Supplemental Materials

When Does Power Disparity Help or Hurt Group Performance?

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Online Supplement for Study 1

Modeling power and power disparity

As early as March (1966) and French (1956), researchers have used mechanical analogies of power. This modeling choice agrees with the theoretical conceptualizations of early scholars in power research, such as Hobbes, Locke, and Dahl, who make an analogy between power and mechanical forces (Clegg, 1989, p. 41). For example, according to Hobbes, power is an "extension and elaboration of metaphors drawn from Galilean mechanics." Similarly, Locke illustrates power in terms of mechanics, as exemplified by the motion of billiard balls (Clegg, 1989, p. 41). Dahl (1963, p. 7) argues that power is "very similar to those on which the idea of force rests in mechanics." Following this tradition, we model an individual's power with her/his mass.

Let an individual j be located at a point with its coordinates given by the $D \times 1$ vector $\mathbf{x}_j$, where D is the dimensionality of the cognitive task space that defines the domain of all solutions in which individuals search for the best solution. S/he has a mass m_j that can represent power as a capacity stemming from her/his hierarchical position. Individual i with mass m_i exerts a force on another individual in relation to her/his power. Because the power of individual j can only be interpreted relative to the power of another individual, considering m_j as individual *j*'s share of the overall power in the group suffices, such that $\sum_{i=1}^{N} m_i = 1$.

This force acts along $\mathbf{x}_i$ and $\mathbf{x}_j$ and causes a change in the position of *i*, such that the individual at $\boldsymbol{x}_i$ is accelerated toward the individual at $\mathbf{x}_j$. That is, the power embedded in the size of the mass is exercised through a force that affects the desired change in the other individual's position. A similar force acts on individual *j* in the opposite direction. Consequently, both individuals move towards $m_i\, x_i + m_j\, x_j$. This point is known as the center of gravity. For a group of *N* individuals, the center of gravity at time *t* is defined as follows:

$$\mathbf{s}^t = \sum_{i=1}^{n} m_i x_i^t \,, \tag{1}$$

Equation (1) implies that the center of the mass will be closer to the individuals with high power and further away from the individuals with low power.

Modeling the problem space

To compare the performances of groups with varying power disparity modes, we need to "produce an arbitrarily large number of statistically identical [tasks] for the simulated agents to solve" (Lazer & Friedman, 2007, p. 673). We utilized the Gaussian landscape generator (Gallagher & Yuan, 2006) to generate task landscapes. The landscape generator consists of a preselected number of multivariate normal distributions (i.e., Gaussian functions) with uniformly distributed means over a fixed *D*-dimensional space and varying covariance matrices. The height of each Gaussian is also random, except for the best one, the value of which is set at ψ^*, and the ratio, *r*, of the best to the second best is $r\psi^*$. Then, this landscape generator is simply defined as the maximum value over all Gaussians. Each position on the landscape has a performance value, $\mathrm{f}(\mathbf{x}_i)$, represented as the height of individual *i*'s position.

Task complexity has been highlighted as an important contingency in power research (Sturm & Antonakis, 2015). Therefore, we compare the groups under varying degrees of complexity. Various meanings have been attributed to complexity (Siggelkow & Rivkin, 2005). We support the view that a complex task is one that "has many plausible solutions although it is difficult at the outset to judge which approach will yield good results" (Lazer & Friedman, 2007: 673). For example, complex tasks with many plausible solutions include the strategic management of a group or an organization, new drug or software development, new video game design, and fraud investigations, as in Study 3. By contrast, a simple task has a single solution that is easy to find. However, for complex tasks, finding good solutions becomes more difficult because of multiple sub-optimal solutions. In our model, task complexity is represented by the landscape's ruggedness.

The Gaussian landscape generator allowed us to create landscapes with varying complexities with a single parameter, defined as the number of Gaussians. The complexity level of each landscape, γ, is defined as the number of Gaussians. Note that when $\gamma = 1$, a rather simple unimodal landscape (only one global optimum with no local optima) is created. The number of peaks, that is, the number of local optima, increases with γ, such that the actual number of local optima will be less than or equal to γ due to the possibly overlapping Gaussian components. Figure S1 illustrates two landscapes with varying complexities. Clearly, finding the optimum in the left panel is much easier than doing so in a complex task, such as that depicted in the right panel of Figure S1, where there are many local optima and irregular ridges.

