

Additional Materials to Support the Paper “A Computationally Efficient and Robust Method to Estimate Exploratory Factor Analysis Models with Correlated Residuals”

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The document contains two sections to support the paper “A Computationally Efficient and Robust Method to Estimate Exploratory Factor Analysis Models with Correlated Residuals”. Section A contains the R code of factor analyzing a correlation matrix of the items of the positive and negative affect scale (PANAS) (Watson, Clark, & Tellegen, 1988). We considered both the conventional exploratory factor analysis (EFA) model and the EFA model with correlated residuals. We extract factors using both the ordinary least squares (OLS) method and the maximum likelihood (ML) method. In addition, we estimated the models using both a computationally efficient and robust method (Zhang, Jiang, Hattori, & Trichtinger, 2022) and an existing confirmatory factor analysis method (Rosseel et al., 2017).

Section B contains 21 tables to report additional results of the empirical illustration (Table B1) and a simulation study (Tables B2 to B21).

Table B1 reports the ML estimates of the EFA model with and the conventional EFA model of the empirical data set. The test statistics for the EFA models with and without correlated residuals are 1141.6 with 146 degrees of freedom and 1578.8 with 151 degrees of freedom, respectively. The RMSEA for the EFA model with correlated residual is 0.064 with the 90% CI (0.061, 0.068); the RMSEA for the EFA model without correlated residual is 0.076 with the 90% CI (0.072, 0.079). The estimates of the five correlated residuals are 0.231, 0.138, 0.152, 0.166, and 0.026, respectively.

Tables B2 - B15 report the population parameters of the simulation study.

Table B16 reports clean convergence rates of oblique models in the simulation study.

Table B17 reports how the EFA model with correlated residuals and the conventional EFA model fit simulated data. The data were simulated with oblique factors.

Tables B18 - B19 report the means and standard deviations of the index K factors and g factors with oblique factors.

Tables B20 - B21 report the mean empirical coverage rates of 95% confidence intervals for the residual correlations and for the factor correlations with oblique factors.

Supplemental Material, Section A

R code of the Empirical Illustration

```

# Step 1: Preliminary =====

## Prepare the correlation matrix of the PANAS scales

v.r = c(0.37, 0.29, 0.51, 0.43, 0.37, 0.37, 0.52, 0.39, 0.30, 0.40,
        0.47, 0.34, 0.23, 0.35, 0.65, 0.40, 0.37, 0.34, 0.46, 0.46,
        0.47, 0.40, 0.41, 0.39, 0.46, 0.50, 0.47, 0.46, 0.38, 0.32,
        0.26, 0.43, 0.40, 0.40, 0.42, 0.33, 0.45, 0.36, 0.30, 0.41,
        0.36, 0.33, 0.39, 0.36, 0.46, -0.07, -0.03, -0.03, -0.03, -0.03,
        0.06, 0.04, 0.01, -0.03, -0.08, -0.05, -0.03, -0.01, -0.03, -0.02,
        0.07, 0.03, 0.01, -0.01, -0.05, 0.61, -0.04, -0.06, -0.02, -0.02,
        -0.04, 0.09, 0.01, 0.00, -0.03, -0.09, 0.41, 0.47, -0.01, -0.03,
        -0.04, -0.01, -0.01, 0.08, 0.06, -0.02, 0.03, -0.03, 0.32, 0.38,
        0.47, -0.12, -0.12, -0.10, -0.04, -0.16, -0.08, -0.06, -0.12, -0.01,
        -0.05, 0.28, 0.29, 0.31, 0.27, -0.01, -0.06, -0.11, -0.03, -0.07,
        0.02, 0.02, -0.07, 0.07, 0.02, 0.23, 0.31, 0.26, 0.32, 0.45,
        0.01, -0.07, -0.08, -0.03, -0.01, 0.04, 0.01, -0.02, -0.04, -0.03,
        0.35, 0.35, 0.31, 0.29, 0.26, 0.35, -0.09, -0.12, -0.16, -0.14,
        -0.08, -0.01, -0.02, -0.09, -0.12, -0.10, 0.33, 0.40, 0.30, 0.32,
        0.27, 0.36, 0.47, -0.11, -0.11, -0.09, -0.09, -0.15, -0.06, -0.04,
        -0.08, -0.09, -0.08, 0.40, 0.42, 0.37, 0.30, 0.48, 0.41, 0.36,
        0.34, -0.11, -0.17, -0.12, -0.06, -0.15, -0.08, -0.05, -0.10, -0.14,
        -0.15, 0.34, 0.38, 0.41, 0.33, 0.39, 0.35, 0.34, 0.34, 0.46)

PANAS20.cor = matrix(0,20,20)
PANAS20.cor[upper.tri(PANAS20.cor)] = v.r
PANAS20.cor = PANAS20.cor + t(PANAS20.cor) + diag(20)

# Step 2: EFA =====
install.packages("EFAutilities") # install the package from CRAN
library(EFAutilities)           # load the package

### EFA models with correlated residuals

# I.cr: a n.cr-by-2 matrix for specifying correlated residuals
n.cr=5
I.cr = matrix(0,n.cr,2)

I.cr[1,1] = 11
I.cr[1,2] = 12
I.cr[2,1] = 13

