

Supplementary Online Materials for
**Cultural Drift, Indirect Minority Influence, Network Structure and Their
Impacts on Cultural Change and Diversity**

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Supplementary Online Material 1

Table S1. Model Parameters

Parameter	Range of Values	Description
<i>Psychological mechanisms</i>		
error rate (ϵ)	from 0 to 1 in increments of .05	Probability that an agent will randomly flip the value of a focal attitude (see Figure 2)
leniency threshold (λ)	from 0 to 1 in increments of .05	Probability that an agent will listen to an ingroup minority and change one of the related attitudes to the minority position when the minority is consistent.
<i>Network characteristics</i>		
network types	complete, scale-free, ring and square lattice, small world, random, modular	(Refer to the multi-agent network systems section and Figure 1)
number of edges per node (m)	1, 2, 4, 8	The number of edges added to a network for each node. It is half of the average node degree.
rewiring probability (p)	0.005, 0.05, 0.125, 0.25, 0.5, 1.0	In the Watts-Strogatz model, probability that an edge will get rewired.
intergroup connection (c)	5%, 10%, 20%, 30%	In a modular network, the fraction of ties that connect nodes from different communities.

Note. Scale-free networks are generated by Barabási and Albert's (1999) preferential attachment algorithm characterized by having a few high-degree nodes (hubs) and many low-degree nodes. Square lattices are generated with two different neighborhood types: Von Neumann neighborhood (4 side neighbors, $m = 2$) and Moore neighborhood (8 side and corner neighbors, $m = 4$). Ring lattices are networks where the nodes can be arranged in a circle with each node connected to its $2m$ nearest neighbors. Small world networks are generated from ring networks by randomly changing the connections of some edges using Watts and Strogatz's (1998) model with various rewiring probabilities. Modular networks are generated to have some level of community structure with varying intergroup connections (Girvan & Newman, 2002).

Supplementary Online Material 2

The Patterns of Social Change and Diversity on Complete Networks

Fig S2.1. The pattern of social change on a complete network ($n=20$). Error rate = 0.55.

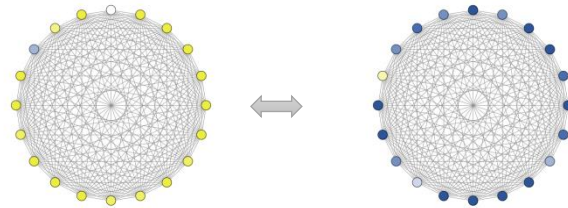


Fig S2.2. The pattern of social change with varying error rates on a complete network ($n = 20$). No cultural change occurs below the error rate of 0.30. An error rate between 0.30 and 0.80, rapid social change occurs. An error rate, larger than 0.80, generates frequent random changes.

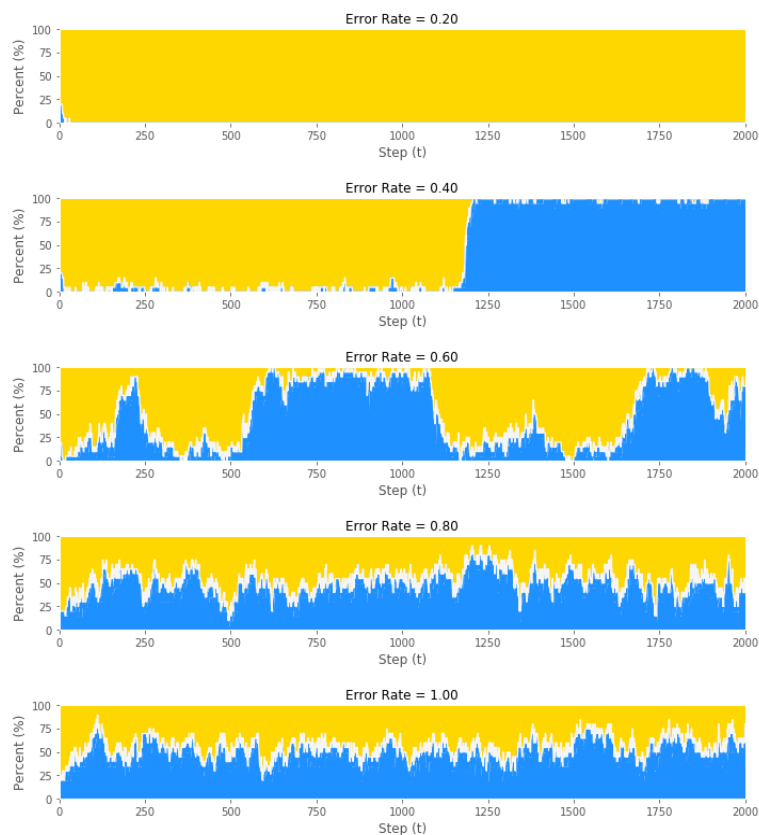
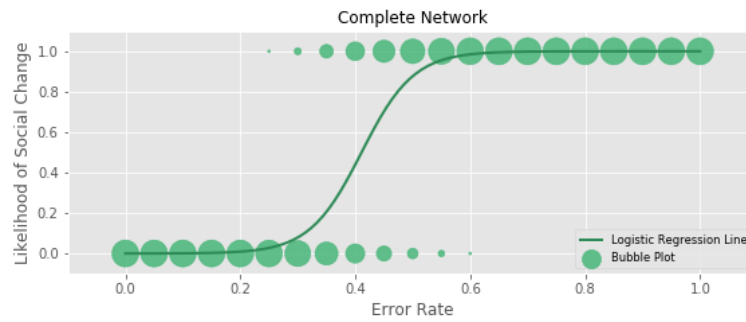
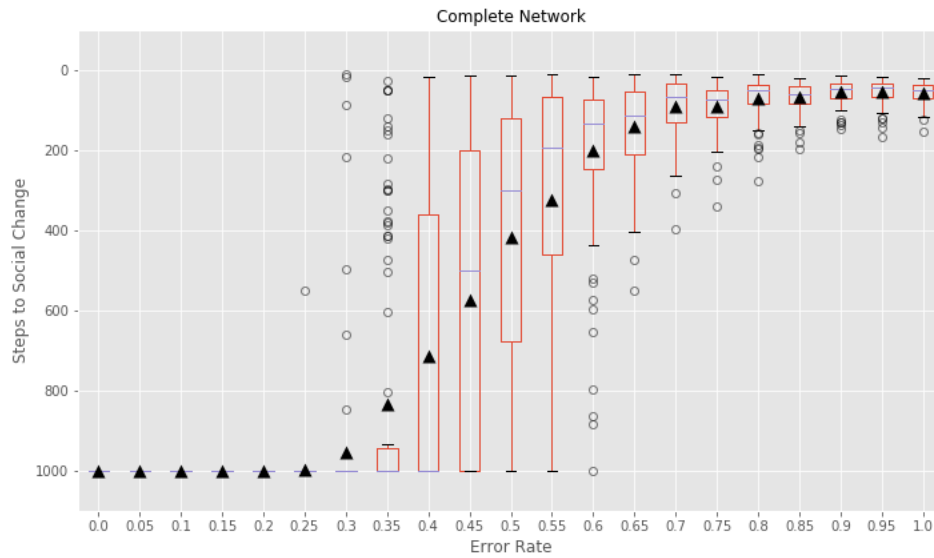


Fig S2.3. The effect of error rate on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of initial minority on a small complete network. The tipping point is 0.35.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

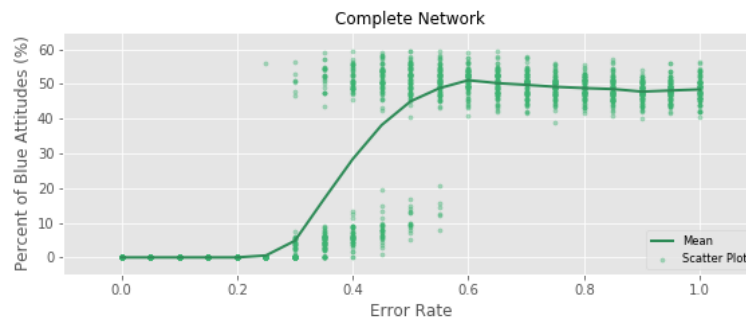
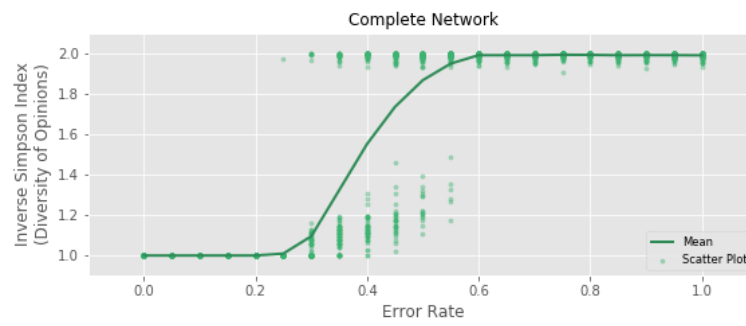


Fig S2.4. Cultural diversity with varying error rates on a small complete network.

(a) Inverse Simpson index



(b) Hamming distance

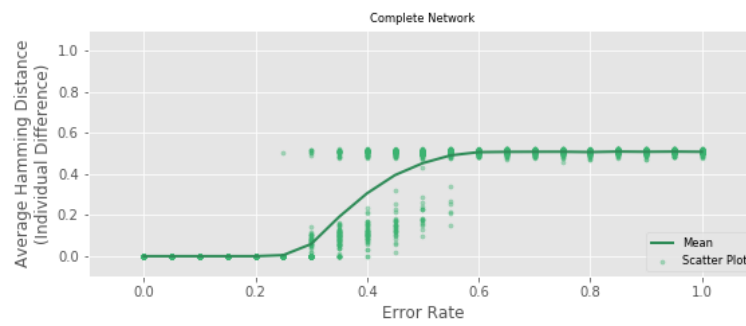


Fig S2.5. The pattern of social change on a small complete network ($n=20$). Leniency threshold = 0.55.



1. 20% initial minority (blue) agents are randomly distributed across the network at step #0.
2. Social change occurs such that the number of blue agents is equal or larger than the number of yellow agents.

Fig S2.6. The pattern of social change with varying leniency thresholds on a complete network ($n = 20$). No cultural change occurs below the leniency threshold of 0.50. A leniency threshold, larger than 0.50, generates gradual social change and small-amplitude high-frequency oscillation.

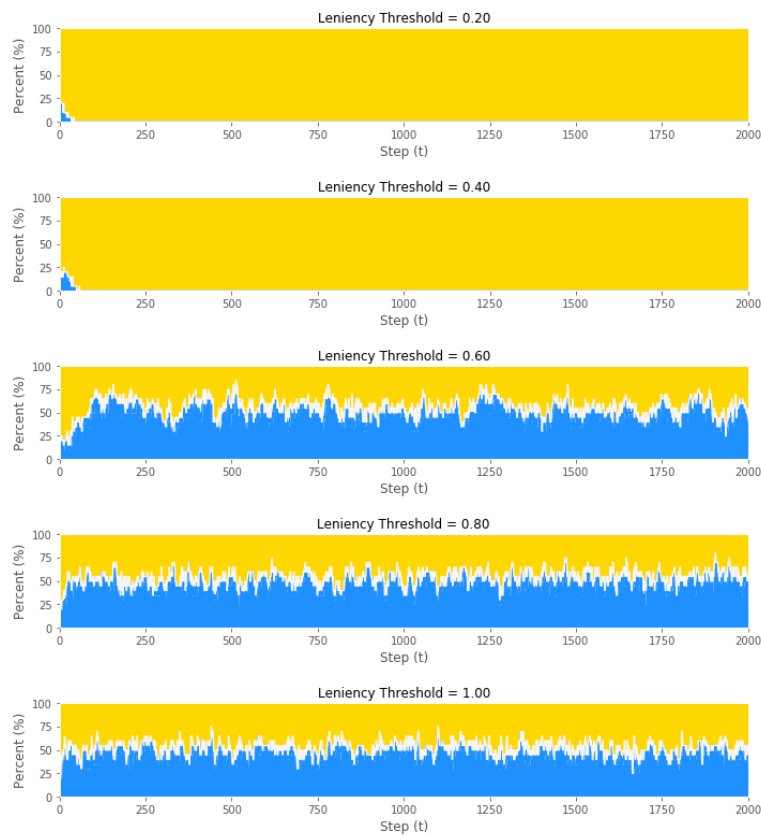
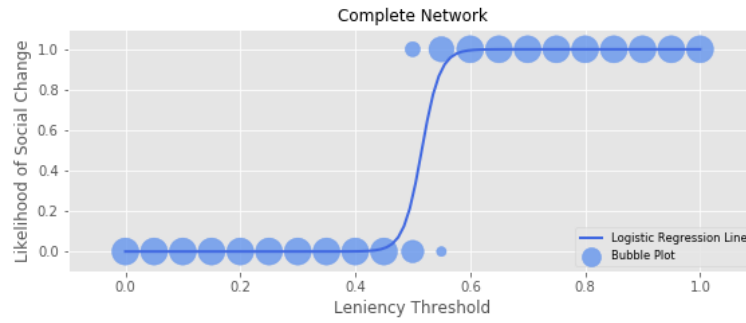
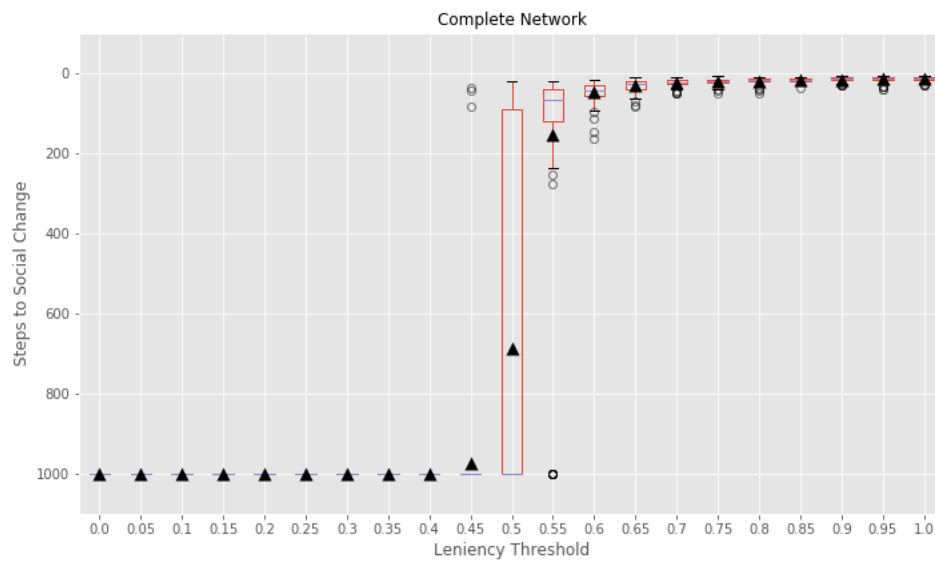


Fig S2.7. The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of initial minority on a small complete network. The tipping point is 0.50.

(a) The likelihood of social change



(b) Steps to social change



(c) The spread of initial minority

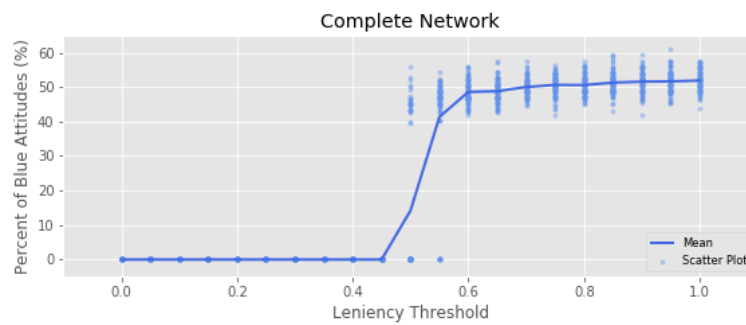
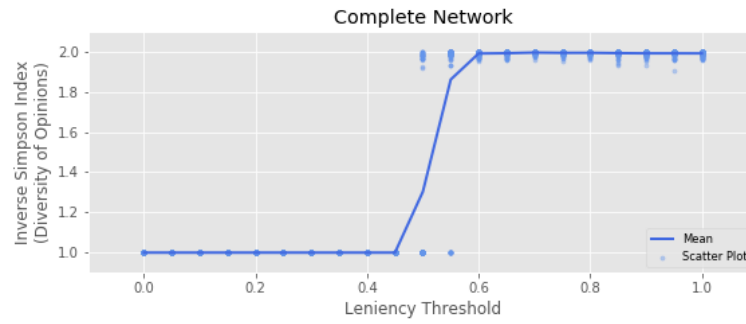
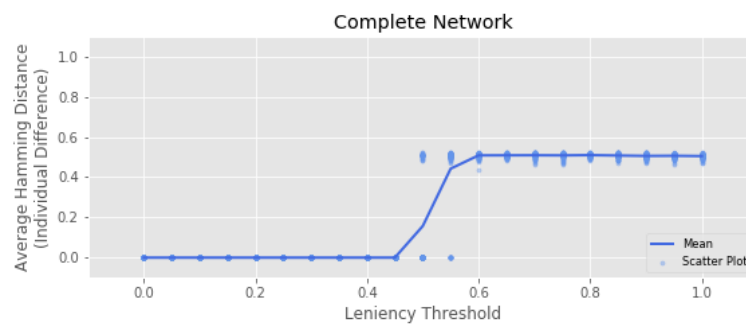


Fig S2.8. Cultural diversity with varying leniency thresholds on a small complete network.

(a) Inverse Simpson index



(b) Hamming distance



Supplementary Online Material 3A

The Patterns of Social Change and Diversity with Varying Error Rates on the Barabási–Albert Network Model

Fig S3A.1. The pattern of social change on a preferential attachment network ($n=200$, $m=1$). Error rate = 0.10.

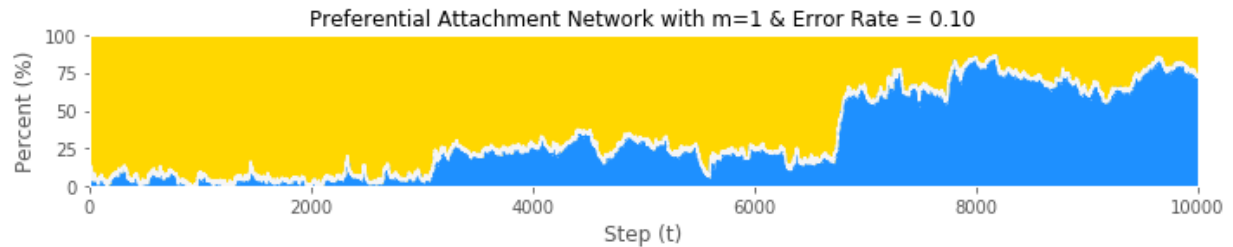
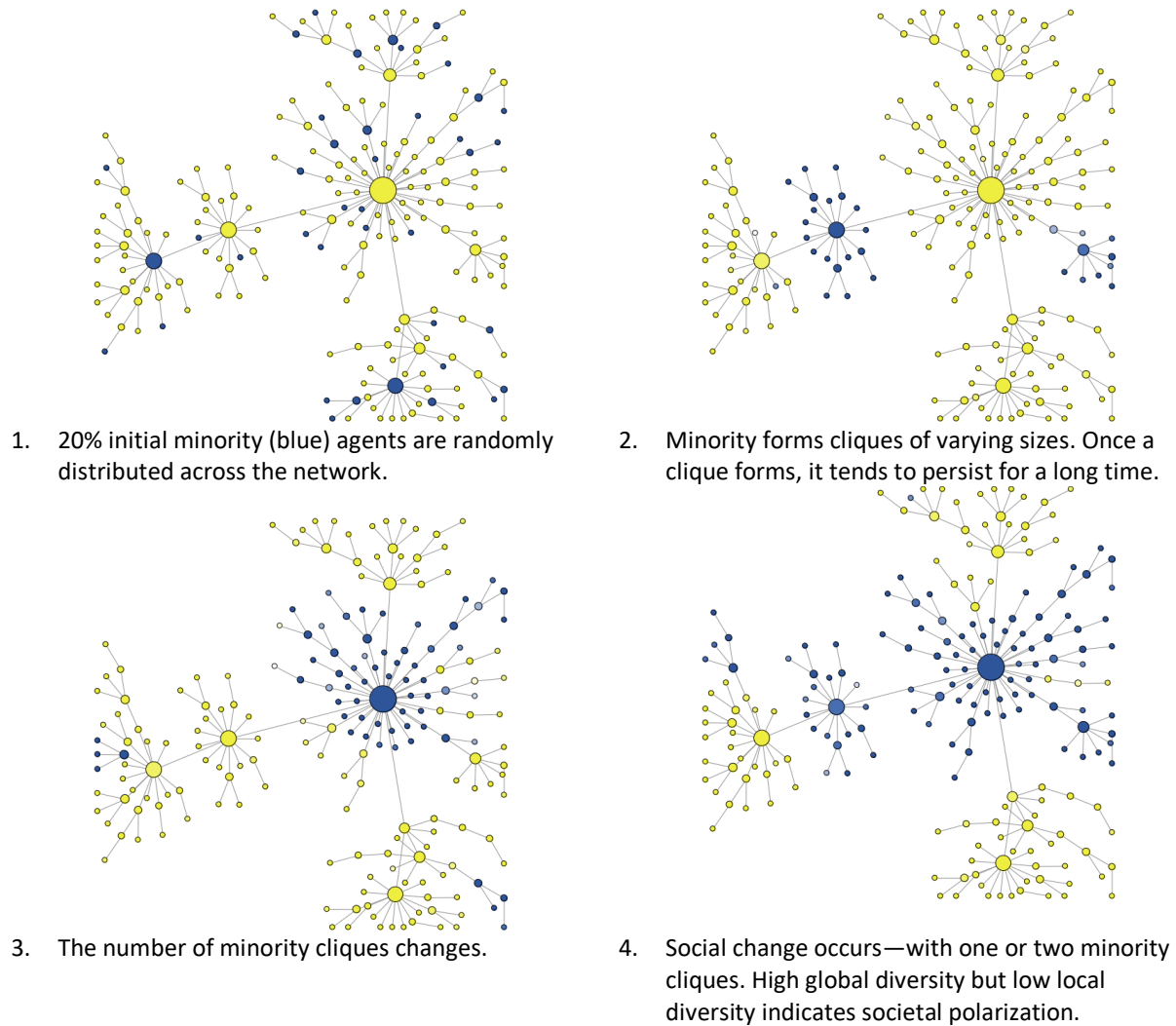
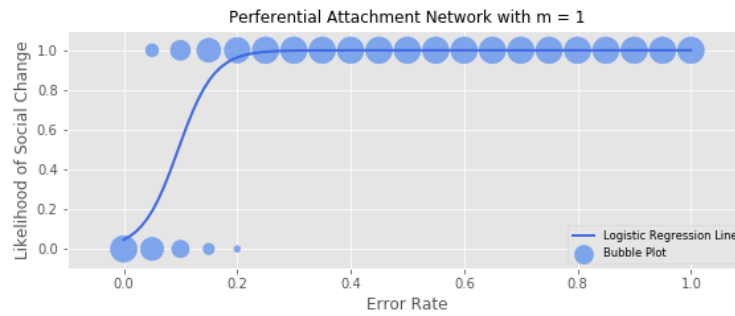
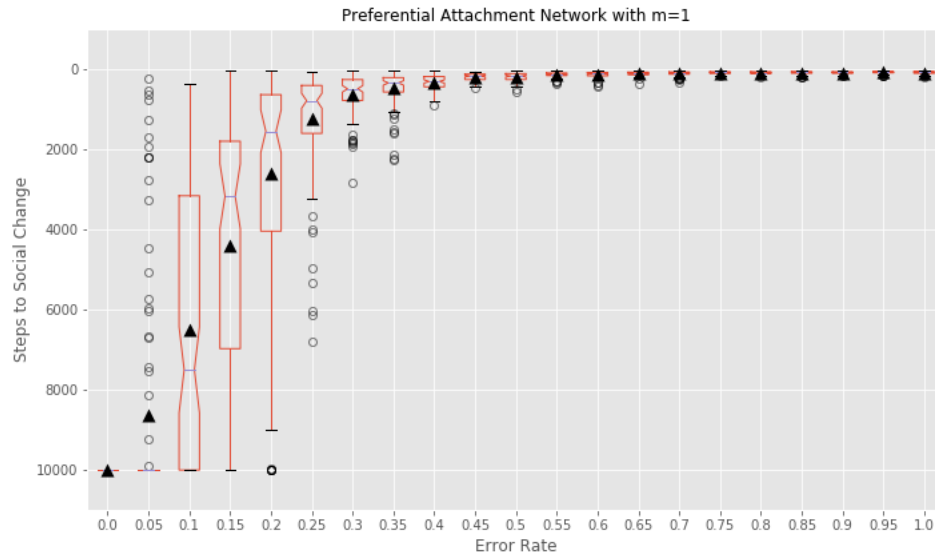


Fig S3A.2. The effect of error rates on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of initial minority on a preferential attachment network ($n=200$, $m=1$). The tipping point is 0.10.

