**Supplemental Materials**

**What to Do When Scalar Invariance Fails: The Extended Alignment Method for Multi-Group Factor Analysis Comparison of Latent Means Across Many Groups**

**by H. Marsh et al., 2016, *Psychological Methods***

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**NOTE: The following materials are designed to go on the online website associated with the journal as Online Supplemental Materials, and are NOT to be part of the published article. They are included in this MS for purposes of review and feedback.**

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**Appendix 1:**

**Quality of PISA Scales**

Much effort went into the development of internationally agreed-upon and comparable scales in the PISA data collections. The PISA assessment framework and items were developed through discussions between international groups of substantive and psychometric experts, followed by rigorous translation, verification, and national adaptions, which were implemented to balance the comparability and ecological validity of measures. Lastly, these measures were piloted and field tested, necessary changes were made, and the surveys were conducted with standardized administration procedures. In an evaluation of earlier PISA scales, where this development was described in more detail, Marsh, Hau, et al. (2006) concluded that the PISA scales were the strongest measure of educational psychology's most useful constructs and demonstrated good support for their psychometric properties, for metric – but not scalar – invariance across 25 countries, and cross-cultural validity in relation to consistent support for convergent and discriminant validity across 25 countries. They argued that the scales were useful for a broad range of educational psychology research: as a set of outcome measures; as a powerful set of intervening variables that facilitates the attainment of many long-term, desirable educational outcomes, and as a basis for mapping other educational psychology constructs in relation to their convergence and divergence with the PISA constructs. Similar to results by Marsh, Hau, et al. (2006) as well as Nagengast and Marsh (2013), the PISA results based on the present investigation show good support for the configural model and even the metric model. We also note that in cross-cultural research (as in other areas also) there are few if any published large-scale studies (i.e., many groups, factors, and items) that support scalar invariance (see earlier discussion).

# Appendix 2:

# Guidelines for Configuring the Alignment-With-CFA (AwC) Approach

The AwC (Alignment-with-CFA) approach can be amended for greater precision. Its model is estimated according to the following steps:

(1) Testing measurement invariance in relation to factor loadings and item intercepts is a precondition for comparing latent factor means across multiple groups. If the scalar model provides good model fit and support for item intercept invariance, alignment or AwC models should not be pursued, and the more parsimonious scalar model can be used for latent mean comparisons.

(2) If item intercept invariance (or even factor loadings) is not supported, and the scalar model provides poor model fit, an alignment analysis should be employed for latent mean comparisons.

(3) If there is a need to conduct an additional analysis that cannot be easily implemented within the alignment framework, but can be estimated with CFA models, then all parameter estimates from the alignment solution should be used as starting values to estimate the AwC model.

(4) Since a total of 2*m* constraints need to be added for the AwC model to be identified, selected parameter estimates are fixed to the values obtained from the alignment solution:

1. The *m* factor variances and *m* factor means are freely estimated. Then, for purposes of identification, a referent indicator (e.g., the first indicator) is selected for each factor, and the factor loading and intercept of this indicator are fixed to its estimated values from the alignment solution (i.e., these values are not allowed to be freely estimated).
2. All other parameter estimates are free in AwC, the same as in the alignment solution.
3. The AwC solution will have the same degrees of freedom, the same chi-square, and goodness of fit statistics as the configural CFA model. However, the AwC solution will have the same parameter estimates as the alignment solution. Standard errors will also be highly similar, but might be slightly inflated, suggesting that caution still needs to be exerted in the interpretation of marginally non-significant results. In this sense, the AwC model is equivalent to the alignment solution. Importantly, the researcher has more flexibility in terms of how to constrain or further modify the AwC model (as it is a true CFA model) than with the alignment model upon which it is based.

# Appendix 3:

# Annotated Mplus Input and Output for the Alignment and Alignment-Within-CFA (AwC) Approaches

TITLE: The alignment model with 30 groups (Model MG4AL see Table 2);

DATA: FILE = “PISA06\_data.dat”;

VARIABLE:

NAMES = SCHOOLID STIDSTD COUNTRY OECD W\_FSTUWT ST16Q01 ST16Q02 ST16Q03 ST16Q04 ST37Q06 Gender SES PV1SCIE;

MISSING=.;

USEVARIABLES ARE ST16Q01-ST19Q06 Gender SES PV1SCIE CONTSCHL;

CLUSTER = CONTSCHL;

! cluster by school;

WEIGHT = W\_FSTUWT;

! W\_FSTUWT is the student-level weighting variable in the PISA database.

classes = c(30);

knownclass = c(COUNTRY=36 40 56 124 203 208 246 250 276 300 348 352 372 380 392 410 442 484 528 554 578 616 620 703 724 752 756 792 826 840);

! Define the 30 multiple groups (countries)

DEFINE: CONTSCHL = (country\*10000) + SCHOOLID;

! Define group to be a unique combination of country (country ID code multiplied by 1000) and school ID;

ANALYSIS: TYPE=MIXTURE COMPLEX;

!Complex: Analysis takes nesting of students with schools into account;

ESTIMATOR = ML;

PROCESSORS = 6;

TITLE: Model MG-MIMICAwC (see Figure 2) 30 groups MIMIC model with the alignment model;

DATA: FILE = “PISA06\_data.dat”;

VARIABLE:

NAMES = SCHOOLID STIDSTD COUNTRY OECD W\_FSTUWT ST16Q01 ST16Q02 ST16Q03 ST16Q04 ST37Q06 Gender SES PV1SCIE;

MISSING=.;

USEVARIABLES ARE ST16Q01-ST19Q06 Gender SES PV1SCIE CONTSCHL;

CLUSTER = CONTSCHL;

! cluster by school;

WEIGHT = W\_FSTUWT;

! W\_FSTUWT is the student-level weighting variable in the PISA database.