```

```

I.cr[2,2] = 14
I.cr[3,1] = 15
I.cr[3,2] = 16
I.cr[4,1] = 17
I.cr[4,2] = 18
I.cr[5,1] = 19
I.cr[5,2] = 20

# Prepare a 20 x 2 target matrix for lambda
MTarget = matrix(0, nrow = 20, ncol = 2)
MTarget[1:10, 1] = 9
MTarget[11:20, 2] = 9

# OLS estimation
efa.ols.cr = efa(covmat = PANAS20.cor, factors = 2, fm = 'ols', n.obs = 1657,
                 I.cr = I.cr, rotation = 'target', MTarget = MTarget)

# ML estimation
efa.ml.cr = efa(covmat = PANAS20.cor, factors = 2, fm = 'ml', n.obs = 1657,
                I.cr = I.cr, rotation = 'target', MTarget = MTarget)

### conventional EFA models

# OLS estimation
efa.ols = efa(covmat = PANAS20.cor, factors = 2, fm = 'ols', n.obs = 1657,
              rotation = 'target', MTarget = MTarget)

# ML estimation
efa.ml = efa(covmat = PANAS20.cor, factors = 2, fm = 'ml', n.obs = 1657,
             rotation = 'target', MTarget = MTarget)

# Print out results
efa.ols.cr
efa.ml.cr
efa.ols
efa.ml

# Examine unique variances (and residual correlations)
efa.ols.cr$Psi
efa.ml.cr$Psi
efa.ols$Psi
efa.ml$Psi

# Step 3: The CFA Method =====

```

```

install.packages("lavaan") # install the package from CRAN
library(lavaan)           # load the package

## Specify the variable names of the correlations matrix
rownames(PANAS20.cor) = paste("V", c(1:20), sep = "")

# We now estimate the EFA model with correlated residuals by re-expressing
# it as a CFA model
modelefa.cr <- '
  # measurement model
  Fa =~ V1 + V2 + V3 + V4 + V5 + V6 + V7 + V8 + V9 + V10 +
        V11 + V12 + V13 + V14 + V15 + V16 + V17 + V18 + V19 + V20
  Fb =~ V2 + V3 + V4 + V5 + V6 + V7 + V8 + V9 + V10 + V11 +
        V12 + V13 + V14 + V15 + V16 + V17 + V18 + V19 + V20
  # latent correlations
  Fb ~~ 0*Fa
  # residual correlations
  V11 ~~ V12
  V13 ~~ V14
  V15 ~~ V16
  V17 ~~ V18
  V19 ~~ V20
'

# We now estimate the conventional EFA model by re-expressing it as a CFA model
modelefa <- '
  # measurement model
  Fa =~ V1 + V2 + V3 + V4 + V5 + V6 + V7 + V8 + V9 + V10 +
        V11 + V12 + V13 + V14 + V15 + V16 + V17 + V18 + V19 + V20
  Fb =~ V2 + V3 + V4 + V5 + V6 + V7 + V8 + V9 + V10 + V11 +
        V12 + V13 + V14 + V15 + V16 + V17 + V18 + V19 + V20
  # latent correlations
  Fb ~~ 0*Fa
'

# OLS estimation of EFA with correlated residuals
cfa.ols.cr <- sem(modelefa.cr, sample.cov=PANAS20.cor, sample.nobs=1657,
                  estimator = 'ULS')

# ML estimation of EFA with correlated residuals
cfa.ml.cr <- sem(modelefa.cr, sample.cov=PANAS20.cor, sample.nobs=1657,
                 estimator = 'ML')

# OLS estimation with conventional EFA

```

```
cfa.ols <- sem(modelefa, sample.cov=PANAS20.cor, sample.nobs=1657, estimator = 'ULS')

# ML estimation with conventional EFA
cfa.ml <- sem(modelefa, sample.cov=PANAS20.cor, sample.nobs=1657, estimator = 'ML')

# Print out results
summary(cfa.ols.cr, standardized = TRUE)
summary(cfa.ml.cr, standardized = TRUE)
summary(cfa.ols, standardized = TRUE)
summary(cfa.ml, standardized = TRUE)

# =====
```

Supplemental Material, Section B

Additional Tables

	EFA_CR		EFA	
	Positive	Negative	Positive	Negative
attentive	0.66 (0.02)	-0.04 (0.02)	0.65 (0.02)	-0.04 (0.02)
interested	0.57 (0.02)	-0.09 (0.02)	0.57 (0.02)	-0.08 (0.02)
alert	0.49 (0.02)	-0.09 (0.03)	0.49 (0.02)	-0.08 (0.02)
excited	0.63 (0.02)	-0.02 (0.02)	0.63 (0.02)	-0.03 (0.02)
enthusiastic	0.73 (0.01)	-0.05 (0.02)	0.73 (0.01)	-0.05 (0.02)
inspired	0.70 (0.02)	0.09 (0.02)	0.70 (0.02)	0.09 (0.02)
proud	0.67 (0.02)	0.06 (0.02)	0.67 (0.02)	0.06 (0.02)
determined	0.67 (0.02)	-0.01 (0.02)	0.67 (0.02)	-0.01 (0.02)
strong	0.59 (0.02)	-0.01 (0.02)	0.59 (0.02)	-0.01 (0.02)
active	0.58 (0.02)	-0.06 (0.02)	0.57 (0.02)	-0.06 (0.02)
distressed	0.04 (0.02)	0.58 (0.02)	0.05 (0.02)	0.63 (0.02)
upset	0.06 (0.02)	0.65 (0.02)	0.06 (0.02)	0.69 (0.02)
hostile	0.04 (0.02)	0.61 (0.02)	0.04 (0.02)	0.63 (0.02)
irritable	0.07 (0.02)	0.54 (0.02)	0.06 (0.02)	0.56 (0.02)
scared	-0.09 (0.02)	0.54 (0.02)	-0.09 (0.02)	0.53 (0.02)
afraid	0.02 (0.02)	0.55 (0.02)	0.01 (0.02)	0.55 (0.02)
shamed	0.04 (0.02)	0.55 (0.02)	0.03 (0.02)	0.57 (0.02)
guilty	-0.07 (0.02)	0.55 (0.02)	-0.07 (0.02)	0.57 (0.02)
nervous	-0.07 (0.02)	0.67 (0.02)	-0.08 (0.02)	0.65 (0.02)
jittery	-0.10 (0.02)	0.63 (0.02)	-0.11 (0.02)	0.61 (0.02)
Positive		-0.08 (0.02)		-0.08 (0.02)