(a) The likelihood of social change



(b) Steps to social change



(c) The spread of initial minority

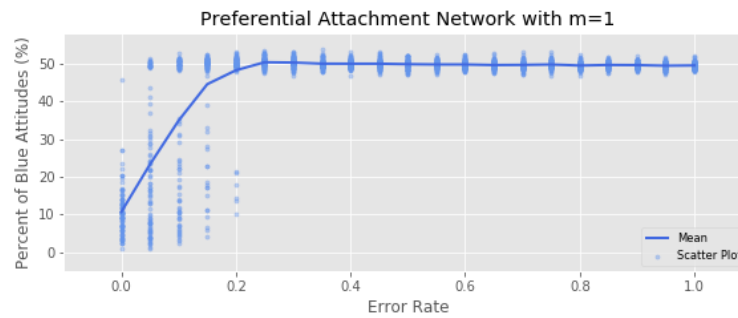
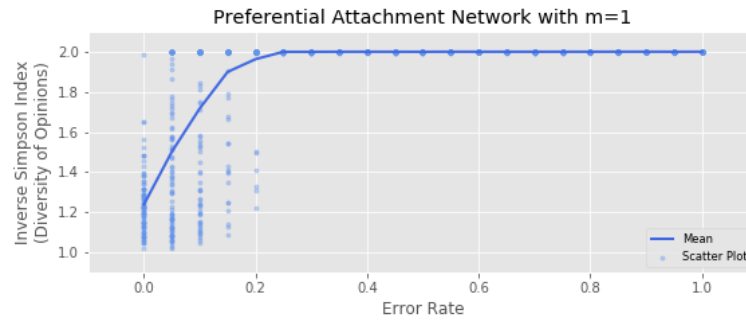


Fig S3A.3. Cultural diversity with varying error rates on preferential attachment networks ($n=200$, $m=1$).

(a) Inverse Simpson index



(b) Hamming distance

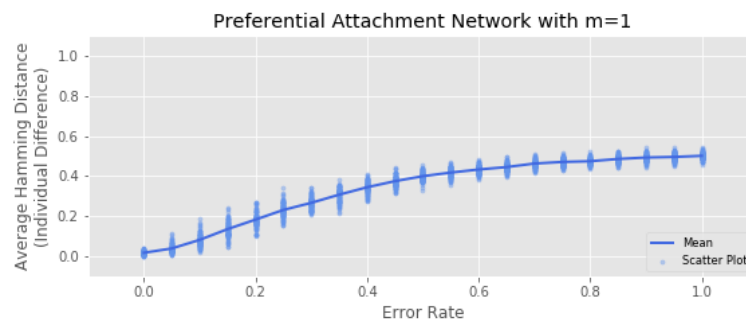
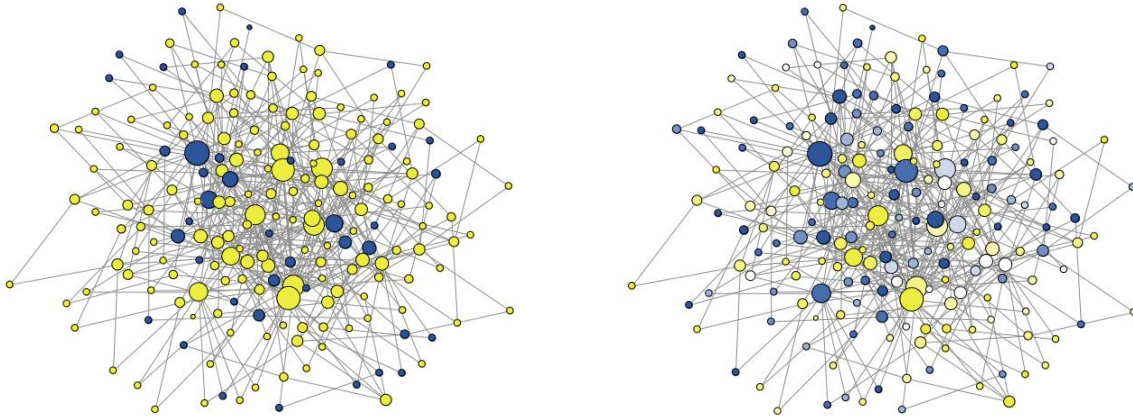


Fig S3A.4. The pattern of social change on a preferential attachment network ($n = 200$, $m = 2$). Error rate = 0.55.



1. 20% initial minority (blue) agents are randomly distributed across the network.
2. Minority size increases, and social change occurs.

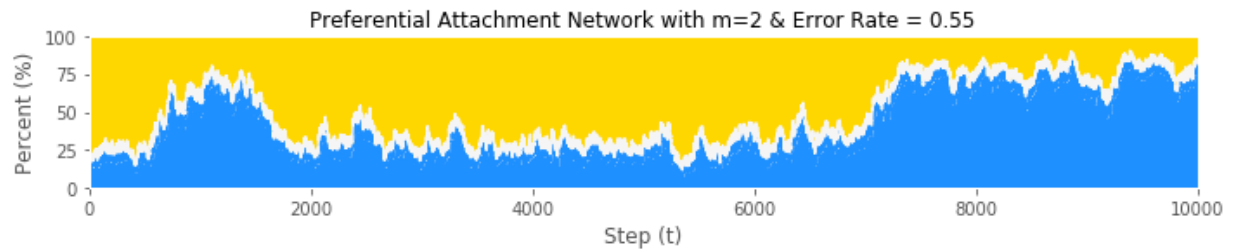
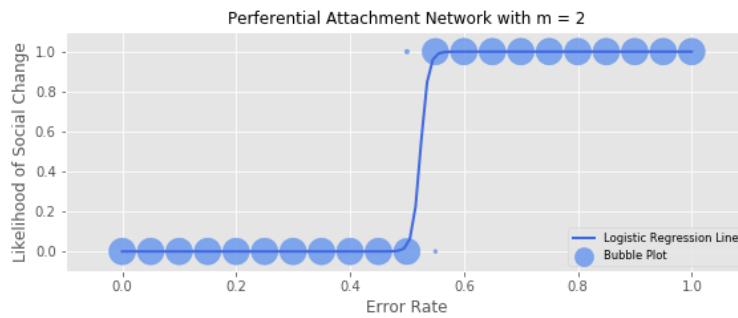
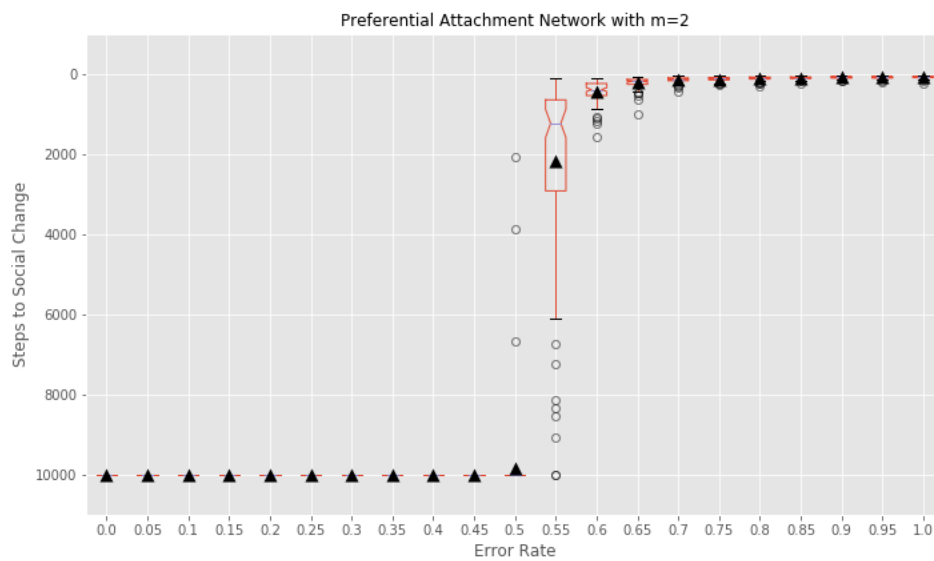


Fig S3A.5. The effect of error rates on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a preferential attachment network ($n=200$, $m=2$). The tipping point is 0.55.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

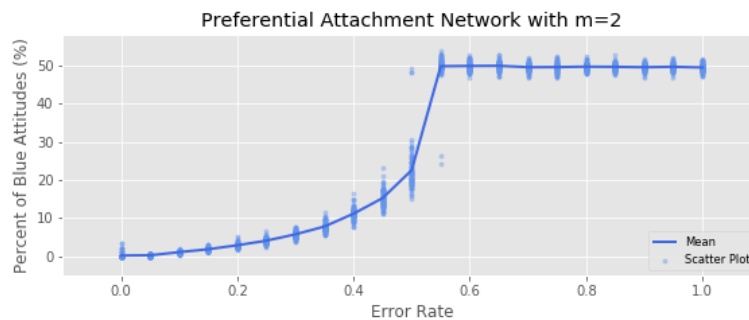
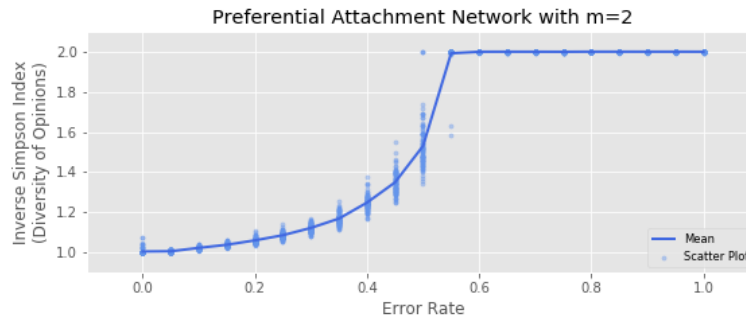


Fig S3A.6. Cultural diversity with varying error rates on preferential attachment networks ($n=200$, $m=2$).

(a) Inverse Simpson index



(b) Hamming distance

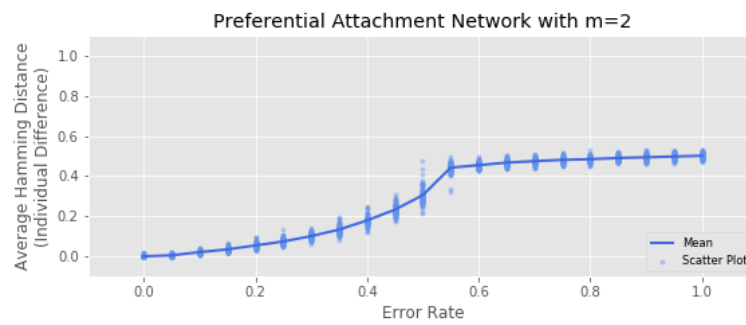
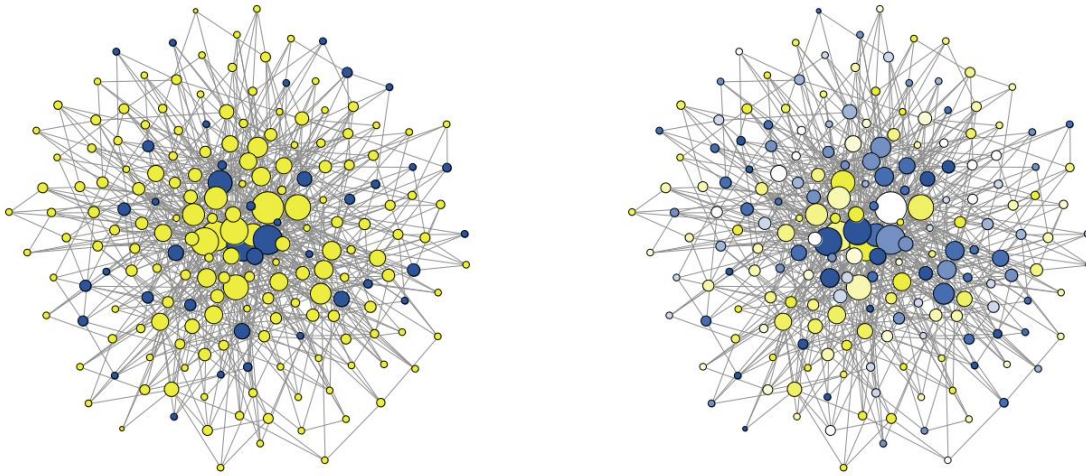


Fig S3A.7. The pattern of social change on a preferential attachment network ($n=200$, $m=4$). Error rate = 0.65.



1. 20% initial minority (blue) agents are randomly distributed across the network.
2. Minority size increases, and social change occurs.

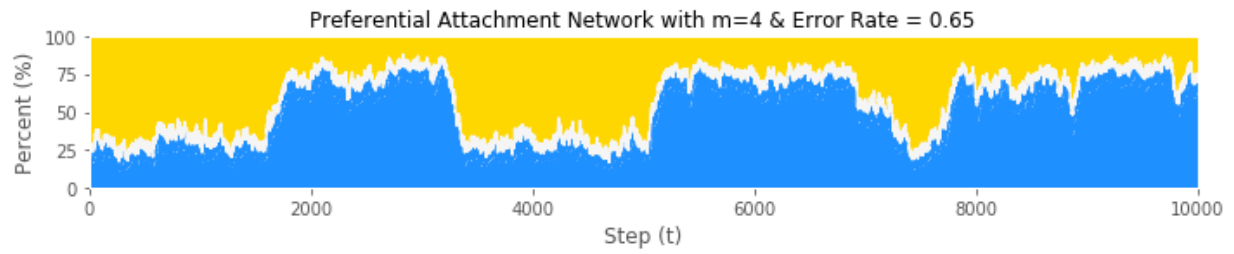
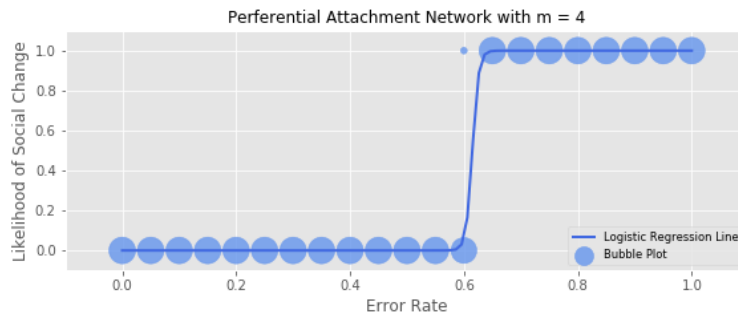
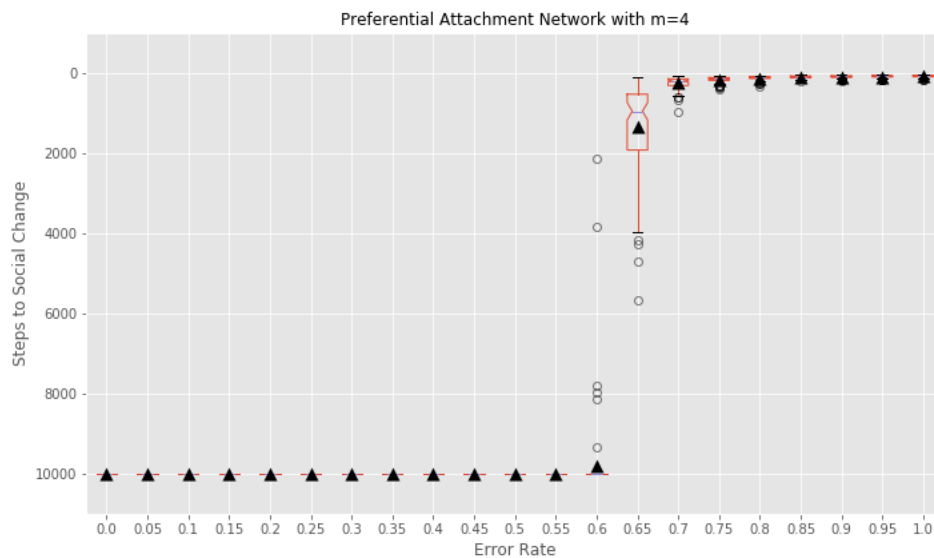


Fig S3A.8. . The effect of error rates on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a preferential attachment network ($n=200$, $m=4$). The tipping point is 0.65.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

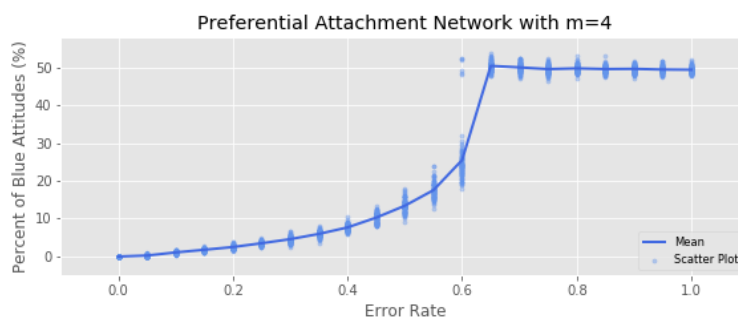
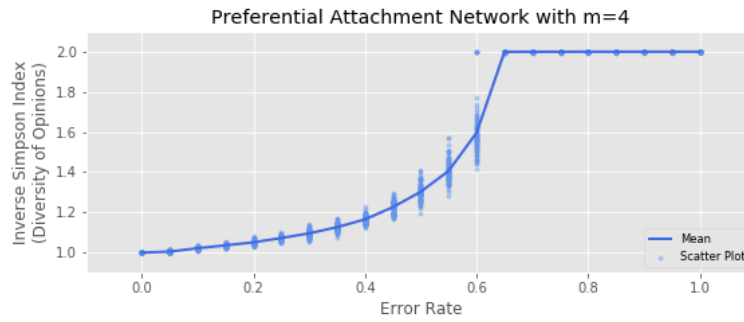


Fig S3A.9. Cultural diversity with varying error rates on preferential attachment networks ($n=200$, $m=4$).

(a) Inverse Simpson index



(b) Hamming distance

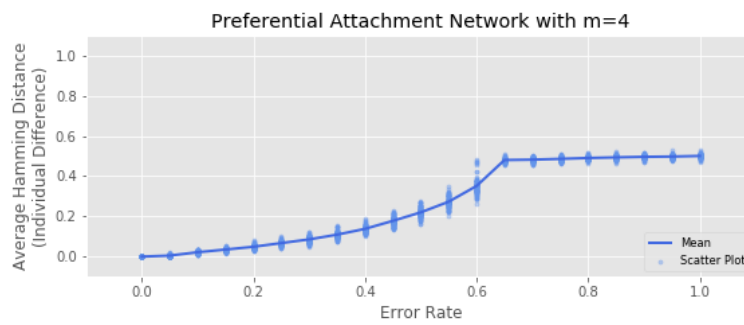
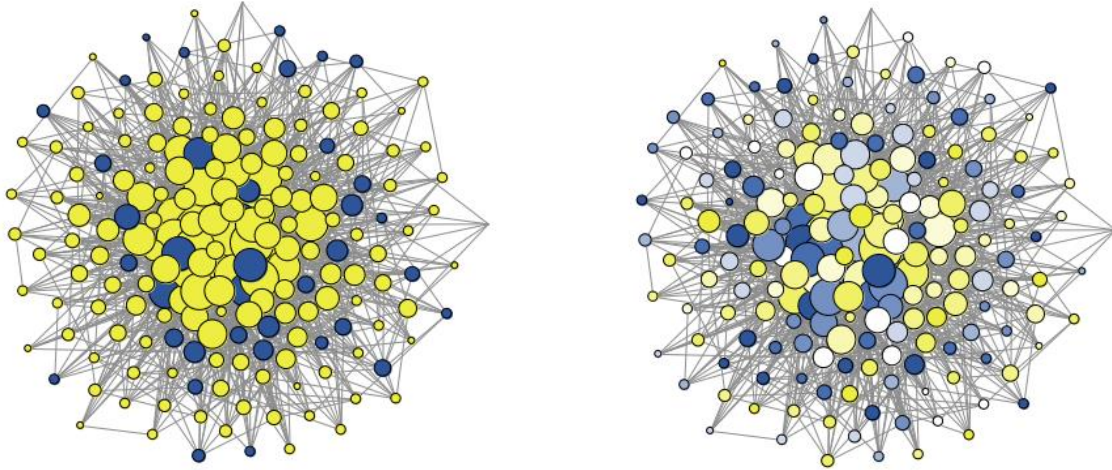


Fig S3A.10. The pattern of social change on a preferential attachment network ($n = 200$, $m = 8$). Error rate = 0.70.



1. 20% initial minority (blue) agents are randomly distributed across the network.
2. Social change occurs.

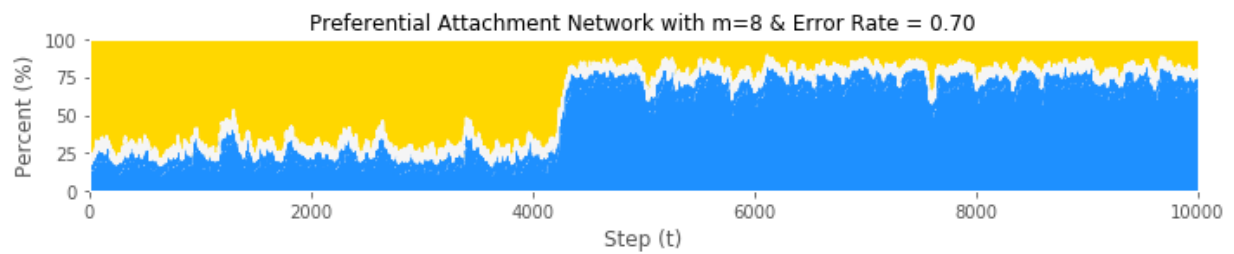
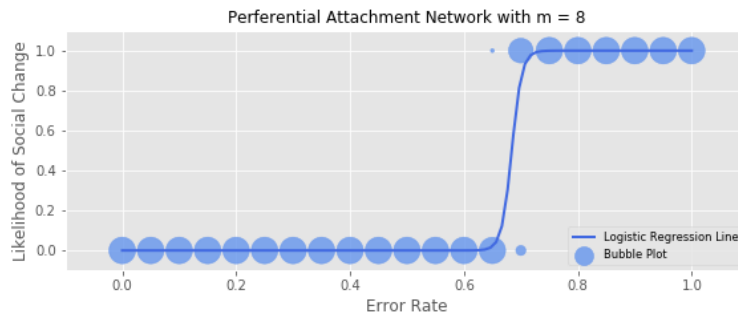
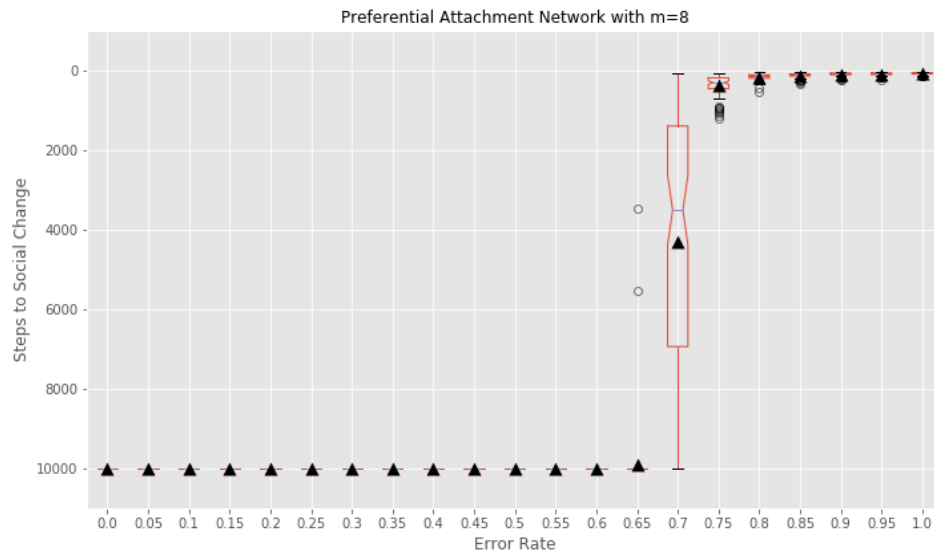


Fig S3A.11. The effect of error rates on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a preferential attachment network ($n=200$, $m=8$). The tipping point is 0.70.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

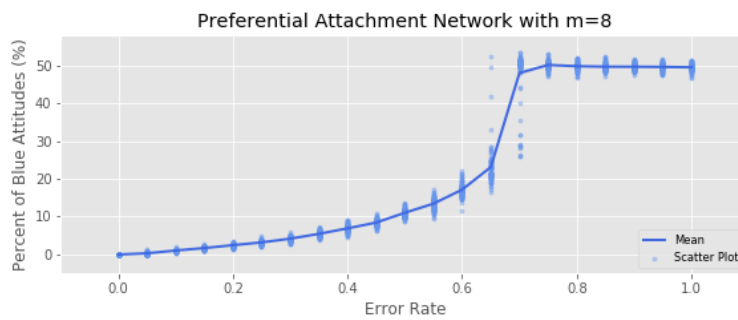
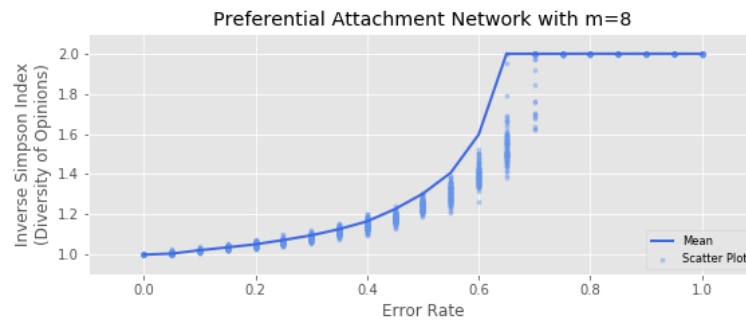
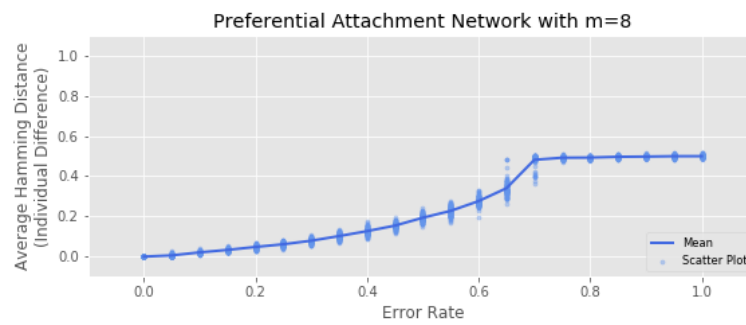


Fig S3A.12. Cultural diversity with varying error rates on a preferential attachment network ($n = 200$, $m = 8$).