GROUPING = country (36 40 56 124 203 208 246 250 276 300 348 352 372 380 392 410 442 484 528 554 578 616 620 703 724 752 756 792 826 840);

! Define the 30 multiple groups (countries)

DEFINE: CONTSCHL = (country\*10000) + SCHOOLID;

! Define group to be a unique combination of country (country ID code multiplied by 1000) and school ID;

ANALYSIS: TYPE= COMPLEX;

!Complex: Analysis takes nesting of students with schools into account;

ESTIMATOR = MLR;

PROCESSORS = 6;

MODEL:

joyscie BY ST16Q01 ST16Q02 ST16Q03 ST16Q04 ST16Q05;

instscie BY ST35Q01 ST35Q02 ST35Q03 ST35Q04 ST35Q05;

scifut BY ST29Q01 ST29Q02 ST29Q03 ST29Q04;

scieeff BY ST17Q01 ST17Q02 ST17Q03 ST17Q04

ST17Q05 ST17Q06 ST17Q07 ST17Q08;

scscie BY ST37Q01 ST37Q02 ST37Q03 ST37Q04 ST37Q05 ST37Q06;

genscie BY ST18Q01 ST18Q02 ST18Q04 ST18Q06 ST18Q09;

perscie BY ST18Q03 ST18Q05 ST18Q07 ST18Q08 ST18Q10;

sciact BY ST19Q01 ST19Q02 ST19Q03 ST19Q04 ST19Q05 ST19Q06;

output:

tech1 tech8 align SVALUES;

! SVALUES: ask for starting values for subsequent AwC model.

<<< Here an example of starting values for the USA group in the output of the alignment model above. >>

%C#30%

joyscie BY st16q01\*0.73192;

joyscie BY st16q02\*0.71737;

MODEL:

joyscie BY ST16Q01 ST16Q02 ST16Q03 ST16Q04 ST16Q05;

instscie BY ST35Q01 ST35Q02 ST35Q03 ST35Q04 ST35Q05;

scifut BY ST29Q01 ST29Q02 ST29Q03 ST29Q04;

scieeff BY ST17Q01 ST17Q02 ST17Q03 ST17Q04

ST17Q05 ST17Q06 ST17Q07 ST17Q08;

scscie BY ST37Q01 ST37Q02 ST37Q03 ST37Q04 ST37Q05 ST37Q06;

genscie BY ST18Q01 ST18Q02 ST18Q04 ST18Q06 ST18Q09;

perscie BY ST18Q03 ST18Q05 ST18Q07 ST18Q08 ST18Q10;

sciact BY ST19Q01 ST19Q02 ST19Q03 ST19Q04 ST19Q05 ST19Q06;

! It is easy to incorporate

covariates in AwC model. (e.g., add

scscie-sciact on Gender SES PV1SCIE

for the MIMIC model (MG-MIMIC4AwC)

! for all parameters, the exact values from an alignment model including all the eight constructs (44 items) as starts values (using \*)

**! For identification purposes, the first item per factor is constrained to its estimated values from the alignment solution, and factor variances and means are free**

<<< Model specifications are shown for the USA groups. All other groups are defined in a similar manner>>

**Model 840:**

joyscie BY st16q01@0.73192;

joyscie BY st16q02\*0.71737;

joyscie BY st16q03\*0.68005;

joyscie BY st16q04\*0.73858;

joyscie BY st16q05\*0.78418;

…..

…..

sciact BY st19q01\*0.44008;

sciact BY st19q02\*0.46243;

sciact BY st19q03\*0.53099;

sciact BY st19q04\*0.32932;

sciact BY st19q05\*0.54562;

sciact BY st19q06\*0.18266;

instscie WITH joyscie\*0.47751;

…..

…..

sciact WITH scscie\*0.47891;

sciact WITH genscie\*0.43133;

sciact WITH perscie\*0.51505;

[ st16q01\*2.63270 ];

[ st16q02\*2.41076 ];

[ st16q03\*2.33300 ];

[ st16q04\*2.72070 ];

[ st16q05\*2.70186 ];

…..

…..

[ st19q01\*1.62389 ];

[ st19q02\*1.12605 ];

[ st19q03\*1.29471 ];

[ st19q04\*1.06963 ];

[ st19q05\*1.33893 ];

[ st19q06\*1.02782 ];

[ joyscie\*0.04410 ];

[ instscie\*0.24782 ];

[ scifut\*0.32264 ];

[ scieeff\*0.09824 ];

[ scscie\*0.19773 ];

[ genscie\*0.20793 ];

[ perscie\*0.25161 ];

[ sciact\*0.75671 ];

st16q01\*0.17500;

st16q02\*0.19479;

st16q03\*0.22249;

st16q04\*0.16762;

st16q05\*0.16281;

…..

…..

st19q01\*0.44733;

st19q02\*0.19859;

st19q03\*0.25967;

joyscie BY st16q03\*0.68005;

joyscie BY st16q04\*0.73858;

joyscie BY st16q05\*0.78418;

…..

…..

sciact BY st19q01@0.44008;

sciact BY st19q02\*0.46243;

sciact BY st19q03\*0.53099;

sciact BY st19q04\*0.32932;

sciact BY st19q05\*0.54562;

sciact BY st19q06\*0.18266;

instscie WITH joyscie\*0.47751;

…..