Table B1

Maximum Likelihood Estimation of Exploratory Factor Analysis with and without residual correlations of twenty affect adjectives (Watson et al, 1988)

	F1	F2	ψ
V1	0.50	-0.02	0.75
V2	0.52	-0.06	0.73
V3	0.55	-0.09	0.69
V4	0.62	-0.01	0.62
V5	0.49	-0.02	0.76
V6	0.61	0.09	0.62
V7	0.62	0.07	0.61
V8	0.58	0.00	0.67
V9	0.57	0.01	0.67
V10	0.46	-0.04	0.79
V11	0.04	0.49	0.76
V12	0.05	0.48	0.76
V13	0.05	0.58	0.66
V14	0.07	0.52	0.72
V15	-0.08	0.59	0.65
V16	0.03	0.55	0.70
V17	0.04	0.58	0.66
V18	-0.08	0.63	0.60
V19	-0.05	0.52	0.72
V20	-0.09	0.59	0.64

Table B2

Population rotated factor loadings and factor correlations of the simulation study: Model I, orthogonal factors, and low communality

	F1	F2	ψ
V1	0.81	-0.03	0.35
V2	0.82	-0.10	0.33
V3	0.84	-0.12	0.29
V4	0.88	-0.01	0.22
V5	0.80	-0.03	0.36
V6	0.87	0.14	0.22
V7	0.88	0.11	0.21
V8	0.86	0.00	0.27
V9	0.85	0.01	0.27
V10	0.78	-0.06	0.39
V11	0.06	0.80	0.36
V12	0.08	0.79	0.36
V13	0.06	0.86	0.26
V14	0.11	0.82	0.32
V15	-0.13	0.86	0.25
V16	0.04	0.84	0.30
V17	0.06	0.86	0.26
V18	-0.11	0.89	0.20
V19	-0.08	0.82	0.32
V20	-0.14	0.86	0.24

Table B3

Population rotated factor loadings and factor correlations of the simulation study: Model I, orthogonal factors, and high communality

	F1	F2	ψ
V1	0.60	-0.03	0.64
V2	0.64	-0.09	0.58
V3	0.73	-0.12	0.46
V4	0.86	-0.02	0.26
V5	0.57	-0.03	0.68
V6	0.85	0.12	0.26
V7	0.87	0.10	0.23
V8	0.77	-0.01	0.40
V9	0.76	0.00	0.42
V10	0.49	-0.04	0.76
V11	0.05	0.57	0.68
V12	0.07	0.55	0.69
V13	0.07	0.78	0.39
V14	0.10	0.65	0.57
V15	-0.11	0.81	0.34
V16	0.05	0.70	0.50
V17	0.07	0.79	0.37
V18	-0.10	0.89	0.20
V19	-0.05	0.65	0.57
V20	-0.12	0.81	0.33

Table B4

Population rotated factor loadings and factor correlations of the simulation study: Model I, orthogonal factors, and wide communality

	F1	F2	ψ
V1	0.51	-0.01	0.75
V2	0.54	-0.06	0.73
V3	0.58	-0.08	0.69
V4	0.62	0.00	0.62
V5	0.49	-0.02	0.76
V6	0.58	0.10	0.62
V7	0.60	0.08	0.61
V8	0.57	0.01	0.67
V9	0.57	0.01	0.67
V10	0.47	-0.03	0.79
V11	0.05	0.48	0.76
V12	0.06	0.47	0.76
V13	0.06	0.56	0.66
V14	0.08	0.50	0.72
V15	-0.08	0.61	0.65
V16	0.04	0.54	0.70
V17	0.05	0.57	0.66
V18	-0.07	0.65	0.60
V19	-0.04	0.53	0.72
V20	-0.08	0.61	0.64
F1		0.27	

Table B5

Population rotated factor loadings and factor correlations of the simulation study: Model I, oblique factors, and low communality

	F1	F2	ψ
V1	0.81	-0.02	0.35
V2	0.84	-0.09	0.33
V3	0.87	-0.12	0.29
V4	0.88	0.01	0.22
V5	0.81	-0.02	0.36
V6	0.83	0.15	0.22
V7	0.85	0.12	0.21
V8	0.85	0.02	0.27
V9	0.84	0.03	0.27
V10	0.79	-0.05	0.39
V11	0.07	0.78	0.36
V12	0.09	0.77	0.36
V13	0.08	0.83	0.26
V14	0.11	0.78	0.32
V15	-0.12	0.89	0.25
V16	0.06	0.82	0.30
V17	0.07	0.84	0.26
V18	-0.10	0.92	0.20
V19	-0.07	0.84	0.32
V20	-0.13	0.90	0.24
F1		0.27	