(a) Inverse Simpson index



(b) Hamming distance



Supplementary Online Material 3B

The Patterns of Social Change and Diversity with Varying Leniency Thresholds on the Barabási–Albert Network Model

Fig S3B.1. The pattern of social change on a preferential attachment network ($n = 200$, $m = 1$). Leniency threshold = 0.50.

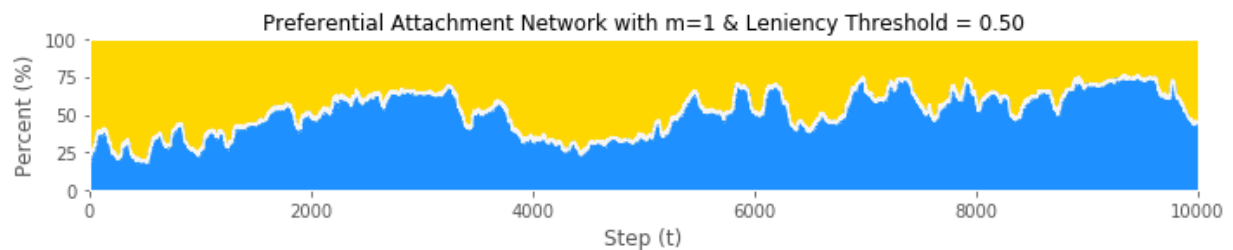
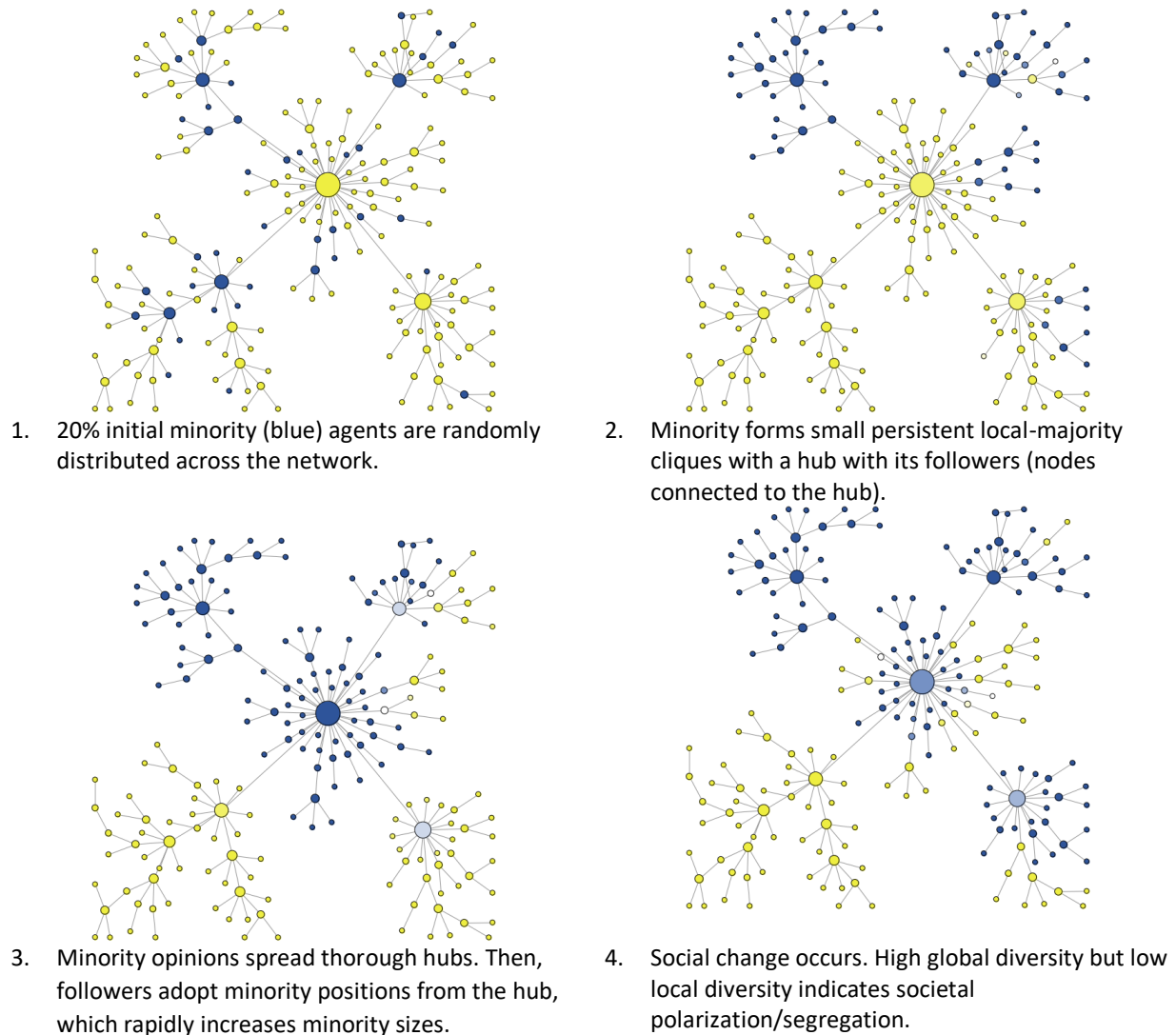
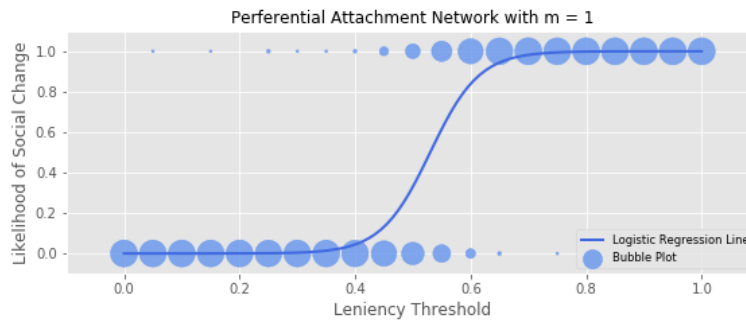
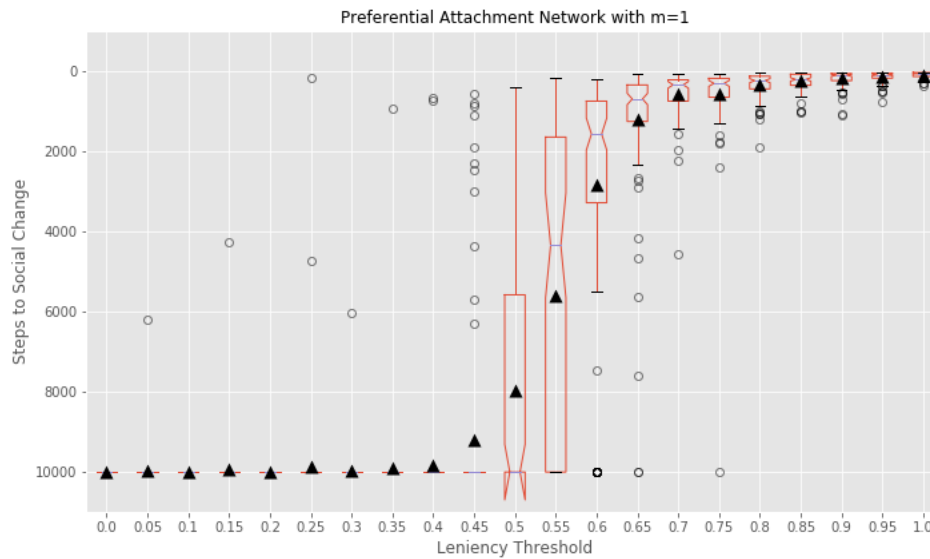


Fig S3B.2. The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of initial minority on a preferential attachment network ($n=200$, $m=1$). The tipping point is around 0.50.

(a) The likelihood of social change



(b) Steps to social change



(c) The spread of initial minority

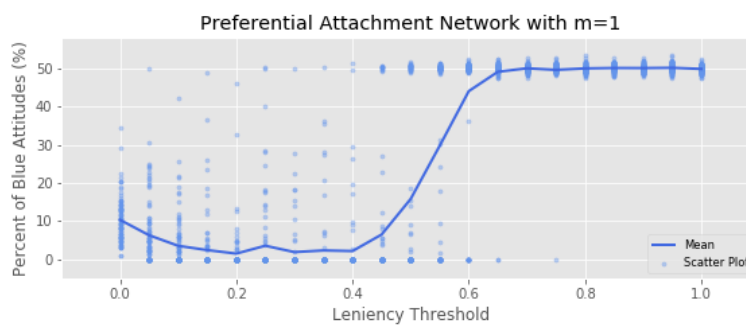
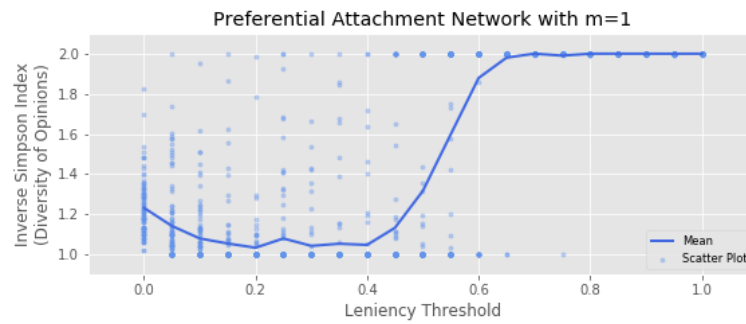


Fig S3B.3. Cultural diversity with varying leniency thresholds on preferential attachment networks (n=200, m=1).

(a) Inverse Simpson index



(b) Hamming distance

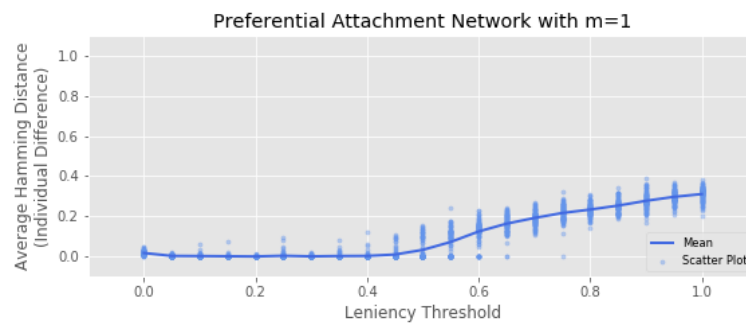
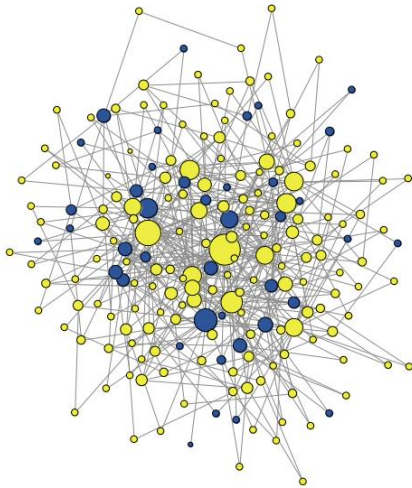
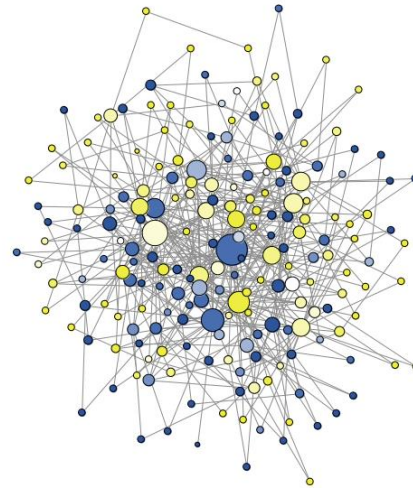


Fig S3B.4. The pattern of social change on a preferential attachment network ($n = 200$, $m = 2$). Leniency threshold = 0.55.



1. 20% initial minority (blue) agents are randomly distributed across the network.



2. Social change occurs.

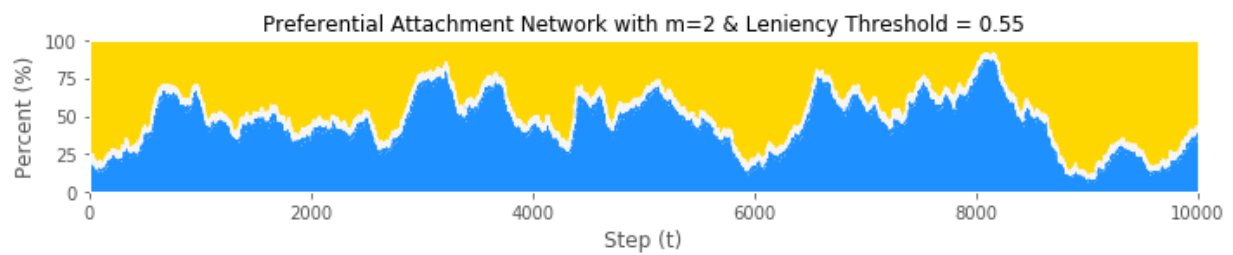
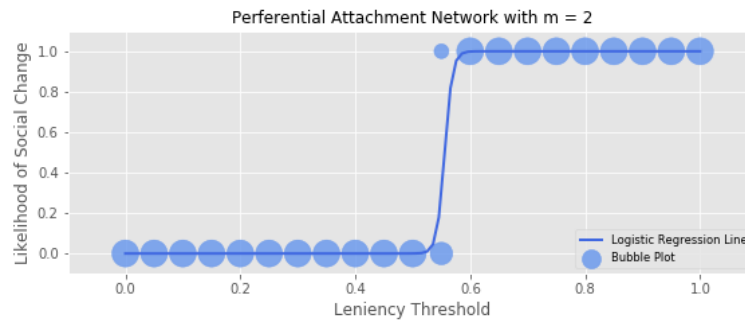
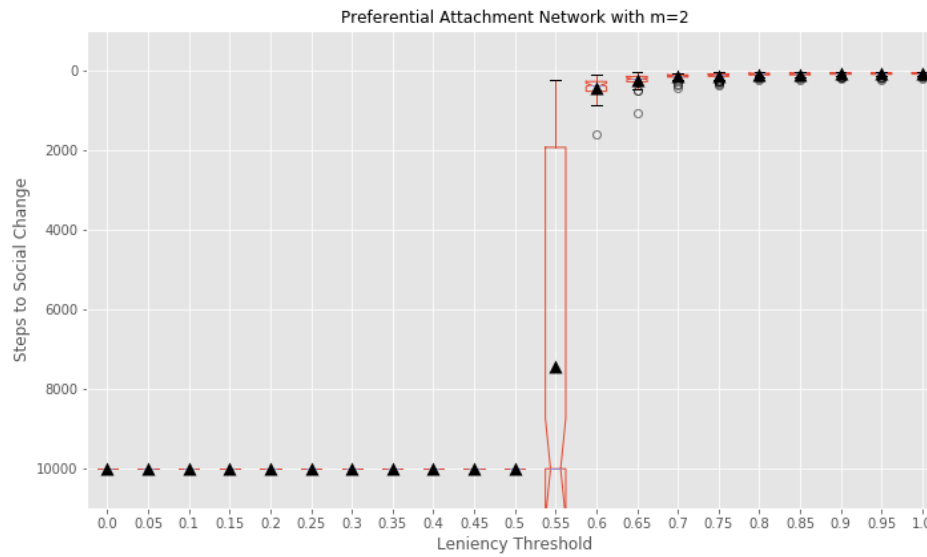


Fig S3B.5. The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a preferential attachment network ($n=200$, $m=2$). The tipping point is 0.55.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

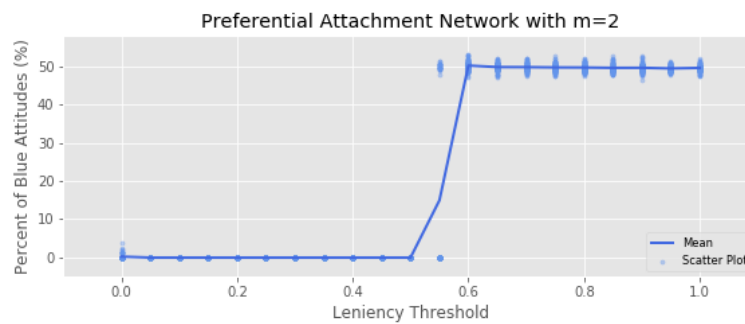
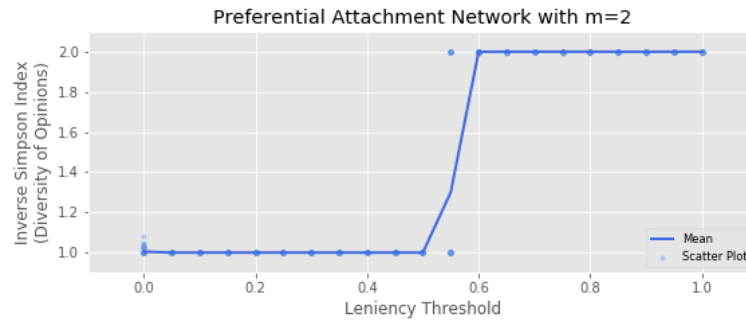


Fig S3B.6. Cultural diversity with varying leniency thresholds on preferential attachment networks (n=200, m=2).

(a) Inverse Simpson index



(b) Hamming distance

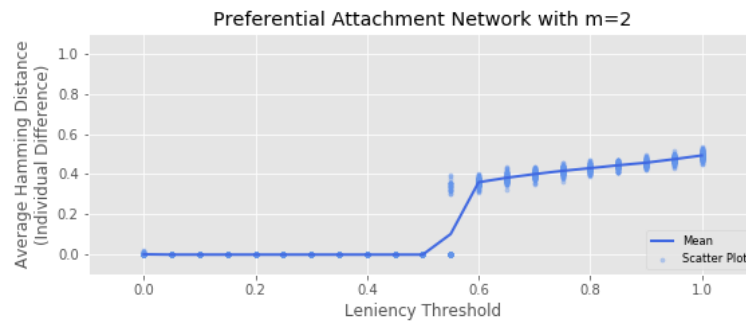
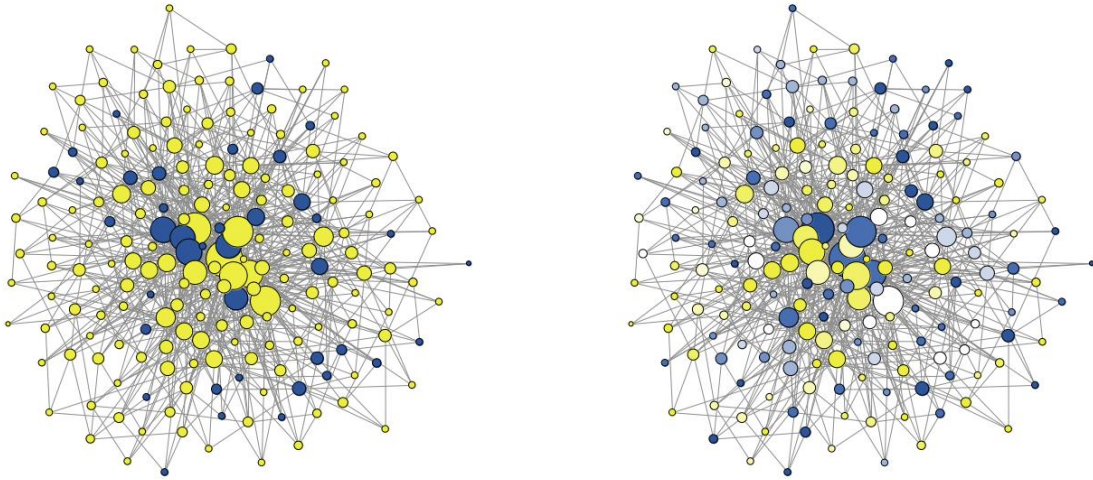


Fig S3B.7. The pattern of social change on a preferential attachment network ($n=200$, $m=4$). Leniency threshold = 0.55.



1. 20% initial minority (blue) agents are randomly distributed across the network.
2. Minority size gradually increases, and social change occurs. Both global and local diversity indices reach maximum level – **Diversity regime**, where equal number of two positions coexists without segregation.

