | -2.38 |
| harmony_around | \| | t2 | -2.09 | 0.10 | -2.28 | -1.90 | -1.62 | 0.07 | -1.77 | -1.48 |
| harmony_around | \| | t3 | 1.01 | 0.06 | 0.90 | 1.12 | 0.91 | 0.05 | 0.80 | 1.02 |
| thoughts_harmony | \| | t1 | -4.53 | 0.31 | -5.18 | -3.97 | -3.51 | 0.20 | -3.91 | -3.15 |
| thoughts_harmony | \| | t2 | -2.19 | 0.14 | -2.49 | -1.92 | -1.75 | 0.10 | -1.96 | -1.56 |
| thoughts_harmony | \| | t3 | 2.25 | 0.14 | 1.99 | 2.54 | 1.53 | 0.09 | 1.35 | 1.72 |
| stable | \| | t1 | -3.30 | 0.18 | -3.66 | -2.96 | -3.23 | 0.17 | -3.57 | -2.93 |
| stable | \| | t2 | -2.11 | 0.10 | -2.31 | -1.92 | -1.97 | 0.11 | -2.18 | -1.77 |
| stable | \| | t3 | 0.15 | 0.05 | 0.04 | 0.25 | 0.57 | 0.07 | 0.44 | 0.70 |
| content | \| | t1 | -4.05 | 0.25 | -4.54 | -3.57 | -3.13 | 0.16 | -3.45 | -2.83 |
| content | \| | t2 | -2.15 | 0.13 | -2.41 | -1.92 | -1.85 | 0.11 | -2.06 | -1.64 |
| content | \| | t3 | 1.83 | 0.11 | 1.63 | 2.06 | 1.08 | 0.08 | 0.93 | 1.25 |
| mind_ease | \| | t1 | -4.06 | 0.26 | -4.59 | -3.60 | -2.79 | 0.14 | -3.09 | -2.51 |
| mind_ease | \| | t2 | -1.61 | 0.11 | -1.83 | -1.40 | -1.20 | 0.09 | -1.38 | -1.04 |
| mind_ease | \| | t3 | 1.94 | 0.13 | 1.71 | 2.20 | 1.93 | 0.11 | 1.72 | 2.15 |
| inner_peace | \| | t1 | -2.81 | 0.14 | -3.10 | -2.54 | -2.71 | 0.13 | -2.97 | -2.47 |
| inner_peace | \| | t2 | -0.85 | 0.06 | -0.98 | -0.73 | -1.13 | 0.07 | -1.28 | -1.00 |
| inner_peace | \| | t3 | 1.77 | 0.09 | 1.61 | 1.95 | 1.21 | 0.07 | 1.07 | 1.35 |
| balance | ~~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| amount_right | ~~ | amount_right | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| harmony_around | ~~ | harmony_around | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| thoughts_harmony | ~~ | thoughts_harmony | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| stable | ~~ | stable | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| content | ~~ | content | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| mind_ease | ~~ | mind_ease | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| inner_peace | ~~ | inner_peace | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| balance | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| amount_right | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| harmony_around | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| thoughts_harmony | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| stable | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| content | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| mind_ease | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| inner_peace | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| bh | ~~ | bh | 1.27 | 0.18 | 0.94 | 1.64 | 0.51 | 0.07 | 0.38 | 0.67 |

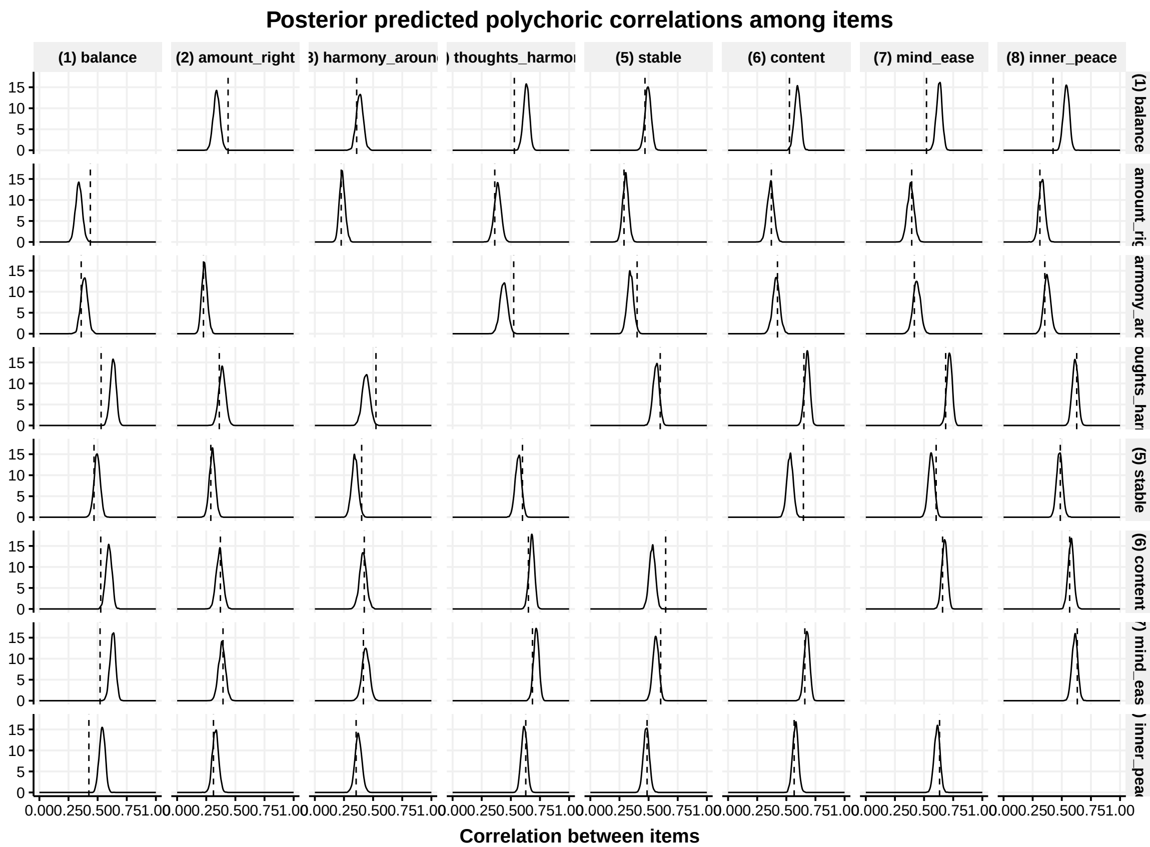



Figure 7: Fixed LR with SEN(0,1.5) posterior distribution of model implied polychoric correlation

*Note.* Vertical dashed lines represent the observed polychoric correlation among items. Polychoric correlations were estimated using the *psych* packages in R.

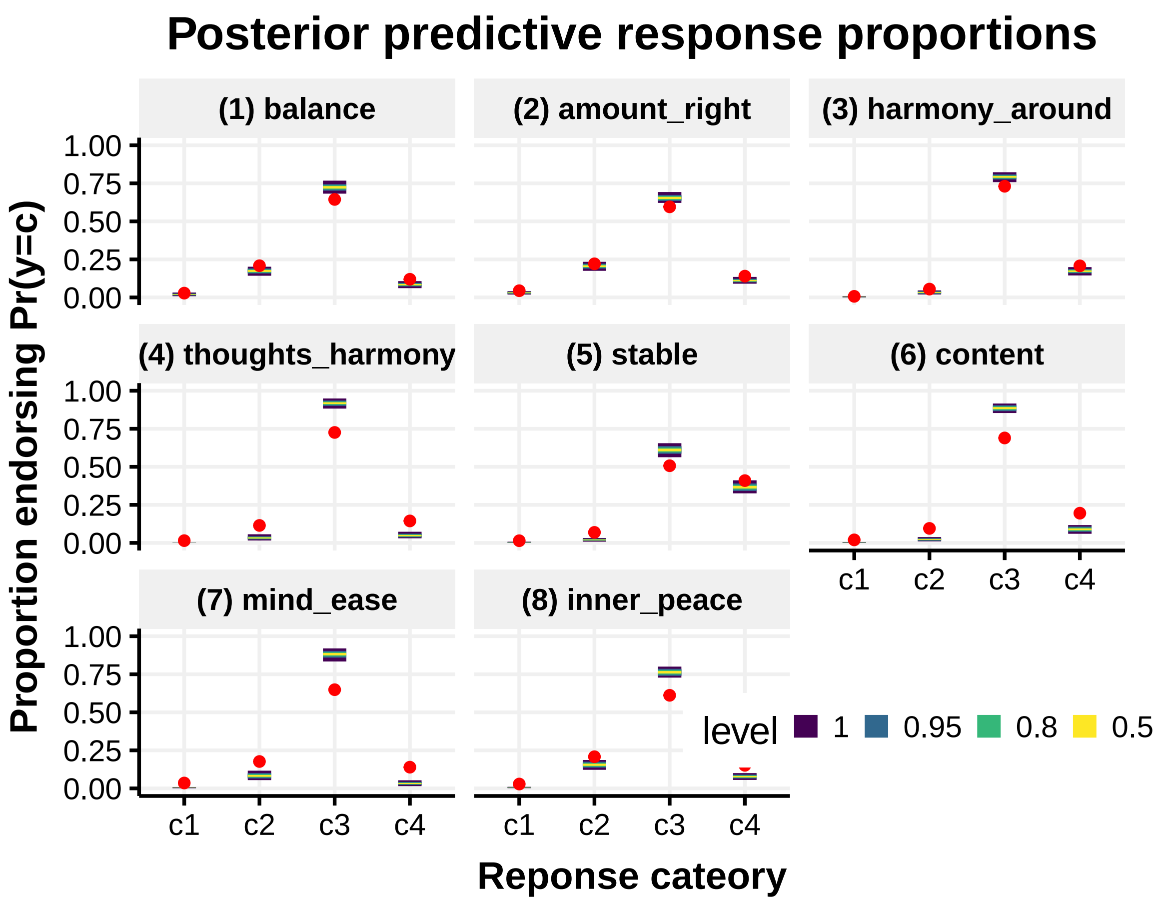



Figure 8: Fixed LR with SEN(0,1.5) PPD of item endorsement proportions across items

*Note.* Dots represent the observed proportion of the sample endorsing the category.

Table 20: Fixed LR with SEN(0,10E5) posterior summary

| | | | Norway | | | | United States | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| lhs | op | rhs | mean | sd | Q.025 | Q.975 | mean | sd | Q.025 | Q.975 |
| bh | =~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| bh | =~ | amount_right | 0.45 | 0.04 | 0.37 | 0.53 | 0.80 | 0.08 | 0.65 | 0.95 |
| bh | =~ | harmony_around | 0.54 | 0.06 | 0.44 | 0.66 | 1.01 | 0.08 | 0.87 | 1.18 |
| bh | =~ | thoughts_harmony | 1.39 | 0.15 | 1.15 | 1.70 | 2.05 | 0.14 | 1.79 | 2.32 |
| bh | =~ | stable | 0.78 | 0.07 | 0.64 | 0.89 | 1.84 | 0.11 | 1.63 | 2.04 |
| bh | =~ | content | 1.19 | 0.12 | 0.98 | 1.43 | 2.04 | 0.16 | 1.74 | 2.30 |
| bh | =~ | mind_ease | 1.43 | 0.14 | 1.21 | 1.72 | 2.15 | 0.13 | 1.91 | 2.41 |
| bh | =~ | inner_peace | 0.92 | 0.08 | 0.79 | 1.06 | 1.51 | 0.13 | 1.28 | 1.74 |
| balance | \| | t1 | -3.51 | 0.23 | -3.98 | -3.14 | -1.99 | 0.07 | -2.13 | -1.84 |
| balance | \| | t2 | -1.36 | 0.08 | -1.54 | -1.21 | -0.68 | 0.05 | -0.78 | -0.58 |
| balance | \| | t3 | 1.95 | 0.09 | 1.78 | 2.14 | 1.30 | 0.07 | 1.18 | 1.43 |
| amount_right | \| | t1 | -2.17 | 0.09 | -2.34 | -1.99 | -1.75 | 0.09 | -1.91 | -1.57 |
| amount_right | \| | t2 | -0.68 | 0.04 | -0.76 | -0.59 | -0.73 | 0.05 | -0.83 | -0.64 |
| amount_right | \| | t3 | 1.36 | 0.06 | 1.25 | 1.47 | 1.09 | 0.06 | 0.98 | 1.20 |
| harmony_around | \| | t1 | -32.37 | 32.19 | -112.06 | -4.14 | -2.68 | 0.14 | -2.95 | -2.43 |
| harmony_around | \| | t2 | -2.13 | 0.09 | -2.31 | -1.95 | -1.64 | 0.07 | -1.79 | -1.51 |
| harmony_around | \| | t3 | 1.02 | 0.06 | 0.91 | 1.12 | 0.91 | 0.05 | 0.81 | 1.02 |
| thoughts_harmony | \| | t1 | -4.64 | 0.36 | -5.38 | -3.98 | -3.64 | 0.20 | -4.01 | -3.28 |
| thoughts_harmony | \| | t2 | -2.19 | 0.15 | -2.49 | -1.93 | -1.81 | 0.10 | -2.03 | -1.62 |
| thoughts_harmony | \| | t3 | 2.21 | 0.15 | 1.94 | 2.51 | 1.55 | 0.09 | 1.38 | 1.73 |
| stable | \| | t1 | -3.33 | 0.20 | -3.83 | -3.02 | -3.23 | 0.14 | -3.53 | -3.00 |
| stable | \| | t2 | -2.13 | 0.10 | -2.34 | -1.96 | -1.95 | 0.10 | -2.16 | -1.77 |
| stable | \| | t3 | 0.14 | 0.05 | 0.04 | 0.24 | 0.54 | 0.07 | 0.41 | 0.68 |
| content | \| | t1 | -4.22 | 0.27 | -4.81 | -3.68 | -3.14 | 0.17 | -3.45 | -2.79 |
| content | \| | t2 | -2.20 | 0.13 | -2.50 | -1.97 | -1.86 | 0.10 | -2.06 | -1.66 |
| content | \| | t3 | 1.84 | 0.10 | 1.64 | 2.05 | 1.07 | 0.09 | 0.89 | 1.24 |
| mind_ease | \| | t1 | -4.24 | 0.28 | -4.82 | -3.81 | -2.83 | 0.14 | -3.13 | -2.58 |
| mind_ease | \| | t2 | -1.66 | 0.11 | -1.90 | -1.45 | -1.22 | 0.09 | -1.39 | -1.05 |
| mind_ease | \| | t3 | 1.97 | 0.12 | 1.74 | 2.21 | 1.92 | 0.11 | 1.71 | 2.16 |
| inner_peace | \| | t1 | -2.86 | 0.16 | -3.16 | -2.54 | -2.73 | 0.11 | -2.93 | -2.50 |
| inner_peace | \| | t2 | -0.86 | 0.06 | -0.98 | -0.75 | -1.15 | 0.08 | -1.32 | -1.01 |
| inner_peace | \| | t3 | 1.76 | 0.08 | 1.60 | 1.92 | 1.20 | 0.07 | 1.06 | 1.34 |
| balance | ~~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| amount_right | ~~ | amount_right | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| harmony_around | ~~ | harmony_around | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| thoughts_harmony | ~~ | thoughts_harmony | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| stable | ~~ | stable | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| content | ~~ | content | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| mind_ease | ~~ | mind_ease | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| inner_peace | ~~ | inner_peace | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| balance | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| amount_right | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| harmony_around | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| thoughts_harmony | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| stable | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| content | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| mind_ease | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| inner_peace | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| bh | ~~ | bh | 1.34 | 0.18 | 1.05 | 1.72 | 0.48 | 0.04 | 0.40 | 0.56 |