…..

sciact WITH scscie\*0.47891;

sciact WITH genscie\*0.43133;

sciact WITH perscie\*0.51505;

[ st16q01@2.63270 ];

[ st16q02\*2.41076 ];

[ st16q03\*2.33300 ];

[ st16q04\*2.72070 ];

[ st16q05\*2.70186 ];

…..

…..

[ st19q01@1.62389 ];

[ st19q02\*1.12605 ];

[ st19q03\*1.29471 ];

[ st19q04\*1.06963 ];

[ st19q05\*1.33893 ];

[ st19q06\*1.02782 ];

[ joyscie\*0.04410 ];

[ instscie\*0.24782 ];

[ scifut\*0.32264 ];

[ scieeff\*0.09824 ];

[ scscie\*0.19773 ];

[ genscie\*0.20793 ];

[ perscie\*0.25161 ];

[ sciact\*0.75671 ];

st16q01\*0.17500;

st16q02\*0.19479;

st16q03\*0.22249;

st16q04\*0.16762;

st16q05\*0.16281;

…..

…..

st19q01\*0.44733;

st19q02\*0.19859;

st19q03\*0.25967;

st19q04\*0.24697;

st19q05\*0.31817;

st19q06\*0.23871;

joyscie\*0.93175;

instscie\*0.70558;

scifut\*0.86945;

scieeff\*1.01327;

scscie\*0.96378;

genscie\*1.08514;

perscie\*0.78113;

sciact\*1.18903;

<<Mplus output>>

MODEL RESULTS:

!unstandardized results for the alignment model above.

JOYSCIE BY

ST16Q01 0.732 0.008 90.750 0.000

ST16Q02 0.717 0.009 79.000 0.000

ST16Q03 0.680 0.008 80.764 0.000

ST16Q04 0.739 0.008 94.243 0.000

ST16Q05 0.784 0.008 102.351 0.000

…..

…..

SCIACT BY

ST19Q01 0.440 0.016 26.978 0.000

ST19Q02 0.462 0.015 31.327 0.000

ST19Q03 0.531 0.024 22.318 0.000

ST19Q04 0.329 0.010 32.620 0.000

ST19Q05 0.546 0.019 28.814 0.000

ST19Q06 0.183 0.012 14.648 0.000

INSTSCIE WITH

JOYSCIE 0.478 0.017 28.129 0.000

…..

…..

SCIACT WITH

JOYSCIE 0.654 0.026 24.893 0.000

INSTSCIE 0.389 0.021 18.376 0.000

SCIFUT 0.545 0.023 23.308 0.000

SCIEEFF 0.474 0.026 18.443 0.000

SCSCIE 0.479 0.025 19.093 0.000

GENSCIE 0.431 0.024 17.732 0.000

PERSCIE 0.515 0.023 22.134 0.000

Means

JOYSCIE 0.044 0.027 1.646 0.100

INSTSCIE 0.248 0.023 10.804 0.000

SCIFUT 0.323 0.022 14.655 0.000

SCIEEFF 0.098 0.037 2.652 0.008

SCSCIE 0.198 0.039 5.054 0.000

st19q04\*0.24697;

st19q05\*0.31817;

st19q06\*0.23871;

joyscie\*0.93175;

instscie\*0.70558;

scifut\*0.86945;

scieeff\*1.01327;

scscie\*0.96378;

genscie\*1.08514;

perscie\*0.78113;

sciact\*1.18903;

<<Mplus output>>

MODEL RESULTS:

!unstandardized results for the AwC model above.

JOYSCIE BY

ST16Q01 0.732 0.000 999.00 999.00

ST16Q02 0.717 0.008 84.657 0.000

ST16Q03 0.680 0.009 75.183 0.000

ST16Q04 0.739 0.009 83.899 0.000

ST16Q05 0.784 0.008 95.966 0.000

…..

…..

SCIACT BY

ST19Q01 0.440 0.000 999.00 999.00 ST19Q02 0.462 0.013 34.882 0.000

ST19Q03 0.531 0.019 27.514 0.000

ST19Q04 0.329 0.014 24.028 0.000

ST19Q05 0.546 0.015 36.337 0.000

ST19Q06 0.183 0.014 12.637 0.000

INSTSCIE WITH

JOYSCIE 0.478 0.017 27.613 0.000

…..

…..

SCIACT WITH

JOYSCIE 0.654 0.024 27.053 0.000

INSTSCIE 0.389 0.020 19.881 0.000

SCIFUT 0.545 0.021 26.262 0.000

SCIEEFF 0.474 0.026 18.162 0.000

SCSCIE 0.479 0.022 21.776 0.000

GENSCI 0.431 0.025 17.379 0.000

PERSCIE 0.515 0.026 19.866 0.000

Means

JOYSCIE 0.044 0.023 1.941 0.052

INSTSCIE 0.248 0.017 14.483 0.000

SCIFUT 0.323 0.014 23.811 0.000

SCIEEFF 0.098 0.030 3.285 0.001

SCSCIE 0.198 0.025 7.926 0.000

GENSCIE 0.208 0.036 5.854 0.000

PERSCIE 0.252 0.028 8.836 0.000

SCIACT 0.757 0.036 20.759 0.000

Intercepts

ST16Q01 2.633 0.000 999.00 999.00

ST16Q02 2.411 0.009 254.053 0.000

ST16Q03 2.333 0.015 153.671 0.000

ST16Q04 2.721 0.013 205.973 0.000

ST16Q05 2.702 0.016 172.449 0.000

…..

…..

