Supplementary Materials for Putting Fractions Together

## Supplement S1. Materials and Procedures for Experiment 1

The following sections present the scripts used during the intervention and the items used for the assessments.

### Intervention Part 1

Throughout, if you are describing and demonstrating something, it should be simultaneous – that is, do each thing at the same time as you describe it, instead of finishing the whole description before starting the demonstration.

#### Introduction

|  |  |
| --- | --- |
| 1 | * “We’re going to play a game to help practice marking fractions on number lines. Here you can see a set of fraction strips which you will use to play the game. Each strip represents a fraction between 1/2 and 1/10. You can click and drag a strip to move it around. Try dragging a 1/2 strip to the middle of the screen.” * **[After this is done]** “Notice how there is still a stack of 1/2 strips left in the original location. You can click on it again to get another 1/2 strip. Try dragging another 1/2 strip to the middle of the screen.” * **[After this is done]** “Great! Press the space bar to continue.” |
| 2 | * “Here are the same fraction strips and a number line from 0 to 1. Notice that the 1/2 strips are half the length from 0 to 1, so two 1/2 strips together equals one. Let me show you.” * **[Click and drag one 1/2 strip so it is just below the number line and its left end is at zero. Click and drag another 1/2 strip so it is just below the line, its left end is flush against the first 1/2 strip, and its right end is at 1.]** * “See? Okay, press the space bar to continue.” |
| 3 | * “Similarly, the 1/3 strips are one third the length from 0 to 1, so three 1/3 strips together equals one. You try dragging down three 1/3 strips just the way I did with the 1/2 strips.” * **[After this is done]** “Great! Press the space bar to continue.” |
| 4 | * “We can use the fraction strips to estimate fractions on a number line. The fraction’s denominator tells you which kind of strip to use, and the numerator tells you how many strips you need. Here, the denominator is 2, so I need 1/2 strips, and the numerator is 1, so I only need one of them. To estimate 1/2, I’ll drag the 1/2 strip to the line with its left end at zero. Then the right end shows the size of 1/2, so I’ll click there. * **[Do it.]** “See? Okay, press the space bar to continue.” |
| 5 | * “Now you try**!” [Participant should drag a 1/3 piece onto the line with its left end at zero, and then click the right end.** * **[After this is done]** “Great! Press the space bar to continue.” |
| 6 | * “Here, the denominator is 4, so I need 1/4 strips, and the numerator is 2, so I need two of them. To estimate 2/4, I’ll drag one 1/4 strip onto the line at zero, and drag the second 1/4 strip right next to the first one. The right end of the last strip shows the size of 2/4, so I’ll click there.” * [Do it.] “See? Okay, press the space bar to continue.” |

|  |  |
| --- | --- |
| 7 | * “Now you try!” **[Participant should drag four 1/6 pieces onto the line, with the leftmost piece having its left end at 0 and each other piece being flush against the previous piece. Then, participant should click on the line at the right end of the last piece.]** * **[After this is done]** “Great! Press the space bar to continue.” |
| 8 | * “Try again with this one!” **[Participant should drag three 1/8 pieces onto the line, with the leftmost piece having its left end at 0 and each other piece being flush against the previous piece. Then, participant should click on the line at the right end of the last piece.]** * **[After this is done]** “Great! Press the space bar to continue.” |
| 9 | * “Now you are going to play a game called Catch the Monster with Fractions. Some monsters have escaped. You need to use your knowledge of fractions to recapture them.” * “On each page, you will see a number line from 0 to 1. A fraction will appear above the line to show where the monster is hiding. Your job is to decide where the fraction belongs on the number line to try and catch the monster. If you click near the monster, you’ll see it caught inside a cage. But if you click too far away from the monster, it will escape!” * “Use the fraction strips to help you, just like we practiced. Ready? Let’s catch some monsters! (Press the space bar to start.)” |

***Feedback: Introduction Part 1***

This feedback should be given ***while*** participant is working, ideally ***before*** they give an answer. If more than one item applies, use the first one that applies. Also, if the participant fails to follow your instructions, then demonstrate by doing what you are telling the participant to do.

|  |  |
| --- | --- |
| Takes wrong type of fraction strip | * “Oops! The denominator is xx, so you need a 1/xxth strip.” |
| Takes wrong number of fraction strips | * “Oops! The numerator is xx, so you need xx strips.” |
| Puts fraction strips in wrong place | * **[First strip]** “This strip’s left end should start at zero.” * **[Other strips]** “This strip’s left end should push right up against the previous strip.” |
| Used fraction strips correctly, but clicked in the wrong location | * **[Only one strip]** “You should click at the strip’s right end.” * **[More than one]** “You should click at the right end of the last strip.” |

#### Practice – Instructions

Read the following when it appears on screen:

You’re doing great! Now we’re going to make things harder. You’ll be able to see the fraction strips, but they won’t move. Try to imagine moving them onto the number line to help mark the right spot.

On the first trial after reading the above, after finishing the trial, read the following:

[ “Great job! You caught the monster!” **OR** “Oops, the monster escaped!” ] “The green bar here shows you the monster’s exact location. The blue bar is your answer. So you can see whether you were close or not.”