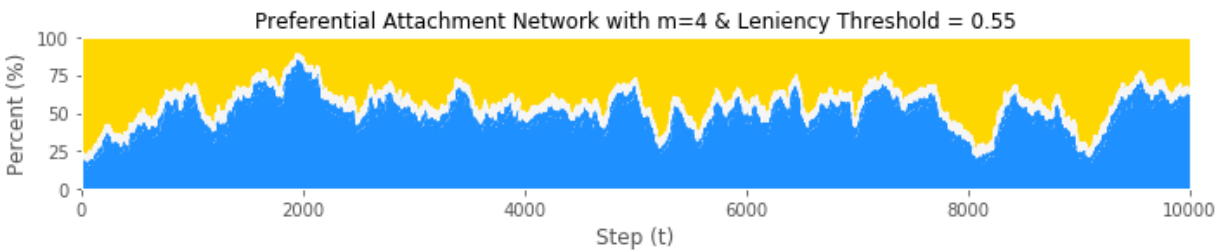
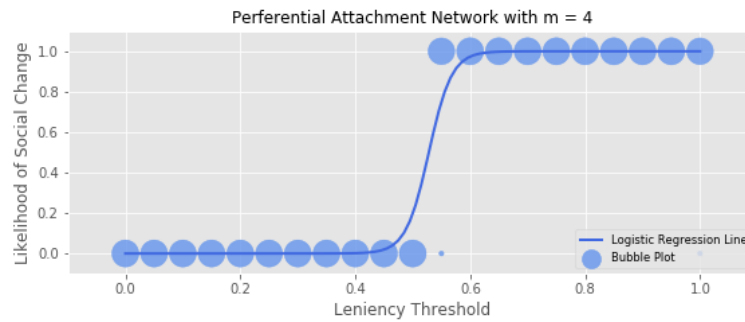
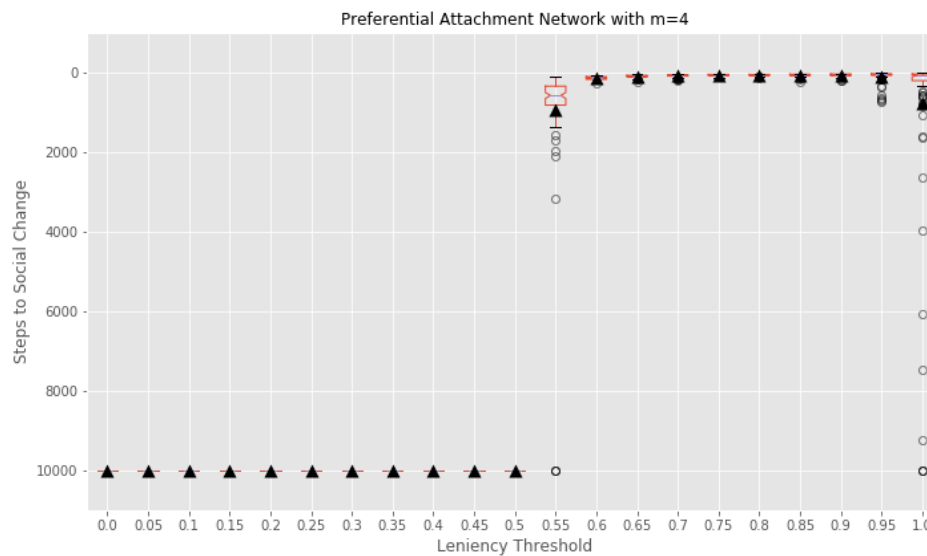


Fig S3B.8. . The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a preferential attachment network ($n=200$, $m=4$). The tipping point is 0.55.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

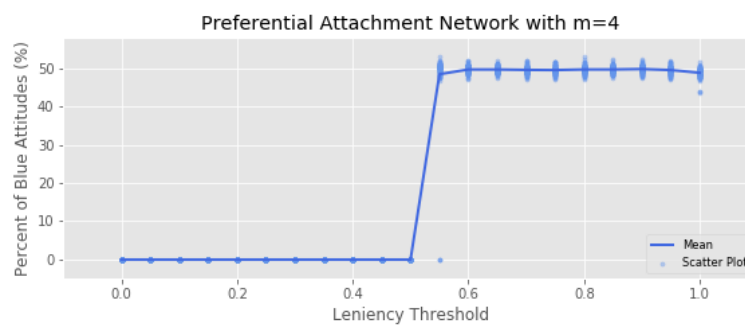
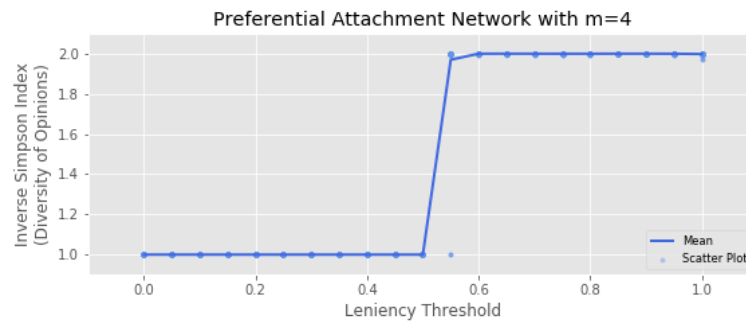


Fig S3B.9. Cultural diversity with varying leniency thresholds on preferential attachment networks (n=200, m=4).

(a) Inverse Simpson index



(b) Hamming distance

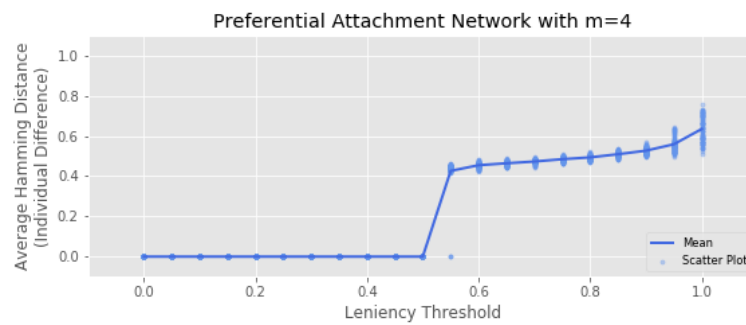
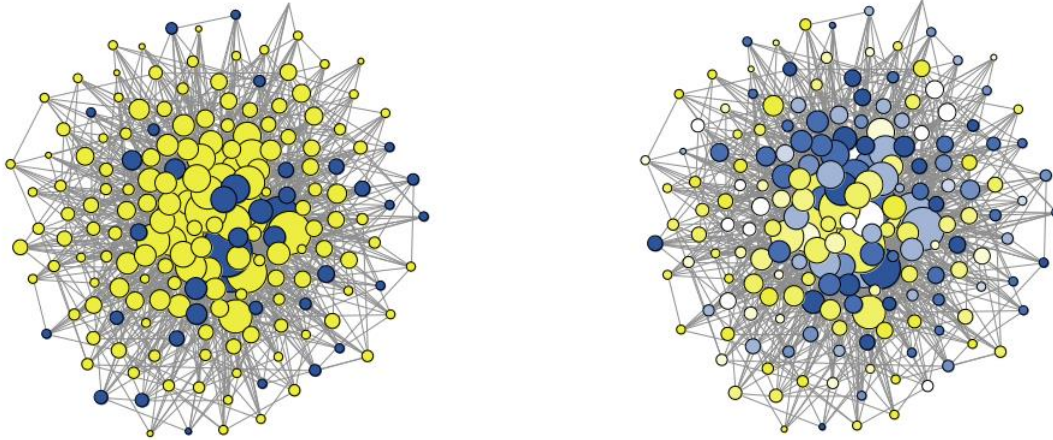


Fig S3B.10. The pattern of social change on a preferential attachment network ($n = 200$, $m = 8$). Leniency threshold = 0.55.



1. 20% initial minority (blue) agents are randomly distributed across the network.
2. Minority size gradually increases, and social change occurs. Both global and local diversity indices reach maximum level – **Diversity regime**, where equal number of two positions coexists without segregation.

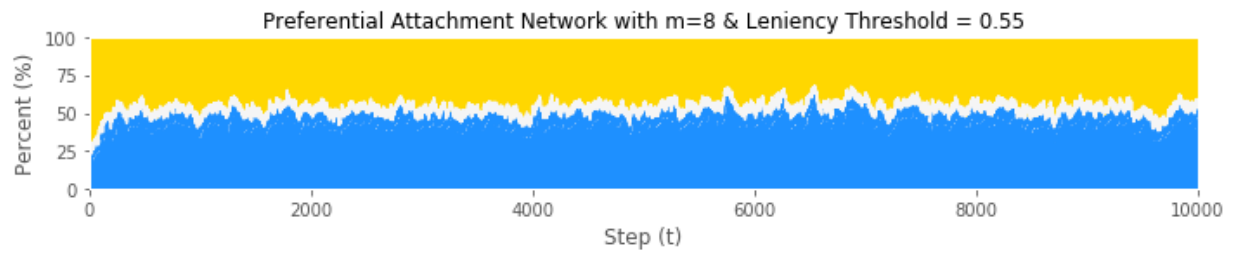
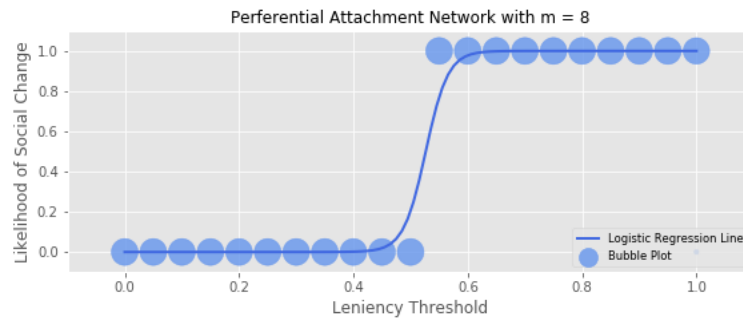
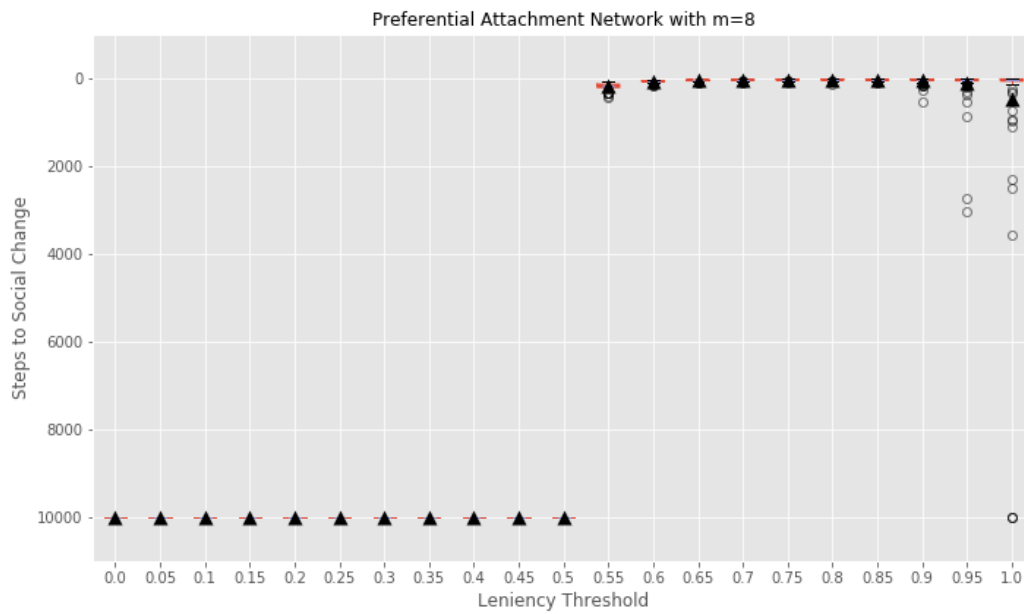


Fig S3B.11. The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a preferential attachment network ($n=200$, $m=8$). The tipping point is 0.55

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

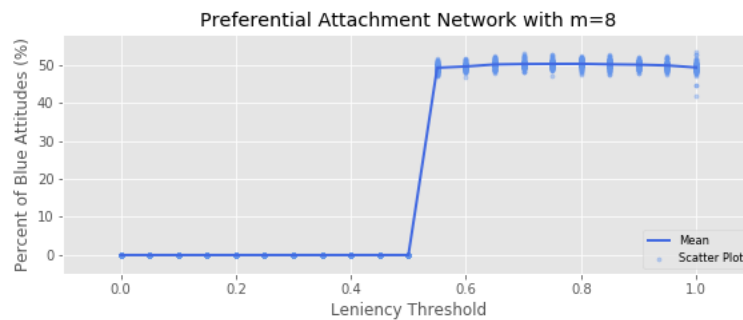
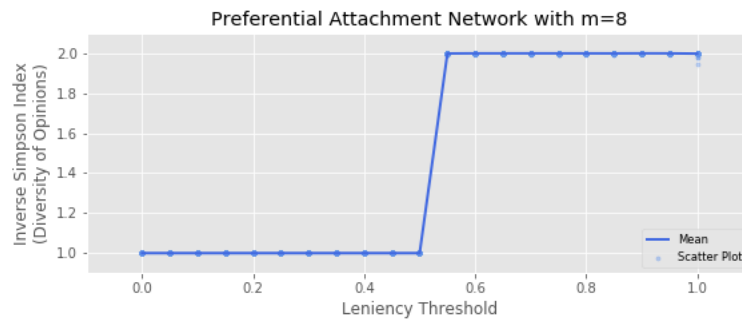
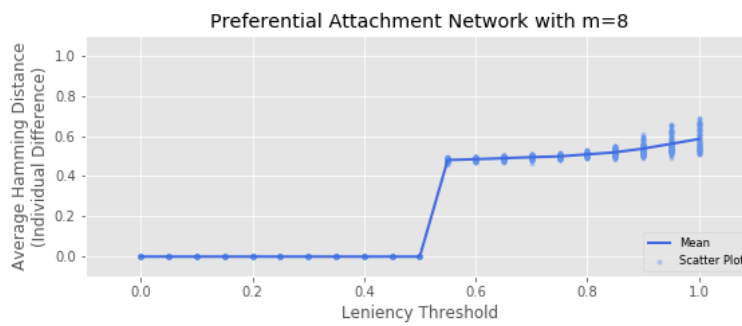


Fig S3B.12. Cultural diversity with varying leniency thresholds on a preferential attachment network ($n = 200$, $m = 8$)

(a) Inverse Simpson Index



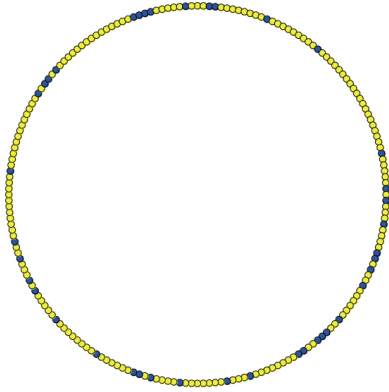
(b) Hamming Distance



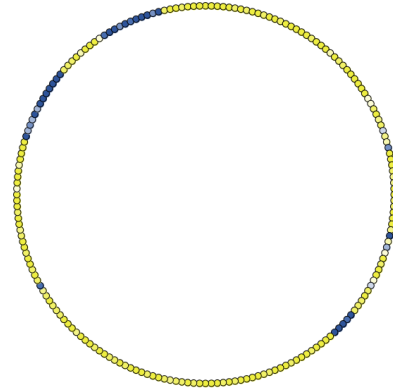
Supplementary Online Material 4A

The Patterns of Social Change and Diversity with Varying Error Rates on Ring Lattices

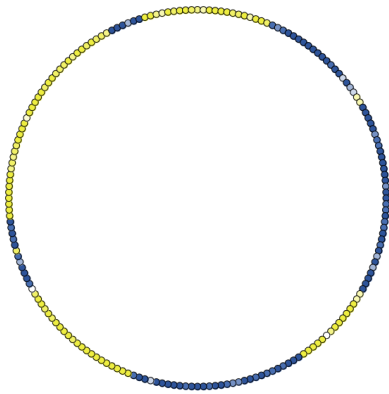
Fig S4A.1. . The pattern of social change on a ring lattice ($n = 200$, $m = 1$). Error rate = 0.30.



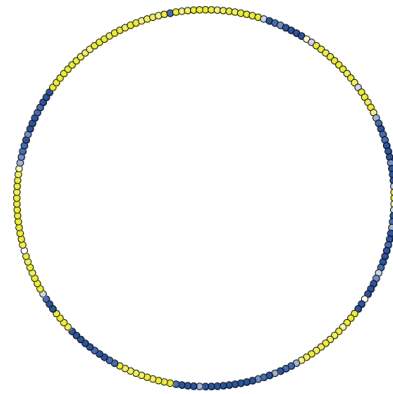
1. 20% initial minority (blue) agents are randomly distributed across the network.



2. Minority form cliques of varying sizes.



3. Social change occurs. There are multiple cliques.



4. As the simulation goes, the system doesn't converge to one position but two positions, distributed in several cliques, take turns – segregation regime.

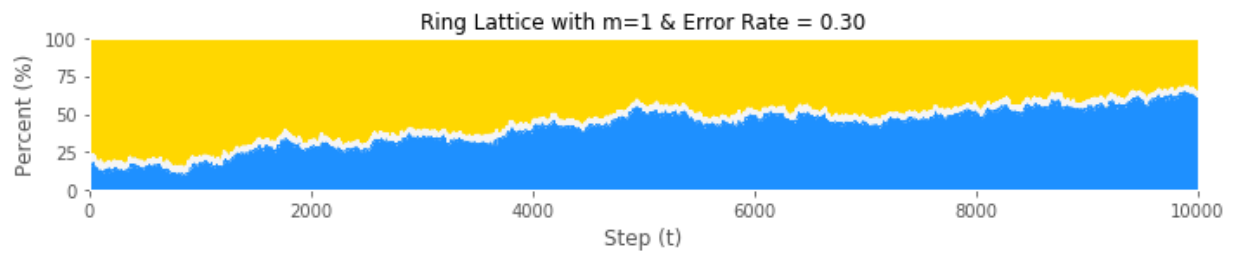
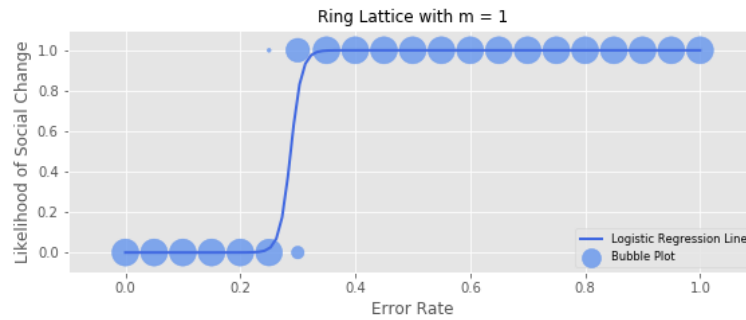
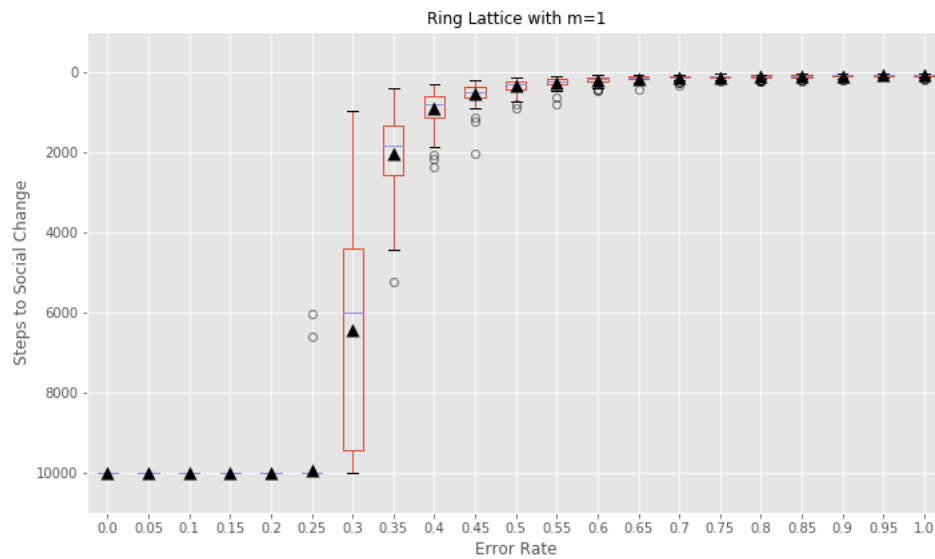


Fig S4A.2. The effect of error rates on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of initial minority on a ring lattice ($n=200$, $m=1$). The tipping point is 0.30.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

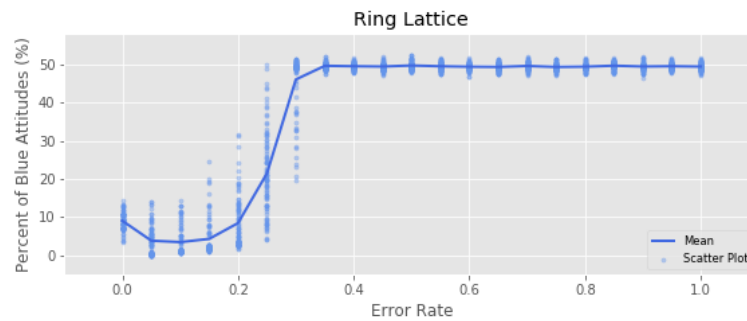
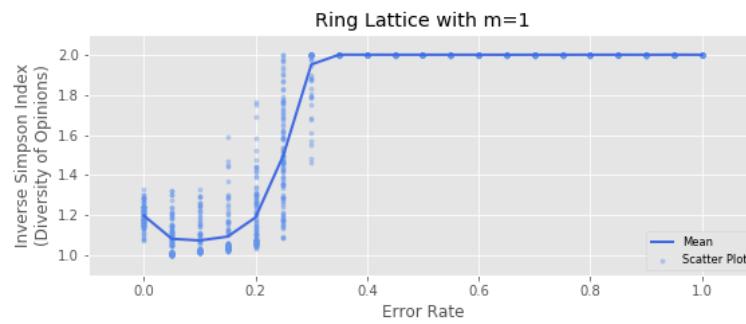


Fig S4A.3. Cultural diversity at varying error rates.

(a) Inverse Simpson index



(b) Hamming distance

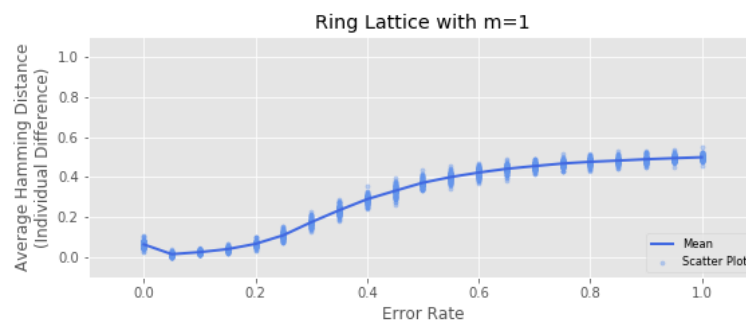
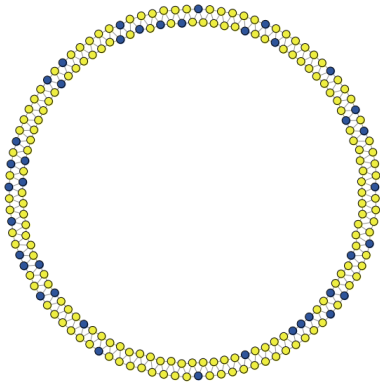
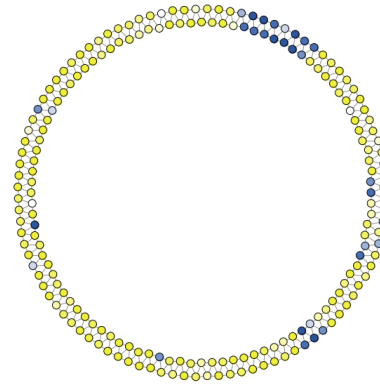


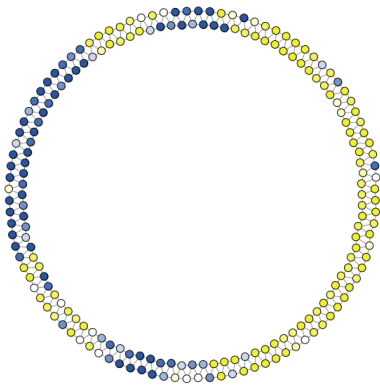
Fig S4A.4. The patterns of social change on a ring lattice ($n=200$, $m = 2$). Error rate = 0.35.



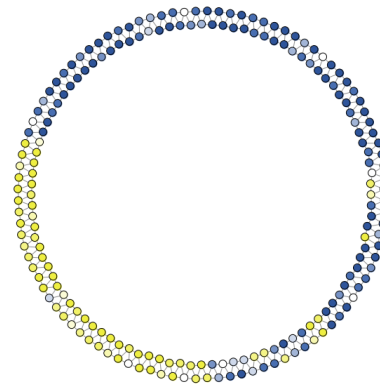
1. 20% initial minority (blue) agents are randomly distributed across the network.



2. Minority opinion holders form cliques with varying sizes.



3. The number of minority cliques changes.



4. When social change occurs, there are typically a few minority cliques exist. Sometimes two groups emerge indicating societal polarization, but often around the boundary of two groups, small cliques split from the larger group.