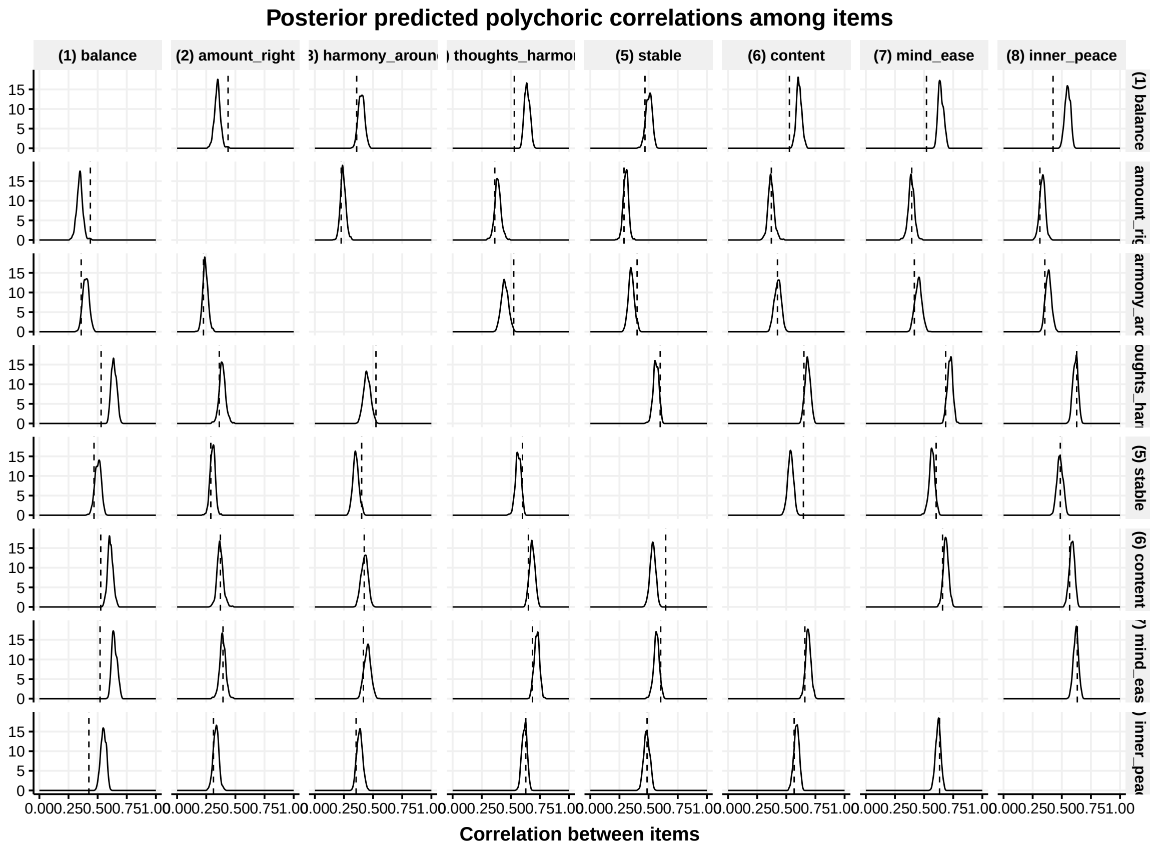



Figure 9: Fixed LR with SEN$(0,10^5)$ posterior distribution of model implied polychoric correlation

*Note.* Vertical dashed lines represent the observed polychoric correlation among items. Polychoric correlations were estimated using the *psych* packages in R.

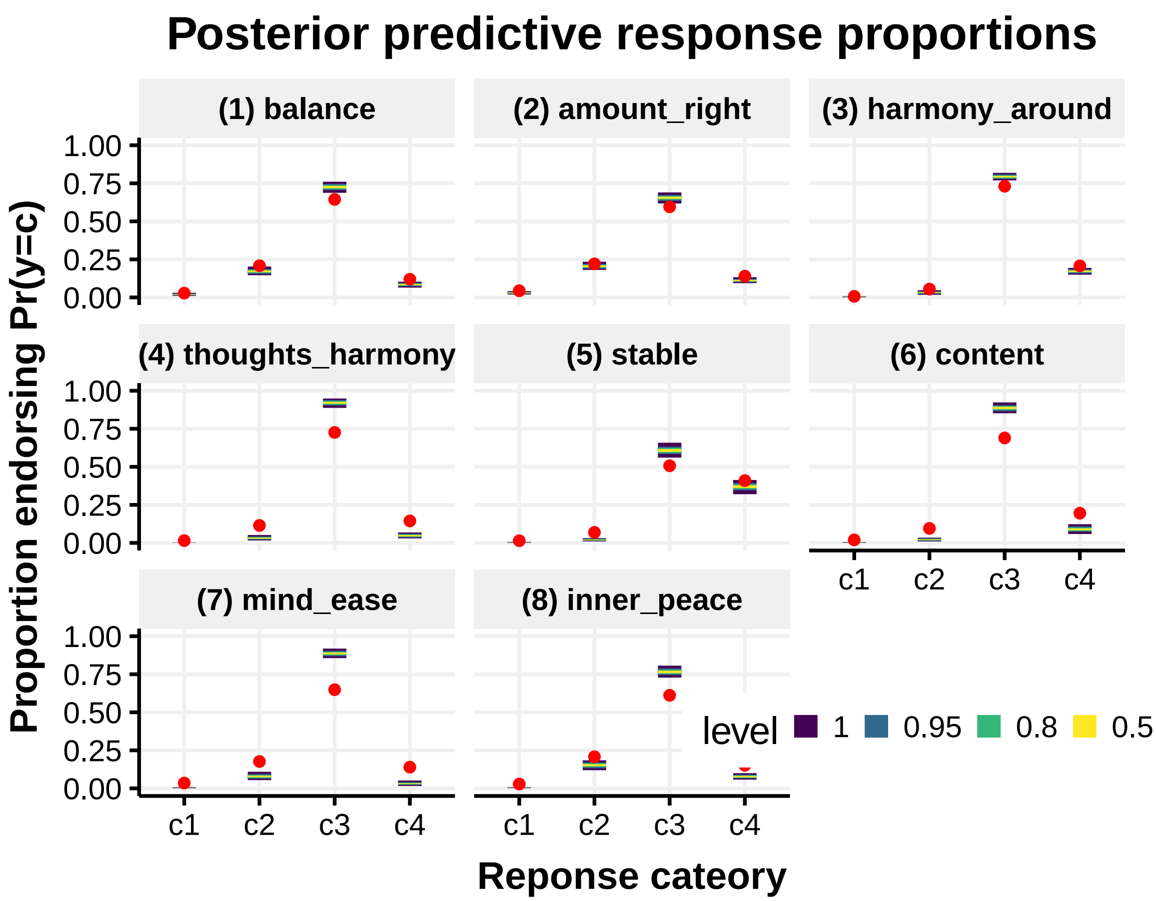



Figure 10: Fixed LR with SEN(0,$10^5$) PPD of item endorsement proportions across items

*Note.* Dots represent the observed proportion of the sample endorsing the category.

### 5.4.3 Results with Thresholds Equal Across Groups

Table 21: Equal thresholds with ID([1,1,1,1]) posterior summary

| | | | Norway | | | | United States | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| lhs | op | rhs | mean | sd | Q.025 | Q.975 | mean | sd | Q.025 | Q.975 |
| bh | =~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| bh | =~ | amount_right | 0.46 | 0.05 | 0.37 | 0.56 | 0.58 | 0.06 | 0.46 | 0.70 |
| bh | =~ | harmony_around | 0.55 | 0.06 | 0.44 | 0.68 | 0.76 | 0.08 | 0.62 | 0.92 |
| bh | =~ | thoughts_harmony | 1.45 | 0.15 | 1.18 | 1.77 | 1.62 | 0.16 | 1.33 | 1.97 |
| bh | =~ | stable | 0.83 | 0.08 | 0.68 | 0.98 | 1.04 | 0.10 | 0.87 | 1.24 |
| bh | =~ | content | 1.19 | 0.12 | 0.98 | 1.44 | 1.69 | 0.16 | 1.40 | 2.02 |
| bh | =~ | mind_ease | 1.40 | 0.14 | 1.15 | 1.69 | 1.54 | 0.15 | 1.28 | 1.86 |
| bh | =~ | inner_peace | 0.95 | 0.09 | 0.78 | 1.14 | 1.05 | 0.10 | 0.87 | 1.25 |
| balance | \| | t1 | -3.40 | 0.17 | -3.73 | -3.09 | -3.40 | 0.17 | -3.73 | -3.09 |
| balance | \| | t2 | -1.29 | 0.07 | -1.45 | -1.16 | -1.29 | 0.07 | -1.45 | -1.16 |
| balance | \| | t3 | 1.90 | 0.10 | 1.72 | 2.10 | 1.90 | 0.10 | 1.72 | 2.10 |
| amount_right | \| | t1 | -2.02 | 0.08 | -2.18 | -1.88 | -2.02 | 0.08 | -2.18 | -1.88 |
| amount_right | \| | t2 | -0.70 | 0.05 | -0.79 | -0.61 | -0.70 | 0.05 | -0.79 | -0.61 |
| amount_right | \| | t3 | 1.37 | 0.06 | 1.26 | 1.49 | 1.37 | 0.06 | 1.26 | 1.49 |
| harmony_around | \| | t1 | -3.45 | 0.19 | -3.85 | -3.11 | -3.45 | 0.19 | -3.85 | -3.11 |
| harmony_around | \| | t2 | -2.09 | 0.09 | -2.29 | -1.91 | -2.09 | 0.09 | -2.29 | -1.91 |
| harmony_around | \| | t3 | 1.01 | 0.05 | 0.91 | 1.12 | 1.01 | 0.05 | 0.91 | 1.12 |
| thoughts_harmony | \| | t1 | -4.52 | 0.29 | -5.13 | -3.99 | -4.52 | 0.29 | -5.13 | -3.99 |
| thoughts_harmony | \| | t2 | -2.16 | 0.14 | -2.46 | -1.90 | -2.16 | 0.14 | -2.46 | -1.90 |
| thoughts_harmony | \| | t3 | 2.20 | 0.14 | 1.94 | 2.48 | 2.20 | 0.14 | 1.94 | 2.48 |
| stable | \| | t1 | -3.32 | 0.15 | -3.63 | -3.02 | -3.32 | 0.15 | -3.63 | -3.02 |
| stable | \| | t2 | -2.13 | 0.09 | -2.32 | -1.95 | -2.13 | 0.09 | -2.32 | -1.95 |
| stable | \| | t3 | 0.16 | 0.05 | 0.06 | 0.26 | 0.16 | 0.05 | 0.06 | 0.26 |
| content | \| | t1 | -3.95 | 0.22 | -4.41 | -3.54 | -3.95 | 0.22 | -4.41 | -3.54 |
| content | \| | t2 | -2.14 | 0.12 | -2.39 | -1.91 | -2.14 | 0.12 | -2.39 | -1.91 |
| content | \| | t3 | 1.79 | 0.10 | 1.59 | 2.00 | 1.79 | 0.10 | 1.59 | 2.00 |
| mind_ease | \| | t1 | -3.71 | 0.21 | -4.13 | -3.32 | -3.71 | 0.21 | -4.13 | -3.32 |
| mind_ease | \| | t2 | -1.62 | 0.10 | -1.83 | -1.44 | -1.62 | 0.10 | -1.83 | -1.44 |
| mind_ease | \| | t3 | 1.93 | 0.12 | 1.71 | 2.17 | 1.93 | 0.12 | 1.71 | 2.17 |
| inner_peace | \| | t1 | -2.75 | 0.13 | -3.01 | -2.51 | -2.75 | 0.13 | -3.01 | -2.51 |
| inner_peace | \| | t2 | -0.87 | 0.06 | -0.99 | -0.76 | -0.87 | 0.06 | -0.99 | -0.76 |
| inner_peace | \| | t3 | 1.75 | 0.09 | 1.58 | 1.92 | 1.75 | 0.09 | 1.58 | 1.92 |
| balance | ~~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 2.50 | 0.26 | 2.03 | 3.04 |
| amount_right | ~~ | amount_right | 1.00 | 0.00 | 1.00 | 1.00 | 1.36 | 0.11 | 1.15 | 1.61 |
| harmony_around | ~~ | harmony_around | 1.00 | 0.00 | 1.00 | 1.00 | 1.51 | 0.15 | 1.24 | 1.84 |
| thoughts_harmony | ~~ | thoughts_harmony | 1.00 | 0.00 | 1.00 | 1.00 | 1.72 | 0.25 | 1.30 | 2.27 |
| stable | ~~ | stable | 1.00 | 0.00 | 1.00 | 1.00 | 0.85 | 0.10 | 0.66 | 1.05 |
| content | ~~ | content | 1.00 | 0.00 | 1.00 | 1.00 | 1.79 | 0.24 | 1.37 | 2.30 |
| mind_ease | ~~ | mind_ease | 1.00 | 0.00 | 1.00 | 1.00 | 1.35 | 0.18 | 1.04 | 1.75 |
| inner_peace | ~~ | inner_peace | 1.00 | 0.00 | 1.00 | 1.00 | 1.27 | 0.13 | 1.04 | 1.55 |
| balance | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.20 | 0.08 | -0.36 | -0.04 |
| amount_right | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.06 | -0.01 | 0.23 |
| harmony_around | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.10 | 0.07 | -0.23 | 0.04 |
| thoughts_harmony | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.17 | 0.10 | -0.02 | 0.36 |
| stable | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.33 | 0.07 | -0.47 | -0.20 |
| content | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.33 | 0.09 | 0.14 | 0.51 |
| mind_ease | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.27 | 0.09 | -0.45 | -0.10 |
| inner_peace | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.39 | 0.07 | 0.25 | 0.53 |
| bh | ~~ | bh | 1.22 | 0.17 | 0.91 | 1.59 | 1.28 | 0.19 | 0.94 | 1.70 |