Table B6

Population rotated factor loadings and factor correlations of the simulation study: Model I, oblique factors, and high communality

	F1	F2	ψ
V1	0.60	-0.02	0.64
V2	0.67	-0.08	0.58
V3	0.76	-0.11	0.46
V4	0.87	-0.01	0.26
V5	0.57	-0.02	0.68
V6	0.82	0.13	0.26
V7	0.84	0.11	0.23
V8	0.77	0.00	0.40
V9	0.76	0.01	0.42
V10	0.50	-0.04	0.76
V11	0.06	0.55	0.68
V12	0.07	0.53	0.69
V13	0.08	0.76	0.39
V14	0.10	0.62	0.57
V15	-0.10	0.83	0.34
V16	0.06	0.69	0.50
V17	0.08	0.77	0.37
V18	-0.09	0.91	0.20
V19	-0.04	0.66	0.57
V20	-0.11	0.84	0.33
F1		0.27	

Table B7

Population rotated factor loadings and factor correlations of the simulation study: Model I, oblique factors, and wide communality

	Comm.	cr1	cr2	cr3	cr4	cr5
Orthogonal	Low	0.30	0.30	0.30	0.30	0.30
	High	0.30	0.13	0.12	0.10	0.12
	Wide	0.30	0.30	0.30	0.10	0.30
Oblique	Low	0.30	0.30	0.30	0.30	0.30
	High	0.30	0.13	0.12	0.10	0.12
	Wide	0.30	0.30	0.30	0.10	0.30

Table B8

Population residual correlations of the simulation study: Model I

	F1	F2	F3	F4	F5	ψ
V1	0.50	-0.01	0.07	-0.04	0.05	0.74
V2	0.48	0.01	-0.01	-0.07	-0.01	0.76
V3	0.61	-0.01	0.01	0.02	-0.02	0.62
V4	0.54	-0.06	-0.01	0.05	-0.07	0.70
V5	0.61	0.05	0.04	0.04	0.03	0.62
V6	0.48	0.02	-0.03	0.04	-0.02	0.76
V7	0.08	0.59	0.00	-0.01	0.00	0.65
V8	0.03	0.58	0.05	-0.02	-0.06	0.66
V9	0.03	0.61	-0.08	0.05	-0.05	0.61
V10	-0.07	0.55	0.06	0.02	0.02	0.69
V11	-0.04	0.58	-0.03	0.03	0.02	0.66
V12	-0.04	0.52	0.04	-0.02	-0.06	0.72
V13	0.01	0.04	0.46	-0.02	-0.07	0.78
V14	-0.03	0.01	0.61	0.09	0.01	0.61
V15	0.02	0.00	0.50	0.03	0.04	0.74
V16	-0.01	0.05	0.56	-0.04	0.00	0.68
V17	0.02	-0.07	0.46	-0.05	0.00	0.78
V18	0.07	0.00	0.60	0.01	-0.01	0.63
V19	-0.01	0.03	0.03	0.51	0.05	0.74
V20	0.05	0.03	0.07	0.59	-0.01	0.64
V21	0.07	0.00	-0.02	0.50	0.01	0.75
V22	-0.03	0.05	0.02	0.49	0.00	0.76
V23	0.03	-0.01	-0.03	0.55	-0.05	0.70
V24	-0.05	-0.04	-0.04	0.50	-0.04	0.75
V25	-0.02	-0.06	0.05	-0.04	0.48	0.76
V26	0.00	-0.06	-0.03	-0.06	0.54	0.70
V27	-0.06	-0.03	0.04	0.05	0.55	0.69
V28	0.00	0.00	-0.03	-0.04	0.47	0.77
V29	0.06	0.02	-0.02	0.00	0.50	0.75
V30	-0.01	-0.01	-0.04	0.03	0.58	0.66

Table B9

Population rotated factor loadings and factor correlations of the simulation study: Model II, orthogonal factors, and low communality

