

## Online Appendix A: r2MLM R function

### **r2MLM R function Description:**

This function reads in raw data and multilevel model (MLM) parameter estimates and outputs all relevant  $R^2$  measures and barchart decompositions (e.g., Figures 6-8). That is, when predictors are cluster-mean-centered, all  $R^2$ 's in Table 1 and decompositions in Figure 1 are outputted. When predictors are not cluster-mean-centered, the total  $R^2$ 's from Table 5, as well as barchart decompositions are outputted. Any number of level-1 and/or level-2 predictors is supported. Any of the level-1 predictors can have random slopes.

### **r2MLM R function Input:**

*data* – Dataset with rows denoting observations and columns denoting variables

*within\_covs* – List of numbers corresponding to the columns in the dataset of the level-1 predictors used in the MLM (if none used, set to NULL)

*between\_covs* – List of numbers corresponding to the columns in the dataset of the level-2 predictors used in the MLM (if none used, set to NULL)

*random\_covs* – List of numbers corresponding to the columns in the dataset of the level-1 predictors that have random slopes in the MLM (if no random slopes, set to NULL)

*gamma\_w* – Vector of fixed slope estimates for all level-1 predictors, to be entered in the order of the predictors listed by *within\_covs* (if none, set to NULL)

*gamma\_b* – Vector of fixed intercept estimate (if applicable; see *has\_intercept* below) and fixed slope estimates for all level-2 predictors, to be entered intercept first (if applicable) followed by level-2 slopes in the order listed by *between\_covs* (if none, set to NULL)

*Tau* – random effect covariance matrix; note that the first row/column denotes the intercept variance and covariances (if intercept is fixed, set all to 0) and each subsequent row/column denotes a given random slope's variance and covariances (to be entered in the order listed by *random\_covs*)

*sigma2* – level-1 residual variance

*has\_intercept* – if set to TRUE, the first element of *gamma\_b* is assumed to be the fixed intercept estimate; if set to FALSE, the first element of *gamma\_b* is assumed to be the first fixed level-2 predictor slope; set to TRUE by default

*clustermeancentered* – if set to TRUE, all level-1 predictors (indicated by the *within\_covs* list) are assumed to be cluster-mean-centered and function will output all decompositions; if set to FALSE, function will output only total decompositions (see Description above); set to TRUE by default