“Notice that the correct fraction strips have appeared so you can see why the green mark is the correct location. There are xx strips because the numerator is xx. And they are xxths because the denominator is xx. The correct answer is at the right end of the last strip.”

Read the following when it appears on screen:

You're doing great! The next part of the game is the hardest because you won't have the fraction strips to help you at all. Try to imagine what the fraction strips look like to help mark the right spot.

#### Practice – Feedback

During practice, give feedback as follows. All feedback should be given AFTER participant clicks on the line but BEFORE they press the space bar.

|  |  |
| --- | --- |
| Takes wrong type of fraction strip | * “Oops! The denominator is xx, so you need a 1/xxth strip.” |
| Takes wrong number of strips | * “Oops! The numerator is xx, so you need xx strips.” |
| Puts fraction strips in wrong place | * **[First strip]** “This strip’s left end should start at zero.” * **[Other strips]** “This strip’s left end should push right up against the previous strip.” |
| Did not use fraction strips and missed the monster – **movable fraction strips only** | * “Try using fraction strips on the next one.” |
| Used fraction strips correctly, but clicked in the wrong location and missed the monster – **movable fraction strips only** | * **[Only one strip]** “You should click at the strip’s right end.” * **[More than one]** “You should click at the right end of the last strip.” |
| Missed the monster and did not look at the feedback for at least 1s | * If you miss the monster, be sure to look at where the right answer was. It can help you get closer next time. |
| Catches the monster | * No feedback needed, even if fraction strips were not used. |

### Intervention Part 2 – Fractions-and-Fraction-Sums Condition

#### Introduction

|  |  |
| --- | --- |
| 1 | * “We are going to play the game again. But it will be a little different this time, because we are going to practice **adding** fractions on number lines.” |
| 2 | * “We can use fraction strips to help estimate the sum of two fractions. I start by building up the first fraction with its left end at zero. Then, I build up the second fraction right next to the first fraction. All of those strips together represent the sum.” * “Here, the problem is 3/8 plus 2/9. Watch me do it. To estimate the sum, First, I get three 1/8 strips to represent the first fraction, 3/8. I click at the right end of the strips to show the size of 3/8.” * “Then, I add two 1/9 strips to represent the second fraction, 2/9. I click at the right end of ***these*** strips to show the size of 3/8 ***plus*** 2/9.” * **[Do it.]** “See? Okay, press the space bar to continue.” |
| 3 | * “Now you try!” * **[Participant should drag two 1/9 pieces onto the line, then click at the right end of these. THEN, they should drag a 1/6 piece onto the line, flush against the previous piece, and click on the line again at the right end of the last piece.]** * **[After this is done]** “Great! Press the space bar to continue.” |
| 4 | * “Try again with this one.” * **[Participant should drag three 1/5 pieces onto the line, then click at the right end of these. THEN, they should drag two more 1/6 pieces onto the line, with each piece being flush against the previous piece, and click on the line at the right end of the last piece.]** * **[After this is done]** “Great! Press the space bar to continue.” |
| 5 | * “Now we’ll play Catch the Monster with Fractions again, with addition problems this time. Use the fraction strips to help you, just like we practiced. Ready? Let’s catch some monsters! (Press the space bar to start.)” |

***Feedback: Introduction Part 2***

|  |  |
| --- | --- |
| Takes wrong type of fraction strip for either fraction | * “Oops! The denominator is xx, so you need a 1/xxth strip.” |
| Takes wrong number of fraction strips for either fraction | * “Oops! The numerator of the **[first/second]** fraction is xx, so you need xx strips.” |
| Puts fraction strips in wrong place for either the first fraction or the sum | * **[First strip]** “This strip’s left end should start at zero.” * **[Other strips]** “This strip’s left end should push right up against the previous strip.” |
| Only clicks once on the line | * “Remember to click again to show the size of the sum.” |
| Clicks in wrong location for either the first fraction or the sum | * **[Only one strip]** “You should click at the strip’s right end.” * **[More than one]** “You should click at the right end of the last strip.” |
| Does not click on the location of the first fraction before starting to build the second fraction | * “Oops! Before you start building the second fraction, click on the line to show the size of the first fraction.” |
| Takes strips for only the first fraction and not the second | * “Oops! You still need to build up the second fraction. You need [numerator] [denominator]ths strips.” |
| If participant clicks twice in the same location and then submits the answer, before they continue to the next trial | * “Remember, first click on the line to show how big the first fraction is. Then click again to show how big the sum of both fractions is.” |

#### Practice – Instructions

Read the following when it appears on screen:

You’re doing great! Now we’re going to make things harder. You’ll be able to see the fraction strips, but they won’t move. Try to imagine moving them onto the number line to help mark the right spot.

Immediately after the above, say the following, which does not appear on screen:

Just like before, please click once on the line to show the size of the first fraction, then click again to show the size of the sum, where you think the monster is. The monster will appear after your second click.

On the first trial after reading the above, after finishing the trial, read the following:

[ “Great job! You caught the monster!” **OR** “Oops, the monster escaped!” ] Notice that the dark gray and the first green mark shows you the size of the first fraction. It’s [name the first fraction], so there are [numerator] [denominator]th strips.