	F1	F2	F3	F4	F5	ψ
V1	0.80	-0.01	0.12	-0.06	0.09	0.34
V2	0.79	0.01	-0.02	-0.11	-0.02	0.36
V3	0.88	-0.01	0.01	0.03	-0.03	0.22
V4	0.82	-0.09	-0.02	0.08	-0.10	0.30
V5	0.87	0.07	0.06	0.06	0.04	0.22
V6	0.79	0.03	-0.04	0.06	-0.03	0.36
V7	0.12	0.86	-0.01	-0.01	0.00	0.25
V8	0.05	0.85	0.07	-0.03	-0.08	0.26
V9	0.05	0.87	-0.12	0.08	-0.07	0.21
V10	-0.10	0.83	0.09	0.03	0.02	0.29
V11	-0.06	0.86	-0.05	0.04	0.02	0.26
V12	-0.06	0.81	0.06	-0.03	-0.09	0.32
V13	0.02	0.07	0.77	-0.04	-0.12	0.38
V14	-0.05	0.02	0.87	0.13	0.01	0.21
V15	0.04	0.01	0.81	0.05	0.07	0.34
V16	-0.02	0.07	0.84	-0.05	0.01	0.28
V17	0.03	-0.12	0.78	-0.08	0.00	0.38
V18	0.10	0.00	0.87	0.01	-0.02	0.23
V19	-0.02	0.05	0.04	0.81	0.08	0.34
V20	0.08	0.05	0.10	0.86	-0.02	0.24
V21	0.11	-0.01	-0.03	0.80	0.02	0.35
V22	-0.05	0.07	0.02	0.80	0.00	0.36
V23	0.04	-0.01	-0.04	0.83	-0.08	0.30
V24	-0.08	-0.06	-0.06	0.80	-0.07	0.35
V25	-0.04	-0.10	0.08	-0.06	0.78	0.36
V26	0.00	-0.08	-0.04	-0.08	0.83	0.30
V27	-0.09	-0.04	0.06	0.07	0.83	0.29
V28	0.00	0.00	-0.06	-0.06	0.79	0.37
V29	0.10	0.03	-0.03	0.01	0.80	0.35
V30	-0.01	-0.02	-0.06	0.05	0.86	0.26

Table B10

Population rotated factor loadings and factor correlations of the simulation study: Model II, orthogonal factors, and high communality

	F1	F2	F3	F4	F5	ψ
V1	0.60	-0.01	0.09	-0.05	0.06	0.62
V2	0.56	0.01	-0.02	-0.08	-0.01	0.69
V3	0.85	-0.01	0.01	0.03	-0.03	0.27
V4	0.70	-0.07	-0.02	0.07	-0.09	0.50
V5	0.84	0.07	0.06	0.06	0.04	0.27
V6	0.56	0.02	-0.03	0.04	-0.02	0.69
V7	0.11	0.80	0.00	-0.01	0.00	0.35
V8	0.05	0.79	0.07	-0.03	-0.07	0.37
V9	0.04	0.86	-0.11	0.08	-0.07	0.23
V10	-0.09	0.72	0.08	0.02	0.02	0.47
V11	-0.06	0.79	-0.04	0.04	0.02	0.37
V12	-0.05	0.65	0.06	-0.03	-0.07	0.57
V13	0.01	0.04	0.50	-0.03	-0.08	0.74
V14	-0.05	0.01	0.86	0.13	0.01	0.24
V15	0.03	0.00	0.60	0.03	0.05	0.63
V16	-0.02	0.06	0.74	-0.05	0.00	0.45
V17	0.02	-0.08	0.51	-0.05	0.00	0.73
V18	0.10	-0.01	0.83	0.01	-0.02	0.30
V19	-0.01	0.04	0.04	0.62	0.07	0.61
V20	0.07	0.05	0.10	0.81	-0.02	0.33
V21	0.08	0.00	-0.02	0.59	0.02	0.64
V22	-0.04	0.06	0.02	0.57	0.01	0.67
V23	0.04	-0.01	-0.03	0.71	-0.06	0.49
V24	-0.06	-0.04	-0.04	0.59	-0.05	0.64
V25	-0.03	-0.07	0.06	-0.05	0.55	0.69
V26	0.00	-0.08	-0.03	-0.08	0.71	0.49
V27	-0.08	-0.04	0.06	0.06	0.72	0.46
V28	0.00	0.00	-0.04	-0.05	0.52	0.72
V29	0.08	0.02	-0.02	0.00	0.59	0.65
V30	-0.01	-0.02	-0.05	0.04	0.79	0.37

Table B11

Population rotated factor loadings and factor correlations of the simulation study: Model II, orthogonal factors, and wide communality

	F1	F2	F3	F4	F5	ψ
V1	0.48	0.00	0.07	-0.04	0.06	0.74
V2	0.51	0.01	-0.02	-0.07	0.00	0.76
V3	0.61	0.00	0.00	0.02	-0.01	0.62
V4	0.56	-0.06	-0.02	0.05	-0.05	0.70
V5	0.56	0.05	0.04	0.04	0.04	0.62
V6	0.48	0.02	-0.03	0.04	-0.01	0.76
V7	0.08	0.57	-0.01	-0.02	0.02	0.65
V8	0.03	0.57	0.05	-0.03	-0.03	0.66
V9	0.04	0.62	-0.09	0.05	-0.03	0.61
V10	-0.07	0.54	0.06	0.01	0.03	0.69
V11	-0.04	0.59	-0.03	0.02	0.04	0.66
V12	-0.04	0.54	0.04	-0.03	-0.04	0.72
V13	0.01	0.03	0.48	-0.02	-0.07	0.78
V14	-0.04	0.01	0.59	0.09	0.01	0.61
V15	0.02	0.00	0.48	0.03	0.04	0.74
V16	-0.02	0.05	0.56	-0.03	0.01	0.68
V17	0.02	-0.08	0.50	-0.05	0.00	0.78
V18	0.06	-0.01	0.58	0.01	0.00	0.63
V19	-0.01	0.03	0.03	0.48	0.06	0.74
V20	0.05	0.03	0.07	0.55	0.00	0.64
V21	0.06	-0.01	-0.01	0.48	0.02	0.75
V22	-0.03	0.04	0.02	0.48	0.01	0.76
V23	0.03	-0.02	-0.02	0.56	-0.04	0.70
V24	-0.06	-0.05	-0.04	0.54	-0.04	0.75
V25	-0.02	-0.04	0.06	-0.03	0.49	0.76
V26	0.01	-0.03	-0.02	-0.05	0.57	0.70
V27	-0.05	0.00	0.05	0.06	0.54	0.69
V28	0.01	0.03	-0.03	-0.03	0.48	0.77
V29	0.07	0.04	-0.02	0.01	0.47	0.75
V30	0.00	0.01	-0.04	0.05	0.58	0.66
F2	0.29					
F3	0.32	0.31				
F4	0.31	0.32	0.30			
F5	0.23	0.20	0.25	0.23		