### **r2MLM R function Code:**

```
r2MLM <- function(data,within_covs,between_covs,random_covs,
  gamma_w,gamma_b,Tau,sigma2,has_intercept=T,clustermeancentered=T){
  if(has_intercept==T){
    if(length(gamma_b)>1) gamma <- c(1,gamma_w,gamma_b[2:length(gamma_b)])
    if(length(gamma_b)==1) gamma <- c(1,gamma_w)
    if(is.null(within_covs)==T) gamma_w <- 0
  }
  if(has_intercept==F){
    gamma <- c(gamma_w,gamma_b)
    if(is.null(within_covs)==T) gamma_w <- 0
    if(is.null(between_covs)==T) gamma_b <- 0
  }
  if(is.null(gamma)) gamma <- 0
  ##compute phi
  phi <- var(cbind(1,data[,c(within_covs)],data[,c(between_covs)]),na.rm=T)
  if(has_intercept==F) phi <- var(cbind(data[,c(within_covs)],data[,c(between_covs)]),na.rm=T)
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if(is.null(within_covs)==T & is.null(between_covs)==T & has_intercept==F) phi <- 0
phi_w <- var(data[,within_covs],na.rm=T)
if(is.null(between_covs)==T) phi_w <- 0
phi_b <- var(cbind(1,data[,between_covs]),na.rm=T)
if(is.null(between_covs)==T) phi_b <- 0
##compute psi and kappa
var_randomcovs <- var(cbind(1,data[,c(random_covs)]),na.rm=T)
if(length(Tau)>1) psi <- matrix(c(diag(Tau)),ncol=1)
if(length(Tau)==1) psi <- Tau
if(length(Tau)>1) kappa <- matrix(c(Tau[lower.tri(Tau)==TRUE]),ncol=1)
if(length(Tau)==1) kappa <- 0
v <- matrix(c(diag(var_randomcovs)),ncol=1)
r <- matrix(c(var_randomcovs[lower.tri(var_randomcovs)==TRUE]),ncol=1)
if(is.null(random_covs)==TRUE){
  v <- 0
  r <- 0
  m <- matrix(1,ncol=1)
}
if(length(random_covs)>0) m <- matrix(c(colMeans(cbind(1,data[,c(random_covs)])),na.rm=T)),ncol=1
##total variance
totalvar_notdecomp <- t(v)%*%psi + 2*(t(r)%*%kappa) + t(gamma)%*%phi%*%gamma + t(m)%*%Tau%*%m + sigma2
totalwithinvar <- (t(gamma_w)%*%phi_w%*%gamma_w) + (t(v)%*%psi + 2*(t(r)%*%kappa)) + sigma2
totalbetweenvar <- (t(gamma_b)%*%phi_b%*%gamma_b) + Tau[1]
totalvar <- totalwithinvar + totalbetweenvar
##total decomps
decomp_fixed_notdecomp <- (t(gamma)%*%phi%*%gamma) / totalvar
decomp_fixed_within <- (t(gamma_w)%*%phi_w%*%gamma_w) / totalvar
decomp_fixed_between <- (t(gamma_b)%*%phi_b%*%gamma_b) / totalvar
decomp_fixed <- decomp_fixed_within + decomp_fixed_between
decomp_varslopes <- (t(v)%*%psi + 2*(t(r)%*%kappa)) / totalvar
decomp_varmeans <- (t(m)%*%Tau%*%m) / totalvar
decomp_sigma <- sigma2/totalvar
##within decomps
decomp_fixed_within_w <- (t(gamma_w)%*%phi_w%*%gamma_w) / totalwithinvar
decomp_varslopes_w <- (t(v)%*%psi + 2*(t(r)%*%kappa)) / totalwithinvar
decomp_sigma_w <- sigma2/totalwithinvar
##between decomps
decomp_fixed_between_b <- (t(gamma_b)%*%phi_b%*%gamma_b) / totalbetweenvar
decomp_varmeans_b <- Tau[1] / totalbetweenvar
#NEW measures
if (clustermeancentered==TRUE){
R2_f <- decomp_fixed
R2_f1 <- decomp_fixed_within
R2_f2 <- decomp_fixed_between
R2_fv <- decomp_fixed + decomp_varslopes
R2_fvm <- decomp_fixed + decomp_varslopes + decomp_varmeans
R2_v <- decomp_varslopes
R2_m <- decomp_varmeans
R2_f_w <- decomp_fixed_within_w
R2_f_b <- decomp_fixed_between_b
R2_fv_w <- decomp_fixed_within_w + decomp_varslopes_w
R2_v_w <- decomp_varslopes_w
R2_m_b <- decomp_varmeans_b
}
if (clustermeancentered==FALSE){
R2_f <- decomp_fixed_notdecomp
R2_fv <- decomp_fixed_notdecomp + decomp_varslopes
R2_fvm <- decomp_fixed_notdecomp + decomp_varslopes + decomp_varmeans
R2_v <- decomp_varslopes
R2_m <- decomp_varmeans
}
if(clustermeancentered==TRUE){
decomp_table <- matrix(c(decomp_fixed_within,decomp_fixed_between,decomp_varslopes,decomp_varmeans,decomp_sigma,
  decomp_fixed_within_w,"NA",decomp_varslopes_w,"NA",decomp_sigma_w,
  "NA",decomp_fixed_between_b,"NA",decomp_varmeans_b,"NA"),ncol=3)
rownames(decomp_table) <- c("fixed", "within", "fixed", "between", "slope variation", "mean variation", "sigma2")
colnames(decomp_table) <- c("total", "within", "between")
R2_table <- matrix(c(R2_f1,R2_f2,R2_v_w,R2_m,R2_f,R2_fv,R2_fvm,
  R2_f_w,"NA",R2_v_w,"NA","NA",R2_fv_w,"NA",
  "NA",R2_f_b,"NA",R2_m_b,"NA","NA","NA"))
}