The light gray shows you the size of the second fraction. It’s [name the first fraction], so there are [numerator] [denominator]th strips. The two added together shows you the size of the sum. The second green mark shows the location of the sum on the number line.”

Read the following when it appears on screen:

You're doing great! The next part of the game is the hardest because you won't have the fraction strips to help you at all. Try to imagine what the fraction strips look like to help mark the right spot.

#### Practice – Feedback

During practice, give feedback as follows. All feedback should be given AFTER participant clicks on the line but BEFORE they press the space bar.

|  |  |
| --- | --- |
| Takes wrong type of fraction strip for either fraction | * “Oops! The denominator is xx, so you need a 1/xxth strip.” |
| Takes wrong number of fraction strips for either fraction | * “Oops! The numerator of the **[first/second]** fraction is xx, so you need xx strips.” |
| Puts fraction strips in wrong place for either the first fraction or the sum | * **[First strip]** “This strip’s left end should start at zero.” * **[Other strips]** “This strip’s left end should push right up against the previous strip.” |
| Only clicks once on the line | * “Remember to click again to show the size of the sum.” |
| Did not use fraction strips and missed the monster – **movable fraction strips only** | * “Try using fraction strips on the next one.” |
| Used strips for only the first fraction and not the second and missed the monster – **movable fraction strips only** | * “Try using fraction strips for both the first and the second fraction next time.” |
| Used fraction strips correctly for both fractions, but clicked in the wrong location and missed the monster – **movable fraction strips only** | * “You should click at the right end of the last strip.” |
| Missed the monster and did not look at the feedback for at least 1s | * If you miss the monster, be sure to look at where the right answer was. It can help you get closer next time. |
| Catches the monster | * No feedback needed, even if fraction strips were not used. |
| If participant clicks twice in the same location and then submits the answer, before they continue to the next trial | * “Remember, first click on the line to show how big the first fraction is. Then click again to show how big the sum of both fractions is.” |

### Intervention Part 2 – Individual-Fractions Condition

#### Introduction

|  |  |
| --- | --- |
| 1 | * “We are going to play the game again. Just like before, we will practice using fraction strips to estimate fractions on a number line. Remember, the fraction’s denominator tells you which kind of strip to use, and the numerator tells you how many strips you need.” |
| 2 | * “Try using fraction strips to show 1/3 on the number line, and then click at the right end to show the size of 1/3.” * **[Participant should drag a 1/3 piece onto the line with its left end at zero, and then click the right end. After this is done:]** “Great! Press the space bar to continue.” |
| 3 | * “Now try it with 4/6!” * **[Participant should drag four 1/6 pieces onto the line, with the leftmost piece having its left end at 0 and each other piece being flush against the previous piece. Then, participant should click on the line at the right end of the last piece. After this is done:]** “Great! Press the space bar to continue.” |
| 4 | * “Try again with 3/8!” **[Participant should drag three 1/8 pieces onto the line, with the leftmost piece having its left end at 0 and each other piece being flush against the previous piece. Then, participant should click on the line at the right end of the last piece.]** * **[After this is done]** “Great! Press the space bar to continue.” |
| 5 | * “Now we’ll play Catch the Monster with Fractions again. Use the fraction strips to help you, just like we practiced. Ready? Let’s catch some monsters! (Press the space bar to start.)” |

***Feedback: Introduction Part 2***

This feedback should be given ***while*** participant is working, ideally ***before*** they give an answer. If more than one item applies, use the first one that applies. Also, if the participant fails to follow your instructions, then demonstrate by doing what you are telling the participant to do.

|  |  |
| --- | --- |
| Takes wrong type of fraction strip | * “Oops! The denominator is xx, so you need a 1/xxth strip.” |
| Takes wrong number of fraction strips | * “Oops! The numerator is xx, so you need xx strips.” |
| Puts fraction strips in wrong place | * **[First strip]** “This strip’s left end should start at zero.” * **[Other strips]** “This strip’s left end should push right up against the previous strip.” |
| Used fraction strips correctly, but clicked in the wrong location | * **[Only one strip]** “You should click at the strip’s right end.” * **[More than one]** “You should click at the right end of the last strip.” |

#### Practice – Instructions

Read the following when it appears on screen:

You’re doing great! Now we’re going to make things harder. You’ll be able to see the fraction strips, but they won’t move. Try to imagine moving them onto the number line to help mark the right spot.

On the first trial after reading the above, after finishing the trial, read the following:

[ “Great job! You caught the monster!” **OR** “Oops, the monster escaped!” ] “The green bar here shows you the monster’s exact location. The blue bar is your answer. So you can see whether you were close or not.”

“Notice that the correct fraction strips have appeared so you can see why the green mark is the correct location. There are xx strips because the numerator is xx. And they are xxths because the denominator is xx. The correct answer is at the right end of the last strip.”

Read the following when it appears on screen:

You're doing great! The next part of the game is the hardest because you won't have the fraction strips to help you at all. Try to imagine what the fraction strips look like to help mark the right spot.