ST19Q01 1.624 0.017 96.907 0.000

ST19Q02 1.126 0.013 85.931 0.000

ST19Q03 1.295 0.016 78.744 0.000

ST19Q04 1.070 0.009 115.829 0.000

ST19Q05 1.339 0.018 72.948 0.000

ST19Q06 1.028 0.013 78.160 0.000

Variances

JOYSCIE 0.932 0.025 36.572 0.000

INSTSCIE 0.706 0.023 31.103 0.000

SCIFUT 0.869 0.021 41.597 0.000

SCIEEFF 1.013 0.040 25.155 0.000

SCSCIE 0.964 0.038 25.124 0.000

GENSCIE 1.085 0.054 20.231 0.000

PERSCIE 0.781 0.031 25.446 0.000

SCIACT 1.189 0.085 13.936 0.000

Residual Variances

ST16Q01 0.175 0.006 27.283 0.000

ST16Q02 0.195 0.006 33.802 0.000

ST16Q03 0.222 0.007 33.757 0.000

ST16Q04 0.168 0.006 25.923 0.000

ST16Q05 0.163 0.008 21.616 0.000

…..

…..

ST19Q01 0.447 0.013 33.577 0.000

ST19Q02 0.199 0.010 19.488 0.000

ST19Q03 0.260 0.011 23.291 0.000

ST19Q04 0.247 0.013 19.398 0.000

ST19Q05 0.318 0.011 27.897 0.000

ST19Q06 0.239 0.022 10.982 0.000

GENSCIE 0.208 0.044 4.710 0.000

PERSCIE 0.252 0.028 8.883 0.000

SCIACT 0.757 0.043 17.650 0.000

Intercepts

ST16Q01 2.633 0.015 172.239 0.000

ST16Q02 2.411 0.016 153.681 0.000

ST16Q03 2.333 0.009 252.438 0.000

ST16Q04 2.721 0.009 293.153 0.000

ST16Q05 2.702 0.010 281.415 0.000

…..

…..

ST19Q01 1.624 0.000 999.00 999.00

ST19Q02 1.126 0.015 77.440 0.000

ST19Q03 1.295 0.019 69.870 0.000

ST19Q04 1.070 0.012 88.178 0.000

ST19Q05 1.339 0.018 74.436 0.000

ST19Q06 1.028 0.014 73.530 0.000

Variances

JOYSCIE 0.932 0.025 37.180 0.000

INSTSCIE 0.706 0.024 29.056 0.000

SCIFUT 0.869 0.019 46.779 0.000

SCIEEFF 1.013 0.055 18.373 0.000

SCSCIE 0.964 0.024 40.116 0.000

GENSCIE 1.085 0.059 18.468 0.000

PERSCIE 0.781 0.046 16.931 0.000

SCIACT 1.189 0.065 18.232 0.000

Residual Variances

ST16Q01 0.175 0.006 27.283 0.000

ST16Q02 0.195 0.006 33.802 0.000

ST16Q03 0.222 0.007 33.757 0.000

ST16Q04 0.168 0.006 25.923 0.000

ST16Q05 0.163 0.008 21.616 0.000

…..

…..

ST19Q01 0.447 0.013 33.577 0.000

ST19Q02 0.199 0.010 19.488 0.000

ST19Q03 0.260 0.011 23.291 0.000

ST19Q04 0.247 0.013 19.398 0.000

ST19Q05 0.318 0.011 27.897 0.000

ST19Q06 0.239 0.022 10.982 0.000

# Appendix 4:

# Correlations Among the Eight Motivational Constructs in PISA 2006

Not surprisingly, all 28 correlations among the eight motivational constructs were positive (Mean[M] *r* = .547) and, due in part to the large sample size, all were statistically significant. The largest correlation was between general value and personal value of science (*r* = .785). Correlations among enjoyment, instrumental motivation and future-oriented motivation in science were substantial (*r* = .590 to .713), and they were all highly correlated with personal value (*r* = .666 to .705). Correlations of self-concept in science with other motivational constructs were moderate (*r* = .399 to .611) and slightly larger than those for self-efficacy in science (*r* = .370 to .518). Correlations involving general value were comparatively smaller (*r* = .386 to .511), except for the aforementioned substantial correlations with personal value. Finally, engagement in extracurricular activities in science was highly correlated with enjoyment (*r* = .639) and personal value (*r* = .592), but less correlated with the other motivation constructs (*r* = .452 to .569).

With respect to correlations relating the motivational constructs to the three covariates, correlations of gender to the eight motivational variables were statistically significant but small (*r* = .024 to .094), and favored boys. This pattern of correlations was similar with those of SES to motivational constructs (*r* = -.030 to .114). However, science achievement was more highly correlated with motivational constructs (*r* = .081 to .372), with the exception that correlation between achievement and extracurricular activities was non-significant (*r* = .012). The strongest relations emerged with science self-efficacy (*r* = .372) and general value of science (*r* = .262). Students who had higher science achievement values tended to report that they would be able to solve a range of scientific problems and ascribe a higher societal value to science. Correlations of achievement to self-concept and general value were somewhat smaller (*r*s = .153 and .149 respectively).