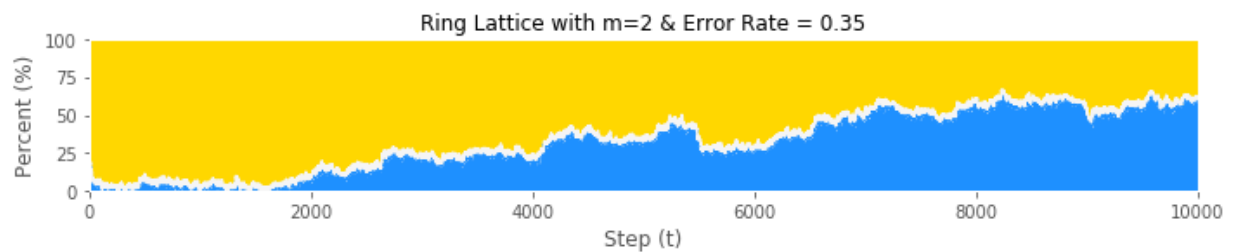
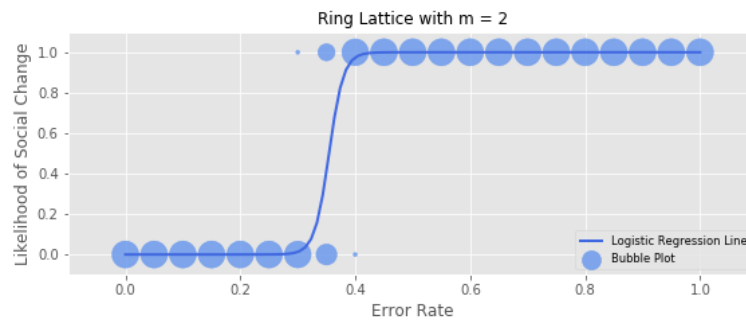
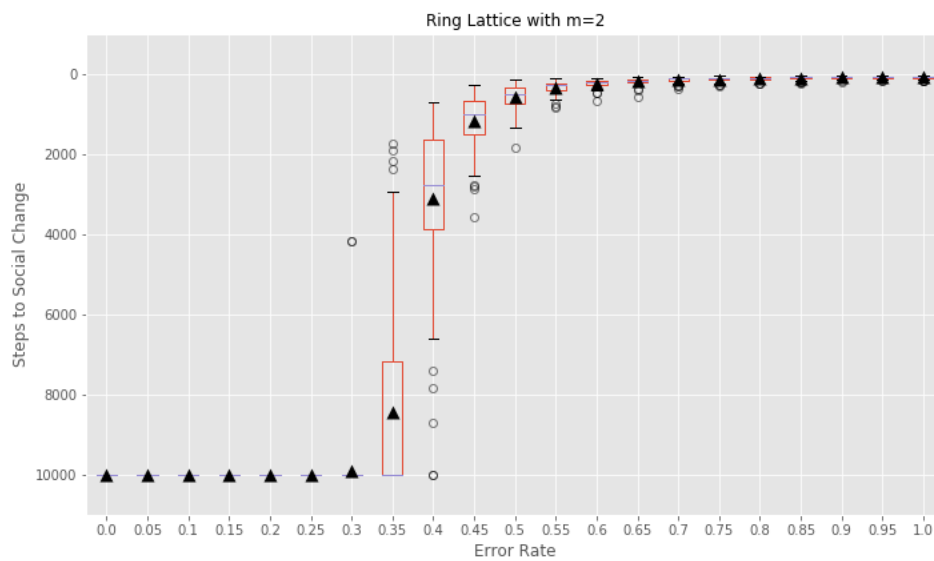


Fig S4A.5. The effect of error rates on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of initial minority on a ring lattice ($n = 200$, $m = 2$). The tipping point is 0.35.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

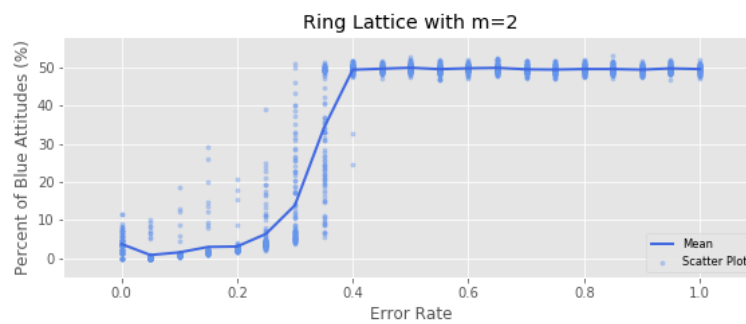
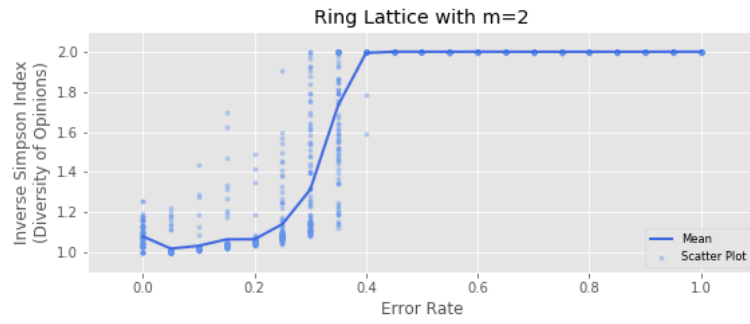


Fig S4A.6. Cultural diversity with varying error rates on ring lattices ($n=200$, $m=2$)

(a) Inverse Simpson index



(b) Hamming distance

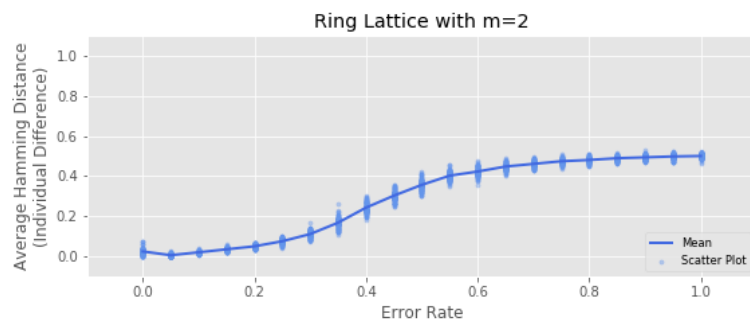
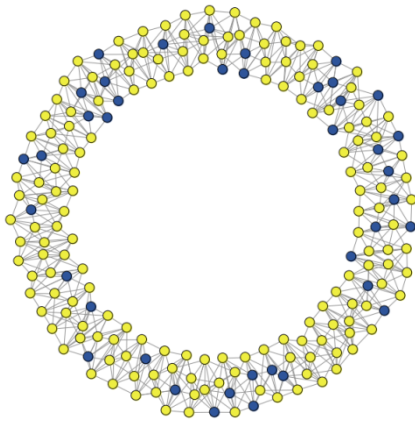
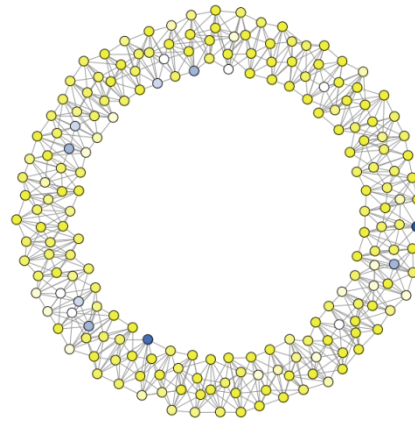


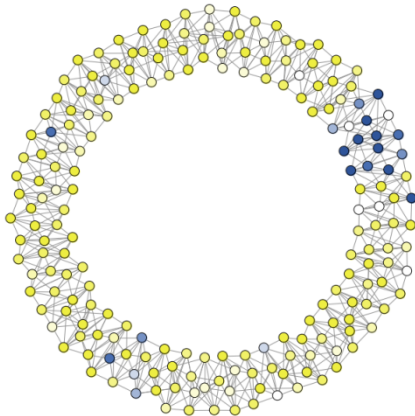
Fig S4A.7. The patterns of social change on a ring lattice ($n=200$, $m = 4$). Error rate = 0.45.



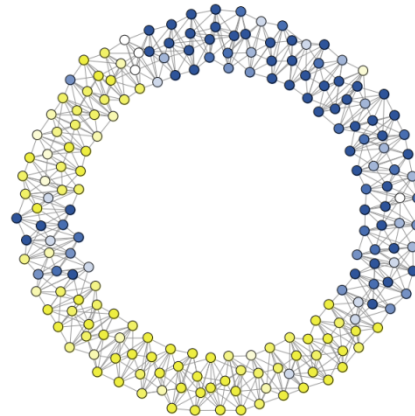
1. 20% initial minority (blue) agents are randomly distributed across the network.



2. Small cliques emerge and disappear



3. Minority opinion holders form cliques with varying sizes.



4. When social change occurs, there are typically a few minority cliques exist. The system doesn't converge to one position easily but two factions take turns - polarization regime.

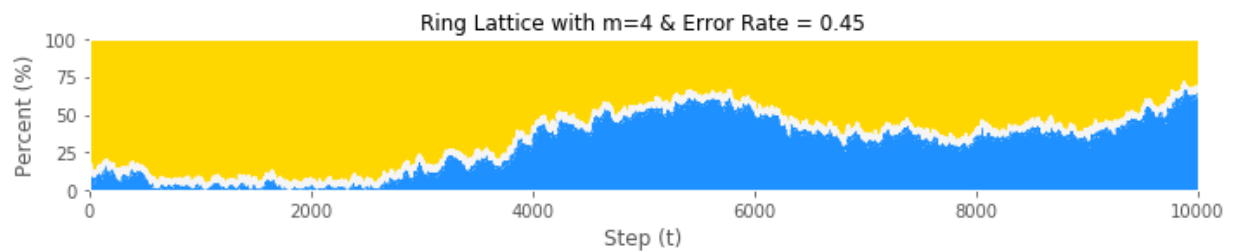
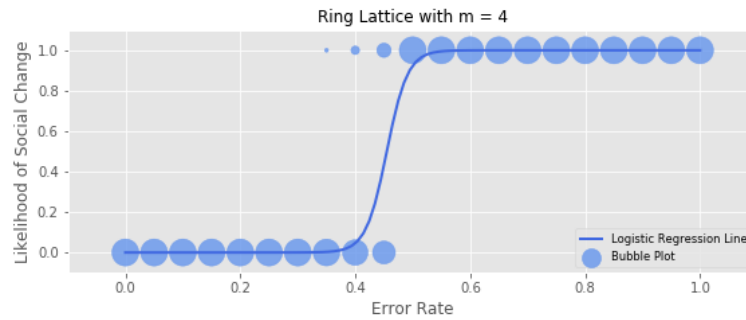
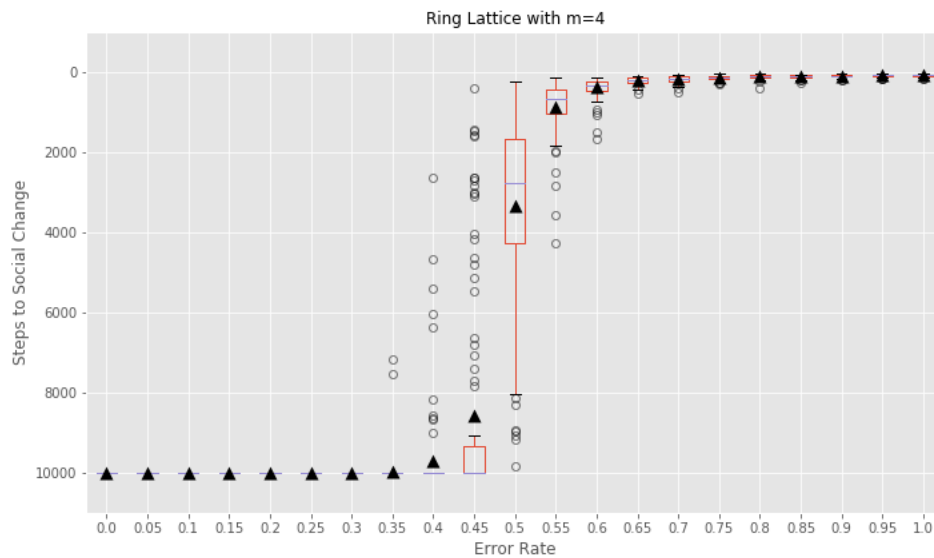


Fig S4A.8. The effect of error rates on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of initial minority on ring lattices ($n=200$, $m=4$). The tipping point is 0.45.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

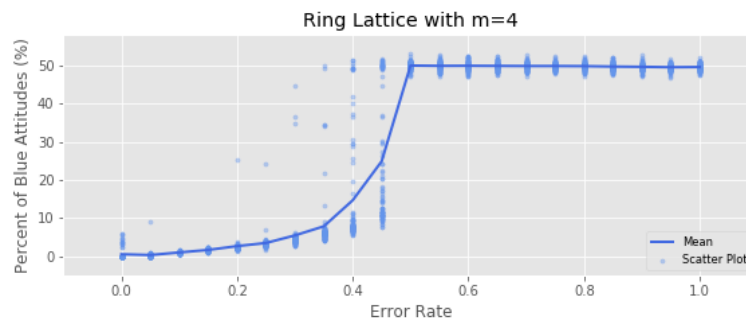
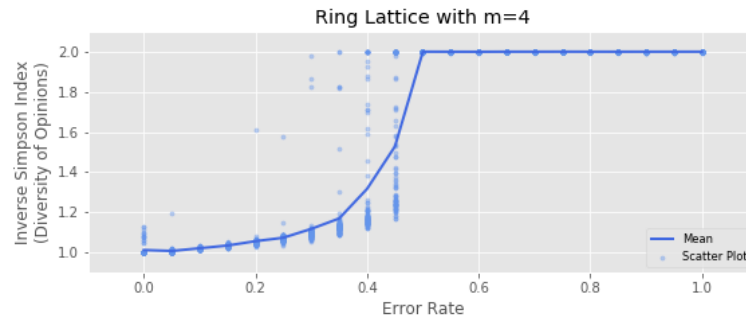


Fig S4A.9. Cultural diversity with varying error rates

(a) Inverse Simpson index



(b) Hamming distance

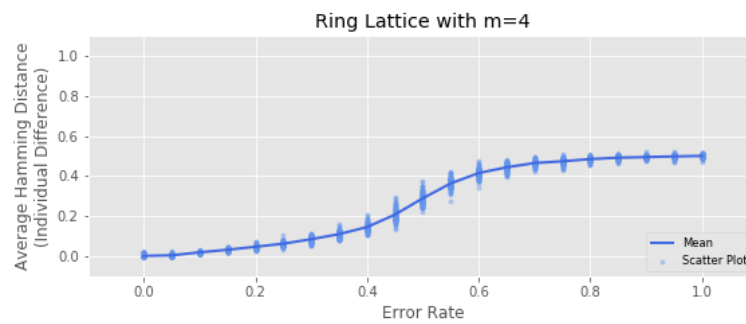
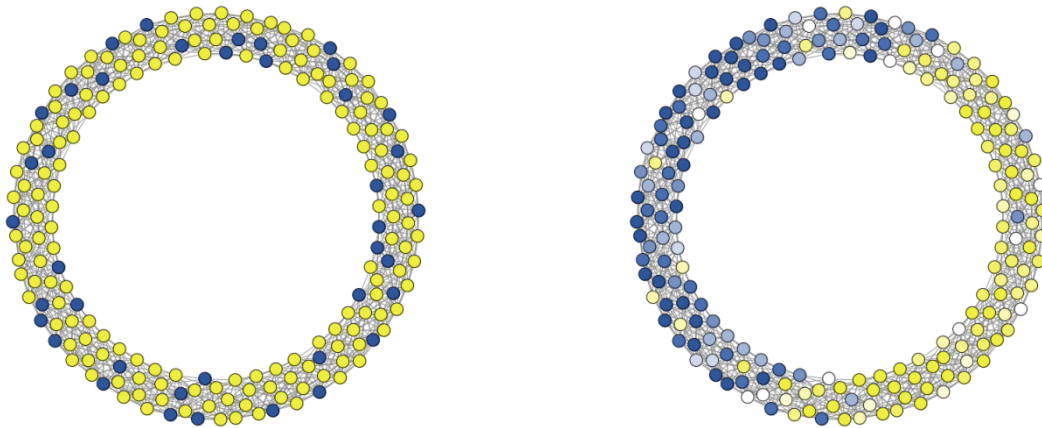


Fig S4A.10. The pattern of social change on a ring lattice ($n = 200$, $m = 8$). Error rate = 0.60



1. 20% initial minority (blue) agents are randomly distributed across the network.
2. As the simulation goes, two factions emerge representing two opposite positions.

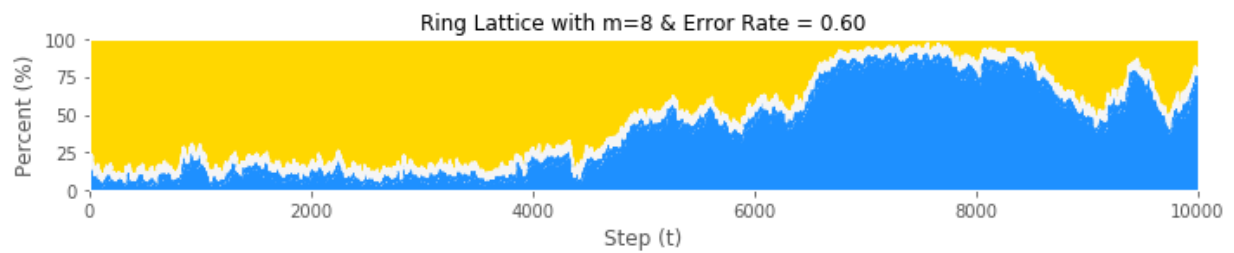
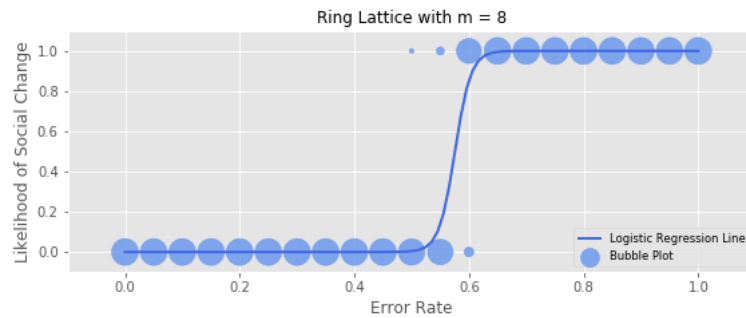
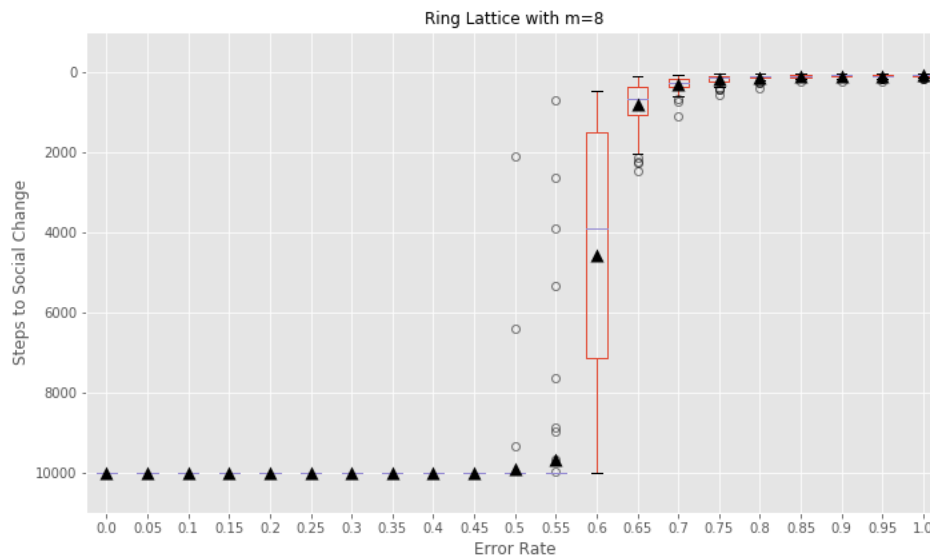


Fig S4A.11. The effect of error rates on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of initial minority on a ring lattice ($n=200$, $m=8$)

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

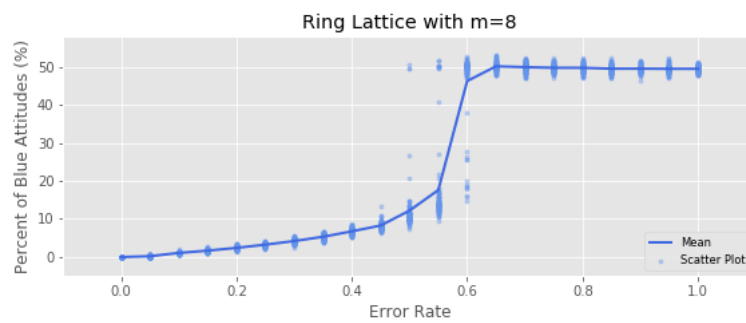
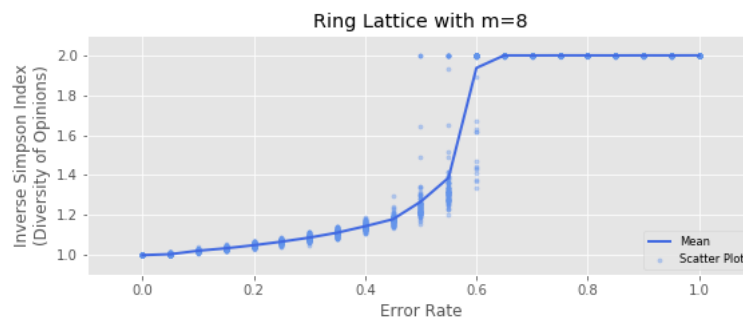
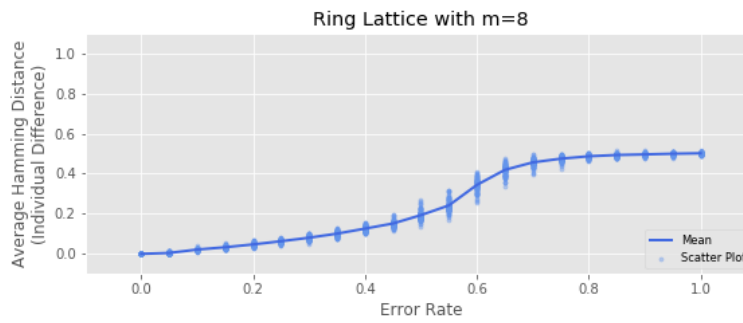


Fig S4A.12. Cultural diversity with varying error rates on a ring lattice ($n=200$, $m=8$)

(a) Inverse Simpson index



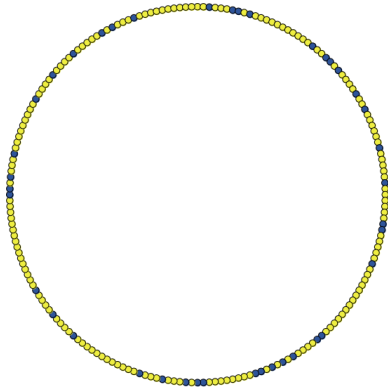
(b) Hamming distance



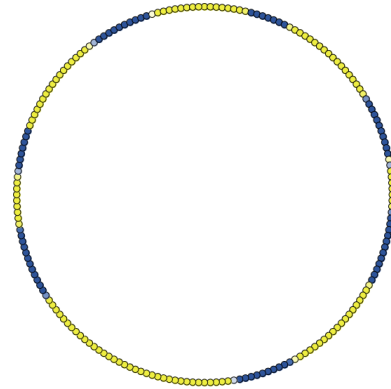
Supplementary Online Material 4B

The Patterns of Social Change and Diversity with Varying Leniency Thresholds on Ring Lattices

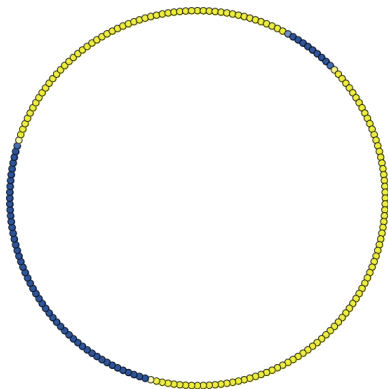
Fig S4B.1. The pattern of social change on a ring lattice ($n = 200$, $m = 1$). Leniency threshold = 0.95.



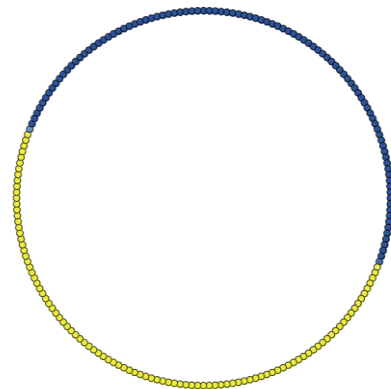
1. 20% initial minority (blue) agents are randomly distributed across the network.



2. Minority forms cliques of varying sizes.



3. The number of minority cliques decreases. Small cliques merge and form a bigger clique.



4. Social change occurs—with one or two minority cliques. High global diversity but low local diversity indicates societal polarization.