#### Practice – Feedback

During practice, give feedback as follows. All feedback should be given AFTER participant clicks on the line but BEFORE they press the space bar.

|  |  |
| --- | --- |
| Takes wrong type of fraction strip | * “Oops! The denominator is xx, so you need a 1/xxth strip.” |
| Takes wrong number of strips | * “Oops! The numerator is xx, so you need xx strips.” |
| Puts fraction strips in wrong place | * **[First strip]** “This strip’s left end should start at zero.” * **[Other strips]** “This strip’s left end should push right up against the previous strip.” |
| Did not use fraction strips and missed the monster – **movable fraction strips only** | * “Try using fraction strips on the next one.” |
| Used fraction strips correctly, but clicked in the wrong location and missed the monster – **movable fraction strips only** | * **[Only one strip]** “You should click at the strip’s right end.” * **[More than one]** “You should click at the right end of the last strip.” |
| Missed the monster and did not look at the feedback for at least 1s | * If you miss the monster, be sure to look at where the right answer was. It can help you get closer next time. |
| Catches the monster | * No feedback needed, even if fraction strips were not used. |

### Items Used in Assessment Tasks

**Number Line Estimation with Individual Fractions**

Set A: 7/9, 4/8, 1/7, 3/10, 6/8, 2/3, 2/8, 1/5, 6/7, 2/9, 3/5, 4/10

Set B: 3/6, 1/9, 6/10, 3/4, 2/5, 2/7, 4/5, 2/6, 2/10, 7/8, 1/6, 5/9

**Fraction Magnitude Comparison**

Set A: 5/8, 4/7, 4/8, 5/6, 4/10, 4/5, 2/3, 2/9, 9/10, 5/5, 3/8, 5/9, 1/7, 8/9, 3/7

Set B: 7/8, 1/6, 2/10, 4/9, 3/4, 5/7, 4/8, 2/5, 3/6, 6/7, 6/6, 2/7, 7/9, 3/10, 6/9

**Number Line Estimation with Equal-Denominator Fraction Sums**

Set A: 2/5+2/5, 1/6+1/6, 4/8+3/8, 1/4+1/4, 2/9+2/9, 3/10+1/10, 1/3+1/3, 1/7+4/7

Set B: 1/3+1/3, 3/10+2/10, 3/9+1/9, 1/4+2/4, 1/5+1/5, 3/8+2/8, 1/7+1/7, 2/6+3/6

**Number Line Estimation with Unequal-Denominator Fraction Sums**

Set A: 2/4+1/3, 1/2+2/5, 1/3+1/7, 1/4+2/7, 1/6+1/10, 1/8+4/6, 1/9+1/2, 3/10+1/8, 1/5+1/9

Set B: 1/6+2/3, 3/7+1/10, 1/3+1/2, 2/9+2/10, 1/5+1/4, 5/9+2/5, 1/8+1/6, 1/2+2/7, 1/4+1/8

## Supplement S2. Materials and Procedures for Experiment 2

The following sections present the scripts used during the intervention and the items used for the assessments.

### Intervention – Fraction-Sums Condition

#### Tutorial – Fraction Sums Condition

1. Now we’re going to play a game to help practice estimating where the sum of two fractions goes on a number line. Here you can see a set of fraction strips which you will use to play the game. Each strip represents a fraction between 1/2 and 1/10 **[point to them]**. You can click and drag a strip to move it around. Try dragging a 1/2 strip to the middle of the screen.

**[After this is done]** Notice how there is still a stack of 1/2 strips left in the original location. You can click on it again to get another 1/2 strip. Try dragging another 1/2 strip to the middle of the screen.

1. Here are the same fraction strips and a number line from 0 to 1. Notice that the 1/2 strips are half the length from 0 to 1, so two 1/2 strips together equals one. Try dragging two 1/2 strips down onto the number line. Put them end to end with the first one starting right at 0. You should see that the two 1/2 strips together exactly equal one.
2. Let’s do the same thing with 1/3. The 1/3 strips are one third the length from 0 to 1, so three 1/3 strips together equals one. Try dragging down three 1/3 strips just like you did with the 1/2 strips.
3. We can use the fraction strips to estimate fractions on a number line. The fraction’s denominator tells you which kind of strip to use, and the numerator tells you how many strips you need.

Here, the denominator is 3 **[point at the denominator]**, so you need 1/3 strips, and the numerator is 1 **[point at the numerator]**, so you only need one of them. To estimate 1/3, drag one 1/3 strip down onto the line with its left end at zero **[point at zero on the line]**.

**[After this is done]** Good! Now, the right end shows the size of 1/3 **[point at the right end of the 1/3 strip on the line]**, so click there.

1. Try again with 2/4!

**[Only if they don’t know what to do, give some or all of this more specific prompt:]**

Here, the denominator is 4 **[point at the denominator]**, so you need 1/4 strips, and the numerator is 2 **[point at the numerator]**, so you need two of them. So drag two 1/4 strips down onto the line starting at zero.

**[After this is done]** Good! Now, the right end shows the size of 2/4 **[point at the right end of the last 1/4 strip on the line]**, so click there.

1. Try again with 4/6!

**[Only if they don’t know what to do, give some or all of this more specific prompt:]**

Here, the denominator is 6 **[point at the denominator]**, so you need 1/6 strips, and the numerator is 4 **[point at the numerator]**, so you need four of them. So drag four 1/6 strips down onto the line starting at zero.