# Appendix 5:

# Annotated Input Files Used in Study 2 (Simulated Data)

# Title: Population Model - Input for the Data Generation (20% large non-invariance)

*! In all input files, statements preceded by ! are annotations.*

*! The Monte Carlo facility is used to generate the data.*

MONTECARLO:

NGROUPS = 15; *! This statement indicates the number of groups.*

NOBSERVATIONS = 15(100); *! This statement indicates the sample size in each group.*

NREPS = 500; *! 500 replications are requested.*

SEED = 4533; ! set seed

REPSAVE = ALL; !*saver all of the data sets generated in a Monte Carlo simulation study*

SAVE = cfa-G15-N100-20-\*.dat; *! This statement identifies the data set to be created.*

*! The following section defines the population model based on the parameters described in Table S2*

*! and Figure 2. The \* symbol precedes specific parameter values.*

*! Factor loadings are noted with BY, regressions with ON, correlations with, means and*

*! intercepts are noted between brackets []; variances and residuals are noted without brackets.*

*ANALYSIS: ESTIMATOR = ML;*

Model population:*!Group 1*

f by y1\*1.00 y2\*1.00 y3\*1.40 y4\*1.00 y5\*1.00;

[y1\*0.00 y2\*0.00 y3\*0.00 y4\*0.00 y5\*0.50];

y1-y5\*1;

f\*1; [f\*0];

# Title: Population Model - Input for the Data Generation (10% large non-invariance)

MONTECARLO:

NGROUPS = 15; *! This statement indicates the number of groups.*

NOBSERVATIONS = 15(100); *! This statement indicates the sample size in each group.*

NREPS = 500; *! 500 replications are requested.*

SEED = 4533; ! set seed

REPSAVE = ALL; !*saver all of the data sets generated in a Monte Carlo simulation study*

SAVE = cfa-G15-N100-10-\*.dat; *! This statement identifies the data set to be created.*

*! The following section defines the population model based on the parameters described in Table S2*

*! and Figure 2. The \* symbol precedes specific parameter values.*

*! Factor loadings are noted with BY, regressions with ON, correlations with, means and*

*! intercepts are noted between brackets []; variances and residuals are noted without brackets.*

*ANALYSIS: ESTIMATOR = ML;*

Model population: *!Group 1*

f by y1\*1.00 y2\*1.00 y3\*1.00 y4\*1.00 y5\*1.00;

[y1\*0.00 y2\*0.00 y3\*0.00 y4\*0.00 y5\*0.50];

y1-y5\*1;

f\*1; [f\*0];

ODEL POPULATION-g2:

f by y1\*1.05 y2\*1.10 y3\*0.90 y4\*0.95 y5\*0.50;

[y1\*-.50 y2\*0.05 y3\*-.10 y4\*0.10 y5\*-.05];

y1-y5\*1;

f\*1.5; [f\*.3];

MODEL POPULATION-g3:

f by y1\*0.95 y2\*0.90 y3\*1.10 y4\*0.30 y5\*1.05;

[y1\*-.05 y2\*0.50 y3\*0.10 y4\*-.05 y5\*0.05];

y1-y5\*1;

f\*1.2; [f\*1];

MODEL POPULATION-g4:

f by y1\*1.00 y2\*1.00 y3\*1.40 y4\*1.00 y5\*1.00;

[y1\*0.00 y2\*0.00 y3\*0.00 y4\*0.00 y5\*0.50];

y1-y5\*1;

f\*1; [f\*0];

MODEL POPULATION-g5:

f by y1\*1.05 y2\*1.10 y3\*0.90 y4\*0.95 y5\*0.50;

[y1\*-.50 y2\*0.05 y3\*-.10 y4\*0.10 y5\*-.05];

y1-y5\*1;

f\*1.5; [f\*.3];

MODEL POPULATION-g6:

f by y1\*0.95 y2\*0.90 y3\*1.10 y4\*0.30 y5\*1.05;

[y1\*-.05 y2\*0.50 y3\*0.10 y4\*-.05 y5\*0.05];

y1-y5\*1;

f\*1.2; [f\*1];

MODEL POPULATION-g7:

f by y1\*1.00 y2\*1.00 y3\*1.40 y4\*1.00 y5\*1.00;

[y1\*0.00 y2\*0.00 y3\*0.00 y4\*0.00 y5\*0.50];

y1-y5\*1;

f\*1; [f\*0];

MODEL POPULATION-g8:

f by y1\*1.05 y2\*1.10 y3\*0.90 y4\*0.95 y5\*0.50;

[y1\*-.50 y2\*0.05 y3\*-.10 y4\*0.10 y5\*-.05];

y1-y5\*1;

f\*1.5; [f\*.3];

MODEL POPULATION-g2:

f by y1\*1.05 y2\*1.10 y3\*0.90 y4\*0.95 y5\*0.50;

[y1\*0.05 y2\*0.05 y3\*-.10 y4\*0.10 y5\*-.05];

y1-y5\*1;

f\*1.5; [f\*.3];

MODEL POPULATION-g3:

f by y1\*0.95 y2\*0.90 y3\*1.10 y4\*1.05 y5\*1.05;

[y1\*-.05 y2\*0.50 y3\*0.10 y4\*-.05 y5\*0.05];

y1-y5\*1;

f\*1.2; [f\*1];

MODEL POPULATION-g4:

f by y1\*1.00 y2\*1.00 y3\*1.40 y4\*1.00 y5\*1.00;

[y1\*0.00 y2\*0.00 y3\*0.00 y4\*0.00 y5\*0.00];

y1-y5\*1;

f\*1; [f\*0];

MODEL POPULATION-g5:

f by y1\*1.05 y2\*1.10 y3\*0.90 y4\*0.95 y5\*0.95;

[y1\*-.50 y2\*0.05 y3\*-.10 y4\*0.10 y5\*-.05];

y1-y5\*1;

f\*1.5; [f\*.3];

MODEL POPULATION-g6:

f by y1\*0.95 y2\*0.90 y3\*1.10 y4\*0.30 y5\*1.05;

[y1\*-.05 y2\*-.05 y3\*0.10 y4\*-.05 y5\*0.05];

y1-y5\*1;

f\*1.2; [f\*1];