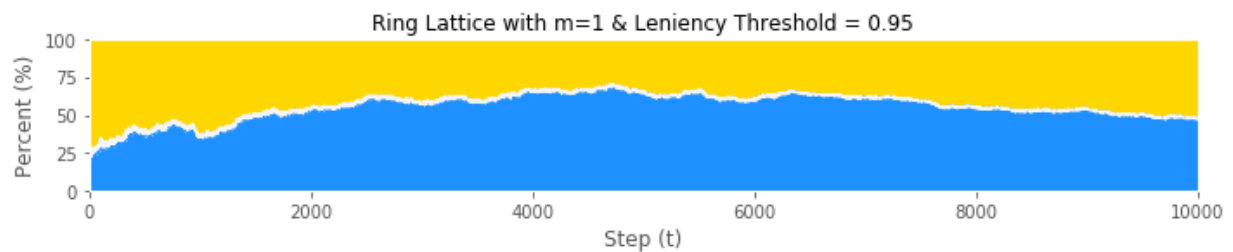
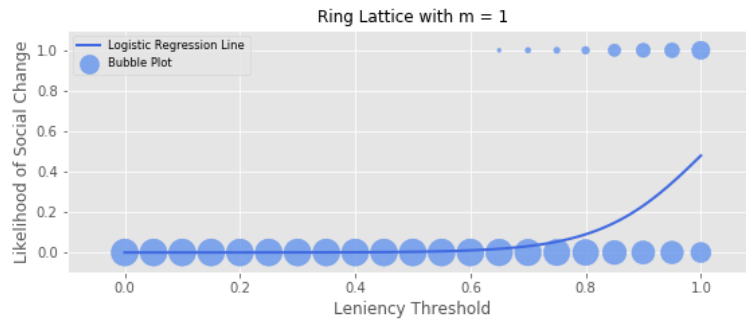
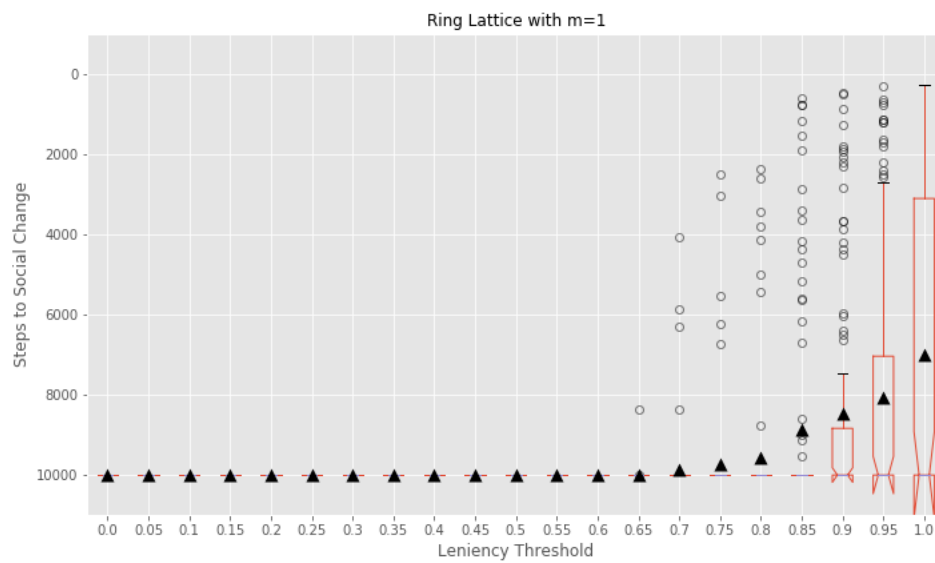


Fig S4B.2. The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of initial minority on a ring lattice ($n=200$, $m=1$)

(a) The likelihood of social change



(b) Steps to social change



(c) The spread of initial minority

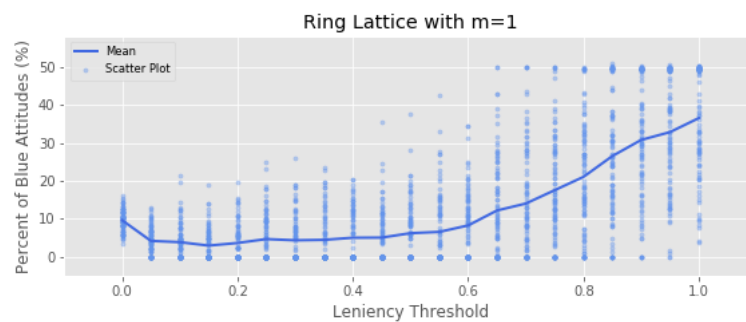
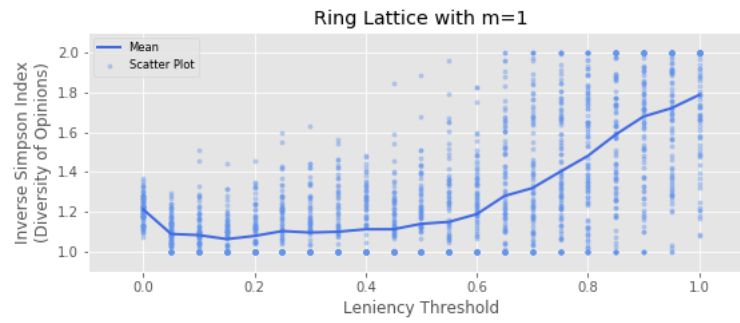


Fig S4B.3. Cultural diversity with varying leniency thresholds on ring lattices ($n=200$, $m=1$).

(a) Inverse Simpson index



(b) Hamming distance

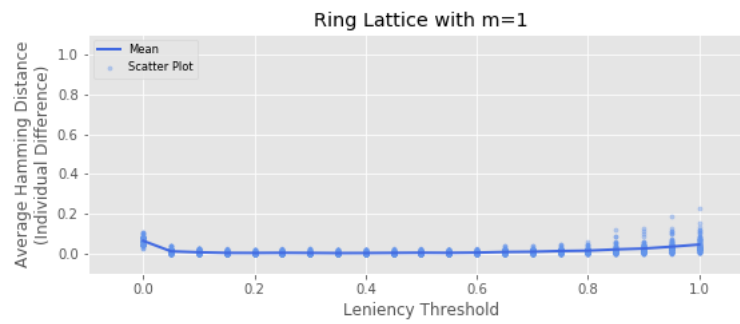
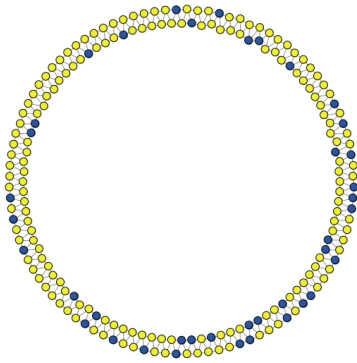
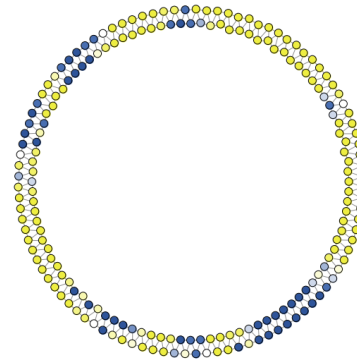


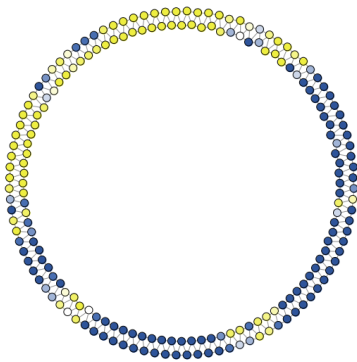
Fig S4B.4. The pattern of social change on a ring lattice ($n = 200$, $m = 2$). Leniency threshold = 0.65.



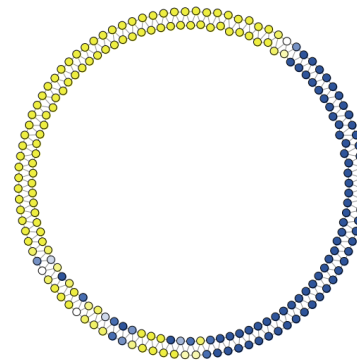
1. 20% initial minority (blue) agents are randomly distributed across the network.



2. Minority forms cliques of varying sizes.



3. The number of minority cliques decreases. Small cliques merge and form a bigger clique.



4. When social change occurs, there are typically a few minority cliques exist. Sometimes two groups emerge indicating societal polarization, but often around the boundary of two groups, small cliques split from the larger group.

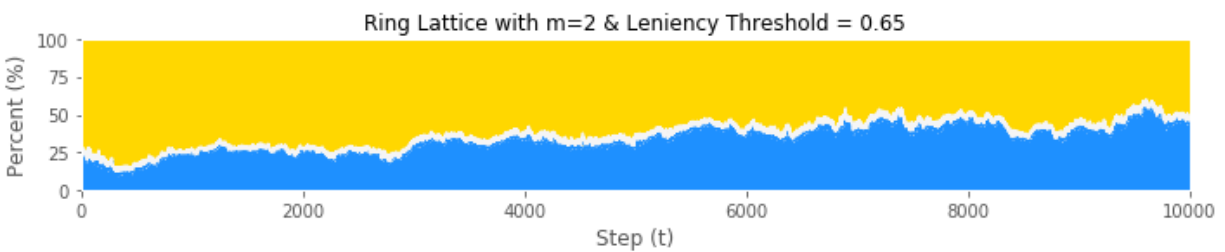
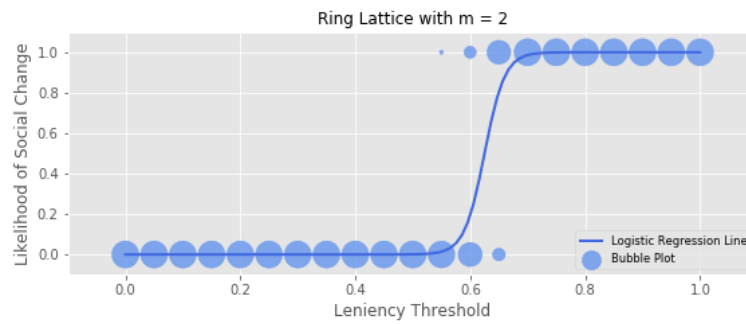
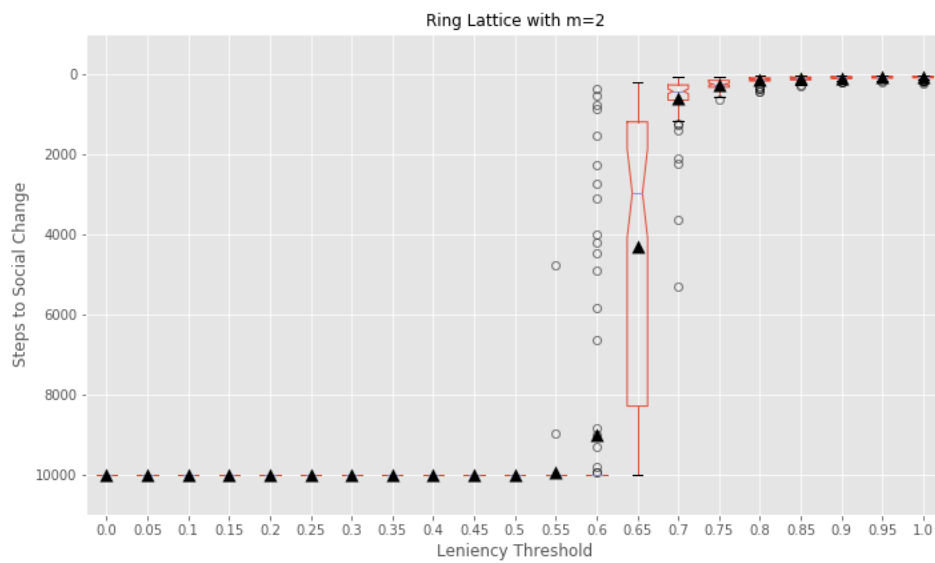


Fig S4B.5. The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a ring lattice ($n=200$, $m=2$). The tipping point is 0.65.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

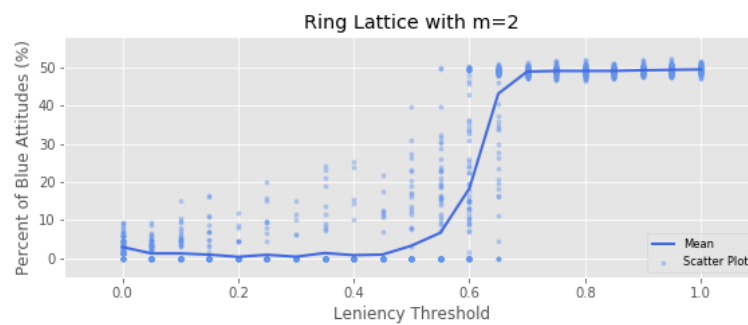
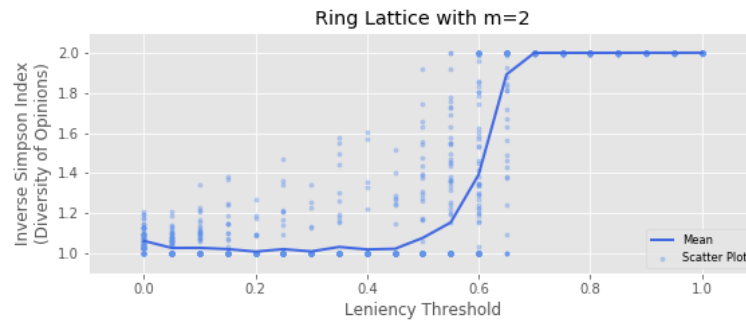


Fig S4B.6. Cultural diversity with varying leniency thresholds on ring lattices ($n=200$, $m=2$).

(a) Inverse Simpson index



(b) Hamming distance

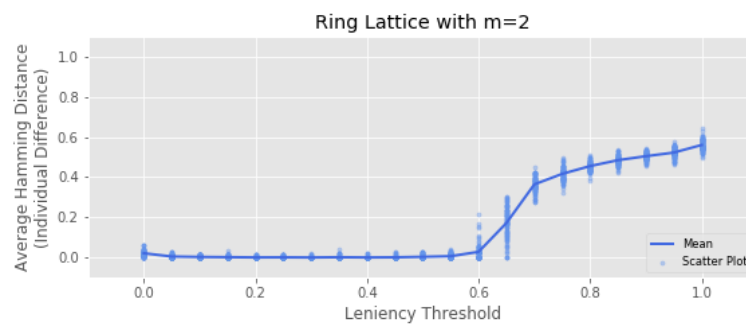
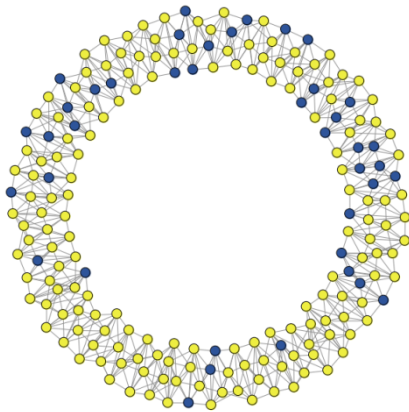
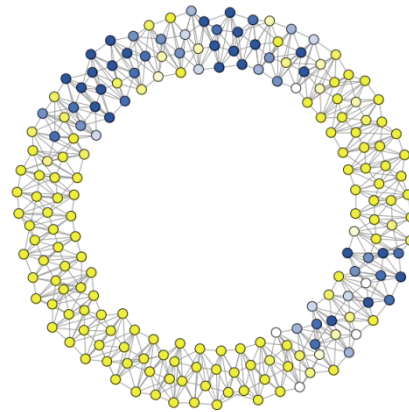


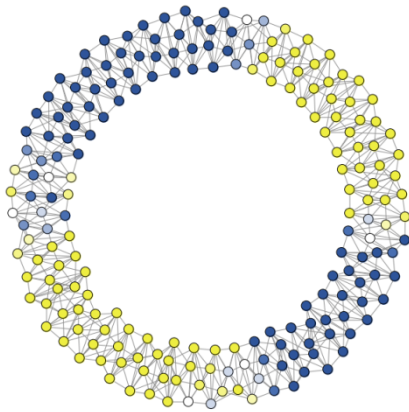
Fig S4B.7. The pattern of social change on a ring lattice ($n=200$, $m=4$). Leniency threshold = 0.55.



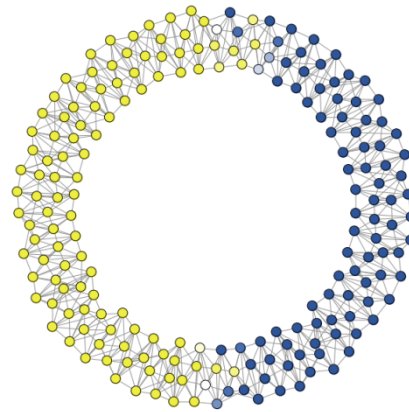
1. 20% initial minority (blue) agents are randomly distributed across the network.



2. Minority forms cliques of varying sizes. The number of minority cliques tends to decrease as the simulation continues.



3. When social change occurs, there are typically a few minority cliques exist. At the boundary of two groups, local diversity is highest. The society oscillates between two positions.



4. As the simulation goes, two segregated factions emerge. However, this can also split into smaller cliques. High global diversity and low local diversity indicates societal polarization.

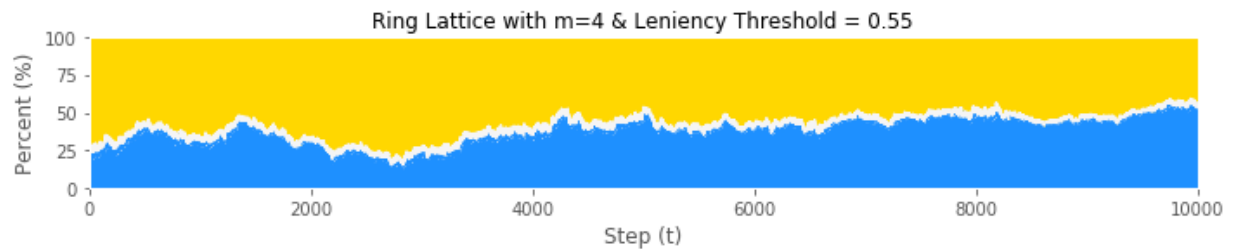
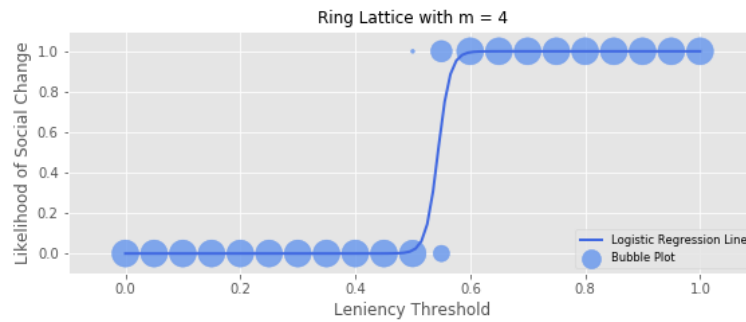
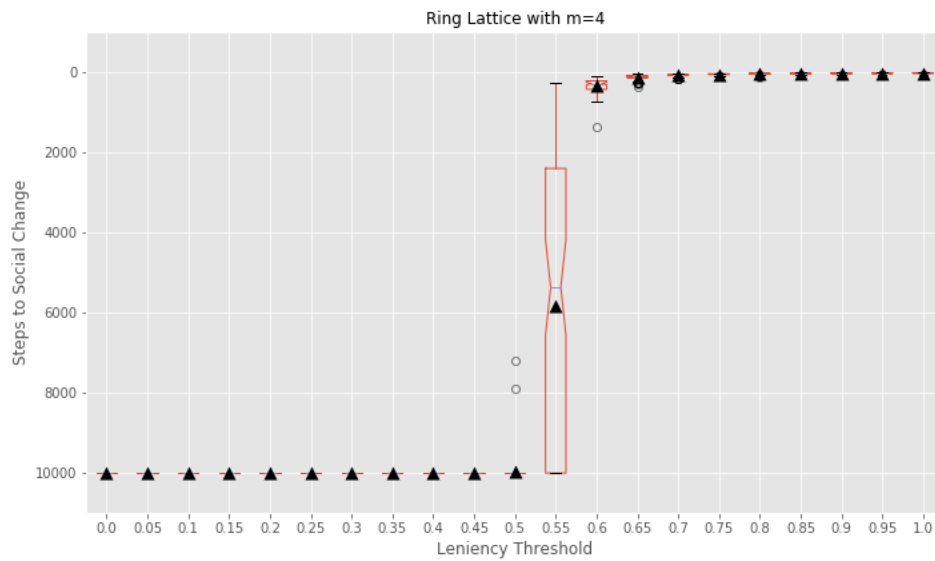


Fig S4B.8. . The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a ring lattice ($n=200$, $m=4$). The tipping point is 0.55.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

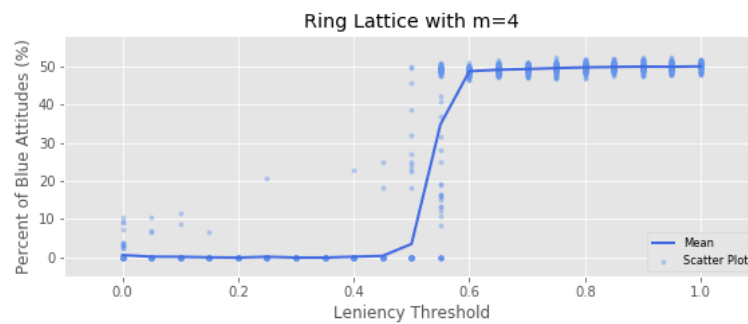
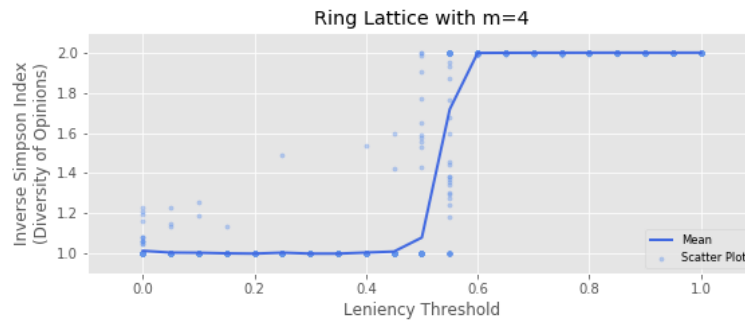


Fig S4B.9. Cultural diversity with varying leniency thresholds on ring lattices ($n=200$, $m=4$).

(a) Inverse Simpson index



(b) Hamming distance

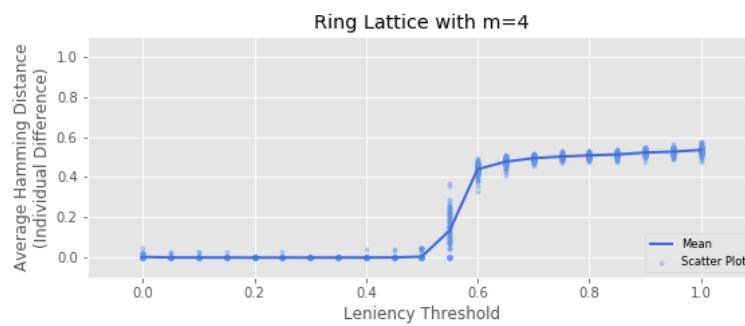
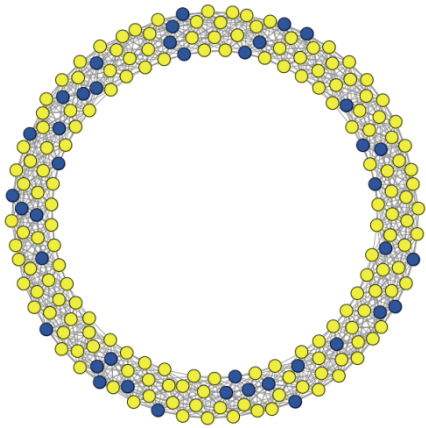
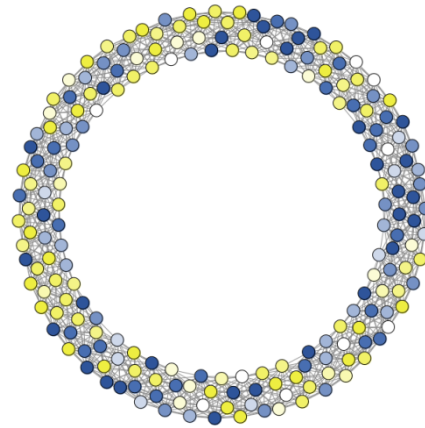


Fig S4B.10. The pattern of social change on a ring lattice ($n = 200$, $m = 8$). Leniency threshold = 0.55.