**[After this is done]** Good! Now, the right end shows the size of 4/6 **[point at the right end of the last 1/6 strip on the line]**, so click there.

1. We can use fraction strips to help estimate the sum of two fractions. You start by building up the first fraction with its left end at zero. Then, you build up the second fraction right next to the first fraction. All of those strips together represent the sum.

Here, the problem is 1/5+1/8. To estimate the sum, first drag one 1/5 strip down onto the number line starting at zero, and then click at the right end of it to show the size of 1/5.

**[After this is done]** Now, add on one 1/8 strip, and click at the right end of that strip to show the size of 1/5 ***plus*** 1/8.

1. Try again with 3/7+1/2.

**[Only if they don’t know what to do, give some or all of this more specific prompt:]**

First, drag three 1/7 strips down onto the line starting at zero, and click at the right end of the last strip to show the size of 3/7.

**[After this is done:]** Now, add on one 1/2 strip, and click at the right end of that strip to show the size of 3/7 ***plus*** 1/2.

1. Try again with 2/9+3/10.

**[Only if they don’t know what to do, give some or all of this more specific prompt:]**

First, drag two 1/9 strips down onto the line starting at zero, and click at the right end of the last strip to show the size of 2/9.

**[After this is done:]** Now, add on three 1/10 strips, and click at the right end of that strip to show the size of 2/9 ***plus*** 3/10.

1. Now you are going to play a game called Catch the Monster with Fractions. Some monsters have escaped, and you need to use your knowledge of fractions to recapture them. On each page, you will see a number line from 0 to 1. A fraction addition problem above the line will show where the monster is hiding.

Your job is to click where the sum of the two fractions belongs on the number line to try and catch the monster. If you click near the monster, you’ll see it caught inside a cage. But if you click too far away from the monster, it will escape! Use the fraction strips to help you, just like we practiced. Ready to catch some monsters?

#### Feedback During Tutorial

This feedback should be given ***while*** participant is working, ***before*** they give an answer. If more than one item applies, use the first one that applies. Also, if the participant fails to follow your instructions, then demonstrate by doing what you are telling the participant to do.

Number 4, marked in bold, is the most common issue, so pay special attention to it.

|  |  |
| --- | --- |
| 1. Takes wrong type of fraction strip | * “Oops! The denominator is xx, so you need a 1/xxth strip.” |
| 1. Takes wrong number of fraction strips | * “Oops! The numerator is xx, so you need xx strips.” |
| 1. Puts fraction strips in wrong place | * **[1st strip]** “This strip’s left end should start at 0.” * **[Others]** “This strip’s left end should push right up against the previous strip.” |
| 1. **[Addition problems]** Did not click on the location of the 1st fraction before starting to build the 2nd fraction | * “Oops! Before you start doing the second fraction, click on the line to show the size of the first fraction.” |
| 1. Used fraction strips correctly, but clicked in the wrong location | * **[Only one strip]** “Click at the strip’s right end.” * **[More]** “Click at the right end of the last strip.” |

#### Practice – Fraction Sums Condition

Read instructions whenever they appear on screen.

#### Feedback During Practice

During practice, give feedback as follows. All feedback should be given AFTER participant clicks on the line but BEFORE they press the space bar. If more than one item applies, use the first one that applies.

Number 4, marked in bold, is the most common issue, so pay special attention to it.

|  |  |
| --- | --- |
| 1. Takes wrong type of fraction strip | * “Oops! The denominator is xx, so you need a 1/xxth strip.” |
| 1. Takes wrong number of fraction strips | * “Oops! The numerator is xx, so you need xx strips.” |
| 1. Puts fraction strips in wrong place | * **[1st strip]** “This strip’s left end should start at 0.” * **[Others]** “This strip’s left end should push right up against the previous strip.” |
| 1. Did not click once for the 1st fraction before starting to build and/or clicking in the location of the 2nd fraction | * “Oops! Next time, before you start doing the second fraction, click on the line to show the size of the first fraction.” |

The following feedback should only be given if the participant missed the monster. Numbers 5-6 only apply when the fraction strips are movable.

|  |  |
| --- | --- |
| 1. Did not use fraction strips | * “Try using fraction strips on the next one.” |
| 1. Used strips for only the first fraction and not the second | * “Try using fraction strips for both the first and the second fraction next time.” |
| 1. Did not look at the feedback for at least 1s | * “If you miss the monster, be sure to look at where the right answer was. It can help you get closer next time.” |

### Intervention – Whole-Number-Sums Condition

#### Tutorial – Whole Number Sums Condition

1. Now we’re going to play a game to help practice estimating where the sum of two numbers goes on a number line. Here you can see a set of buttons which you will use to play the game. Each button is labeled with a number between 1 and 500 **[point to them]**. Clicking a button will make a number strip appear below. The bar to the right of each button shows you the size of the strip that will appear when you click it. Try clicking one of the buttons to see what happens.

**[After this is done]** You can use the buttons to create as many strips as you want. Click on one of the buttons again and see what happens.