MODEL POPULATION-g7:

f by y1\*1.00 y2\*1.00 y3\*1.00 y4\*1.00 y5\*1.00;

[y1\*0.00 y2\*0.00 y3\*0.00 y4\*0.00 y5\*0.50];

y1-y5\*1;

f\*1; [f\*0];

MODEL POPULATION-g8:

f by y1\*1.05 y2\*1.10 y3\*0.90 y4\*0.95 y5\*0.50;

[y1\*0.05 y2\*0.05 y3\*-.10 y4\*0.10 y5\*-.05];

y1-y5\*1;

f\*1.5; [f\*.3];

MODEL POPULATION-g9:

f by y1\*0.95 y2\*0.90 y3\*1.10 y4\*0.30 y5\*1.05;

[y1\*-.05 y2\*0.50 y3\*0.10 y4\*-.05 y5\*0.05];

y1-y5\*1;

f\*1.2; [f\*1];

MODEL POPULATION-g10:

f by y1\*1.00 y2\*1.00 y3\*1.40 y4\*1.00 y5\*1.00;

[y1\*0.00 y2\*0.00 y3\*0.00 y4\*0.00 y5\*0.50];

y1-y5\*1;

f\*1; [f\*0];

MODEL POPULATION-g11:

f by y1\*1.05 y2\*1.10 y3\*0.90 y4\*0.95 y5\*0.50;

[y1\*-.50 y2\*0.05 y3\*-.10 y4\*0.10 y5\*-.05];

y1-y5\*1;

f\*1.5; [f\*.3];

MODEL POPULATION-g12:

f by y1\*0.95 y2\*0.90 y3\*1.10 y4\*0.30 y5\*1.05;

[y1\*-.05 y2\*0.50 y3\*0.10 y4\*-.05 y5\*0.05];

y1-y5\*1;

f\*1.2; [f\*1];

MODEL POPULATION-g13:

f by y1\*1.00 y2\*1.00 y3\*1.40 y4\*1.00 y5\*1.00;

[y1\*0.00 y2\*0.00 y3\*0.00 y4\*0.00 y5\*0.50];

y1-y5\*1;

f\*1; [f\*0];

MODEL POPULATION-g14:

f by y1\*1.05 y2\*1.10 y3\*0.90 y4\*0.95 y5\*0.50;

[y1\*-.50 y2\*0.05 y3\*-.10 y4\*0.10 y5\*-.05];

y1-y5\*1;

f\*1.5; [f\*.3];

MODEL POPULATION-g15:

f by y1\*0.95 y2\*0.90 y3\*1.10 y4\*0.30 y5\*1.05;

[y1\*-.05 y2\*0.50 y3\*0.10 y4\*-.05 y5\*0.05];

y1-y5\*1;

f\*1.2; [f\*1];

OUTPUT: tech9;

MODEL POPULATION-g9:

f by y1\*0.95 y2\*0.90 y3\*1.10 y4\*1.05 y5\*1.05;

[y1\*-.05 y2\*0.50 y3\*0.10 y4\*-.05 y5\*0.05];

y1-y5\*1;

f\*1.2; [f\*1];

MODEL POPULATION-g10:

f by y1\*1.00 y2\*1.00 y3\*1.40 y4\*1.00 y5\*1.00;

[y1\*0.00 y2\*0.00 y3\*0.00 y4\*0.00 y5\*0.00];

y1-y5\*1;

f\*1; [f\*0];

MODEL POPULATION-g11:

f by y1\*1.05 y2\*1.10 y3\*0.90 y4\*0.95 y5\*0.95;

[y1\*-.50 y2\*0.05 y3\*-.10 y4\*0.10 y5\*-.05];

y1-y5\*1;

f\*1.5; [f\*.3];

MODEL POPULATION-g12:

f by y1\*0.95 y2\*0.90 y3\*1.10 y4\*0.30 y5\*1.05;

[y1\*-.05 y2\*-.05 y3\*0.10 y4\*-.05 y5\*0.05];

y1-y5\*1;

f\*1.2; [f\*1];

MODEL POPULATION-g13:

f by y1\*1.00 y2\*1.00 y3\*1.00 y4\*1.00 y5\*1.00;

[y1\*0.00 y2\*0.00 y3\*0.00 y4\*0.00 y5\*0.50];

y1-y5\*1;

f\*1; [f\*0];

MODEL POPULATION-g14:

f by y1\*1.05 y2\*1.10 y3\*0.90 y4\*0.95 y5\*0.50;

[y1\*0.05 y2\*0.05 y3\*-.10 y4\*0.10 y5\*-.05];

y1-y5\*1;

f\*1.5; [f\*.3];

MODEL POPULATION-g15:

f by y1\*0.95 y2\*0.90 y3\*1.10 y4\*1.05 y5\*1.05;

[y1\*-.05 y2\*0.50 y3\*0.10 y4\*-.05 y5\*0.05];

y1-y5\*1;

f\*1.2; [f\*1];

OUTPUT: tech9;

**TITLE: Alignment Model**

*! The following statement is used to identify the data file.*

DATA: FILE = cfa-G15-N100-20-22.dat ; *Here, the data file is labeled* cfa-G15-N100-20-199.dat, which is the 199th replication for 100 of sample size with 20% large non-invariance

VARIABLE: NAMES = y1-y5 GROUP;

USEVARIABLES = y1-y5;

Classes = c(15); ! *number of groups*

knownclass = c(GROUP = 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15);

ANALYSIS:

TYPE = MIXTURE;

ESTIMATOR = MLR;

alignment = fixed;