Table 22: Equal thresholds with ID([1,3,8,4]) posterior summary

| | | | Norway | | | | United States | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| lhs | op | rhs | mean | sd | Q.025 | Q.975 | mean | sd | Q.025 | Q.975 |
| bh | =~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| bh | =~ | amount_right | 0.47 | 0.05 | 0.38 | 0.57 | 0.58 | 0.06 | 0.47 | 0.71 |
| bh | =~ | harmony_around | 0.55 | 0.06 | 0.44 | 0.68 | 0.76 | 0.08 | 0.62 | 0.92 |
| bh | =~ | thoughts_harmony | 1.40 | 0.14 | 1.14 | 1.69 | 1.57 | 0.15 | 1.29 | 1.88 |
| bh | =~ | stable | 0.83 | 0.08 | 0.69 | 1.00 | 1.05 | 0.10 | 0.87 | 1.26 |
| bh | =~ | content | 1.18 | 0.11 | 0.98 | 1.43 | 1.68 | 0.15 | 1.39 | 1.99 |
| bh | =~ | mind_ease | 1.38 | 0.13 | 1.14 | 1.67 | 1.52 | 0.14 | 1.26 | 1.81 |
| bh | =~ | inner_peace | 0.95 | 0.09 | 0.79 | 1.14 | 1.05 | 0.10 | 0.87 | 1.24 |
| balance | \| | t1 | -3.38 | 0.17 | -3.72 | -3.06 | -3.38 | 0.17 | -3.72 | -3.06 |
| balance | \| | t2 | -1.29 | 0.07 | -1.44 | -1.15 | -1.29 | 0.07 | -1.44 | -1.15 |
| balance | \| | t3 | 1.86 | 0.10 | 1.68 | 2.05 | 1.86 | 0.10 | 1.68 | 2.05 |
| amount_right | \| | t1 | -2.03 | 0.08 | -2.19 | -1.88 | -2.03 | 0.08 | -2.19 | -1.88 |
| amount_right | \| | t2 | -0.71 | 0.04 | -0.80 | -0.62 | -0.71 | 0.04 | -0.80 | -0.62 |
| amount_right | \| | t3 | 1.36 | 0.06 | 1.25 | 1.47 | 1.36 | 0.06 | 1.25 | 1.47 |
| harmony_around | \| | t1 | -3.43 | 0.19 | -3.82 | -3.07 | -3.43 | 0.19 | -3.82 | -3.07 |
| harmony_around | \| | t2 | -2.07 | 0.09 | -2.26 | -1.89 | -2.07 | 0.09 | -2.26 | -1.89 |
| harmony_around | \| | t3 | 1.00 | 0.05 | 0.90 | 1.11 | 1.00 | 0.05 | 0.90 | 1.11 |
| thoughts_harmony | \| | t1 | -4.36 | 0.26 | -4.92 | -3.87 | -4.36 | 0.26 | -4.92 | -3.87 |
| thoughts_harmony | \| | t2 | -2.08 | 0.13 | -2.36 | -1.85 | -2.08 | 0.13 | -2.36 | -1.85 |
| thoughts_harmony | \| | t3 | 2.09 | 0.13 | 1.85 | 2.36 | 2.09 | 0.13 | 1.85 | 2.36 |
| stable | \| | t1 | -3.31 | 0.15 | -3.61 | -3.01 | -3.31 | 0.15 | -3.61 | -3.01 |
| stable | \| | t2 | -2.12 | 0.10 | -2.31 | -1.93 | -2.12 | 0.10 | -2.31 | -1.93 |
| stable | \| | t3 | 0.16 | 0.05 | 0.06 | 0.26 | 0.16 | 0.05 | 0.06 | 0.26 |
| content | \| | t1 | -3.89 | 0.21 | -4.32 | -3.50 | -3.89 | 0.21 | -4.32 | -3.50 |
| content | \| | t2 | -2.10 | 0.12 | -2.35 | -1.88 | -2.10 | 0.12 | -2.35 | -1.88 |
| content | \| | t3 | 1.73 | 0.10 | 1.55 | 1.93 | 1.73 | 0.10 | 1.55 | 1.93 |
| mind_ease | \| | t1 | -3.64 | 0.19 | -4.06 | -3.28 | -3.64 | 0.19 | -4.06 | -3.28 |
| mind_ease | \| | t2 | -1.60 | 0.10 | -1.80 | -1.42 | -1.60 | 0.10 | -1.80 | -1.42 |
| mind_ease | \| | t3 | 1.86 | 0.11 | 1.65 | 2.07 | 1.86 | 0.11 | 1.65 | 2.07 |
| inner_peace | \| | t1 | -2.75 | 0.12 | -2.99 | -2.50 | -2.75 | 0.12 | -2.99 | -2.50 |
| inner_peace | \| | t2 | -0.88 | 0.06 | -1.00 | -0.76 | -0.88 | 0.06 | -1.00 | -0.76 |
| inner_peace | \| | t3 | 1.72 | 0.08 | 1.57 | 1.88 | 1.72 | 0.08 | 1.57 | 1.88 |
| balance | ~~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 2.45 | 0.25 | 1.99 | 2.97 |
| amount_right | ~~ | amount_right | 1.00 | 0.00 | 1.00 | 1.00 | 1.36 | 0.11 | 1.15 | 1.59 |
| harmony_around | ~~ | harmony_around | 1.00 | 0.00 | 1.00 | 1.00 | 1.48 | 0.15 | 1.21 | 1.79 |
| thoughts_harmony | ~~ | thoughts_harmony | 1.00 | 0.00 | 1.00 | 1.00 | 1.58 | 0.22 | 1.21 | 2.07 |
| stable | ~~ | stable | 1.00 | 0.00 | 1.00 | 1.00 | 0.84 | 0.10 | 0.66 | 1.04 |
| content | ~~ | content | 1.00 | 0.00 | 1.00 | 1.00 | 1.71 | 0.22 | 1.32 | 2.17 |
| mind_ease | ~~ | mind_ease | 1.00 | 0.00 | 1.00 | 1.00 | 1.29 | 0.16 | 1.01 | 1.65 |
| inner_peace | ~~ | inner_peace | 1.00 | 0.00 | 1.00 | 1.00 | 1.24 | 0.12 | 1.02 | 1.49 |
| balance | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.21 | 0.08 | -0.37 | -0.05 |
| amount_right | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.06 | -0.01 | 0.21 |
| harmony_around | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.10 | 0.07 | -0.23 | 0.04 |
| thoughts_harmony | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.09 | -0.02 | 0.33 |
| stable | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.33 | 0.07 | -0.46 | -0.20 |
| content | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.31 | 0.09 | 0.14 | 0.49 |
| mind_ease | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.28 | 0.09 | -0.45 | -0.11 |
| inner_peace | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.37 | 0.07 | 0.24 | 0.51 |
| bh | ~~ | bh | 1.18 | 0.17 | 0.89 | 1.55 | 1.26 | 0.19 | 0.93 | 1.67 |