```

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    .ncol=3)
rownames(R2_table) <- c("f1","f2","v","m","f","fv","fvm")
colnames(R2_table) <- c("total","within","between")
}
##barchart
if(clustermeancentered==TRUE){
contributions_stacked <- matrix(c(decomp_fixed_within,decomp_fixed_between,decomp_varslopes,decomp_varmeans,decomp_sigma,
decomp_fixed_within_w,0,decomp_varslopes_w,0,decomp_sigma_w,
0,decomp_fixed_between_b,0,decomp_varmeans_b,0),5,3)
colnames(contributions_stacked) <- c("total","within","between")
rownames(contributions_stacked) <- c("fixed slopes (within)",
"fixed slopes (between)",
"slope variation (within)",
"intercept variation (between)",
"residual (within)")
barplot(contributions_stacked, main="Decomposition", horiz=FALSE,
ylim=c(0,1),col=c("darkred","steelblue","darkred","midnightblue","white"),ylab="proportion of variance",
density=c(NA,NA,30,40,NA),angle=c(0,45,0,135,0),xlim=c(0,1),width=c(.3,.3))
legend(.30,-.1,legend=rownames(contributions_stacked),fill=c("darkred","steelblue","darkred","midnightblue","white"),
cex=.7, pt.cex = 1,xpd=T,density=c(NA,NA,30,40,NA),angle=c(0,45,0,135,0))
}
if(clustermeancentered==FALSE){
decomp_table <- matrix(c(decomp_fixed_notdecomp,decomp_varslopes,decomp_varmeans,decomp_sigma),ncol=1)
rownames(decomp_table) <- c("fixed","slope variation","mean variation","sigma2")
colnames(decomp_table) <- c("total")
R2_table <- matrix(c(R2_f,R2_v,R2_m,R2_fv,R2_fvm),ncol=1)
rownames(R2_table) <- c("f","v","m","fv","fvm")
colnames(R2_table) <- c("total")
##barchar
contributions_stacked <- matrix(c(decomp_fixed_notdecomp,decomp_varslopes,decomp_varmeans,decomp_sigma),4,1)
colnames(contributions_stacked) <- c("total")
rownames(contributions_stacked) <- c("fixed slopes",
"slope variation",
"intercept variation",
"residual")
barplot(contributions_stacked, main="Decomposition", horiz=FALSE,
ylim=c(0,1),col=c("darkblue","darkblue","darkblue","white"),ylab="proportion of variance",
density=c(NA,30,40,NA),angle=c(0,0,135,0),xlim=c(0,1),width=c(.6))
legend(.30,-.1,legend=rownames(contributions_stacked),fill=c("darkblue","darkblue","darkblue","white"),
cex=.7, pt.cex = 1,xpd=TRUE,density=c(NA,30,40,NA),angle=c(0,0,135,0))
}
Output <- list(noquote(decomp_table),noquote(R2_table))
names(Output) <- c("Decompositions", "R2s")
return(Output)
}

```

### ***r2MLM R function Example Input:***

#NOTE: estimates in the input represent hypothetical results for a random slope model with two level-1 predictors and two level-2 predictors  
#in practice a user would have previously obtained these input estimates by fitting their model in MLM software  
#additionally, the input consists of hypothetical predictor data, whereas in practice a user would read-in their actual data