1. 20% initial minority (blue) agents are randomly distributed across the network.



2. Social change occurs. Both global and local diversity indices reach maximum level – Diversity regime. There are equal number of two positions coexist without segregation.

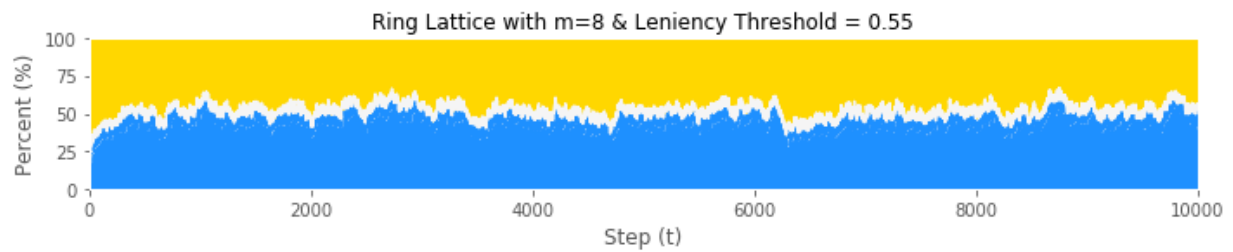
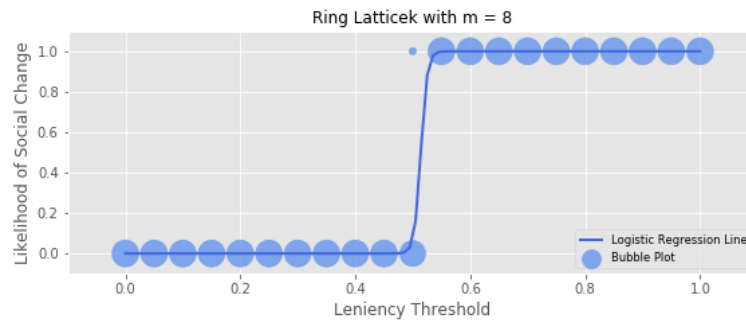
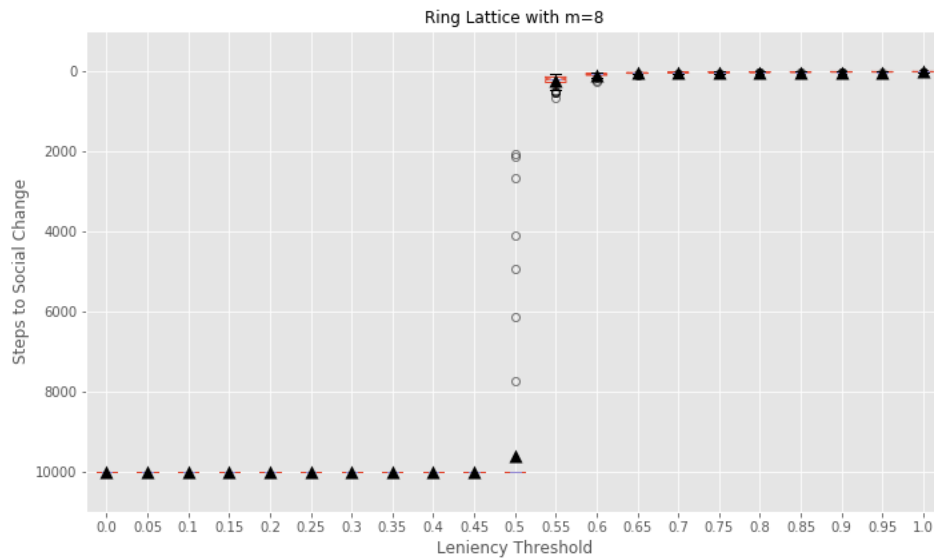


Fig S4B.11. The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a ring lattice ($n=200$, $m=8$). The tipping point is 0.55.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

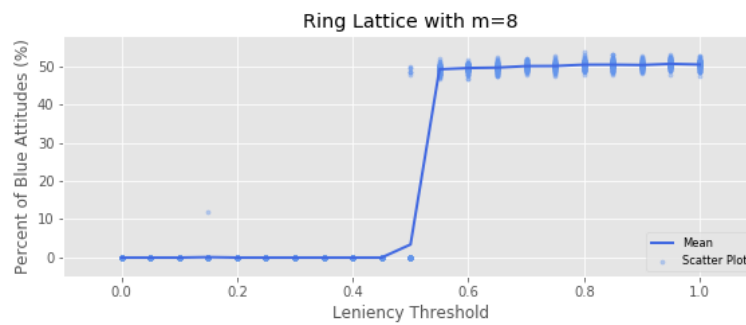
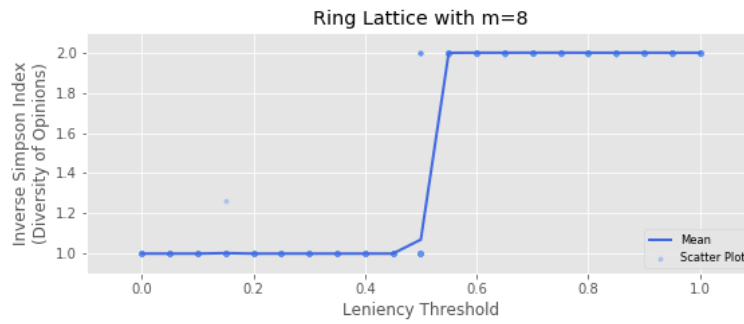
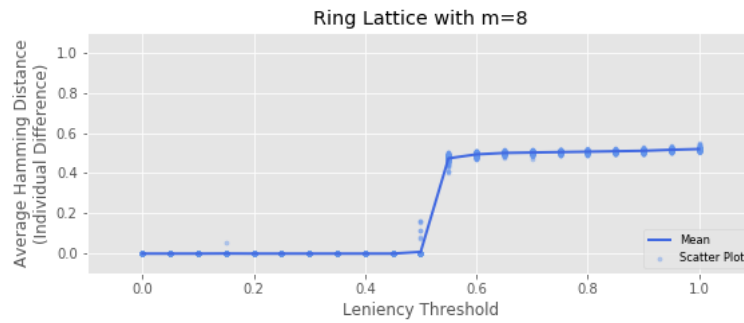


Fig S4B.12. Cultural diversity with varying leniency thresholds on a ring lattice ($n = 200$, $m = 8$)

(a) Inverse Simpson index



(b) Hamming distance



Supplementary Online Material 5A

The Patterns of Social Change and Diversity with Varying Error rates on Square Lattices

Fig S5A.1. The pattern of social change on a square lattice ($n = 196$, $m = 2$). Error rate = 0.50.

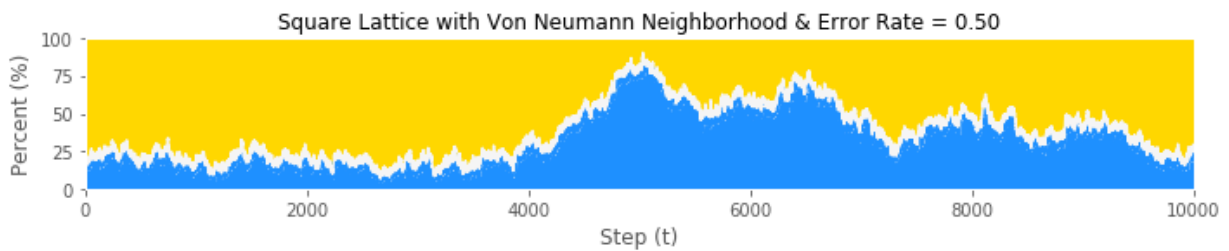
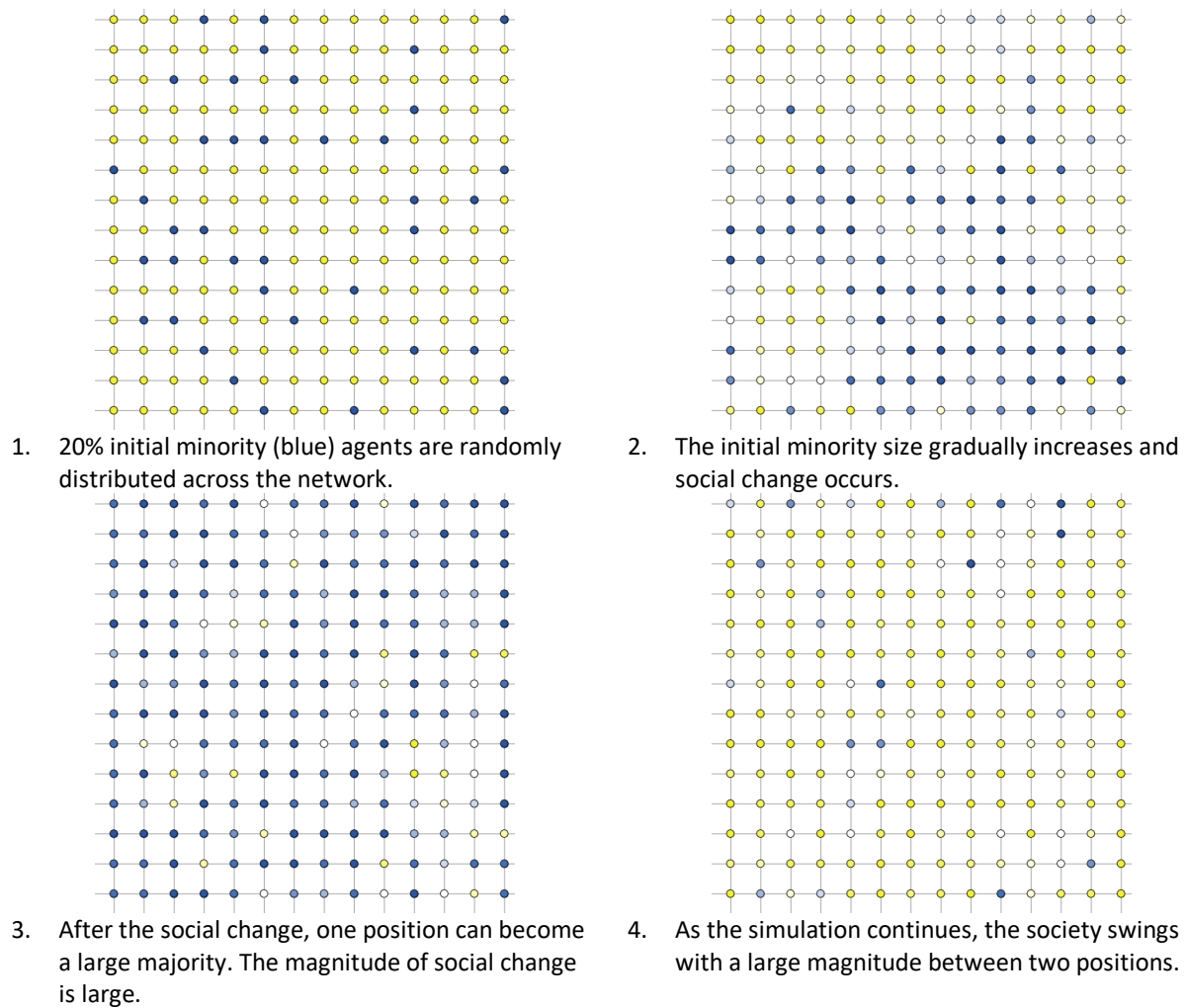
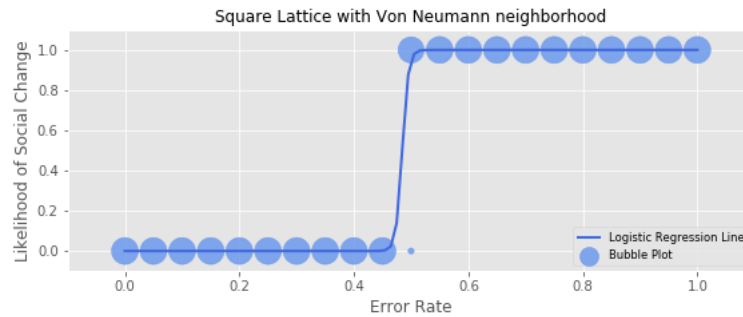
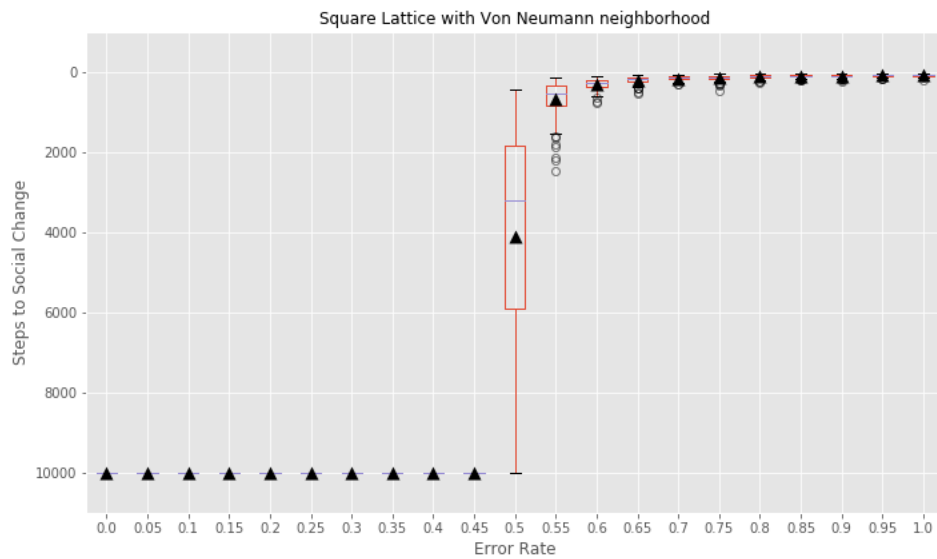


Fig S5A.2. The effect of error rates on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a square lattice ($n=196$, $m=2$). The tipping point is 0.50.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

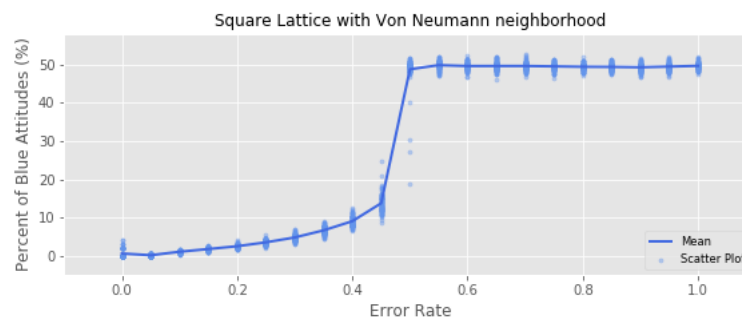
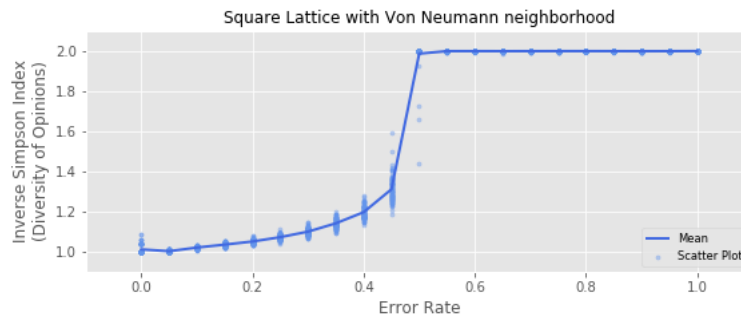


Fig S5A.3. Cultural diversity with varying error rates on square lattices ($n=196$, $m=2$).

(a) Inverse Simpson index



(b) Hamming distance

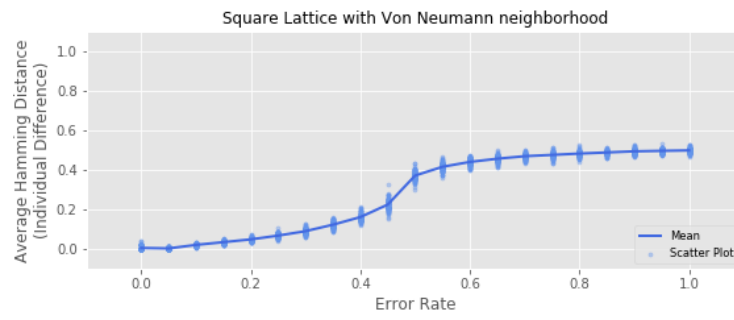


Fig S5A.4. The pattern of social change on a square lattice ($n=196$, $m=4$). Error rate = 60

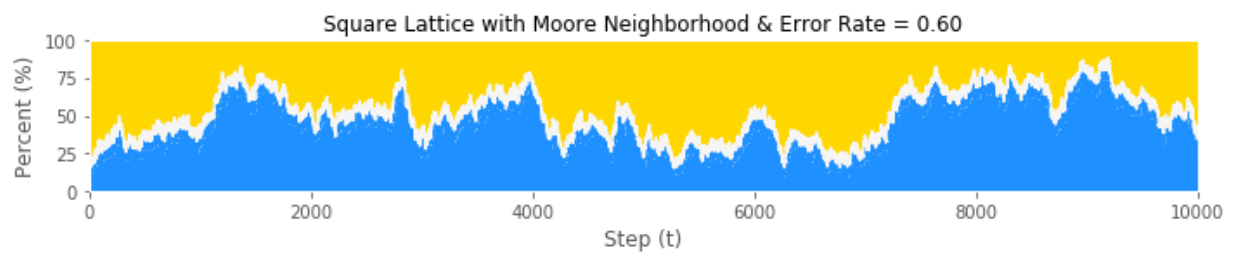
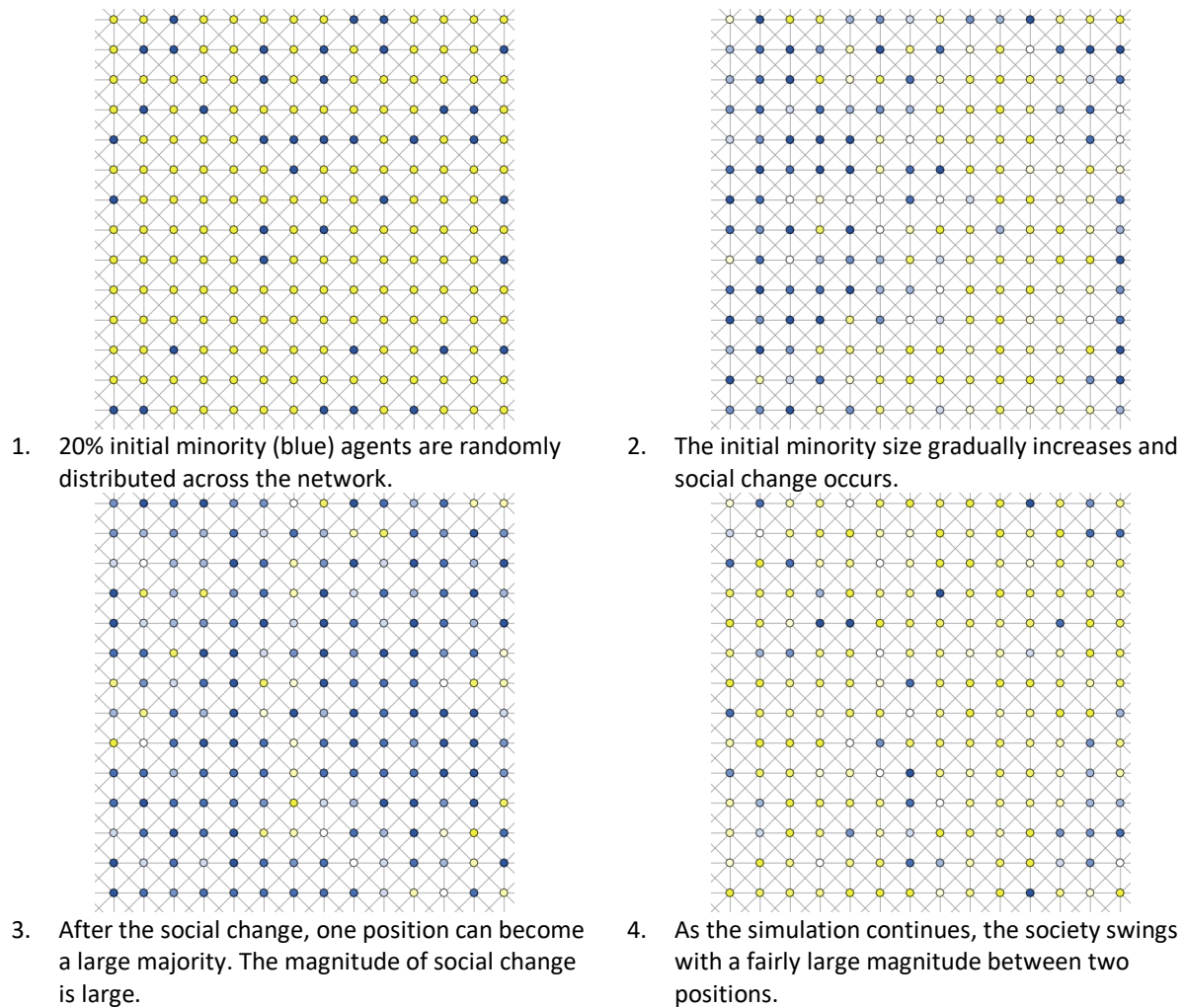
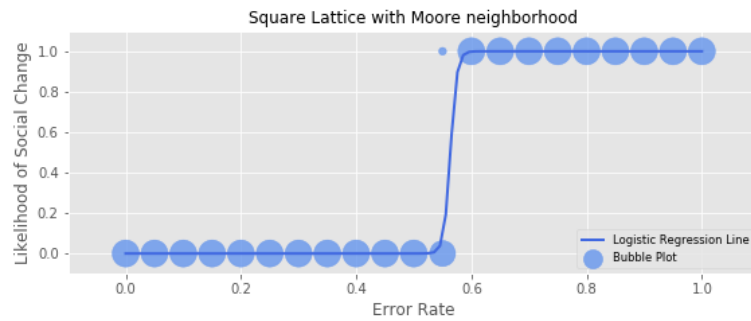
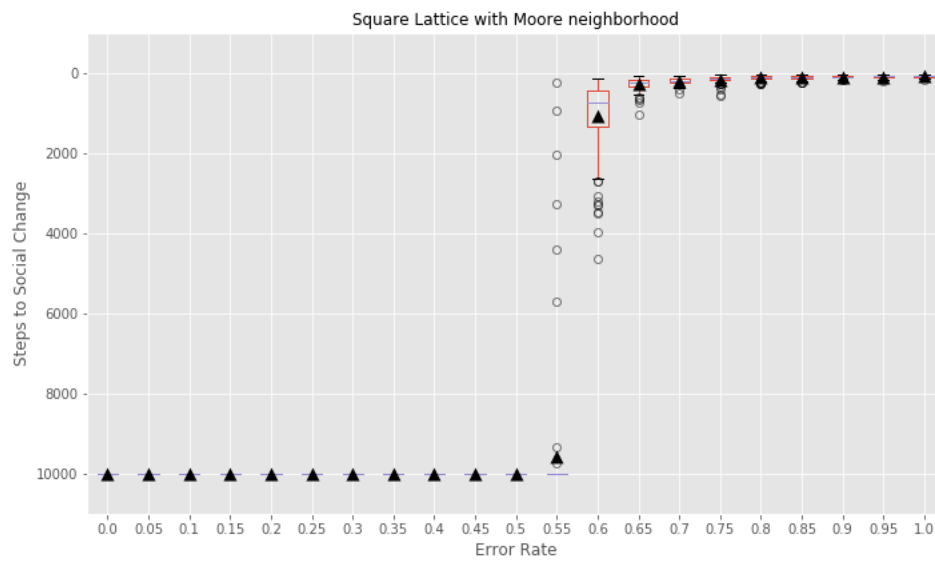


Fig S5A.5. The effect of error rates on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a square lattice ($n=196$, $m=4$). The tipping point is 0.6.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

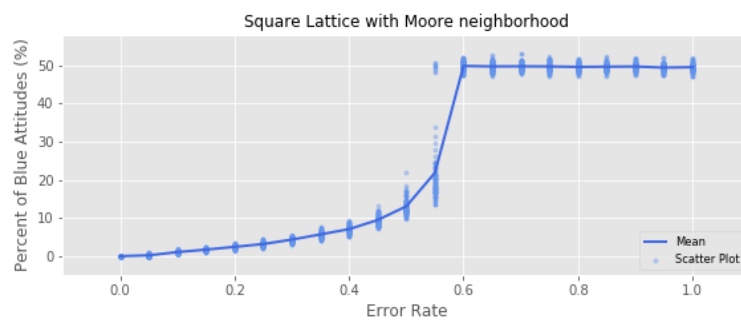
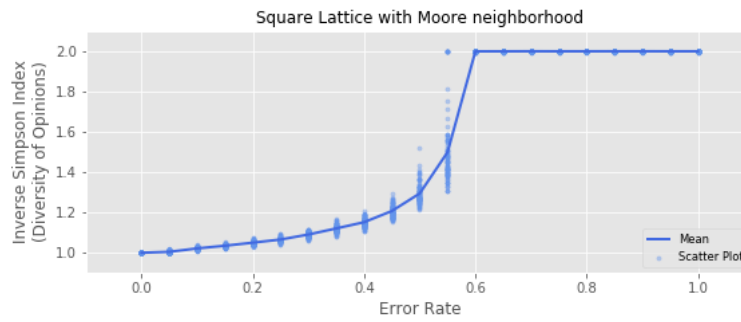
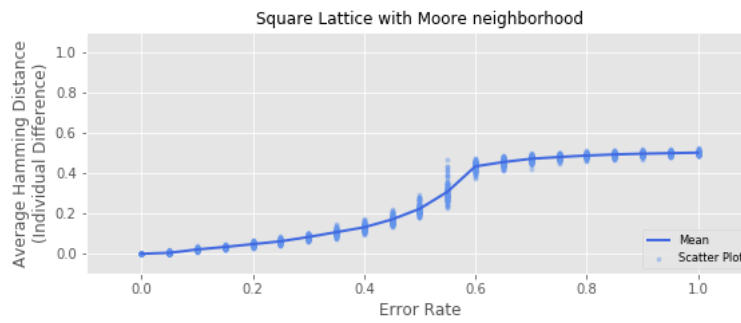


Fig S5A.6. Cultural diversity with varying error rates on square lattices ($n=196$, $m=4$)

(a) Inverse Simpson index



(b) Hamming distance



Supplementary Online Material 5B

The Patterns of Social Change and Diversity with Varying Leniency Thresholds on Square Lattices

Fig S5B.1. The pattern of social change on a square lattice ($n = 200$, $m = 2$). Leniency threshold = 0.55.