Table 23: Equal thresholds with SEN(0,1.5) posterior summary

| | | | Norway | | | | United States | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| lhs | op | rhs | mean | sd | Q.025 | Q.975 | mean | sd | Q.025 | Q.975 |
| bh | =~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| bh | =~ | amount_right | 0.46 | 0.05 | 0.37 | 0.57 | 0.57 | 0.06 | 0.47 | 0.70 |
| bh | =~ | harmony_around | 0.55 | 0.06 | 0.43 | 0.67 | 0.75 | 0.08 | 0.61 | 0.91 |
| bh | =~ | thoughts_harmony | 1.44 | 0.15 | 1.18 | 1.77 | 1.62 | 0.16 | 1.35 | 1.96 |
| bh | =~ | stable | 0.82 | 0.08 | 0.68 | 0.98 | 1.04 | 0.10 | 0.87 | 1.24 |
| bh | =~ | content | 1.20 | 0.11 | 0.99 | 1.43 | 1.70 | 0.15 | 1.42 | 2.02 |
| bh | =~ | mind_ease | 1.39 | 0.13 | 1.15 | 1.67 | 1.54 | 0.15 | 1.27 | 1.86 |
| bh | =~ | inner_peace | 0.94 | 0.09 | 0.78 | 1.13 | 1.05 | 0.10 | 0.87 | 1.25 |
| balance | \| | t1 | -3.40 | 0.16 | -3.74 | -3.11 | -3.40 | 0.16 | -3.74 | -3.11 |
| balance | \| | t2 | -1.30 | 0.07 | -1.45 | -1.16 | -1.30 | 0.07 | -1.45 | -1.16 |
| balance | \| | t3 | 1.91 | 0.10 | 1.72 | 2.10 | 1.91 | 0.10 | 1.72 | 2.10 |
| amount_right | \| | t1 | -2.02 | 0.08 | -2.19 | -1.87 | -2.02 | 0.08 | -2.19 | -1.87 |
| amount_right | \| | t2 | -0.70 | 0.05 | -0.80 | -0.62 | -0.70 | 0.05 | -0.80 | -0.62 |
| amount_right | \| | t3 | 1.37 | 0.06 | 1.26 | 1.50 | 1.37 | 0.06 | 1.26 | 1.50 |
| harmony_around | \| | t1 | -3.44 | 0.19 | -3.84 | -3.10 | -3.44 | 0.19 | -3.84 | -3.10 |
| harmony_around | \| | t2 | -2.10 | 0.09 | -2.29 | -1.91 | -2.10 | 0.09 | -2.29 | -1.91 |
| harmony_around | \| | t3 | 1.01 | 0.06 | 0.90 | 1.12 | 1.01 | 0.06 | 0.90 | 1.12 |
| thoughts_harmony | \| | t1 | -4.51 | 0.28 | -5.09 | -4.00 | -4.51 | 0.28 | -5.09 | -4.00 |
| thoughts_harmony | \| | t2 | -2.17 | 0.14 | -2.47 | -1.92 | -2.17 | 0.14 | -2.47 | -1.92 |
| thoughts_harmony | \| | t3 | 2.21 | 0.14 | 1.95 | 2.50 | 2.21 | 0.14 | 1.95 | 2.50 |
| stable | \| | t1 | -3.33 | 0.15 | -3.64 | -3.03 | -3.33 | 0.15 | -3.64 | -3.03 |
| stable | \| | t2 | -2.14 | 0.10 | -2.34 | -1.95 | -2.14 | 0.10 | -2.34 | -1.95 |
| stable | \| | t3 | 0.16 | 0.05 | 0.06 | 0.26 | 0.16 | 0.05 | 0.06 | 0.26 |
| content | \| | t1 | -3.97 | 0.22 | -4.43 | -3.57 | -3.97 | 0.22 | -4.43 | -3.57 |
| content | \| | t2 | -2.16 | 0.12 | -2.41 | -1.93 | -2.16 | 0.12 | -2.41 | -1.93 |
| content | \| | t3 | 1.80 | 0.10 | 1.61 | 2.01 | 1.80 | 0.10 | 1.61 | 2.01 |
| mind_ease | \| | t1 | -3.71 | 0.20 | -4.13 | -3.34 | -3.71 | 0.20 | -4.13 | -3.34 |
| mind_ease | \| | t2 | -1.63 | 0.10 | -1.84 | -1.44 | -1.63 | 0.10 | -1.84 | -1.44 |
| mind_ease | \| | t3 | 1.93 | 0.12 | 1.73 | 2.17 | 1.93 | 0.12 | 1.73 | 2.17 |
| inner_peace | \| | t1 | -2.75 | 0.13 | -3.02 | -2.50 | -2.75 | 0.13 | -3.02 | -2.50 |
| inner_peace | \| | t2 | -0.87 | 0.06 | -1.00 | -0.75 | -0.87 | 0.06 | -1.00 | -0.75 |
| inner_peace | \| | t3 | 1.75 | 0.09 | 1.59 | 1.92 | 1.75 | 0.09 | 1.59 | 1.92 |
| balance | ~~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 2.53 | 0.26 | 2.05 | 3.08 |
| amount_right | ~~ | amount_right | 1.00 | 0.00 | 1.00 | 1.00 | 1.37 | 0.12 | 1.15 | 1.61 |
| harmony_around | ~~ | harmony_around | 1.00 | 0.00 | 1.00 | 1.00 | 1.51 | 0.15 | 1.24 | 1.83 |
| thoughts_harmony | ~~ | thoughts_harmony | 1.00 | 0.00 | 1.00 | 1.00 | 1.73 | 0.25 | 1.30 | 2.29 |
| stable | ~~ | stable | 1.00 | 0.00 | 1.00 | 1.00 | 0.85 | 0.10 | 0.67 | 1.06 |
| content | ~~ | content | 1.00 | 0.00 | 1.00 | 1.00 | 1.82 | 0.24 | 1.38 | 2.33 |
| mind_ease | ~~ | mind_ease | 1.00 | 0.00 | 1.00 | 1.00 | 1.36 | 0.17 | 1.05 | 1.73 |
| inner_peace | ~~ | inner_peace | 1.00 | 0.00 | 1.00 | 1.00 | 1.27 | 0.13 | 1.04 | 1.55 |
| balance | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.20 | 0.08 | -0.37 | -0.04 |
| amount_right | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.06 | -0.00 | 0.23 |
| harmony_around | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.10 | 0.07 | -0.24 | 0.04 |
| thoughts_harmony | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.17 | 0.10 | -0.01 | 0.36 |
| stable | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.34 | 0.07 | -0.47 | -0.20 |
| content | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.33 | 0.09 | 0.14 | 0.52 |
| mind_ease | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.27 | 0.09 | -0.44 | -0.10 |
| inner_peace | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.39 | 0.07 | 0.25 | 0.53 |
| bh | ~~ | bh | 1.24 | 0.17 | 0.94 | 1.61 | 1.29 | 0.19 | 0.95 | 1.71 |

Table 24: Equal thresholds with SEN(0,10E5) posterior summary

| | | | Norway | | | | United States | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| lhs | op | rhs | mean | sd | Q.025 | Q.975 | mean | sd | Q.025 | Q.975 |
| bh | =~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| bh | =~ | amount_right | 0.46 | 0.05 | 0.37 | 0.56 | 0.57 | 0.06 | 0.46 | 0.69 |
| bh | =~ | harmony_around | 0.55 | 0.06 | 0.44 | 0.67 | 0.76 | 0.08 | 0.61 | 0.92 |
| bh | =~ | thoughts_harmony | 1.49 | 0.16 | 1.20 | 1.81 | 1.66 | 0.17 | 1.36 | 2.01 |
| bh | =~ | stable | 0.82 | 0.08 | 0.67 | 0.98 | 1.04 | 0.10 | 0.86 | 1.25 |
| bh | =~ | content | 1.21 | 0.11 | 1.00 | 1.44 | 1.71 | 0.16 | 1.43 | 2.05 |
| bh | =~ | mind_ease | 1.40 | 0.14 | 1.15 | 1.68 | 1.55 | 0.15 | 1.28 | 1.87 |
| bh | =~ | inner_peace | 0.93 | 0.09 | 0.76 | 1.12 | 1.04 | 0.10 | 0.86 | 1.24 |
| balance | \| | t1 | -3.46 | 0.17 | -3.81 | -3.16 | -3.46 | 0.17 | -3.81 | -3.16 |
| balance | \| | t2 | -1.33 | 0.08 | -1.48 | -1.18 | -1.33 | 0.08 | -1.48 | -1.18 |
| balance | \| | t3 | 1.90 | 0.10 | 1.72 | 2.11 | 1.90 | 0.10 | 1.72 | 2.11 |
| amount_right | \| | t1 | -2.03 | 0.08 | -2.20 | -1.87 | -2.03 | 0.08 | -2.20 | -1.87 |
| amount_right | \| | t2 | -0.71 | 0.05 | -0.80 | -0.62 | -0.71 | 0.05 | -0.80 | -0.62 |
| amount_right | \| | t3 | 1.37 | 0.06 | 1.25 | 1.48 | 1.37 | 0.06 | 1.25 | 1.48 |
| harmony_around | \| | t1 | -3.51 | 0.19 | -3.92 | -3.15 | -3.51 | 0.19 | -3.92 | -3.15 |
| harmony_around | \| | t2 | -2.12 | 0.10 | -2.33 | -1.93 | -2.12 | 0.10 | -2.33 | -1.93 |
| harmony_around | \| | t3 | 1.01 | 0.05 | 0.90 | 1.12 | 1.01 | 0.05 | 0.90 | 1.12 |
| thoughts_harmony | \| | t1 | -4.72 | 0.31 | -5.37 | -4.15 | -4.72 | 0.31 | -5.37 | -4.15 |
| thoughts_harmony | \| | t2 | -2.26 | 0.15 | -2.56 | -1.98 | -2.26 | 0.15 | -2.56 | -1.98 |
| thoughts_harmony | \| | t3 | 2.24 | 0.15 | 1.96 | 2.55 | 2.24 | 0.15 | 1.96 | 2.55 |
| stable | \| | t1 | -3.36 | 0.16 | -3.68 | -3.07 | -3.36 | 0.16 | -3.68 | -3.07 |
| stable | \| | t2 | -2.16 | 0.10 | -2.36 | -1.98 | -2.16 | 0.10 | -2.36 | -1.98 |
| stable | \| | t3 | 0.15 | 0.05 | 0.05 | 0.25 | 0.15 | 0.05 | 0.05 | 0.25 |
| content | \| | t1 | -4.07 | 0.22 | -4.52 | -3.65 | -4.07 | 0.22 | -4.52 | -3.65 |
| content | \| | t2 | -2.21 | 0.12 | -2.46 | -1.96 | -2.21 | 0.12 | -2.46 | -1.96 |
| content | \| | t3 | 1.80 | 0.10 | 1.61 | 2.01 | 1.80 | 0.10 | 1.61 | 2.01 |
| mind_ease | \| | t1 | -3.80 | 0.22 | -4.24 | -3.39 | -3.80 | 0.22 | -4.24 | -3.39 |
| mind_ease | \| | t2 | -1.68 | 0.11 | -1.89 | -1.47 | -1.68 | 0.11 | -1.89 | -1.47 |
| mind_ease | \| | t3 | 1.94 | 0.12 | 1.72 | 2.17 | 1.94 | 0.12 | 1.72 | 2.17 |
| inner_peace | \| | t1 | -2.78 | 0.13 | -3.05 | -2.53 | -2.78 | 0.13 | -3.05 | -2.53 |
| inner_peace | \| | t2 | -0.89 | 0.06 | -1.01 | -0.77 | -0.89 | 0.06 | -1.01 | -0.77 |
| inner_peace | \| | t3 | 1.74 | 0.08 | 1.58 | 1.92 | 1.74 | 0.08 | 1.58 | 1.92 |
| balance | ~~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 2.57 | 0.27 | 2.09 | 3.13 |
| amount_right | ~~ | amount_right | 1.00 | 0.00 | 1.00 | 1.00 | 1.37 | 0.12 | 1.15 | 1.61 |
| harmony_around | ~~ | harmony_around | 1.00 | 0.00 | 1.00 | 1.00 | 1.54 | 0.15 | 1.27 | 1.86 |
| thoughts_harmony | ~~ | thoughts_harmony | 1.00 | 0.00 | 1.00 | 1.00 | 1.84 | 0.27 | 1.37 | 2.44 |
| stable | ~~ | stable | 1.00 | 0.00 | 1.00 | 1.00 | 0.86 | 0.10 | 0.69 | 1.07 |
| content | ~~ | content | 1.00 | 0.00 | 1.00 | 1.00 | 1.87 | 0.24 | 1.44 | 2.39 |
| mind_ease | ~~ | mind_ease | 1.00 | 0.00 | 1.00 | 1.00 | 1.40 | 0.19 | 1.08 | 1.83 |
| inner_peace | ~~ | inner_peace | 1.00 | 0.00 | 1.00 | 1.00 | 1.28 | 0.13 | 1.04 | 1.56 |
| balance | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.22 | 0.08 | -0.38 | -0.06 |
| amount_right | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.06 | -0.01 | 0.22 |
| harmony_around | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.11 | 0.07 | -0.25 | 0.03 |
| thoughts_harmony | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.10 | -0.04 | 0.35 |
| stable | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.35 | 0.07 | -0.48 | -0.21 |
| content | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.31 | 0.09 | 0.13 | 0.50 |
| mind_ease | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.29 | 0.09 | -0.46 | -0.12 |
| inner_peace | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.07 | 0.23 | 0.52 |
| bh | ~~ | bh | 1.26 | 0.18 | 0.95 | 1.65 | 1.32 | 0.20 | 0.97 | 1.75 |