Modeling group search

The objective of the particle swarm optimization (PSO) is to search for the optimum of a fitness function over a *D*-dimensional search space through a group of several individuals. Note that our aim is not to develop a superior optimization algorithm but to simulate group interaction and to compare the performance of different power disparity models from a search perspective. In the PSO, each individual moves at a velocity that is updated in each period and remembers the best position that s/he has ever visited. Individuals in the search space are attracted to the best location found individually and the best location found by any of the group members. The success of the PSO depends on the number of individuals participating in the search, the landscape's complexity, and a few tuning parameters (Poli, Kennedy, & Blackwell, 2007).

Let us briefly formalize the PSO. We refer the reader to Clerc and Kennedy (2002) for further details. At time period (iteration) $t \in \{1, \ldots, T\}$, the best performance reached thus far by individual *i* due to her/his local search is defined as $pbest_i^t$, with the solution located at $\mathbf{p}_i^t$. The value of the best performance found thus far by the group is denoted $gbest_i^t$ at position $\mathbf{g}^t$. For the maximization of $\mathrm{f}(\mathbf{x}_i)$ over $\mathbf{x}$, $pbest_i^t$ and $gbest_i^t$ are non-decreasing because they are updated only if a better solution is found. Individual *i* changes her/his position according to the velocity vector $\mathbf{v}_i^t = (v_{i1}^t, v_{i2}^t, \ldots, v_{iD}^t)$. In the PSO, each individual's velocity and location are updated at the time period $t + 1$ as follows:

$$\mathbf{v}_i^{t+1} = \phi \mathbf{v}_i^t + c_1 \varphi_1 (\mathbf{p}_i^t - \mathbf{x}_i^t) + c_2 \varphi_2 (\mathbf{g}^t - \mathbf{x}_i^t) \quad (2)$$

$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mathbf{v}_i^{t+1} \quad (3)$$

where ϕ is the inertia weight set to some predefined value. The second term on the right-hand side of (2) refers to the acceleration due to individual *i*'s local search, and the third term is the acceleration due to the group search. Parameters c_1 and c_2 are the acceleration constants for local and group searches, respectively. To add randomness to the group search, φ_1 and φ_2 are independent and uniformly distributed between 0 and 1. Together with c_1 and c_2, they govern the strength by which an individual is attracted to his/her best location, $\mathbf{p}_i^t$, and to the overall best location, $\mathbf{g}^t$, found thus far by the group. At iteration *t*, the velocity is updated according to its current velocity affected by inertia and to the best previously found positions by the individual and the group, which is multiplied by the acceleration constants and random terms. The individual's position is then updated using her/his current position and newly updated velocity. The selection of parameters is discussed in a later section.

Equation (3) shows that an individual movement's is determined by his/her current velocity and position, the best solution found from his/her local search, and the best solution found by the group. To include power and power differences in the search, we extend the standard PSO to include a third element, which is movement due to power differences, as formulated in (1). Hence, when (1) is inserted into (2), the velocity update in the PSO algorithm becomes

$$\mathbf{v}_i^{t+1} = \underbrace{\phi\mathbf{v}_i^t}_{\text{Inertia}} + \underbrace{c_1\varphi_1(\mathbf{p}_i^t - \mathbf{x}_i^t)}_{\substack{\text{Attraction towards}\\ \text{the personal best}}} + \underbrace{c_2\varphi_2(\mathbf{g}^t - \mathbf{x}_i^t)}_{\substack{\text{Attraction towards}\\ \text{the best found by}\\ \text{the group}}} + \underbrace{c_3\varphi_3(\mathbf{s}^t - \mathbf{x}_i^t)}_{\substack{\text{Attraction to the}\\ \text{center of gravity}\\ \text{of the entire group}}}, \tag{4}$$

where $\mathbf{s}^t$ is center of gravity; c_3 is the acceleration constant of the acceleration due to the total gravitational force exerted on an individual; and φ_3 is uniformly distributed between 0 and 1. Parameters c_1, c_2 and c_3 have real behavioral interpretations, such that they serve as weights for personal, social, and power cues, respectively. They affect the extent to which each cue affects individual searches.

Note that (4) is in line with the decision-making paradigms proposed by Eisenhardt and Zbaracki (1992): An individual's movement is influenced by (a) the best solution that s/he has found thus far as a result of his/her personal search and the best solution found by any group member (i.e., the bounded rationality paradigm); (b) the social influence due to power differences, which is proportional to power (i.e., the politics and power paradigm); and (c) there is still room for pseudo-randomness in the search (i.e., the garbage can paradigm).