1. Here are the same buttons and a number line from 0 to 1000. So, ten of the 100 strips should exactly cover the whole line. To see that this is really true, click the “100” button ten times.
2. Similarly, ten of the 10 strips should be the same length as one of the 100 strips. To see that this is really true, click the “10” button ten times.
3. We can use the buttons and the strips to estimate where a number should go on the number line. Just click the “100” button as many times as the “hundreds” digit, click the “10” button as many times as the “tens” digit, and click the “1” button as many times as the “ones” digit.

Here, the number is 341 **[point at the number]**, so you should click the 100 button 3 times, then click the 10 button four times, and finally click the 1 button once.

**[After this is done.]** Good! Now, the right end of the last strip shows the size of 341 on the number line **[point at the right end of the last strip on the line]**, so click there.

1. Try again with 512!

**[Only if they don’t know what to do, give some or all of this more specific prompt:]**

You should click the 100 button 5 times, then click the 10 button once, and finally click the 1 button twice.

**[After this is done]** Good! Now, the right end shows the size of 512 **[point at the right end of the last number strip on the line]**, so click there.

1. Try again with 720!

**[Only if they don’t know what to do, give some or all of this more specific prompt:]**

You should click the 100 button seven times, then click the 10 button twice. The ones digit is 0, so you don’t need to click the 1 button.

**[After this is done]** Good! Now, the right end shows the size of 720 **[point at the right end of the last number strip on the line]**, so click there.

1. We can use number strips to help estimate the sum of two numbers. You start by building up the first number with its left end at zero. Then, you build up the second number right next to the first number. All of those strips together represent the sum.

Here, the problem is 211+125. To estimate the sum, first, click the ‘100’ button twice, then click the ‘10’ button once, then click then ‘1’ button once, and finally click at the right end of the last strip to show the size of 211.

**[After this is done.**] Now, click the ‘100’ button once, then click the ‘10’ button twice, then click the ‘1’ button five times, and finally click at the right end of the last strip to show the size of 211 ***plus*** 125.

1. Try again with 422+503.

**[Only if they don’t know what to do, give some or all of this more specific prompt:]**

First, click the ‘100’ button four times, then click the ‘10’ button twice, then click the ‘1’ button twice, and finally click at the right end of the last strip to show the size of 422.

**[After this is done:]** Now, click the ‘100’ button five times and the ‘1’ button 3 times, and click at the right end of the last strip to show the size of 422 ***plus*** 503.

1. Try again with 230+305.

**[Only if they don’t know what to do, give some or all of this more specific prompt:]**

First, click the ‘100’ button twice and the ‘10’ button three times, then click at the right end of the last strip to show the size of 230.

**[After this is done:]** Now, click the ‘100’ button three times and the ‘1’ button five times, then click at the right end of the last strip to show the size of 230 ***plus*** 305.

1. Now you are going to play a game called Catch the Monster. Some monsters have escaped, and you need to use your knowledge of numbers to recapture them. On each page, you will see a number line from 0 to 1000. An addition problem above the line will show where the monster is hiding.

Your job is to click where the sum of the two numbers belongs on the number line to try and catch the monster. If you click near the monster, you’ll see it caught inside a cage. But if you click too far away from the monster, it will escape! Use the buttons and number strips to help you, just like we practiced. Ready to catch some monsters?

#### Feedback During Tutorial

This feedback should be given ***while*** participant is working, ***before*** they give an answer. If more than one item applies, use the first one that applies. Also, if the participant fails to follow your instructions, then demonstrate by doing what you are telling the participant to do.

Number 4, marked in bold, is the most common issue, so pay special attention to it.

|  |  |
| --- | --- |
| 1. Clicks the ‘100’ button the wrong number of times | * “Oops! The ‘100s’ digit is xx **[point to it]**, so click the ‘100’ button xx times.” |
| 1. Clicks the ‘10’ button the wrong number of times | * “Oops! The ‘10s’ digit is xx **[point to it]**, so click the ‘10’ button xx times.” |
| 1. Clicks the ‘1’ button the wrong number of times | * “Oops! The ‘1s’ digit is xx **[point to it]**, so click the ‘1’ button xx times.” |
| 1. **[Addition problems]** Did not click on the location of the 1st number before starting to build the 2nd number | * “Oops! Before you start doing the second number, click on the line to show the size of the first number.” |
| 1. Used number strips correctly, but clicked in the wrong location | * **[Only one strip]** “Click at the strip’s right end.” * **[More]** “Click at the right end of the last strip.” |

***Note:*** Do not correct the participant if they use a shortcut that is correct, such as clicking the ‘500’ button once instead of clicking the ‘100’ button five times.

#### Practice – Whole Number Sums Condition

Read instructions whenever they appear on screen.

#### Feedback During Practice

During practice, give feedback as follows. All feedback should be given AFTER participant clicks on the line but BEFORE they press the space bar. If more than one item applies, use the first one that applies.