### 5.4.4 Results with Informative Priors in Non-reference Group

Table 25: Informed latent response with ID([1,1,1,1]) posterior summary

| | | | Norway | | | | United States | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| lhs | op | rhs | mean | sd | Q.025 | Q.975 | mean | sd | Q.025 | Q.975 |
| bh | =~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| bh | =~ | amount_right | 0.46 | 0.05 | 0.37 | 0.56 | 0.94 | 0.29 | 0.51 | 1.68 |
| bh | =~ | harmony_around | 0.54 | 0.06 | 0.43 | 0.66 | 1.05 | 0.34 | 0.50 | 1.86 |
| bh | =~ | thoughts_harmony | 1.44 | 0.14 | 1.18 | 1.74 | 1.75 | 0.65 | 0.71 | 3.29 |
| bh | =~ | stable | 0.80 | 0.08 | 0.66 | 0.96 | 1.84 | 0.61 | 0.83 | 3.36 |
| bh | =~ | content | 1.20 | 0.12 | 0.99 | 1.43 | 1.96 | 0.65 | 0.84 | 3.41 |
| bh | =~ | mind_ease | 1.43 | 0.14 | 1.18 | 1.73 | 2.07 | 0.69 | 0.90 | 3.68 |
| bh | =~ | inner_peace | 0.94 | 0.09 | 0.78 | 1.13 | 1.61 | 0.48 | 0.81 | 2.82 |
| balance | \| | t1 | -3.40 | 0.19 | -3.79 | -3.06 | -1.67 | 0.41 | -2.45 | -0.85 |
| balance | \| | t2 | -1.32 | 0.08 | -1.48 | -1.17 | -0.52 | 0.25 | -1.02 | -0.02 |
| balance | \| | t3 | 1.92 | 0.10 | 1.73 | 2.12 | 1.19 | 0.33 | 0.55 | 1.82 |
| amount_right | \| | t1 | -2.13 | 0.09 | -2.32 | -1.95 | -1.68 | 0.38 | -2.45 | -0.89 |
| amount_right | \| | t2 | -0.67 | 0.05 | -0.77 | -0.57 | -0.66 | 0.26 | -1.19 | -0.14 |
| amount_right | \| | t3 | 1.36 | 0.06 | 1.24 | 1.49 | 1.15 | 0.31 | 0.56 | 1.77 |
| harmony_around | \| | t1 | -4.26 | 0.71 | -6.06 | -3.29 | -2.25 | 0.54 | -3.23 | -1.05 |
| harmony_around | \| | t2 | -2.08 | 0.10 | -2.28 | -1.89 | -1.33 | 0.38 | -2.04 | -0.54 |
| harmony_around | \| | t3 | 1.01 | 0.05 | 0.91 | 1.12 | 0.93 | 0.30 | 0.32 | 1.52 |
| thoughts_harmony | \| | t1 | -4.53 | 0.32 | -5.18 | -3.93 | -2.54 | 0.79 | -3.96 | -1.00 |
| thoughts_harmony | \| | t2 | -2.16 | 0.14 | -2.46 | -1.90 | -1.20 | 0.44 | -2.05 | -0.34 |
| thoughts_harmony | \| | t3 | 2.22 | 0.15 | 1.96 | 2.52 | 1.24 | 0.42 | 0.41 | 2.03 |
| stable | \| | t1 | -3.31 | 0.19 | -3.71 | -2.96 | -2.57 | 0.68 | -3.77 | -1.16 |
| stable | \| | t2 | -2.10 | 0.10 | -2.31 | -1.91 | -1.51 | 0.45 | -2.31 | -0.58 |
| stable | \| | t3 | 0.15 | 0.05 | 0.05 | 0.26 | 0.61 | 0.27 | 0.07 | 1.14 |
| content | \| | t1 | -4.07 | 0.26 | -4.62 | -3.60 | -2.44 | 0.66 | -3.60 | -1.12 |
| content | \| | t2 | -2.13 | 0.13 | -2.40 | -1.90 | -1.38 | 0.43 | -2.18 | -0.53 |
| content | \| | t3 | 1.82 | 0.11 | 1.61 | 2.04 | 0.99 | 0.33 | 0.32 | 1.60 |
| mind_ease | \| | t1 | -4.07 | 0.26 | -4.61 | -3.58 | -2.23 | 0.60 | -3.28 | -0.95 |
| mind_ease | \| | t2 | -1.59 | 0.11 | -1.81 | -1.39 | -0.91 | 0.34 | -1.53 | -0.25 |
| mind_ease | \| | t3 | 1.93 | 0.13 | 1.69 | 2.20 | 1.66 | 0.47 | 0.70 | 2.52 |
| inner_peace | \| | t1 | -2.81 | 0.14 | -3.10 | -2.54 | -2.38 | 0.52 | -3.32 | -1.24 |
| inner_peace | \| | t2 | -0.85 | 0.06 | -0.97 | -0.72 | -0.94 | 0.30 | -1.52 | -0.35 |
| inner_peace | \| | t3 | 1.77 | 0.09 | 1.61 | 1.94 | 1.19 | 0.32 | 0.55 | 1.80 |
| balance | ~~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 0.77 | 0.29 | 0.25 | 1.32 |
| amount_right | ~~ | amount_right | 1.00 | 0.00 | 1.00 | 1.00 | 1.02 | 0.36 | 0.39 | 1.90 |
| harmony_around | ~~ | harmony_around | 1.00 | 0.00 | 1.00 | 1.00 | 0.84 | 0.32 | 0.23 | 1.46 |
| thoughts_harmony | ~~ | thoughts_harmony | 1.00 | 0.00 | 1.00 | 1.00 | 0.61 | 0.32 | 0.11 | 1.21 |
| stable | ~~ | stable | 1.00 | 0.00 | 1.00 | 1.00 | 0.74 | 0.32 | 0.17 | 1.35 |
| content | ~~ | content | 1.00 | 0.00 | 1.00 | 1.00 | 0.70 | 0.31 | 0.16 | 1.29 |
| mind_ease | ~~ | mind_ease | 1.00 | 0.00 | 1.00 | 1.00 | 0.72 | 0.31 | 0.15 | 1.31 |
| inner_peace | ~~ | inner_peace | 1.00 | 0.00 | 1.00 | 1.00 | 0.86 | 0.30 | 0.28 | 1.45 |
| balance | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.23 | -0.39 | 0.51 |
| amount_right | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.23 | -0.38 | 0.52 |
| harmony_around | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.24 | -0.35 | 0.60 |
| thoughts_harmony | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.25 | -0.41 | 0.59 |
| stable | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 | 0.24 | -0.33 | 0.59 |
| content | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.24 | -0.35 | 0.58 |
| mind_ease | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.24 | -0.39 | 0.56 |
| inner_peace | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.23 | -0.36 | 0.54 |
| bh | ~~ | bh | 1.25 | 0.18 | 0.94 | 1.62 | 0.39 | 0.15 | 0.12 | 0.72 |

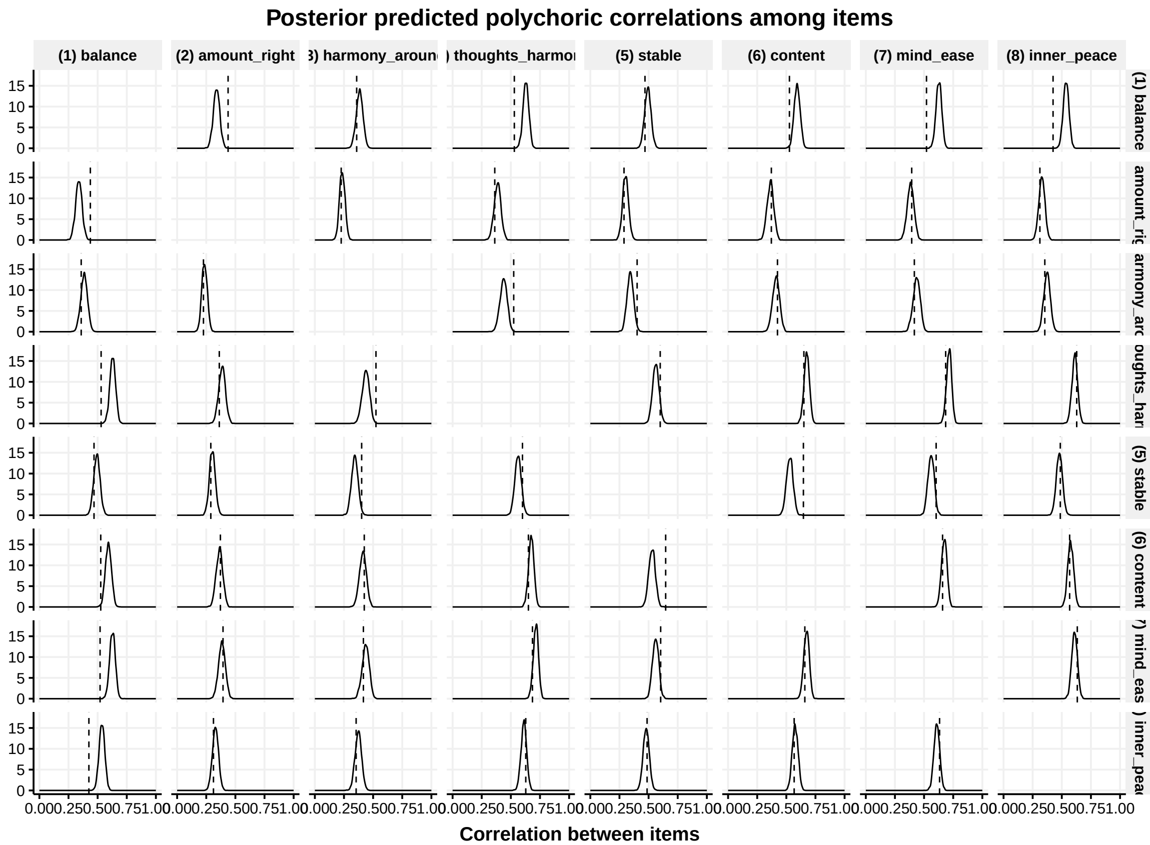



Figure 11: Informed latent response with ID([1,1,1,1]) posterior distribution of model implied polychoric correlation

*Note.* Vertical dashed lines represent the observed polychoric correlation among items. Polychoric correlations were estimated using the *psych* packages in R.

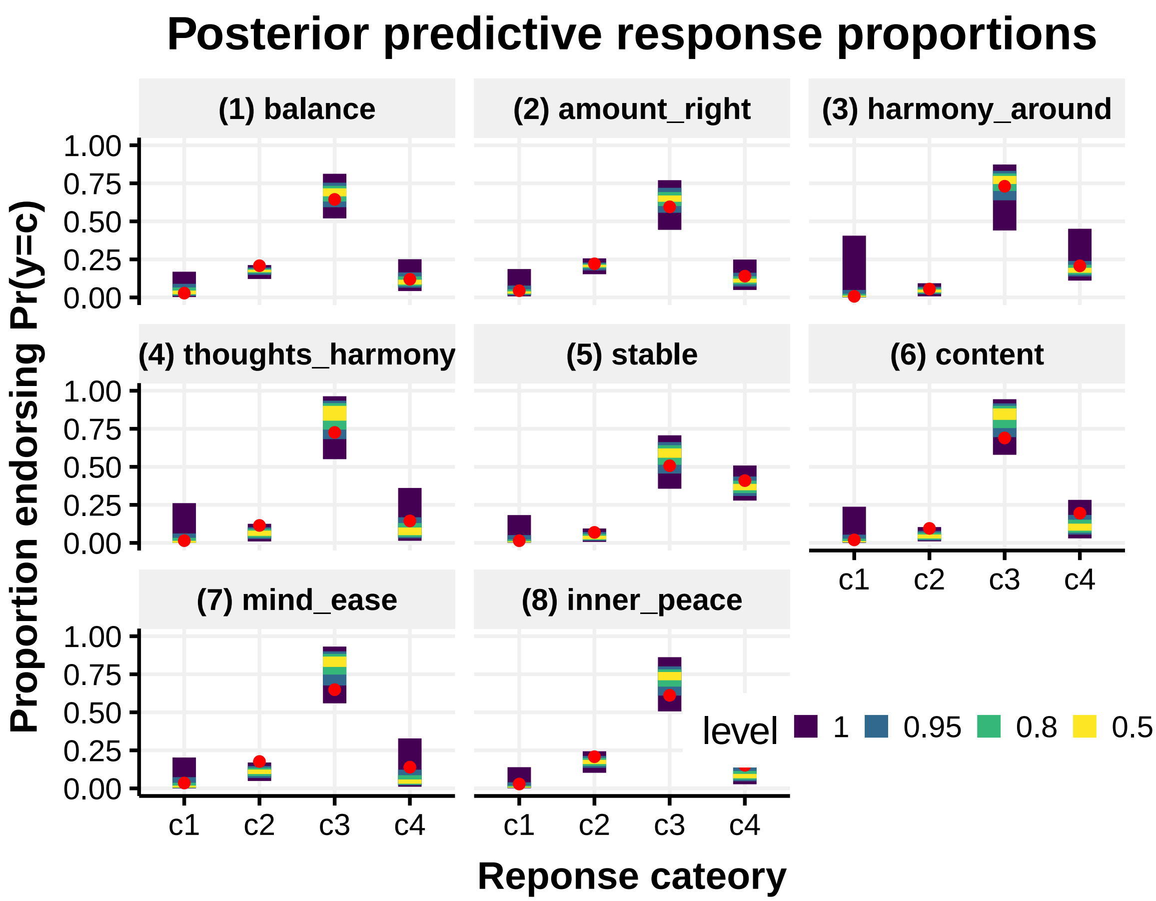



Figure 12: Informed latent response with ID([1,1,1,1]) PPD of item endorsement proportions across items
*Note.* Dots represent the observed proportion of the sample endorsing the category.