Table B12

Population rotated factor loadings and factor correlations of the simulation study: Model II, oblique factors, and low communality

	F1	F2	F3	F4	F5	ψ
V1	0.76	0.00	0.11	-0.06	0.09	0.34
V2	0.83	0.02	-0.03	-0.12	0.00	0.36
V3	0.87	-0.01	0.01	0.03	-0.01	0.22
V4	0.86	-0.09	-0.03	0.08	-0.08	0.30
V5	0.80	0.07	0.05	0.06	0.05	0.22
V6	0.78	0.04	-0.04	0.06	-0.01	0.36
V7	0.12	0.83	-0.01	-0.02	0.03	0.25
V8	0.05	0.84	0.07	-0.04	-0.05	0.26
V9	0.05	0.89	-0.13	0.07	-0.04	0.21
V10	-0.10	0.82	0.09	0.02	0.05	0.29
V11	-0.06	0.87	-0.05	0.03	0.05	0.26
V12	-0.06	0.84	0.06	-0.05	-0.06	0.32
V13	0.01	0.06	0.80	-0.04	-0.11	0.38
V14	-0.06	0.01	0.84	0.14	0.02	0.21
V15	0.03	0.01	0.76	0.05	0.07	0.34
V16	-0.03	0.07	0.84	-0.05	0.02	0.28
V17	0.02	-0.12	0.83	-0.07	0.00	0.38
V18	0.09	-0.01	0.84	0.02	0.00	0.23
V19	-0.01	0.04	0.05	0.76	0.09	0.34
V20	0.07	0.04	0.10	0.79	0.00	0.24
V21	0.10	-0.01	-0.02	0.77	0.04	0.35
V22	-0.05	0.07	0.03	0.78	0.02	0.36
V23	0.04	-0.02	-0.03	0.85	-0.06	0.30
V24	-0.09	-0.08	-0.06	0.87	-0.06	0.35
V25	-0.03	-0.07	0.09	-0.04	0.80	0.36
V26	0.01	-0.05	-0.04	-0.07	0.87	0.30
V27	-0.08	-0.01	0.08	0.09	0.81	0.29
V28	0.01	0.04	-0.06	-0.05	0.80	0.37
V29	0.12	0.07	-0.03	0.02	0.76	0.35
V30	0.00	0.02	-0.05	0.07	0.85	0.26
F2	0.28					
F3	0.31	0.30				
F4	0.31	0.31	0.30			
F5	0.24	0.19	0.25	0.23		

Table B13

Population rotated factor loadings and factor correlations of the simulation study: Model II, oblique factors, and high communality

	F1	F2	F3	F4	F5	ψ
V1	0.58	0.00	0.08	-0.05	0.07	0.62
V2	0.59	0.01	-0.02	-0.09	0.00	0.69
V3	0.85	-0.01	0.00	0.02	-0.01	0.27
V4	0.73	-0.07	-0.03	0.06	-0.07	0.50
V5	0.78	0.07	0.05	0.05	0.05	0.27
V6	0.55	0.03	-0.03	0.04	-0.01	0.69
V7	0.11	0.77	0.00	-0.03	0.03	0.35
V8	0.04	0.78	0.07	-0.04	-0.04	0.37
V9	0.05	0.88	-0.12	0.06	-0.04	0.23
V10	-0.09	0.71	0.08	0.01	0.04	0.47
V11	-0.06	0.80	-0.04	0.03	0.05	0.37
V12	-0.05	0.67	0.06	-0.04	-0.05	0.57
V13	0.01	0.04	0.52	-0.03	-0.07	0.74
V14	-0.05	0.01	0.83	0.12	0.02	0.24
V15	0.02	0.00	0.57	0.03	0.05	0.63
V16	-0.03	0.06	0.75	-0.05	0.01	0.45
V17	0.02	-0.09	0.54	-0.06	0.00	0.73
V18	0.09	-0.01	0.81	0.00	-0.01	0.30
V19	-0.01	0.03	0.04	0.58	0.07	0.61
V20	0.06	0.03	0.09	0.75	0.00	0.33
V21	0.07	-0.01	-0.02	0.57	0.03	0.64
V22	-0.04	0.05	0.02	0.56	0.02	0.67
V23	0.03	-0.02	-0.03	0.73	-0.04	0.49
V24	-0.07	-0.06	-0.05	0.65	-0.04	0.64
V25	-0.02	-0.05	0.07	-0.04	0.56	0.69
V26	0.01	-0.04	-0.03	-0.07	0.74	0.49
V27	-0.07	0.00	0.07	0.07	0.71	0.46
V28	0.01	0.03	-0.04	-0.04	0.54	0.72
V29	0.09	0.05	-0.02	0.01	0.56	0.65
V30	0.01	0.02	-0.05	0.06	0.79	0.37
F2	0.29					
F3	0.32	0.31				
F4	0.32	0.32	0.32			
F5	0.23	0.20	0.26	0.24		