Consequently, the PSO algorithm in the simulation model is constructed as follows:

- Until the final period T is reached, repeat.
- For each individual $i = 1,\ldots, N$, do the following:
    - Pick random numbers, $\varphi_1, \varphi_2, \varphi_3 \sim U(0,1)$;
    - Update the individual's velocity according to (4); and
    - Update the individuals' position according to (3).
    - If $\mathrm{f}(\mathbf{x}_i^t) > \mathrm{f}(\mathbf{p}_i^t)$, do the following:
        - Update the individual's best position $\mathbf{p}_i^t \leftarrow \mathbf{x}_i^t$.
    - If $\mathrm{f}(\mathbf{x}_i^t) > \mathrm{f}(\mathbf{g}^t)$, do the following:
        - Update the group's best position $\mathbf{g}^t \leftarrow \mathbf{x}_i^t$.

Parameters used in the simulation experiments

Online Supplement for Study 3

Complete set of survey items

***Group performance*.** Survey question**:** **"**Please compare the performance of your team [X] with the performance of teams that performed similar tasks." Scale: 1 = much worse; 3 = average; 5 = much better; Cronbach's alpha = 0.85; Rwg_j = 0.95; ICC1 = 0.23; ICC2 = 0.39.

1. Efficiency
2. Quality
3. Cooperation
4. Speed
5. Overall performance

***Team leader's competence*.** Survey question**:** "Please indicate to what extent you agree with the following statements about your team leader [X]." Scale: 1 = strongly disagree; 3 = neutral; 5 = strongly agree; Cronbach's alpha = 0.96; Rwg_j = 0.92; ICC1 = 0.23; ICC2 = 0.82.

My team leader…

1. … is a good role model for creative thinking.
2. … consistently seeks new ideas and ways to develop new solutions.
3. … generates groundbreaking ideas.
4. … suggests new ways to achieve goals or objectives.
5. … is a good source of creative ideas.
6. … often has new and innovative ideas.
7. … suggests new ways of performing work tasks.
8. … develops adequate plans and schedules for the implementation of new ideas.

***Job complexity*.** Survey question**:** "Please indicate to what extent you agree with the following statements about your work." Scale: 1 = strongly disagree; 3 = neutral; 5 = strongly agree; Cronbach's alpha = 0.88; Rwg_j = 0.88; ICC1 = 0.05; ICC2 = 0.42.

1. The job requires that I only do one task or activity at a time (reverse scored).
2. The tasks on the job are simple and uncomplicated (reverse scored).
3. The job comprises relatively uncomplicated tasks (reverse scored).
4. The job involves performing relatively simple tasks (reverse scored).

***Tenure*.** Survey question**:** "How long have you worked at [X]?

Education level. Survey question: "What is your highest level of education?" List of options: 1 = primary school; 2 = high school; 3 = vocational school; 4 = university education; 5 = graduate school or higher.

Goal-focused leadership. Survey question**:** "Please indicate to what extent you agree with the following statements about your team leader [X]." Scale: 1 = strongly disagree; 3 = neutral; 5 = strongly agree; Cronbach's alpha = 0.89; Rwg_j = 0.88; ICC1 = 0.19; ICC2 = 0.77.

My team leader…

1. ... provides direction and defines priorities.
2. ... clarifies specific roles and responsibilities.
3. ... translates strategies into understandable objectives and plans.
4. ... links the team's mission to the mission of the company overall.
5. ... follows up to make sure the job gets done.

The results of the supplementary analyses

We tested two additional extensions of our model: a three-way interaction among power disparity, competence and job complexity, and the curvilinear effect of power. First, job complexity may manifest itself in competing solutions, which can increase the need for a leader with high competence (cf. Aime, Humphrey, DeRue, & Paul, 2014). Second, our literature review outlined the conflicting propositions of functionalist and conflict theories of power. Although we proposed leader competence to integrate these views, an alternative explanation could be that an optimum level of power disparity exists, and this disparity produces coordination benefits up to a certain threshold. However, after that threshold, conflict may crowd out the coordination benefits. The results in Table S2 exclude both the three-way interaction and the curvilinear effect of power disparity as alternative explanations.