Table 26: Informed latent response with ID([1,3,8,4]) posterior summary

| | | | Norway | | | | United States | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| lhs | op | rhs | mean | sd | Q.025 | Q.975 | mean | sd | Q.025 | Q.975 |
| bh | =~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| bh | =~ | amount_right | 0.47 | 0.05 | 0.38 | 0.57 | 0.94 | 0.23 | 0.58 | 1.50 |
| bh | =~ | harmony_around | 0.55 | 0.06 | 0.43 | 0.67 | 1.02 | 0.27 | 0.58 | 1.63 |
| bh | =~ | thoughts_harmony | 1.39 | 0.13 | 1.15 | 1.68 | 1.49 | 0.45 | 0.80 | 2.55 |
| bh | =~ | stable | 0.81 | 0.08 | 0.67 | 0.97 | 1.77 | 0.49 | 0.95 | 2.86 |
| bh | =~ | content | 1.18 | 0.11 | 0.98 | 1.41 | 1.77 | 0.51 | 0.94 | 2.94 |
| bh | =~ | mind_ease | 1.40 | 0.14 | 1.15 | 1.69 | 1.68 | 0.49 | 0.90 | 2.82 |
| bh | =~ | inner_peace | 0.95 | 0.09 | 0.78 | 1.14 | 1.52 | 0.40 | 0.84 | 2.44 |
| balance | \| | t1 | -3.39 | 0.19 | -3.76 | -3.03 | -1.82 | 0.36 | -2.50 | -1.11 |
| balance | \| | t2 | -1.31 | 0.08 | -1.48 | -1.16 | -0.68 | 0.22 | -1.11 | -0.26 |
| balance | \| | t3 | 1.87 | 0.09 | 1.69 | 2.06 | 1.00 | 0.27 | 0.49 | 1.53 |
| amount_right | \| | t1 | -2.14 | 0.09 | -2.31 | -1.96 | -1.80 | 0.31 | -2.43 | -1.18 |
| amount_right | \| | t2 | -0.68 | 0.05 | -0.77 | -0.58 | -0.79 | 0.22 | -1.23 | -0.36 |
| amount_right | \| | t3 | 1.34 | 0.06 | 1.22 | 1.46 | 1.01 | 0.25 | 0.54 | 1.49 |
| harmony_around | \| | t1 | -4.26 | 0.67 | -5.98 | -3.30 | -2.19 | 0.44 | -3.03 | -1.29 |
| harmony_around | \| | t2 | -2.06 | 0.09 | -2.24 | -1.88 | -1.30 | 0.31 | -1.92 | -0.67 |
| harmony_around | \| | t3 | 1.00 | 0.05 | 0.89 | 1.10 | 0.88 | 0.25 | 0.42 | 1.36 |
| thoughts_harmony | \| | t1 | -4.41 | 0.29 | -5.03 | -3.88 | -2.23 | 0.55 | -3.37 | -1.26 |
| thoughts_harmony | \| | t2 | -2.09 | 0.13 | -2.36 | -1.84 | -1.09 | 0.32 | -1.75 | -0.52 |
| thoughts_harmony | \| | t3 | 2.11 | 0.13 | 1.87 | 2.37 | 0.99 | 0.30 | 0.44 | 1.61 |
| stable | \| | t1 | -3.32 | 0.19 | -3.71 | -2.97 | -2.40 | 0.54 | -3.39 | -1.35 |
| stable | \| | t2 | -2.09 | 0.10 | -2.29 | -1.90 | -1.37 | 0.36 | -2.06 | -0.68 |
| stable | \| | t3 | 0.15 | 0.05 | 0.04 | 0.25 | 0.64 | 0.23 | 0.20 | 1.09 |
| content | \| | t1 | -4.02 | 0.25 | -4.53 | -3.55 | -2.20 | 0.53 | -3.25 | -1.22 |
| content | \| | t2 | -2.10 | 0.12 | -2.35 | -1.88 | -1.25 | 0.35 | -1.94 | -0.59 |
| content | \| | t3 | 1.76 | 0.10 | 1.56 | 1.96 | 0.87 | 0.27 | 0.36 | 1.42 |
| mind_ease | \| | t1 | -4.01 | 0.25 | -4.53 | -3.55 | -1.95 | 0.48 | -2.93 | -1.08 |
| mind_ease | \| | t2 | -1.56 | 0.10 | -1.78 | -1.37 | -0.88 | 0.28 | -1.44 | -0.36 |
| mind_ease | \| | t3 | 1.85 | 0.12 | 1.64 | 2.10 | 1.19 | 0.36 | 0.55 | 1.95 |
| inner_peace | \| | t1 | -2.82 | 0.14 | -3.09 | -2.56 | -2.34 | 0.47 | -3.23 | -1.41 |
| inner_peace | \| | t2 | -0.86 | 0.06 | -0.98 | -0.74 | -0.98 | 0.27 | -1.53 | -0.47 |
| inner_peace | \| | t3 | 1.73 | 0.08 | 1.57 | 1.90 | 1.02 | 0.27 | 0.49 | 1.55 |
| balance | ~~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 0.74 | 0.25 | 0.31 | 1.21 |
| amount_right | ~~ | amount_right | 1.00 | 0.00 | 1.00 | 1.00 | 0.99 | 0.26 | 0.51 | 1.55 |
| harmony_around | ~~ | harmony_around | 1.00 | 0.00 | 1.00 | 1.00 | 0.77 | 0.25 | 0.31 | 1.26 |
| thoughts_harmony | ~~ | thoughts_harmony | 1.00 | 0.00 | 1.00 | 1.00 | 0.43 | 0.20 | 0.15 | 0.92 |
| stable | ~~ | stable | 1.00 | 0.00 | 1.00 | 1.00 | 0.67 | 0.25 | 0.24 | 1.15 |
| content | ~~ | content | 1.00 | 0.00 | 1.00 | 1.00 | 0.56 | 0.24 | 0.19 | 1.05 |
| mind_ease | ~~ | mind_ease | 1.00 | 0.00 | 1.00 | 1.00 | 0.47 | 0.22 | 0.15 | 0.98 |
| inner_peace | ~~ | inner_peace | 1.00 | 0.00 | 1.00 | 1.00 | 0.76 | 0.26 | 0.30 | 1.27 |
| balance | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.11 | 0.20 | -0.49 | 0.27 |
| amount_right | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.07 | 0.20 | -0.46 | 0.31 |
| harmony_around | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.20 | -0.30 | 0.49 |
| thoughts_harmony | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.20 | -0.35 | 0.42 |
| stable | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.20 | -0.19 | 0.60 |
| content | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.20 | -0.30 | 0.49 |
| mind_ease | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.07 | 0.21 | -0.48 | 0.33 |
| inner_peace | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.21 | -0.41 | 0.39 |
| bh | ~~ | bh | 1.21 | 0.16 | 0.91 | 1.55 | 0.37 | 0.13 | 0.16 | 0.67 |

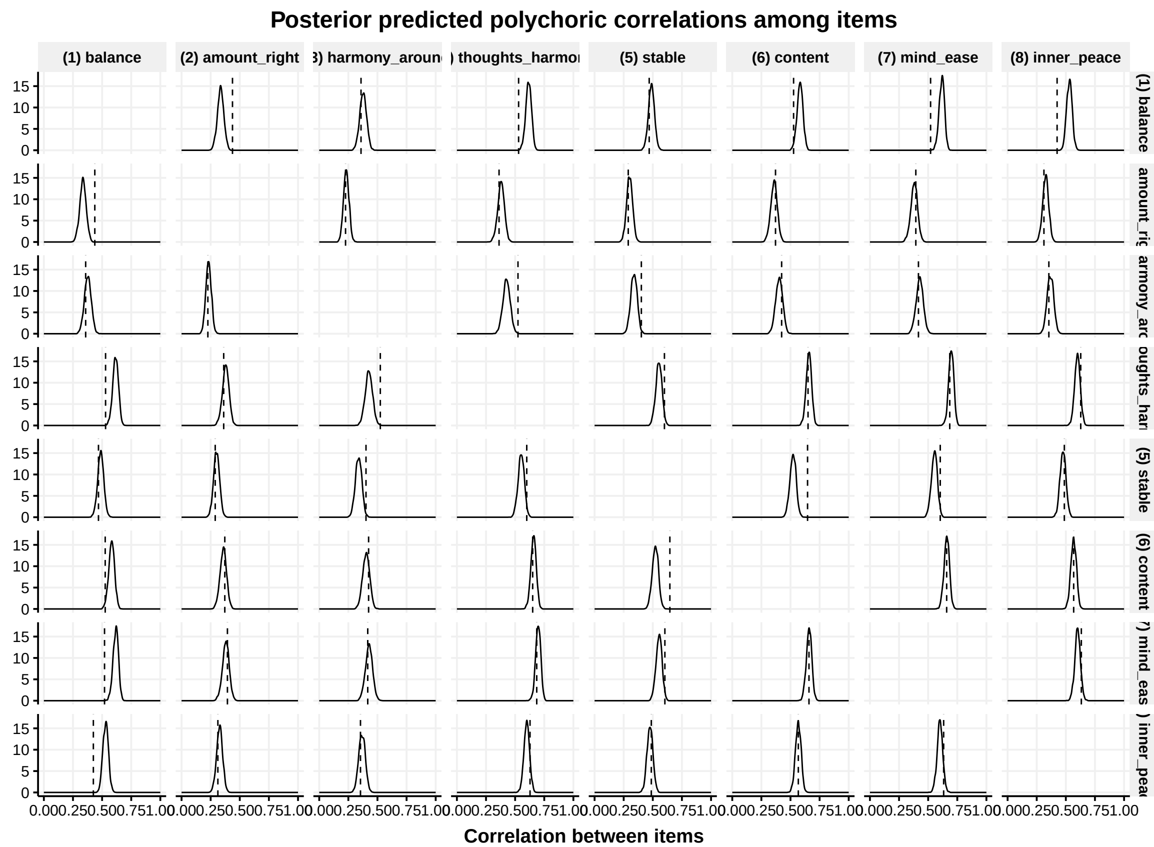



Figure 13: Informed latent response with ID([1,3,8,4]) posterior distribution of model implied polychoric correlation

*Note.* Vertical dashed lines represent the observed polychoric correlation among items. Polychoric correlations were estimated using the *psych* packages in R.

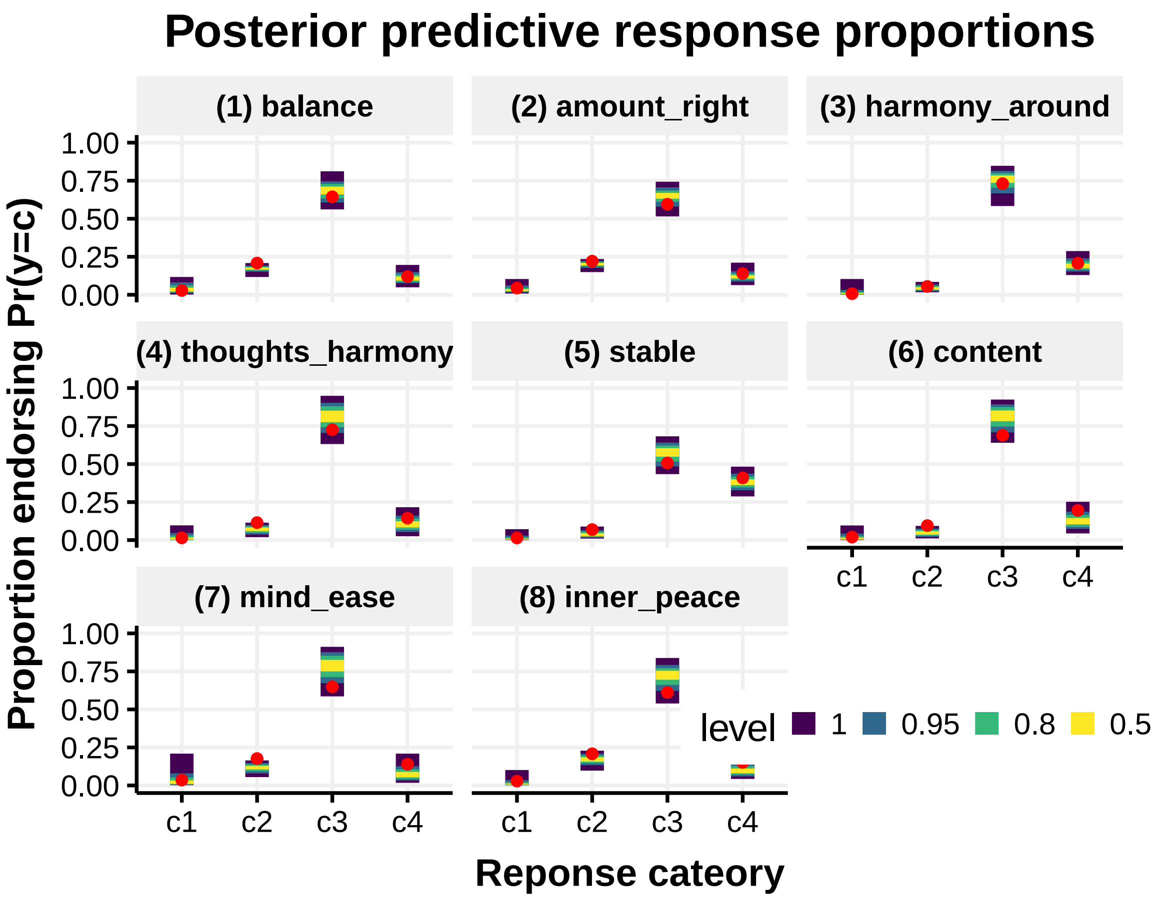



Figure 14: Informed latent response with ID([1,3,8,4]) PPD of item endorsement proportions across items
*Note.* Dots represent the observed proportion of the sample endorsing the category.