Table B14

Population rotated factor loadings and factor correlations of the simulation study: Model II, oblique factors, and wide communality

	Comm.	cr1	cr2	cr3	cr4	cr5	cr6	cr7	cr8	cr9	cr10	cr11	cr12	cr13	cr14	cr15
Orthogonal	Low	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
	High	0.30	0.11	0.11	0.12	0.11	0.13	0.11	0.14	0.12	0.12	0.30	0.15	0.15	0.14	0.13
	Wide	0.30	0.13	0.14	0.30	0.12	0.30	0.12	0.30	0.15	0.30	0.30	0.30	0.30	0.30	0.30
Oblique	Low	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
	High	0.30	0.11	0.11	0.12	0.11	0.13	0.11	0.14	0.12	0.12	0.30	0.15	0.15	0.14	0.13
	Wide	0.30	0.13	0.14	0.30	0.12	0.30	0.12	0.30	0.15	0.30	0.30	0.30	0.30	0.30	0.30

Table B15

Population residual correlations of the simulation study: Model II

Comm.	Model I								
	EFA_cr			EFA		CFA_cr		CFA	
	N	OLS	ML	OLS	ML	OLS	ML	OLS	ML
Low	100	100	99.9	100	100	48.4	36.9	39.3	32.8
	300	100	100	100	100	29.4	21.8	22.3	21.4
	1000	100	100	100	100	11.9	10.5	5.5	8.4
High	100	100	100	100	100	22.6	16.9	24.6	15.9
	300	100	100	100	100	4.9	5	8.1	3.6
	1000	100	100	100	100	0.1	0.1	1.7	0.1
Wide	100	100	100	100	100	42.4	29.9	34.2	27
	300	100	100	100	100	24.7	16.4	15.1	14.7
	1000	100	100	100	100	6.3	3.9	2.4	2.4
Model II									
Low	100	94.3	29.9	100	93.5	0.1	13	0	39
	300	100	97.1	100	100	0	24.9	0	31.8
	1000	100	100	100	100	0	1.8	0	22.3
High	100	100	99.6	100	100	0.1	39	0.1	43.9
	300	100	100	100	100	0	50.1	0	43.2
	1000	100	100	100	100	0	43.9	0	29.2
Wide	100	98.7	82.7	100	95.3	0.1	32.7	0	35.3
	300	100	100	100	100	0.1	42.4	0.1	36
	1000	100	100	100	100	0	30.2	0	33.1

Table B16

Clean Convergence Rates of oblique models. We leave out the % sign to save space. The shaded entries indicate convergence rates less than 50%. “N”: sample size; “OLS”: ordinary least squares; “ML”: maximum likelihood.

Comm.	Model I				
	EFA_cr			EFA	
	N	OLS	ML	OLS	ML
Low	100	0.02 (13.8, 0.3)	0.03 (20.5, 0.2)	0.06 (87, 27.4)	0.08 (99.9, 90.7)
	300	0.01 (7, 0)	0.01 (7.5, 0)	0.07 (100, 92.4)	0.08 (100, 100)
	1000	0.01 (5.7, 0)	0 (4.7, 0)	0.07 (100, 100)	0.08 (100, 100)
High	100	0 (0, 0)	0.03 (22.9, 0.3)	0 (0, 0)	0.12 (100, 100)
	300	0 (0, 0)	0.01 (6.5, 0)	0.04 (100, 0)	0.11 (100, 100)
	1000	0 (0.8, 0)	0 (5.7, 0)	0.05 (100, 5.4)	0.11 (100, 100)
Wide	100	0 (0, 0)	0.03 (20.3, 0.2)	0.01 (3.4, 0)	0.12 (100, 100)
	300	0 (0.3, 0)	0.01 (8.8, 0)	0.06 (100, 30.5)	0.12 (100, 100)
	1000	0 (2.4, 0)	0 (4.7, 0)	0.07 (100, 100)	0.12 (100, 100)
	Model II				
	N	OLS	ML	OLS	ML
Low	100	0.01 (6, 0)	*	0.06 (98.9, 39.2)	0.1 (100, 100)
	300	0.01 (7.3, 0)	0.01 (11.1, 0)	0.07 (100, 100)	0.09 (100, 100)
	1000	0 (6.2, 0)	0 (6.1, 0)	0.08 (100, 100)	0.09 (100, 100)
High	100	0 (0, 0)	0.04 (63, 0.9)	0 (0, 0)	0.13 (100, 100)
	300	0 (0, 0)	0.01 (15.8, 0)	0.05 (100, 0)	0.12 (100, 100)
	1000	0 (0.5, 0)	0 (6.6, 0)	0.06 (100, 100)	0.12 (100, 100)
Wide	100	0 (0, 0)	0.04 (59, 0.4)	0.01 (8.7, 0)	0.12 (100, 100)
	300	0 (0.2, 0)	0.01 (15.1, 0)	0.06 (100, 100)	0.11 (100, 100)
	1000	0 (2.5, 0)	0 (6.9, 0)	0.07 (100, 100)	0.11 (100, 100)

