**Academic Buoyancy in High School: A Cross-lagged Multilevel Modeling Approach Exploring Reciprocal Effects with Perceived School Support, Motivation, and Engagement**

**Online Supplementary Material**

**Longitudinal Measurement Invariance**

 To examine longitudinal invariance in our measurement structure across the two measurement points assessed one year apart, we conducted a series of increasingly constrained CFA (Streba, 2017; Vandenberg & Lance, 2000; Widaman et al., 2010). For Models 1 – 7, we conducted four CFAs where:

1. All factor means, variances, and covariances were freely estimated
2. The factor loadings of parallel items were constrained to be equal across time points (i.e., factor loading of academic buoyancy item 1 in 2018 was set to be equal to academic buoyancy item 1 in 2019)
3. The intercepts of parallel items were constrained to be equal across time points
4. The residuals of parallel items were constrained to be equal across time points.

We examined these increasingly restrictive models for changes in model fit to determine longitudinal measurement invariance. Changes in CFI less than or equal to .01 and changes in RMSEA less than or equal to .015 were considered indicative of longitudinal measurement invariance (Chen, 2007; Cheung & Rensvold, 2002). As shown in Table S1, there was evidence of longitudinal measurement invariance.

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| Table S1*Fit Indices for Longitudinal Measurement Invariance* |
|  | 1. Configural
 | 1. Metric
 | 1. Scalar
 | 1. Residual
 |
|  | RMSEA | CFI | RMSEA | CFI | RMSEA | CFI | RMSEA | CFI |
| Learning Support | .040 | .938 | .039+ | .938 | .039+ | .937 | .038+ | .938 |
| Teacher Relational Support | .040 | .935 | .040+ | .935 | .039+ | .934 | .038+ | .936 |
| Sense of Belonging | .044 | .925 | .043+ | .926 | .043+ | .925 | .041+ | .927 |
| Classroom Management | .047 | .905 | .046+ | .905 | .045+ | .904 | .044+ | .906 |
| Perseverance | .058 | .927 | .057+ | .926 | .056+ | .925 | .054+ | .926 |
| Perceived Competence | .060 | .932 | .059+ | .932 | .058+ | .930 | .055+ | .933 |
| Valuing | .049 | .936 | .046+ | .936 | .049+ | .932 | .047+ | .932 |
| *Notes.* CFI = comparative fit index; RMSEA = root mean square error of approximation.+ΔRMSEA ≤ .015ΔCFI ≤ .01 |

**Additional Analyses**

Table S2 shows the standardized beta estimates for associations between covariates (gender, grade, SES, and prior achievement) and each substantive variable in the analyses.

Table S3 shows a comparison of the model results from the original school belonging and academic buoyancy model (Model 3).

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| Table S2*Standardized Beta Coefficients for Covariates Involved in Cross-lagged Models (Models 1 –7)* |
|  |  | Learning Support (LS; Model 1) | Teacher Relational Support (RS; Model 2) | School Belonging (SB; Model 3) | Classroom Management(CM; Model 4) |
|  |  | T1 AB | T2 AB | T1 LS | T2 LS | T1 AB | T2 AB | T1 RS | T2 RS | T1 AB | T2 AB | T1 BS | T2 BS | T1 AB | T2 AB | T1 CM | T2 CM |
| Gender |  | **-.17** | **-.16** | -.01 | **-.01** | **-.17** | **-.16** | **-.02** | **-.02** | **-.17** | **-.16** | **-.06** | **-.04** | **-.17** | **-.16** | **-.07** | **-.04** |
|  | SE | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 |
| Grade |  | **-.14** | **.02** | **-.08** | **.10** | **-.14** | **.03** | **-.14** | **.12** | **-.14** | **.03** | **-.11** | **.06** | **-.14** | **.02** | **-.10** | **.14** |
|  | SE | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 |
| SES |  | **.18** | **.04** | **.20** | **.05** | **.17** | **.03** | **.19** | **.04** | **.17** | **.03** | **.21** | **.05** | **.17** | **.03** | **.19** | **.04** |
|  | SE | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 |
| Prior Achievement |  | **-.02** | .01 | **.04** | **.07** | **-.02** | -.01 | .01 | **.04** | **-.03** | -.01 | **.02** | **.06** | **-.03** | .01 | **-.07** | **.01** |
| SE | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 |
|  |  | Perseverance (PR; Model 5) | Perceived Competence (PC; Model 6) | Valuing (VA; Model 7) |  |
|  |  | T1 AB | T2 AB | T1 PR | T2 PR | T1 AB | T2 AB | T1 PC | T2 PC | T1 AB | T2 AB | T1 VA | T2 VA |  |  |  |  |
| Gender |  | **-.17** | **-.15** | .01 | **-.03** | **-.16** | **-.15** | **-.01** | **-.01** | **-.16** | **-.15** | **-.03** | .01 |  |  |  |  |
|  | SE | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 |  |  |  |  |
| Grade |  | **-.13** | **.03** | **-.07** | **.04** | **-.13** | **.03** | **-.09** | **.02** | **-.13** | **.03** | **-.21** | -.01 |  |  |  |  |
|  | SE | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 |  |  |  |  |
| SES |  | **.16** | **.03** | **.24** | **.06** | **.16** | **.03** | **.20** | **.07** | **.16** | **.03** | **.15** | **.05** |  |  |  |  |
|  | SE | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 |  |  |  |  |
| Prior Achievement |  | **-.04** | -.01 | **.11** | **.04** | **-.04** | -.01 | **.18** | **.14** | **-.04** | -.01 | **-.05** | -.01 |  |  |  |  |
| SE | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 |  |  |  |  |
| *Notes*. AB = Academic Buoyancy; T1 = Time 1; T2 = Time 2; SES = socio-economic status; SE = standard error; Gender: 0 = male; 1 = female. Bolded paths are statistically significant. Estimates between 0 and .004 are presented as .01; estimates between -.004 and 0 are presented as -.01. Estimates for School Belonging (Model 3) are for the revised model with a fixed path. Results of the original Model 3 are presented in Table S3 in online supplementary materials. |