Table 27: Informed latent response with SEN(0,1.5) posterior summary

| | | | Norway | | | | United States | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| lhs | op | rhs | mean | sd | Q.025 | Q.975 | mean | sd | Q.025 | Q.975 |
| bh | =~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| bh | =~ | amount_right | 0.45 | 0.05 | 0.37 | 0.55 | 0.89 | 0.23 | 0.51 | 1.43 |
| bh | =~ | harmony_around | 0.53 | 0.06 | 0.43 | 0.65 | 1.04 | 0.28 | 0.45 | 1.63 |
| bh | =~ | thoughts_harmony | 1.43 | 0.14 | 1.18 | 1.73 | 1.86 | 0.55 | 0.86 | 3.01 |
| bh | =~ | stable | 0.79 | 0.07 | 0.66 | 0.94 | 1.80 | 0.53 | 0.76 | 2.89 |
| bh | =~ | content | 1.19 | 0.11 | 0.97 | 1.44 | 2.00 | 0.61 | 0.83 | 3.35 |
| bh | =~ | mind_ease | 1.39 | 0.13 | 1.16 | 1.67 | 2.17 | 0.58 | 1.16 | 3.42 |
| bh | =~ | inner_peace | 0.92 | 0.08 | 0.77 | 1.09 | 1.51 | 0.44 | 0.69 | 2.45 |
| balance | \| | t1 | -3.43 | 0.19 | -3.84 | -3.09 | -1.80 | 0.38 | -2.56 | -1.01 |
| balance | \| | t2 | -1.34 | 0.08 | -1.50 | -1.19 | -0.56 | 0.26 | -1.09 | -0.04 |
| balance | \| | t3 | 1.94 | 0.10 | 1.76 | 2.15 | 1.27 | 0.32 | 0.62 | 1.88 |
| amount_right | \| | t1 | -2.12 | 0.09 | -2.31 | -1.95 | -1.73 | 0.42 | -2.54 | -0.91 |
| amount_right | \| | t2 | -0.67 | 0.05 | -0.77 | -0.57 | -0.69 | 0.29 | -1.28 | -0.12 |
| amount_right | \| | t3 | 1.36 | 0.06 | 1.24 | 1.47 | 1.18 | 0.35 | 0.57 | 1.90 |
| harmony_around | \| | t1 | -3.98 | 0.48 | -5.09 | -3.25 | -2.46 | 0.56 | -3.53 | -1.15 |
| harmony_around | \| | t2 | -2.09 | 0.10 | -2.28 | -1.90 | -1.48 | 0.39 | -2.19 | -0.65 |
| harmony_around | \| | t3 | 1.01 | 0.06 | 0.91 | 1.12 | 0.95 | 0.30 | 0.31 | 1.52 |
| thoughts_harmony | \| | t1 | -4.54 | 0.30 | -5.18 | -4.00 | -2.95 | 0.72 | -4.17 | -1.40 |
| thoughts_harmony | \| | t2 | -2.19 | 0.14 | -2.49 | -1.94 | -1.43 | 0.42 | -2.14 | -0.56 |
| thoughts_harmony | \| | t3 | 2.25 | 0.14 | 1.98 | 2.54 | 1.41 | 0.39 | 0.60 | 2.15 |
| stable | \| | t1 | -3.30 | 0.19 | -3.70 | -2.97 | -2.78 | 0.68 | -3.89 | -1.25 |
| stable | \| | t2 | -2.11 | 0.10 | -2.32 | -1.91 | -1.66 | 0.45 | -2.46 | -0.71 |
| stable | \| | t3 | 0.15 | 0.05 | 0.05 | 0.25 | 0.57 | 0.28 | 0.00 | 1.14 |
| content | \| | t1 | -4.09 | 0.24 | -4.59 | -3.62 | -2.74 | 0.67 | -3.79 | -1.12 |
| content | \| | t2 | -2.17 | 0.12 | -2.43 | -1.93 | -1.59 | 0.44 | -2.34 | -0.59 |
| content | \| | t3 | 1.84 | 0.11 | 1.64 | 2.05 | 1.04 | 0.32 | 0.35 | 1.61 |
| mind_ease | \| | t1 | -4.03 | 0.25 | -4.56 | -3.59 | -2.54 | 0.54 | -3.56 | -1.35 |
| mind_ease | \| | t2 | -1.60 | 0.11 | -1.83 | -1.39 | -1.05 | 0.33 | -1.70 | -0.41 |
| mind_ease | \| | t3 | 1.93 | 0.12 | 1.71 | 2.16 | 1.88 | 0.45 | 0.96 | 2.74 |
| inner_peace | \| | t1 | -2.81 | 0.14 | -3.11 | -2.53 | -2.43 | 0.55 | -3.44 | -1.19 |
| inner_peace | \| | t2 | -0.85 | 0.06 | -0.98 | -0.73 | -0.97 | 0.32 | -1.56 | -0.36 |
| inner_peace | \| | t3 | 1.77 | 0.09 | 1.60 | 1.96 | 1.19 | 0.35 | 0.46 | 1.89 |
| balance | ~~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 0.86 | 0.27 | 0.39 | 1.43 |
| amount_right | ~~ | amount_right | 1.00 | 0.00 | 1.00 | 1.00 | 1.09 | 0.45 | 0.40 | 2.04 |
| harmony_around | ~~ | harmony_around | 1.00 | 0.00 | 1.00 | 1.00 | 0.96 | 0.35 | 0.21 | 1.74 |
| thoughts_harmony | ~~ | thoughts_harmony | 1.00 | 0.00 | 1.00 | 1.00 | 0.78 | 0.31 | 0.17 | 1.39 |
| stable | ~~ | stable | 1.00 | 0.00 | 1.00 | 1.00 | 0.82 | 0.32 | 0.17 | 1.42 |
| content | ~~ | content | 1.00 | 0.00 | 1.00 | 1.00 | 0.84 | 0.32 | 0.16 | 1.47 |
| mind_ease | ~~ | mind_ease | 1.00 | 0.00 | 1.00 | 1.00 | 0.91 | 0.32 | 0.28 | 1.61 |
| inner_peace | ~~ | inner_peace | 1.00 | 0.00 | 1.00 | 1.00 | 0.89 | 0.33 | 0.22 | 1.52 |
| balance | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.24 | -0.42 | 0.52 |
| amount_right | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.26 | -0.43 | 0.55 |
| harmony_around | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.23 | -0.39 | 0.54 |
| thoughts_harmony | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.24 | -0.38 | 0.55 |
| stable | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.25 | -0.41 | 0.56 |
| content | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.24 | -0.38 | 0.55 |
| mind_ease | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.25 | -0.42 | 0.53 |
| inner_peace | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.25 | -0.39 | 0.59 |
| bh | ~~ | bh | 1.30 | 0.18 | 0.98 | 1.71 | 0.44 | 0.15 | 0.20 | 0.79 |

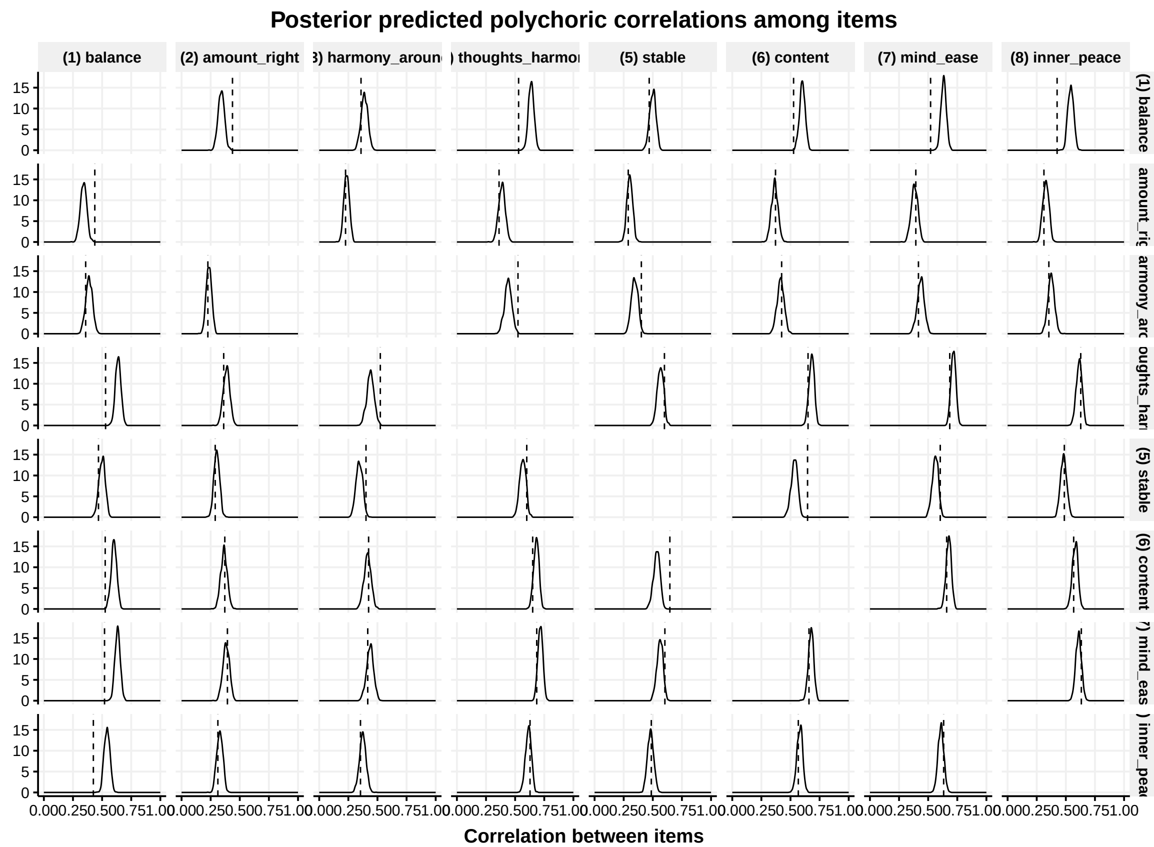



Figure 15: Informed latent response with SEN(0,1.5) posterior distribution of model implied polychoric correlation

*Note.* Vertical dashed lines represent the observed polychoric correlations among items. Polychoric correlations were estimated using the *psych* packages in R.

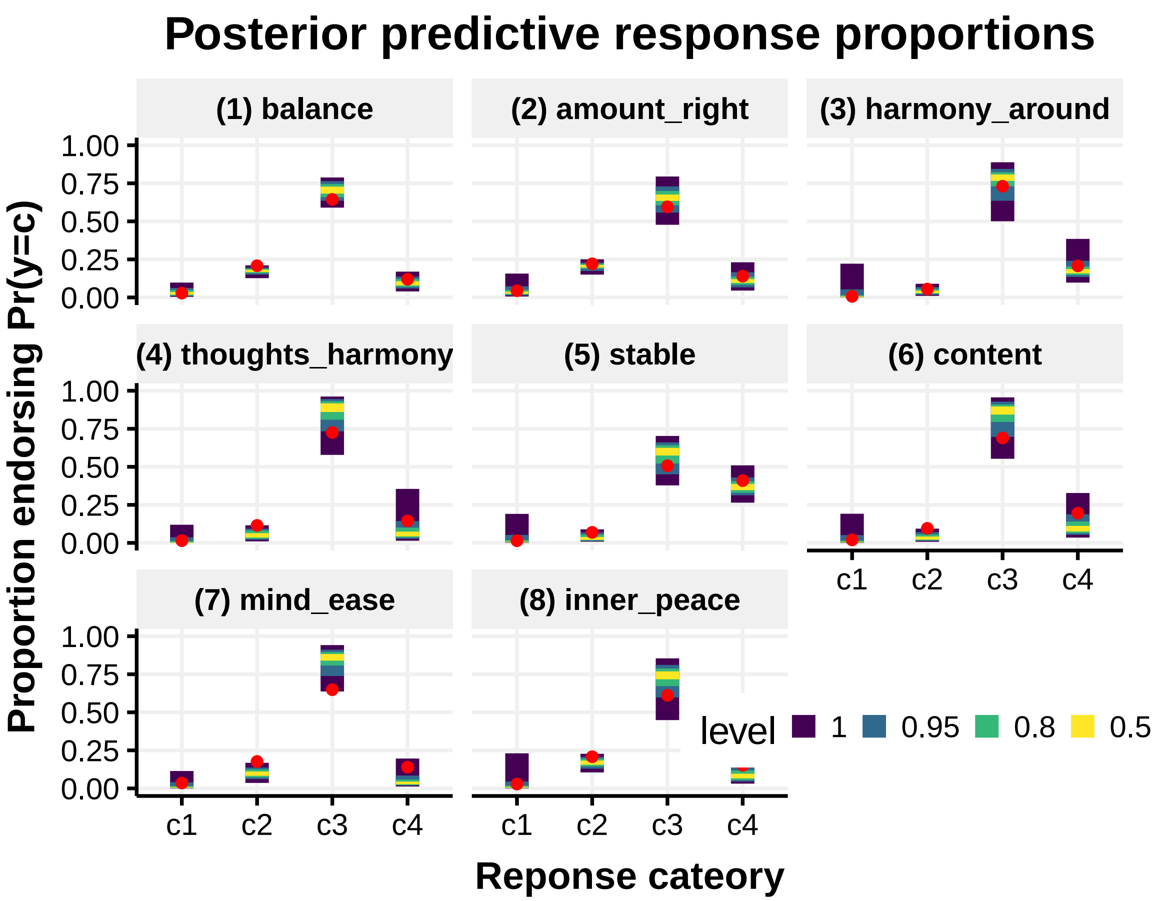



Figure 16: Informed latent response with SEN(0,1.5) PPD of item endorsement proportions across items

*Note.* Dots represent the observed proportion of the sample endorsing the category.