ALGORITHM = INTEGRATION;

PROCESSORS = 2;

MODEL: %OVERALL%

F by Y1-Y5;

*! Specific sections of output are requested.*

OUTPUT: SVALUES align;

**Title: ICM-CFA (Configural, metric, and scalar invariance model)**

*! The following statement is used to identify the data file.*

DATA: FILE = cfa-G15-N100-20-199.dat ; *Here, the data file is labeled* cfa-G15-N100-20-199.dat, which is the 199th replication for 100 of sample size with 20% large non-invariance

VARIABLE: NAMES = y1-y5 GROUP;

USEVARIABLES = y1-y5;

GROUPING = GROUP (1 2 3 4 5 6 7 8 9 10 11 12 13 14 15);

ANALYSIS:

ESTIMATOR = ML; *!Maximum Likelihood (ML) estimation is used.*

Model = configural metric, and scalar;

MODEL:

f by y1\* y2 y3 y4 y5; ! freely estimate the first factor loading

[y1 y2 y3 y4 y5];

f@1; [f@0];! *For identification purposes, factor variance and latent mean are fixed to be 1 and 0 in the first group.*

*! Specific sections of output are requested.*

Output: sampstat standardized SVALUES stdyx tech4;

**Title: ICM-CFA (Partial invariance model)**

*! The following statement is used to identify the data file.*

DATA: FILE = cfa-G15-N100-20-199.dat ; *Here, the data file is labeled* cfa-G15-N100-20-199.dat, which is the 199th replication for 100 of sample size with 20% large non-invariance

VARIABLE: NAMES = y1-y5 GROUP;

USEVARIABLES = y1-y5;

GROUPING = GROUP (1 2 3 4 5 6 7 8 9 10 11 12 13 14 15);

ANALYSIS:

ESTIMATOR = ML;

MODEL:

f by y1\* y2 y3 y4 y5;

[y1 y2 y3 y4 y5];

y1-y5\*1;

f@1;

[f@0];

MODEL 1:

[Y5]; [Y1]; ! *freely estimate intercept Y1 and Y5 according to the strategy of partial invariance described in the main text*

f@1; [f@0]; *For identification purposes, factor variance and latent mean are fixed to be 1 and 0 in the first group.*

MODEL 2:

f; [f];! *Freely estimate factor variance and latent mean from Group 2 to Group 15*

MODEL 3:

F BY Y4;

f; [f];

MODEL 4:

[Y2];

[Y4];

[Y3];

F BY Y2;

f; [f];

MODEL 5:

F BY Y5;

[Y1];

F BY Y3;

f; [f];

MODEL 6:

F BY Y4;

f; [f];

MODEL 7:

[Y2];

[Y4];

f; [f];

MODEL 8:

F BY Y5;

f; [f];

MODEL 9:

F BY Y4;

f; [f];

MODEL 10:

[Y2];

[Y4];

f; [f];

MODEL 11:

F BY Y5;

F BY Y3;

f; [f];

MODEL 12:

F BY Y4;

f; [f];

MODEL 13:

[Y2];

[Y4];

[Y3];

F BY Y3;

f; [f];

MODEL 14:

f; [f];

MODEL 15:

F BY Y4;

f; [f];

OUTPUT: MODINDICES(5) tech9 SVALUES;

**The previous population model is used to generate Validation data, except for changing the seed.**

**Title: Cross-validation Alignment Model**

DATA: FILE = cfa-G15-N100-20-V199.dat ; *Here, the data file is labeled* cfa-G15-N100-20-V199.dat, which is the 199th replication of the validation data for 100 of sample size with 20% large non-invariance

VARIABLE: NAMES = y1-y5 GROUP;

USEVARIABLES = y1-y5;

GROUPING = GROUP (1 2 3 4 5 6 7 8 9 10 11 12 13 14 15);

ANALYSIS: ESTIMATOR = ML;

*! The previous Alignment model is re-expressed using CFA based on validation data.*

*! The model section uses the exact values of the non-standardized loadings, intercepts, residual*

*! variances, and factor variances estimated from the previous model (using @).*

*!Only latent mean are freely estimated.*

*!But for identification purposes, latent mean are fixed to be 0 in the first group.*

MODEL:

f by y1\* y2-y5;

f@1;

MODEL 1:

F BY Y1@1.135;

F BY Y2@0.941;

F BY Y3@1.398;

F BY Y4@0.908;

F BY Y5@1.092;

[F@0];

[Y1@0.124];

[Y2@0.183];

[Y3@0.113];

[Y4@0.1];

[Y5@0.598];

F@1;

Y1@0.914;

Y2@1.249;

Y3@0.916;

Y4@1.171;

Y5@0.988;

MODEL 2:

F BY Y1@1.108;

F BY Y2@1.111;

F BY Y3@1.086;

F BY Y4@0.993;

F BY Y5@0.66;

[F]; ! *freely estimate latent mean*

[Y1@-0.353];

[Y2@0.442];

[Y3@0.044];

[Y4@0.355];

[Y5@0.098];

F@0.925;

Y1@0.997;

Y2@1.106;

Y3@0.895;

Y4@0.966;

Y5@1.17;

………..

………..