References

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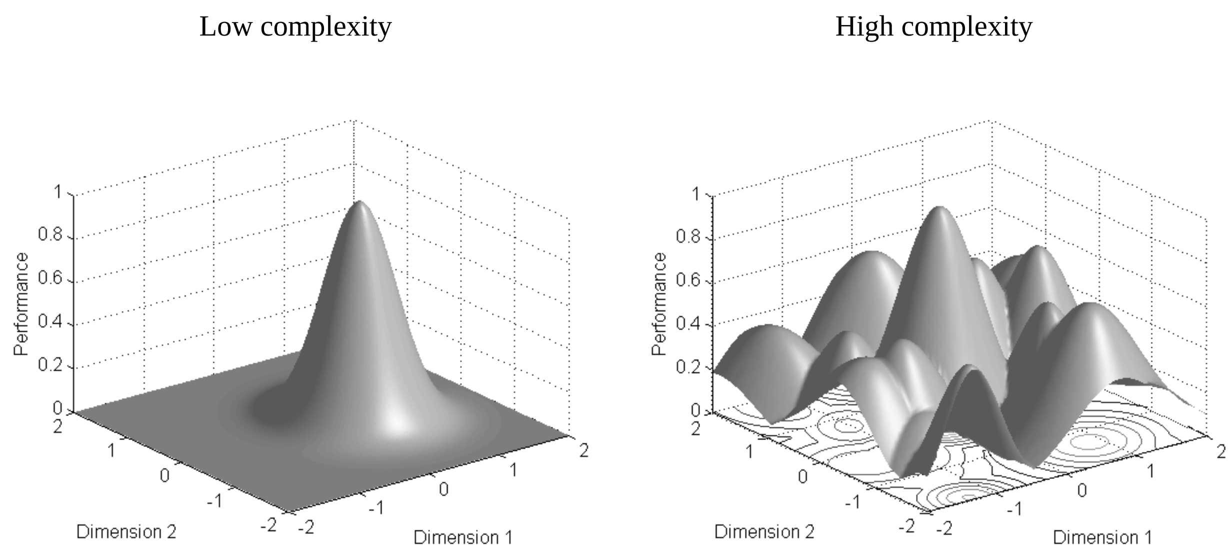

Figure S1. Example task landscapes created using the Gaussian landscape generator[a]

[a] The global optimum is assigned a value of 1, and the second highest optimum is assigned a value of 0.75. A low complexity landscape (left) is created using $\gamma = 1$ components, and $\gamma = 20$ components are used for the high complexity landscape (right).

Table S1

Parameters Used in the Simulation Experiments

Parameters	Main Experiment	Sensitivity Analyses		
		Complexity	Group size	c_3
Varying factors				
Complexity level (γ)	10	1,…,21	10	10
No. of individuals (N)	10	10	5,…,25	10
High power (m^{high})	0.1,…,0.9	1	1	1
Acceleration coefficient (c_3)	0.6	0.6	0.6	0,…,0.9
Fixed factors				
Dimensionality (D)	5	5	5	5
Range of search space	[-2,2]	[-2,2]	[-2,2]	[-2,2]
Value of global optimum (ψ^*)	1	1	1	1
Value of highest local optimum (r)	0.75	0.75	0.75	0.75
No. of landscapes per complexity level	1000	1000	1000	1000
No. of iterations (T)	100	100	100	100
Maximum velocity (v^{max})	2	2	2	2
Acceleration coefficients (c_1, c_2)	0.6	0.6	0.6	0.6
Inertia weight (ϕ)	0.5	0.5	0.5	0.5
Bounce method[a]	Bounce-back	Bounce-back	Bounce-back	Bounce-back

[a] When the individual reaches the limits of the search space, s/he bounces back and his/her velocity dampens.

Table S2

Additional Analyses of the Three-way Interaction Effect and Curvilinear Relationships in Study 3

	Model 4 (IV)		Model 5 (IV)		Model 6 (OLS)	
	b	SE	b	SE	b	SE
Control variables						
Constant	3.39	.07***	3.40	.07***	3.44	.08***
Job complexity	.11	.07	.10	.07	.11	.08
Education level diversity	.07	.07	.09	.07	.08	.08
Tenure diversity	.03	.06	.04	.06	.04	.07
Main effects						
Power disparity	.09	.08	.11	.08	.13	.09
Competence	-.05	.09	-.02	.09		
Interaction curvilinear effects						
Power disparity × Power holder's task competence	.10	.14				
Power disparity × Power holder's task competence × Job complexity	.03	.11	.12	.08		
Power disparity^2					-.07	.06
Weak instruments test	21.76***		12.87***		—	
Wu-Hausman test	.20		.12		—	

Notes: *N* = 46 groups. **p* < 0.10; ***p* < 0.05; ****p* < .01. *b* = standardized regression coefficient; SE = robust standard errors; IV = instrumental variable regression; OLS = ordinary least squares regression.