Number 4, marked in bold, is the most common issue, so pay special attention to it.

|  |  |
| --- | --- |
| 1. Clicks the ‘100’ button the wrong number of times | * “Oops! The ‘100s’ digit is xx **[point to it]**, so click the ‘100’ button xx times.” |
| 1. Clicks the ‘10’ button the wrong number of times | * “Oops! The ‘10s’ digit is xx **[point to it]**, so click the ‘10’ button xx times.” |
| 1. Clicks the ‘1’ button the wrong number of times | * “Oops! The ‘1s’ digit is xx **[point to it]**, so click the ‘1’ button xx times.” |
| 1. Did not click once for the 1st number before starting to build and/or clicking in the location of the 2nd number | * “Oops! Next time, before you start doing the second number, click on the line to show the size of the first number.” |

The following feedback should only be given if the participant missed the monster. Numbers 5-6 only apply when the number strips are movable.

|  |  |
| --- | --- |
| 1. Did not use number strips | * “Try using the buttons to help you on the next one.” |
| 1. Used strips for only the first number and not the second | * “Try using the buttons to build both the first and the second numbers next time.” |
| 1. Did not look at the feedback for at least 1s | * “If you miss the monster, be sure to look at where the right answer was. It can help you get closer next time.” |

### Items Used in Assessment Tasks

**Fraction Sum Comparison-to-One**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Design Cell | | | |
| Set | Less than 0.5 | Less than 0.8 but greater than 0.5 | Greater than 1.2 but less than 1.5 | Greater than 1.5 |
| Set A | 2/10+1/8  1/4+1/10  3/10+2/9 | 3/8+1/3  6/10+1/6  2/3+1/8 | 5/6+2/5  8/9+3/8  4/5+1/2 | 3/4+5/9  6/7+3/4  8/9+5/6 |
| Set B | 2/8+1/9  1/5+1/6  3/10+1/7 | 1/2+1/5  4/7+1/6  4/6+1/9 | 7/8+2/5  8/9+3/7  3/4+2/3 | 5/6+3/4  7/8+4/5  9/10+6/7 |

**Fraction Sum Comparison-to-One with Think-Aloud**

Set A: 2/10+1/8 and 8/9+5/6

Set B: 2/8+1/9 and 9/10+6/7

**Number Line Estimation with Unequal-Denominator Fraction Sums**

Set A: 2/4+1/3, 1/2+2/5, 1/3+1/7, 1/4+2/7, 1/6+1/10, 1/8+4/6, 1/9+1/2, 3/10+1/8, 1/5+1/9

Set B: 1/6+2/3, 3/7+1/10, 1/3+1/2, 2/9+2/10, 1/5+1/4, 5/9+2/5, 1/8+1/6, 1/2+2/7, 1/4+1/8

## Supplement S3. Descriptive Statistics Broken Down by Grade for Experiment 1

Table S3.1. Mean (standard deviation) pretest and midtest or posttest performance and change in performance by condition for fourth graders in Experiment 1. EMM denotes Estimated Marginal Mean; EMMs were were derived from the ANCOVAs described in the main text and are adjusted for the covariate (pretest). PAE denotes percent absolute error.

|  |  |  |
| --- | --- | --- |
|  | Individual-Fractions-and-Sums Condition | Individual-Fractions Condition |
|  | Number Line Estimation of Individual Fractions – PAE | |
| Pretest | 10.5 *(8.2)* | 10.3 *(7.6)* |
| Midtest | 7.4 *(5.0)* | *7.0 (4.7)* |
| Change | -3.1 *(5.4)* | -3.3 *(4.9)* |
| EMM of Change | -4.0 | -4.3 |
|  | Fraction Magnitude Comparison – % Correct | |
| Pretest | 71.1 *(18.8)* | 73.8 *(17.0)* |
| Midtest | 75.6 *(18.5)* | 71.6 *(27.5)* |
| Change | 4.4 *(13.3)* | -2.2 *(25.9)* |
| EMM of Change | 2.8 | -2.7 |
|  | Number Line Estimation of Equal-Denominator Sums – PAE | |
| Pretest | 20.9 *(8.6)* | 16.5 *(13.9)* |
| Posttest | 11.6 *(8.1)* | 11.7 *(11.6)* |
| Change | -9.3 *(12.0)* | -4.8 *(7.6)* |
| EMM of Change | -6.8 | -4.8 |
|  | Number Line Estimation of Unequal-Denominator Sums – PAE | |
| Pretest | 26.3 *(8.5)* | 23.9 *(11.4)* |
| Posttest | 9.7 *(6.5)* | 14.9 *(10.0)* |
| Change | -16.6 *(8.8)* | -8.9 *(7.2)* |
| EMM of Change | -14.0 | -8.0 |

Table S3.2. Mean (standard deviation) pretest and midtest or posttest performance and change in performance by condition for fifth graders in Experiment 1. EMM denotes Estimated Marginal Mean; EMMs were were derived from the ANCOVAs described in the main text and are adjusted for the covariate (pretest). PAE denotes percent absolute error.