```

data <- matrix(NA,100,4)
xs <- mvtnorm(n=100,mu=c(0,0),Sigma=matrix(c(2,.75,.75,1.5),2,2))
ws <- mvtnorm(n=10,mu=c(0,2),Sigma=matrix(c(1,.5,.5,2),2,2))
data[,1:2] <- xs
for (i in seq(10)){
  data[(10*(i-1)+1):(i*10),3] <- ws[i,1]
  data[(10*(i-1)+1):(i*10),4] <- ws[i,2]
  data[(10*(i-1)+1):(i*10),1] <- data[(10*(i-1)+1):(i*10),1] - mean(data[(10*(i-1)+1):(i*10),1])
  data[(10*(i-1)+1):(i*10),2] <- data[(10*(i-1)+1):(i*10),2] - mean(data[(10*(i-1)+1):(i*10),2])
}
r2MLM(data,within_covs=c(1,2),between_covs=c(3,4),random_covs=c(1,2),
gamma_w=c(2.5,-1),gamma_b=c(1,.25,1.5),Tau=matrix(c(4,1,.75,1,1,.25,.75,.25,.5),3,3),sigma2=10)

```

**Online Appendix B: Empirical example results**

Online Appendix B Table 1. Parameter estimates and standards errors from Empirical Example 1

<b><i>Fixed effects</i></b>	<b>Est</b>	<b>SE</b>	<b><i>t</i></b>
intercept	51.19	0.86	59.71*
school-mean-centered homework	1.88	0.84	2.22*
school-mean-centered parental education	1.81	0.39	4.62**
school-mean homework	0.70	1.53	0.45
school-mean parental education	3.18	1.01	3.15**

  

<b><i>Variance components</i></b>	<b>Est</b>	<b>SE</b>	<b><i>z</i> †</b>
variance of intercept	13.61	5.27	2.58**
variance of school-mean-centered homework	14.36	5.25	2.74**
variance of school-mean-centered parental education	1.17	1.43	1.43
covariance of intercept with school-mean-centered homework	-0.14	3.75	-0.04
covariance of intercept with school-mean-centered parental education	-2.88	1.92	-1.50
covariance of school-mean-centered homework with school-mean-centered parental education	-0.16	1.72	-0.09
variance of level-1 residual	49.32	3.26	15.13**

*Notes:* Results obtained from SAS Proc Mixed. \*significant,  $p < .05$ ; \*\*significant,  $p < .01$

†  $z$ -tests of variance components are conservative; one way to address this is to employ the alpha-correction approach of Fitzmaurice et al. (2011, p. 209) which uses  $\alpha = .10$  instead of  $\alpha = .05$ . For a discussion of other alternatives, see Rights and Sterba (2016).

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**ONLINE APPENDIX B: Empirical example results**

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Online Appendix B Table 2. Parameter estimates and standards errors from Empirical Example 2

<b><i>Fixed effects</i></b>	<b>Est</b>	<b>SE</b>	<b><i>t</i></b>
Intercept	41.04	0.31	134.29**
school-mean-centered verbal IQ	2.24	0.09	26.14**
school-mean-centered SES	0.17	0.02	10.77**
school-mean-centered verbal IQ * SES	-0.01	0.01	-1.63
school-mean verbal IQ	3.36	0.37	9.13**
school-mean SES	0.07	0.05	1.34
school-mean verbal IQ * SES	-0.09	0.04	-2.37*

  

<b><i>Variance components</i></b>	<b>Est</b>	<b>SE</b>	<b><i>z</i> †</b>
variance of intercept	8.03	1.35	5.95**
variance of school-mean-centered verbal IQ	0.23	0.11	2.09**
covariance of intercept with school-mean-centered verbal IQ	-0.92	0.30	-3.06*
variance of level-1 residual	39.18	1.22	32.04**

*Notes:* Results obtained from SAS Proc Mixed. \*significant,  $p < .05$ ; \*\*significant,  $p < .01$

†  $z$ -tests of variance components are conservative; one way to address this is to employ the alpha-correction approach of Fitzmaurice et al. (2011, p. 209) which uses  $\alpha = .10$  instead of  $\alpha = .05$ . For a discussion of other alternatives, see Rights and Sterba (2016).

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**ONLINE APPENDIX B: Empirical example results**

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Online Appendix B Table 3. Parameter estimates and standards errors from Empirical Example 3

<b><i>Fixed effects</i></b>	<b>Est</b>	<b>SE</b>	<b><i>t</i></b>
intercept	5.08	0.08	65.29**
class-mean-centered extraversion	0.45	0.02	25.67**
class-mean-centered sex	1.23	0.04	33.68**
teacher experience	0.06	0.01	5.22**
class-mean-centered extraversion * teacher experience	-0.03	0.01	-9.50**

  

<b><i>Variance components</i></b>	<b>Est</b>	<b>SE</b>	<b><i>z</i> †</b>
variance of intercept	0.58	0.09	6.68**
variance of class-mean-centered extraversion	0.01	0.01	1.26
covariance of intercept with class-mean- centered extraversion	-0.01	0.01	-0.91
variance of level-1 residual	39.18	0.02	29.97**

Notes: Results obtained from SAS Proc Mixed. \*significant,  $p < .05$ ; \*\*significant,  $p < .01$

†  $z$ -tests of variance components are conservative; one way to address this is to employ the alpha-correction approach of Fitzmaurice et al. (2011, p. 209) which uses  $\alpha = .10$  instead of  $\alpha = .05$ . For a discussion of other alternatives, see Rights and Sterba (2016).