Table 28: Informed latent response with SEN(0,10E5) posterior summary

| | | | Norway | | | | United States | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| lhs | op | rhs | mean | sd | Q.025 | Q.975 | mean | sd | Q.025 | Q.975 |
| bh | =~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| bh | =~ | amount_right | 0.46 | 0.06 | 0.36 | 0.57 | 0.77 | 0.32 | 0.32 | 1.36 |
| bh | =~ | harmony_around | 0.52 | 0.06 | 0.38 | 0.62 | 1.06 | 0.28 | 0.48 | 1.53 |
| bh | =~ | thoughts_harmony | 1.40 | 0.14 | 1.20 | 1.78 | 2.12 | 0.69 | 0.98 | 3.22 |
| bh | =~ | stable | 0.79 | 0.07 | 0.68 | 0.93 | 1.62 | 0.41 | 0.88 | 2.18 |
| bh | =~ | content | 1.20 | 0.10 | 1.01 | 1.39 | 2.21 | 0.64 | 0.99 | 2.96 |
| bh | =~ | mind_ease | 1.41 | 0.13 | 1.19 | 1.70 | 2.26 | 0.78 | 1.07 | 3.54 |
| bh | =~ | inner_peace | 0.92 | 0.07 | 0.79 | 1.06 | 1.88 | 0.93 | 0.66 | 3.46 |
| balance | \| | t1 | -3.61 | 0.21 | -4.05 | -3.28 | -2.22 | 0.91 | -4.18 | -1.15 |
| balance | \| | t2 | -1.35 | 0.09 | -1.53 | -1.18 | -0.74 | 0.37 | -1.55 | -0.12 |
| balance | \| | t3 | 1.95 | 0.09 | 1.77 | 2.15 | 1.51 | 0.56 | 0.66 | 2.77 |
| amount_right | \| | t1 | -2.17 | 0.08 | -2.33 | -2.01 | -1.64 | 0.33 | -2.33 | -1.08 |
| amount_right | \| | t2 | -0.67 | 0.05 | -0.76 | -0.58 | -0.63 | 0.23 | -1.05 | -0.23 |
| amount_right | \| | t3 | 1.37 | 0.06 | 1.26 | 1.49 | 1.18 | 0.20 | 0.72 | 1.63 |
| harmony_around | \| | t1 | -144.16 | 97.56 | -298.06 | -4.08 | -3.12 | 0.68 | -5.22 | -2.17 |
| harmony_around | \| | t2 | -2.12 | 0.10 | -2.33 | -1.94 | -1.95 | 0.45 | -3.31 | -1.25 |
| harmony_around | \| | t3 | 1.01 | 0.06 | 0.90 | 1.13 | 0.96 | 0.33 | 0.37 | 1.58 |
| thoughts_harmony | \| | t1 | -4.57 | 0.34 | -5.38 | -3.99 | -3.93 | 0.47 | -5.09 | -3.29 |
| thoughts_harmony | \| | t2 | -2.21 | 0.15 | -2.56 | -1.99 | -1.98 | 0.27 | -2.58 | -1.47 |
| thoughts_harmony | \| | t3 | 2.23 | 0.13 | 2.00 | 2.50 | 1.65 | 0.33 | 1.13 | 2.48 |
| stable | \| | t1 | -3.45 | 0.18 | -3.77 | -3.03 | -3.01 | 0.70 | -4.02 | -1.69 |
| stable | \| | t2 | -2.16 | 0.08 | -2.30 | -1.98 | -1.82 | 0.46 | -2.58 | -0.97 |
| stable | \| | t3 | 0.14 | 0.06 | 0.03 | 0.24 | 0.57 | 0.22 | 0.16 | 1.00 |
| content | \| | t1 | -4.24 | 0.28 | -5.03 | -3.90 | -3.57 | 0.58 | -4.49 | -2.30 |
| content | \| | t2 | -2.22 | 0.12 | -2.47 | -2.03 | -2.10 | 0.36 | -2.74 | -1.17 |
| content | \| | t3 | 1.86 | 0.10 | 1.67 | 2.06 | 1.27 | 0.48 | 0.49 | 2.13 |
| mind_ease | \| | t1 | -4.22 | 0.14 | -4.52 | -3.89 | -3.12 | 0.92 | -5.75 | -2.26 |
| mind_ease | \| | t2 | -1.61 | 0.10 | -1.79 | -1.42 | -1.32 | 0.43 | -2.52 | -0.82 |
| mind_ease | \| | t3 | 1.95 | 0.12 | 1.71 | 2.17 | 2.20 | 0.61 | 1.56 | 4.07 |
| inner_peace | \| | t1 | -2.81 | 0.12 | -3.06 | -2.62 | -3.32 | 0.52 | -4.21 | -2.30 |
| inner_peace | \| | t2 | -0.87 | 0.07 | -1.01 | -0.74 | -1.40 | 0.26 | -1.92 | -0.88 |
| inner_peace | \| | t3 | 1.78 | 0.08 | 1.63 | 1.93 | 1.43 | 0.39 | 0.77 | 2.36 |
| balance | ~~ | balance | 1.00 | 0.00 | 1.00 | 1.00 | 1.41 | 1.09 | 0.43 | 4.00 |
| amount_right | ~~ | amount_right | 1.00 | 0.00 | 1.00 | 1.00 | 1.01 | 0.27 | 0.65 | 1.76 |
| harmony_around | ~~ | harmony_around | 1.00 | 0.00 | 1.00 | 1.00 | 1.37 | 0.67 | 0.67 | 3.49 |
| thoughts_harmony | ~~ | thoughts_harmony | 1.00 | 0.00 | 1.00 | 1.00 | 1.19 | 0.31 | 0.85 | 2.04 |
| stable | ~~ | stable | 1.00 | 0.00 | 1.00 | 1.00 | 0.94 | 0.39 | 0.36 | 1.58 |
| content | ~~ | content | 1.00 | 0.00 | 1.00 | 1.00 | 1.35 | 0.53 | 0.70 | 2.34 |
| mind_ease | ~~ | mind_ease | 1.00 | 0.00 | 1.00 | 1.00 | 1.38 | 0.92 | 0.79 | 4.41 |
| inner_peace | ~~ | inner_peace | 1.00 | 0.00 | 1.00 | 1.00 | 1.46 | 0.49 | 0.69 | 2.52 |
| balance | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.19 | -0.33 | 0.40 |
| amount_right | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.18 | -0.28 | 0.42 |
| harmony_around | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.08 | 0.23 | -0.46 | 0.40 |
| thoughts_harmony | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | 0.21 | -0.38 | 0.39 |
| stable | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.19 | -0.30 | 0.43 |
| content | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.29 | -0.48 | 0.54 |
| mind_ease | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.16 | -0.29 | 0.35 |
| inner_peace | ~1 | | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | 0.20 | -0.38 | 0.48 |
| bh | ~~ | bh | 1.36 | 0.22 | 1.06 | 1.85 | 0.75 | 0.57 | 0.22 | 2.10 |

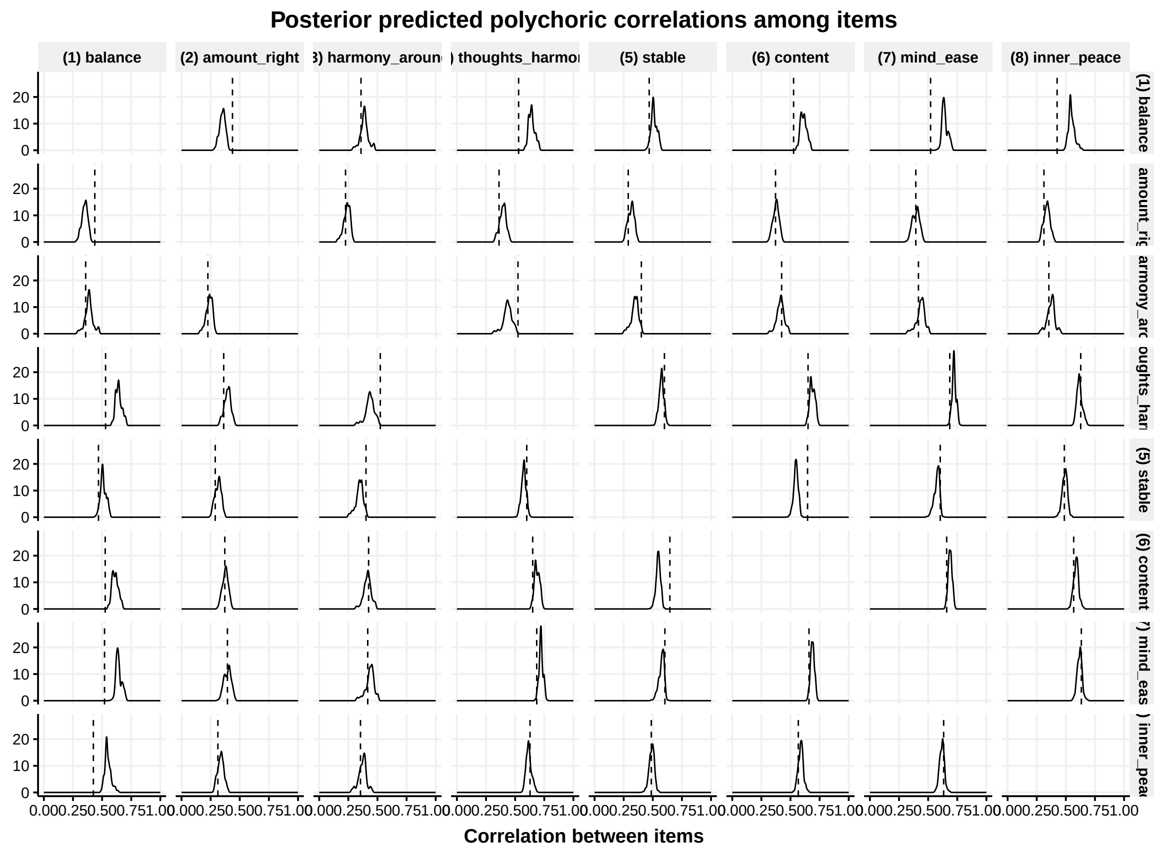



Figure 17: Informed latent response with SEN(0,10E5) posterior distribution of model implied polychoric correlation

*Note.* Vertical dashed lines represent the observed polychoric correlation among items. Polychoric correlations were estimated using the *psych* packages in R.

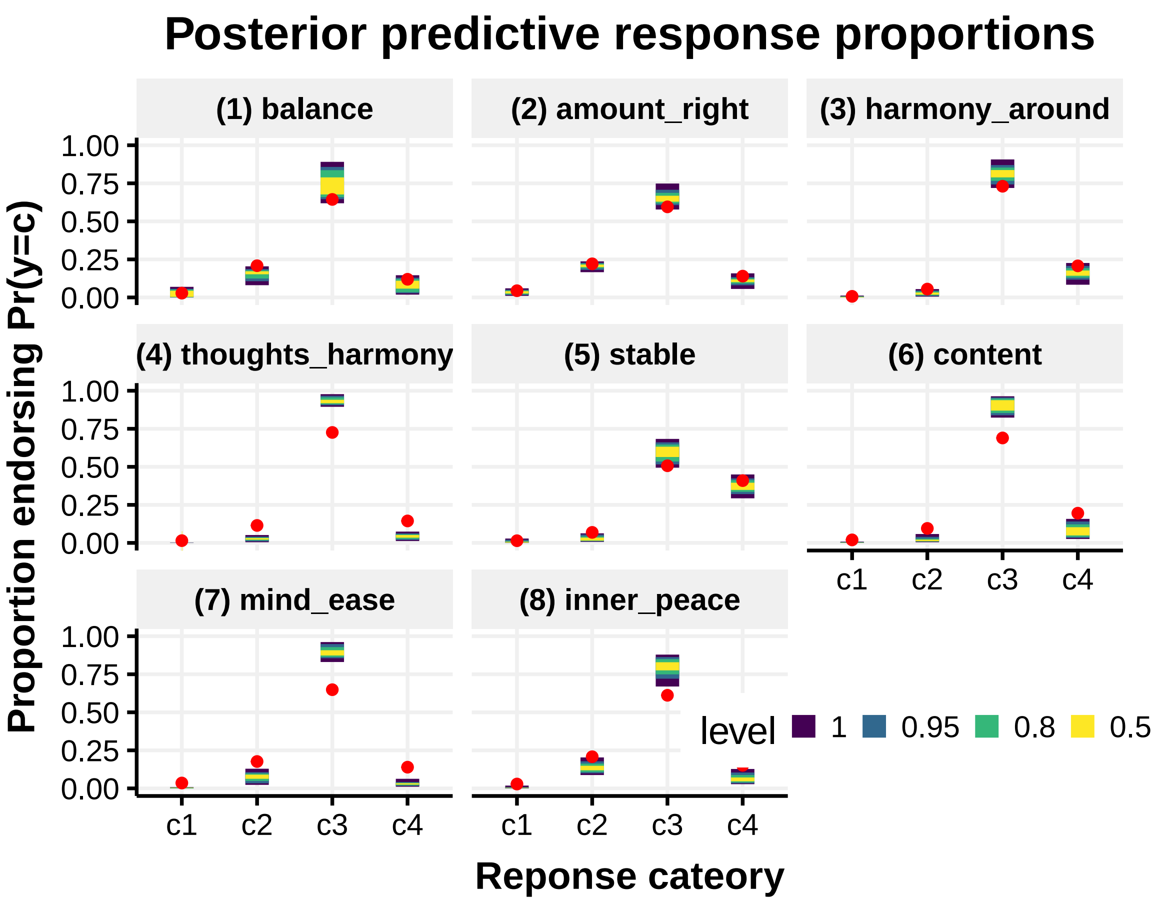



Figure 18: Informed latent response with SEN(0,10E5) PPD of item endorsement proportions across items

*Note.* Dots represent the observed proportion of the sample endorsing the category.

## References


Goodrich, B. (2017). Truncated multivariate normal variates in stan. Accessed on [2023-12-18] from https://global.discourse-cdn.com/standard14/uploads/mc_stan/original/2X/7/7f43a0b7a1035803d5590068f0fbb24bead61abc.pdf.

Padgett, R. N. (2022). *Misclassification errors informed by response time in item factor analysis*. PhD thesis, Baylor University, Waco, TX.

Rosseel, Y. (2012). Lavaan: An R package for structural equation modeling and more. *Journal of Statistical Software*, 48(2):1–36.

Wu, H. and Estabrook, R. (2016). Identification of confirmatory factor analysis models of different levels of invariance for ordered categorical outcomes. *Psychometrika*, 81(4):1014–1045.