|  |  |  |
| --- | --- | --- |
|  | Individual-Fractions-and-Sums Condition | Individual-Fractions Condition |
|  | Number Line Estimation of Individual Fractions – PAE | |
| Pretest | 11.1 *(9.4)* | 14.7 *(11.3)* |
| Midtest | 5.4 *(3.1)* | *6.8 (5.4)* |
| Change | -5.7 *(8.2)* | -7.9 *(10.2)* |
| EMM of Change | -6.1 | -5.7 |
|  | Fraction Magnitude Comparison – % Correct | |
| Pretest | 82.7 *(19.7)* | 71.7 *(20.3)* |
| Midtest | 89.8 *(13.6)* | 77.5 *(16.8)* |
| Change | 7.1 *(14.0)* | 5.8 *(19.5)* |
| EMM of Change | 10.2 | 4.5 |
|  | Number Line Estimation of Equal-Denominator Sums – PAE | |
| Pretest | 12.7 *(8.6)* | 16.5 *(9.5)* |
| Posttest | 6.4 *(3.1)* | 8.1 *(6.8)* |
| Change | -6.4 *(6.7)* | -8.4 *(8.5)* |
| EMM of Change | -8.5 | -8.4 |
|  | Number Line Estimation of Unequal-Denominator Sums – PAE | |
| Pretest | 18.9 *(12.5)* | 21.0 *(10.2)* |
| Posttest | 6.9 *(3.8)* | 12.8 *(7.9)* |
| Change | -12.0 *(12.1)* | -8.2 *(8.8)* |
| EMM of Change | -14.3 | -9.1 |

## Supplement S4. Descriptive Statistics Broken Down by Grade for Experiment 2

Table S4.1. Mean (standard deviation) pretest and posttest performance and change in performance by condition fifth graders in Experiment 2. EMM denotes Estimated Marginal Mean; EMMs were were derived from the ANCOVAs described in the main text and are adjusted for the covariate (pretest). PAE denotes percent absolute error.

|  |  |  |
| --- | --- | --- |
|  | Fraction-Sums Condition | Whole-Number-Sums Condition |
|  | Fraction Sum Magnitude Comparison – % Correct | |
| Pretest | 65.9 *(24.7)* | 77.7 *(18.1)* |
| Posttest | 83.3 *(23.1)* | 79.5 *(17.2)* |
| Change | 17.4 *(22.8)* | 1.9 *(15.0)* |
| EMM of Change | 15.0 | 4.6 |
|  | Fraction Sum Magnitude Comparison with Think-Aloud – % Correct | |
| Pretest | 79.5 *(33.3)* | 81.8 *(24.6)* |
| Posttest | 88.6 *(21.4)* | 81.8 *(24.6)* |
| Change | 9.1 *(42.6)* | 0.0 *(21.8)* |
| EMM of Change | 11.7 | 4.3 |
|  | Number Line Estimation of Unequal-Denominator Sums – PAE | |
| Pretest | 26.9 *(11.7)* | 25.7 *(9.8)* |
| Posttest | 11.7 *(5.8)* | 21.8 *(9.7)* |
| Change | -15.2 *(12.4)* | -3.9 *(7.5)* |
| EMM of Change | -13.1 | -2.6 |

Table S4.2. Mean (standard deviation) pretest and posttest performance and change in performance by condition sixth graders in Experiment 2. EMM denotes Estimated Marginal Mean; EMMs were were derived from the ANCOVAs described in the main text and are adjusted for the covariate (pretest). PAE denotes percent absolute error.

|  |  |  |
| --- | --- | --- |
|  | Fraction-Sums Condition | Whole-Number-Sums Condition |
|  | Fraction Sum Magnitude Comparison – % Correct | |
| Pretest | 65.6 *(21.4)* | 77.0 *(23.2)* |
| Posttest | 78.9 *(19.5)* | 76.4 *(26.4)* |
| Change | 13.3 *(19.9)* | -0.6 *(17.7)* |
| EMM of Change | 10.8 | 1.8 |
|  | Fraction Sum Magnitude Comparison with Think-Aloud – % Correct | |
| Pretest | 69.0 *(28.1)* | 75.9 *(32.1)* |
| Posttest | 82.8 *(24.2)* | 77.8 *(25.3)* |
| Change | 13.8 *(32.4)* | 1.9 *(25.9)* |
| EMM of Change | 8.6 | 1.8 |
|  | Number Line Estimation of Unequal-Denominator Sums – PAE | |
| Pretest | 25.3 *(10.4)* | 18.9 *(9.3)* |
| Posttest | 9.9 *(3.9)* | 16.8 *(8.2)* |
| Change | -15.4 *(9.1)* | -2.0 *(6.6)* |
| EMM of Change | -14.5 | -5.6 |

## Supplement S5. Details on Excluded Participants in Experiment 2

One child (a fifth grade participant in the fraction-sums condition) was excluded from all analyses because he requested to stop participating in the experiment during the intervention. Two children were excluded from analyses of the fraction sum comparison-to-one with think-aloud and number line estimation with unequal-denominator fraction sums tasks, one (a sixth grader in the whole-number-sums condition) because he ran out of time on the posttest and the other (a sixth grader in the fraction-sums condition) because she requested to stop participating in the experiment during the fraction sum comparison-to-one with think-aloud task on the posttest. Finally, one child (a sixth grader in the whole-number-sums condition) was excluded from analyses of the fraction sum comparison-to-one with think-aloud task because the experimenter did not read the instructions for this task correctly at pretest.