…………

MODEL 15:

F BY Y1@1.192;

F BY Y2@1.073;

F BY Y3@1.104;

F BY Y4@0.498;

F BY Y5@1.032;

[F]; ! *freely estimate latent mean*

[Y1@-0.199];

[Y2@0.463];

[Y3@0.196];

[Y4@-0.148];

[Y5@0.048];

F@0.632;

Y1@0.716;

Y2@0.82;

Y3@1.251;

Y4@0.915;

[Y5@1.223](mailto:Y5@1.223);

**Title: Cross-validation Scalar Model**

DATA: FILE = cfa-G15-N100-20-V199.dat ; *Here, the data file is labeled* cfa-G15-N100-20-V199.dat, which is the 199th replication of the validation data for 100 of sample size with 20% large non-invariance

VARIABLE: NAMES = y1-y5 GROUP;

USEVARIABLES = y1-y5;

GROUPING = GROUP (1 2 3 4 5 6 7 8 9 10 11 12 13 14 15);

ANALYSIS:

ESTIMATOR = ML;

*! The previous scalar model is re-expressed using CFA based on validation data.*

*! The model section uses the exact values of the non-standardized loadings, intercepts, residual*

*! variances, and factor variances estimated from the previous model (using @).*

*!Only latent mean are freely estimated.*

*!But for identification purposes, latent mean are fixed to be 0 in the first group.*

MODEL:

f by y1\* y2-y5;

f@1;

MODEL 1:

F BY Y1@1.215;

F BY Y2@1.22;

F BY Y3@1.285;

F BY Y4@0.684;

F BY Y5@0.905;

[F@0];

[Y1@0.013];

[Y2@0.396];

[Y3@0.232];

[Y4@0.049];

[Y5@0.385];

F@1;

Y1@0.842;

Y2@1.25;

Y3@1.065;

Y4@1.265;

[Y5@1.171](mailto:Y5@1.171);

MODEL 2:

[F]; ! *freely estimate latent mean*

F@0.744;

Y1@1.011;

Y2@1.176;

Y3@0.882;

Y4@1.32;

[Y5@1.193](mailto:Y5@1.193);

………..

………..

…………

MODEL 14:

[F]; ! *freely estimate latent mean*

F@0.848;

Y1@1.081;

Y2@1.187;

Y3@0.998;

Y4@1.845;

[Y5@1.069](mailto:Y5@1.069);

MODEL 15:

[F]; ! *freely estimate latent mean*

F@0.532;

Y1@0.763;

Y2@0.847;

Y3@1.192;

Y4@0.964;

[Y5@1.311](mailto:Y5@1.311);

**Title: Cross-validation partial Model**

DATA: FILE = cfa-G15-N100-20-V199.dat ; *Here, the data file is labeled* cfa-G15-N100-20-V199.dat, which is the 199th replication of the validation data for 100 of sample size with 20% large non-invariance

VARIABLE: NAMES = y1-y5 GROUP;

USEVARIABLES = y1-y5;

GROUPING = GROUP (1 2 3 4 5 6 7 8 9 10 11 12 13 14 15);

ANALYSIS: ESTIMATOR = ML;

*! The previous partial solution specific to the 199th replication is re-expressed using CFA based on validation data.*

*! The model section uses the exact values of the non-standardized loadings, intercepts, residual*

*! variances, and factor variances estimated from the previous model (using @).*

*!Only latent mean are freely estimated.*

*!But for identification purposes, latent mean are fixed to be 0 in the first group.*

MODEL:

f by y1\* y2-y5;

f@1;

MODEL 1:

F BY Y1@1.167;

F BY Y2@1.12;

F BY Y3@1.209;

F BY Y4@1.024;

F BY Y5@1.012;

[F@0];

[Y1@-0.013];

[Y2@0.094];

[Y3@-0.212];

[Y4@0.013];

[Y5@0.479];

F@1;

Y1@0.875;

Y2@1.168;

Y3@1.167;

Y4@1.112;

[Y5@1.062](mailto:Y5@1.062);

MODEL 2:

F BY Y1@1.167;

F BY Y2@1.12;

F BY Y3@1.209;

F BY Y4@1.024;

F BY Y5@1.012;

[F]; ! *freely estimate latent mean*

[Y1@-0.509];

[Y2@0.094];

[Y3@-0.212];

[Y4@0.013];

[Y5@-0.064];

F@0.748;

Y1@1.048;

Y2@1.197;

Y3@0.893;

Y4@0.988;

Y5@1.141;

………

………

………

MODEL 15:

F BY Y1@1.167;

F BY Y2@1.12;

F BY Y3@1.209;

F BY Y4@0.32;

F BY Y5@1.012;

[F]; ! *freely estimate latent mean*

[Y1@-0.509];

[Y2@0.094];

[Y3@-0.212];

[Y4@0.013];

[Y5@-0.064];

F@0.619;

Y1@0.745;

Y2@0.791;

Y3@1.234;

Y4@0.958;

[Y5@1.25](mailto:Y5@1.25);

OUTPUT: MODINDICES(5) SVALUES;

**Appendix 6:**

**Boxplots of Deviations Factor loading and Intercepts for Items from Selected PISA Scales Based on Alignment and scalar Models**

Self-Concept: Factor Loadings General Value: Factor Loadings

 

Self-Concept: Intercepts General Value: Intercepts

 

**Appendix 7:**

**Largest Modification Indices (MI) and Expected Parameter Change (EPC) for the Basic AwC**



*Note.* Modification Indices (MI) and Expected Parameter Change (EPC) are not available in the alignment approach, but can be obtained for the basic alignment-within-CFA (AwC) model. The 63 (out of 44 items x 30 groups = 1,320 factor loadings and 1,320 intercepts) StdYX\_EPC values greater than .2 (in absolute value) are highlighted in yellow. ModV1 and modV2 = specific items associated with MI. Parameters refer to factor loadings (indicated by “BY”) as an operator or as item intercepts (indicated by variable names in brackets). Group refers to the 30 different OECD countries.