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| Table S3*Standardized Estimates from Original School Belonging and Academic Buoyancy Cross-lagged Model*  |
|  |  | T2 Academic Buoyancy | T2 School Belonging  |
| *Level 2* |
| T1 Academic Buoyancy |  | .60\*\*\* | -.09 |
| SE | .09 | .07 |
|  | 95% CI | [.43, .77] | [-.23, .05] |
| T1 School Belonging |  | .38\*\*\* | 1.028\*\*\* |
| SE | .09 | .06 |
|  | 95% CI | [.21, .55] | [.92, 1.14] |
| *Level 1* |
| T1 Academic Buoyancy |  | .41\*\*\* | .06\*\*\* |
| SE | .01 | .01 |
|  | 95% CI | [.39, .43] | [.05, .07] |
| T1 School Belonging |  | .07\*\*\* | .46\*\*\* |
| SE | .01 | .01 |
|  | 95% CI | [.06, .08] | [.44, .48] |
| *Notes*. SE = standard error; T1 = Time 1; T2 = Time 2; CI = confidence interval; Model fit statistics: χ2 = 52615.142, df = 372, *p* ≤ .001; CFI = .925, RMSEA = .044. Estimates between 0 and .004 are presented as .01; estimates between -.004 and 0 are presented as -.01. \*\*\**p* ≤ .001; \*\* *p* ≤ .01; \**p* ≤ .05  |

**Multi-group SEMs**

Multi-group SEMs (MGSEM) examined potential model invariance across both school and student demographic subgroups. For each demographic variable (listed below), we investigated overall model invariance by comparing changes in model fit indices across two estimated models. The first model was an unconstrained model. The unconstrained model estimated an SEM for each subgroup. For example, the cross-lagged SEM examining teacher relational support and academic buoyancy was estimated for small and large schools. Measurement parameters (e.g., factor loadings) across these models were constrained to be equal. However, all structural paths (covariates predicting substantive variables, auto-regressive paths, and cross-lagged paths) and correlations among covariates and substantive variables were freely estimated across each group. Thus, it was possible for large schools to have a different standardized estimate of the path between teacher relational support and academic buoyancy than small schools. The second model was a constrained model. In addition to holding the measurement parameters constant across subgroups, the constrained model also constrained the structural and correlated paths to be equal across subgroups. Minimal changes in model fit between the unconstrained and constrained models were considered evidence of invariance. Changes in RMSEA ≤ .015 and changes in CFI ≤ .01 were indicative of model invariance (Chen, 2007; Cheung & Rensvold, 2002).

The unconstrained and constrained models were run for each cross-lagged SEM (7 SEMs) across six school demographic variables and four student demographic variables. These demographic groups included:

1. School size: small schools (< 900 student enrolments; 214 schools) vs. large schools (≥ 900 student enrolments; 78 schools).
2. School location: major cities (172 schools) vs. regional and remote schools (110 schools).
3. School gender composition: co-educational (260 schools) vs. single-sex schools (32 schools).
4. School selectiveness: comprehensive (264 schools) vs. partially or fully selective schools (28 schools).
5. School SES: Low SES (82 schools) vs. mid-low SES (103 schools) vs. mid-high SES (58 schools) vs. high SES (49 schools).
6. School average ability: Significantly below average (63 schools) vs. average (181 schools) vs. significantly above average (38 schools).
7. Student gender: male students (35239 students) vs. female students (36392 students).
8. Students’ year in school: Year 7 (18988 students) vs. Year 8 (15891 students) vs. Year 9 (15133 students) vs. Year 10 (11838 students) vs. Year 11 (10011 students)
9. Student’ SES: Low SES (15259 students) vs. mid-low SES (16526 students) vs. mid-high SES (16740 students) vs. high SES (18639 students).
10. Students’ prior achievement: Significantly below average (22337 students) vs. average (23639 students) vs. significantly above average ability (22276 students).