Table B17

Model fit of EFA with correlated residuals and conventional EFA for oblique models. Each entry include the mean of the RMSEA across the 1,000 simulated samples and the empirical rejection rates of of perfect fit and close fit. The two empirical rejection rates are in parentheses. “N”: sample size; “OLS”: ordinary least squares; “ML”: maximum likelihood; “”: the clean convergence rate lower than 50%*

Communality	Model I								
	EFA_cr						EFA		
	OLS			ML		OLS		ML	
	N	M	SD	M	SD	M	SD	M	SD
Low	100	0.98	0.01	0.97	0.01	0.98	0.01	0.97	0.01
	300	0.99	0.00	0.99	0.00	0.99	0.00	0.99	0.00
	1000	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
High	100	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	300	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	1000	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
Wide	100	0.99	0.00	0.99	0.00	0.99	0.00	0.99	0.00
	300	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	1000	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
Model II									
Low	100	0.89	0.03	*	*	0.91	0.03	0.89	0.04
	300	0.96	0.01	0.25	0.19	0.97	0.01	0.96	0.01
	1000	0.99	0.00	0.26	0.19	0.99	0.00	0.99	0.00
High	100	0.99	0.00	0.24	0.19	0.99	0.00	0.98	0.00
	300	1.00	0.00	0.24	0.18	1.00	0.00	0.99	0.00
	1000	1.00	0.00	0.23	0.15	1.00	0.00	1.00	0.00
Wide	100	0.96	0.01	0.29	0.22	0.97	0.01	0.96	0.02
	300	0.99	0.00	0.29	0.21	0.99	0.00	0.98	0.00
	1000	1.00	0.00	0.24	0.13	1.00	0.00	0.99	0.00

Table B18

The mean and standard deviation of the index K for factor recovery for oblique models.

“N”: sample size; “OLS”: ordinary least squares; “ML”: maximum likelihood; “”: the clean convergence rate lower than 50%.*

Communality	Model I								
	EFA_cr						EFA		
	OLS			ML		OLS		ML	
	N	M	SD	M	SD	M	SD	M	SD
Low	100	0.09	0.01	0.10	0.02	0.09	0.01	0.10	0.02
	300	0.05	0.01	0.05	0.01	0.05	0.01	0.06	0.01
	1000	0.03	0.00	0.03	0.00	0.03	0.00	0.03	0.00
High	100	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01
	300	0.03	0.00	0.03	0.00	0.03	0.00	0.03	0.00
	1000	0.02	0.00	0.01	0.00	0.02	0.00	0.02	0.00
Wide	100	0.07	0.01	0.07	0.01	0.07	0.01	0.07	0.01
	300	0.04	0.01	0.04	0.01	0.04	0.01	0.04	0.01
	1000	0.02	0.00	0.02	0.00	0.02	0.00	0.03	0.00
Model II									
Low	100	0.12	0.02	*	*	0.12	0.01	0.13	0.02
	300	0.07	0.01	0.30	0.05	0.07	0.01	0.07	0.01
	1000	0.04	0.00	0.29	0.04	0.04	0.00	0.05	0.00
High	100	0.06	0.01	0.45	0.07	0.06	0.01	0.07	0.01
	300	0.03	0.00	0.45	0.06	0.03	0.00	0.05	0.01
	1000	0.02	0.00	0.45	0.05	0.02	0.00	0.04	0.01
Wide	100	0.09	0.01	0.36	0.07	0.08	0.01	0.09	0.01
	300	0.05	0.00	0.36	0.07	0.05	0.00	0.07	0.01
	1000	0.03	0.00	0.37	0.04	0.03	0.00	0.06	0.00

Table B19

The mean and standard deviation of the index g for factor recovery for oblique models.

“N”: sample size; “OLS”: ordinary least squares; “ML”: maximum likelihood; “”: the clean convergence rate lower than 50%.*

Comm.	Model I		
	N	OLS	ML
Low	100	94.74	95.36
	300	95.04	95.10
	1000	94.72	95.12
High	100	93.64	93.22
	300	94.60	94.82
	1000	94.88	95.14
Wide	100	93.78	94.22
	300	95.16	94.64
	1000	94.98	94.68
	Model II		
	N	OLS	ML
Low	100	97.34	*
	300	96.34	97.61
	1000	95.19	95.72
High	100	93.27	94.33
	300	94.35	94.63
	1000	94.97	94.92
Wide	100	95.08	96.53
	300	95.23	95.23
	1000	94.85	95.10

Table B20

The mean empirical coverage rates of 95% confidence intervals for residual correlations for oblique models. “N”: sample size; “OLS”: ordinary least squares; “ML”: maximum likelihood; “”: the clean convergence rate lower than 50%.*

Comm.	Model I		
	N	OLS	ML
Low	100	94.00	94.20
	300	94.90	94.50
	1000	94.60	94.10
High	100	96.20	93.80
	300	93.50	95.60
	1000	95.80	95.00
Wide	100	95.60	94.00
	300	94.40	94.30
	1000	93.20	94.90
	Model II		
	N	OLS	ML
Low	100	87.24	*
	300	90.53	57.19
	1000	93.17	40.86
High	100	94.96	73.94
	300	94.83	53.86
	1000	94.92	33.84
Wide	100	93.58	75.19
	300	94.45	57.69
	1000	94.81	37.21

Table B21

The mean empirical coverage rates of 95% confidence intervals for factor correlations. “N”: sample size; “OLS”: ordinary least squares; “ML”: maximum likelihood; “”: the clean convergence rate lower than 50%.*

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