For school demographic variables (variables a – f), MGSEM were examined using the same SEMs analysed in the main analyses. That is, all multilevel models (Models 1 – 4) were examined using multi-level, multi-group analyses and all single-level models were examined using single-level multi-group analyses split across school demographic variables. For student demographic variables (variables g – j), all MGSEM were analysed using single-level models. This is because multigroup analyses require independent groups and it is not possible to create independent groups in a two-level model using a within-level variable.

Results of the MGSEM demonstrated evidence of overall model invariance across the demographic factors, with minimal changes in model fit across the unconstrained and constrained models. In one analysis, there was a change in CFI larger than .01. More precisely, for the school belonging model across school average ability groups, the unconstrained model (RMSEA = .041; CFI = .937) was different than the constrained model (RMSEA = .036, ΔRMSEA = .005; CFI = .948, ΔCFI = .011). However, this marginal difference suggested that the constrained model was a better fit to the data than the unconstrained model, providing further evidence that pooling these schools together was more appropriate than estimating separate models for different school ability levels. A full account of model fit changes is provided in Table S4.

It is noted that these tests investigate overall model invariance, enabling us to determine if the models, overall, are similar or different across demographic groups. There may be smaller structural differences across demographics groups (e.g., a single path that is different for large vs. small schools). These additional analyses may be an important direction for future research.

Table S4

*Changes in Model Fit from Multi-group SEM Analyses*

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Learning Support | Teacher Relational Support | School Belonging | Classroom Management | Perseverance | Perceived Competence | Valuing |
| School Size | RMSEA | .037 | .036+ | .037 | .036+ | .040 | .040+ | .041 | .040+ | .055 | .052+ | .056 | .053+ | .046 | .044+ |
| CFI | .945 | .946 | .942 | .944 | .937 | .937 | .923 | .923 | .927 | .930 | .932 | .935 | .940 | .942 |
| School Location | RMSEA | .041 | .040+ | .040 | .040+ | .404 | .044+ | .047 | .045+ | .057 | .055+ | .058 | .055+ | .048 | .046+ |
| CFI | .938 | .939 | .937 | .937 | .937 | .929 | .910 | .912 | .928 | .928 | .932 | .933 | .938 | .939 |
| School Gender Composition | RMSEA | .034 | .032+ | .033 | .032+ | .036 | .035+ | .037 | .036+ | .056 | .052+ | .056 | .052+ | .047 | .045+ |
| CFI | .953 | .956 | .953 | .954 | .945 | .946 | .936 | .937 | .917 | .922 | .925 | .929 | .934 | .937 |
| School Selectiveness | RMSEA | .037 | .037+ | .038 | .037+ | .040 | .040+ | .042 | .041+ | .056 | .053+ | .057 | .053+ | .047 | .045+ |
| CFI | .947 | .947 | .944 | .945 | .937 | .937 | .925 | .925 | .924 | .926 | .930 | .936 | .937 | .940 |
| School SES | RMSEA | .046 | .045+ | .046 | .044+ | .049 | .048+ | .052 | .050+ | .056 | .053+ | .058 | .053+ | .047 | .045+ |
| CFI | .922 | .923 | .918 | .920 | .915 | .916 | .889 | .891 | .931 | .932 | .934 | .937 | .940 | .942 |
| School Average Ability | RMSEA | .036 | .034+ | .033 | .032+ | .041 | .036+ | .038 | .037+ | .055 | .051+ | .056 | .052+ | .046 | .044+ |
| CFI | .951 | .955 | .954 | .955 | .937 | .948 | .938 | .939 | .929 | .932 | .935 | .938 | .938 | .939 |
| Student Gender | RMSEA | .048 | .047+ | .048 | .047+ | .051 | .051+ | .054 | .053+ | .059 | .058+ | .061 | .058+ | .051 | .050+ |
| CFI | .944 | .943 | .944 | .944 | .941 | .941 | .928 | .927 | .932 | .932 | .937 | .937 | .939 | .939 |
| Student Year in School | RMSEA | .047 | .046+ | .047 | .046+ | .051 | .050+ | .054 | .052+ | .059 | .057+ | .059 | .056+ | .051 | .049+ |
| CFI | .943 | .942 | .941 | .940 | .937 | .936 | .920 | .919 | .930 | .928 | .933 | .932 | .937 | .937 |
| Student SES | RMSEA | .049 | .048+ | .049 | .048+ | .052 | .051+ | .055 | .054+ | .061 | .059+ | .062 | .059+ | .052 | .051+ |
| CFI | .941 | .940 | .939 | .939 | .933 | .932 | .918 | .917 | .924 | .922 | .927 | .926 | .935 | .935 |
| Student Prior Achievement | RMSEA | .049 | .048+ | .049 | .048+ | .053 | .052+ | .056 | .054+ | .061 | .059+ | .063 | .060+ | .052 | .051+ |
| CFI | .941 | .940 | .940 | .939 | .936 | .935 | .921 | .920 | .927 | .926 | .920 | .920 | .937 | .936 |
| *Notes.* CFI = comparative fit index; RMSEA = root mean square error of approximation; SES = socio-economic status.+ΔRMSEA ≤ .015ΔCFI ≤ .01 |