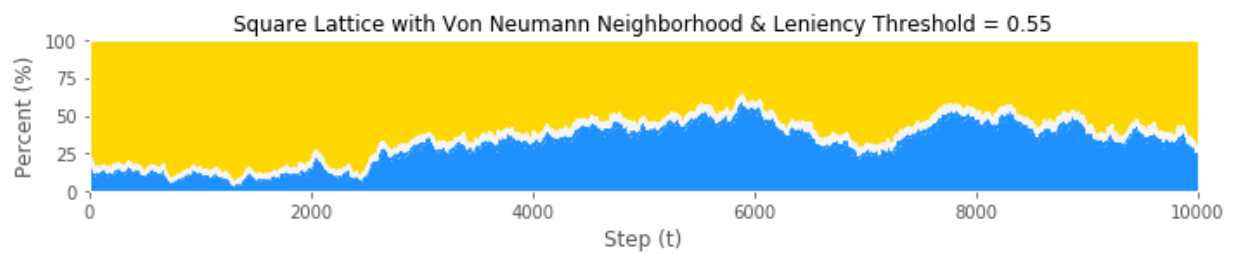
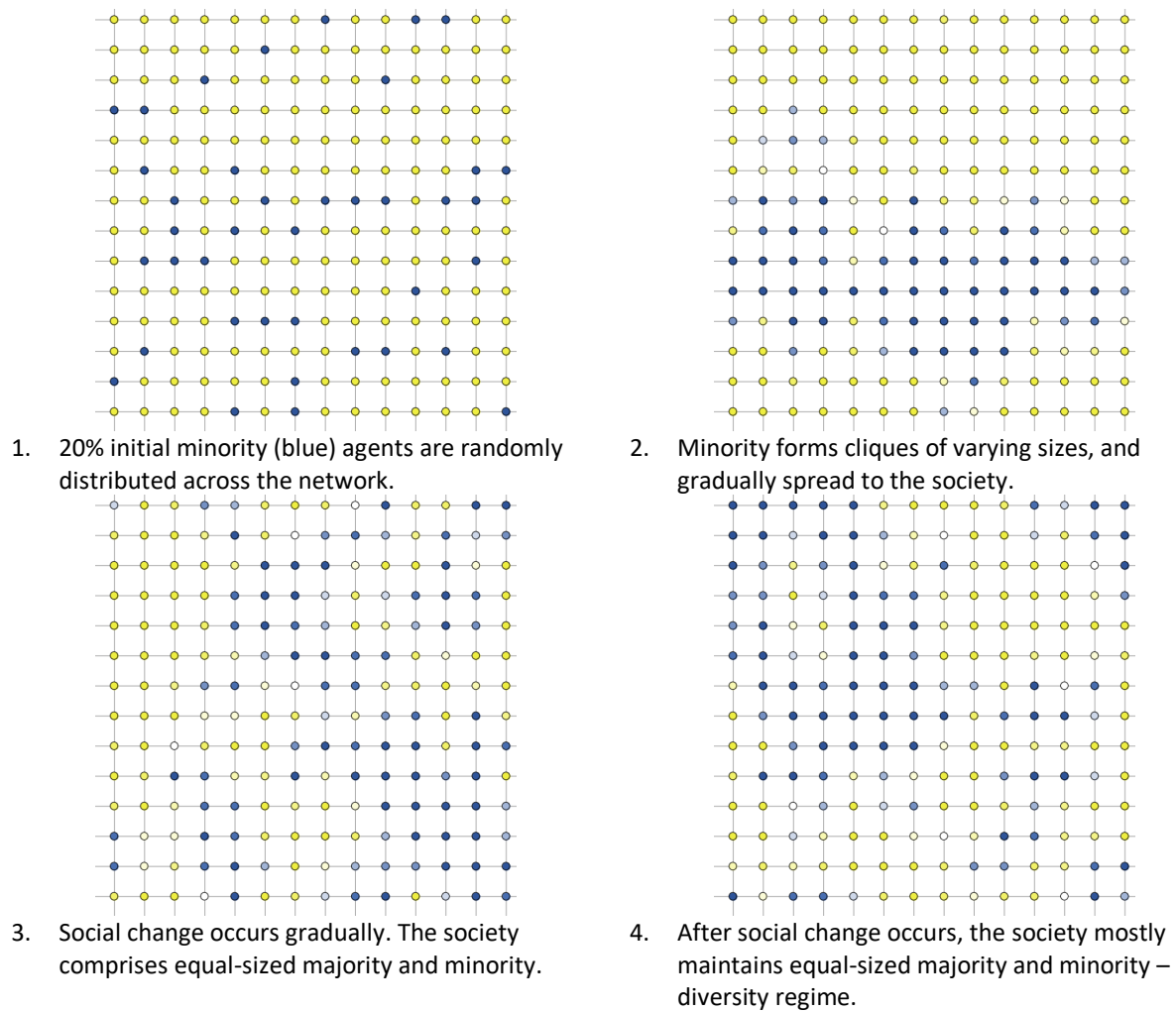
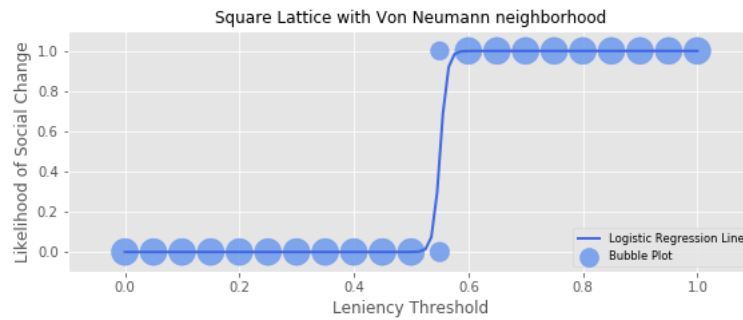
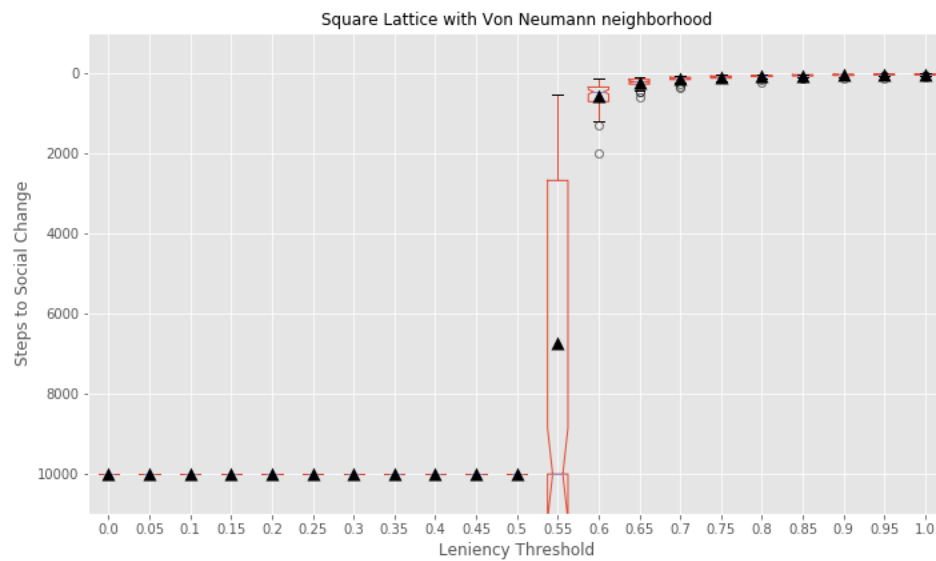


Fig S5B.2. The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a square lattice ($n=196$, $m=2$). The tipping point is 0.55.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

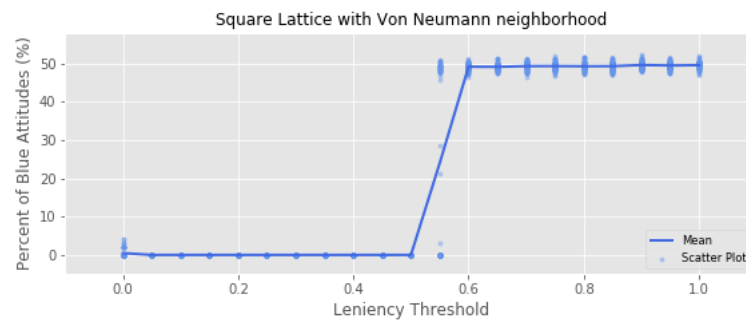


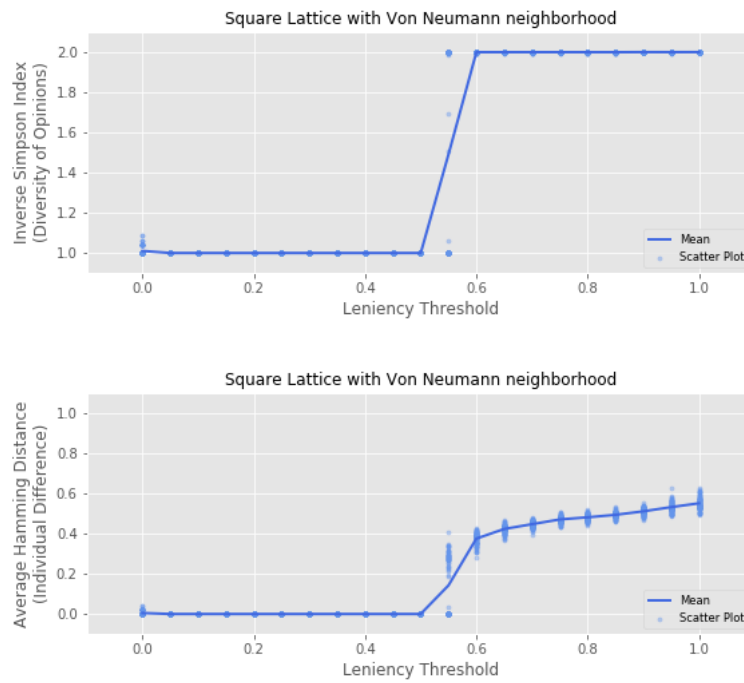
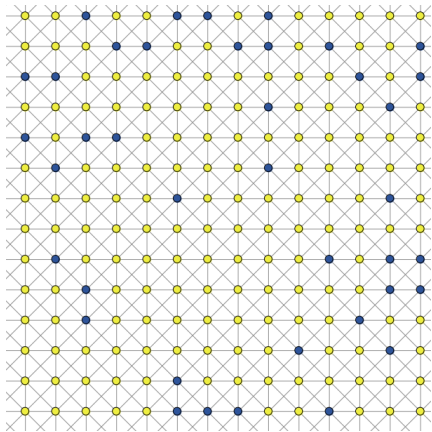
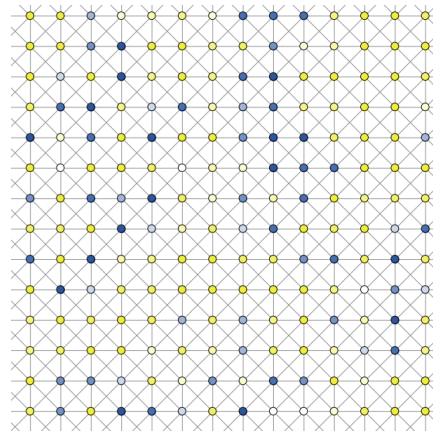
Fig S5B.3. Cultural diversity with varying leniency thresholds on square lattices ($n=196$, $m=2$).

Fig S5B.4. The pattern of social change on a square lattice ($n=196$, $m=4$). Leniency threshold = 0.55



1. 20% initial minority (blue) agents are randomly distributed across the network.



2. Minority size increases gradually. There are many small cliques form. Agents are connected to both similar and different agents. The society achieves a diversity regime.

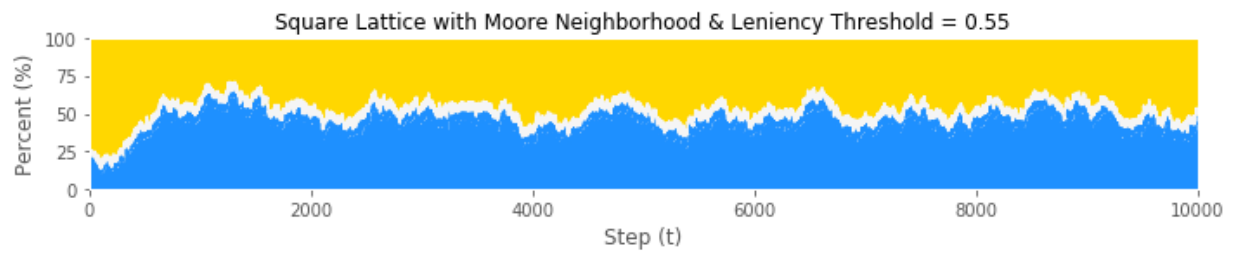
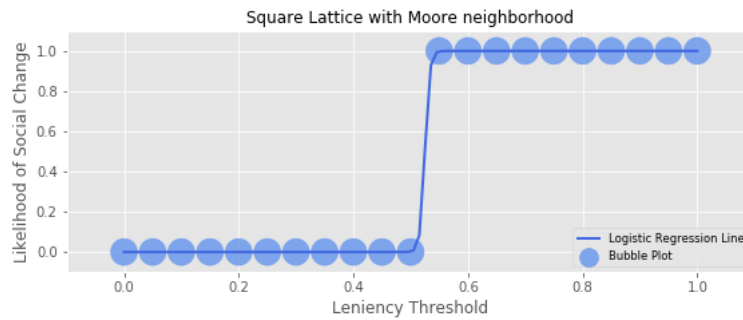
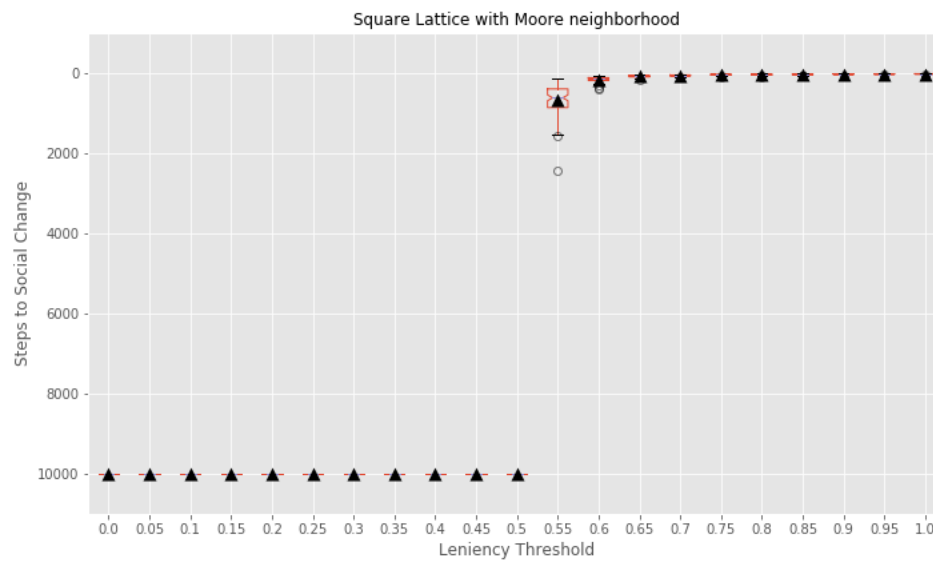


Fig S5B.5. The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a square lattice ($n=196$, $m=4$). The tipping point is 0.55.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

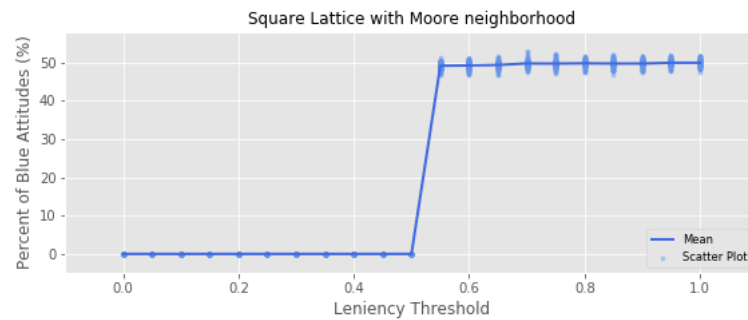
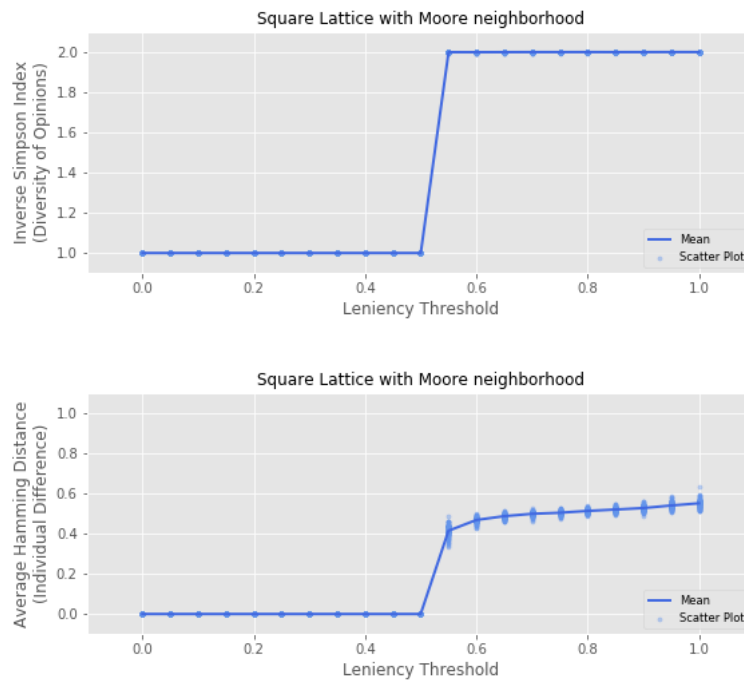
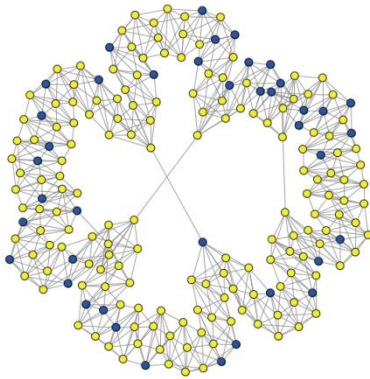


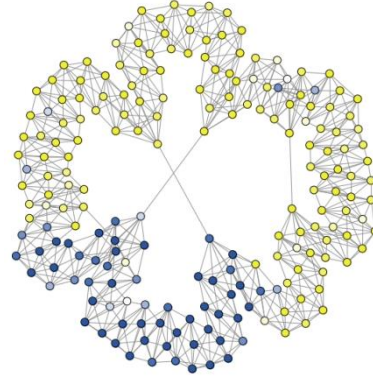
Fig S5B.6. Cultural diversity with varying leniency thresholds on square lattices ($n=196$, $m=4$).

Supplementary Online Material 6A
The Patterns of Social Change and Diversity with Varying Error Rates
on the Watts-Strogatz Small world networks (N=200, m=4)

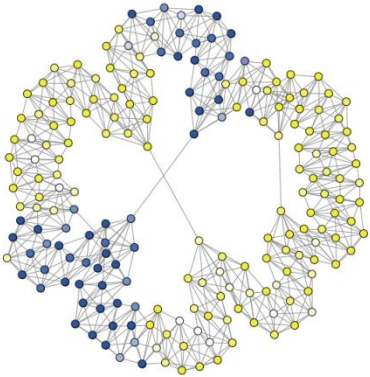
Fig S6A.1. The pattern of social change on a small world network ($p = 0.005$). Error Rate = 0.45.



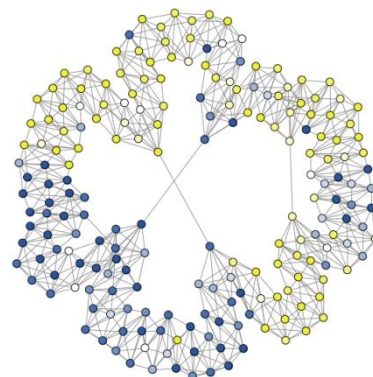
1. 20% initial minority (blue) agents are randomly distributed across the network.



2. Minority forms cliques of varying sizes.



3. In small world networks, minority opinions not just spread via clustered ties but via long ties formed by rewiring. When two minority cliques connected via a long tie, they reinforce each other and become a persistent large minority group.



4. Social change occurs. —with typically one large clique and several small cliques. High global diversity but low local diversity indicates societal polarization/segregation.

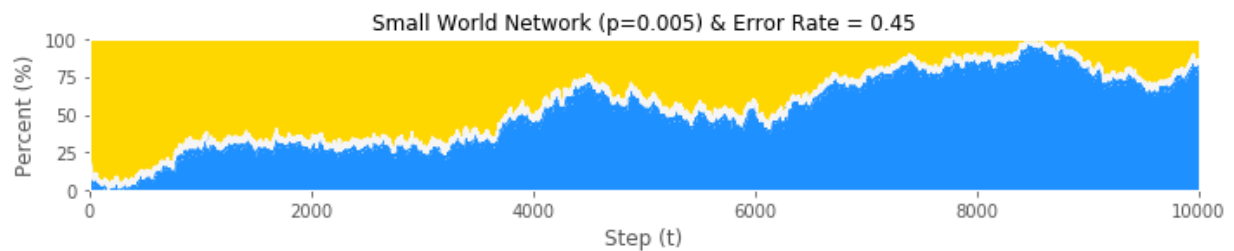
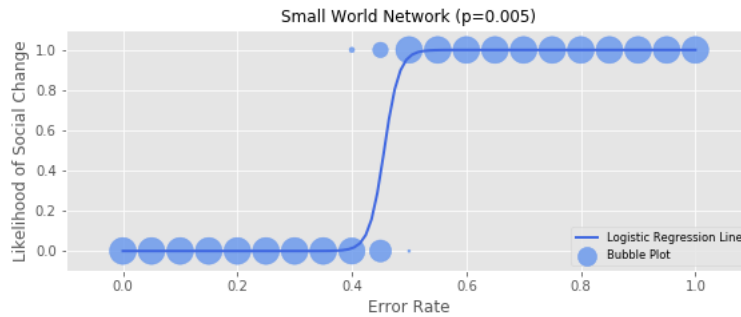
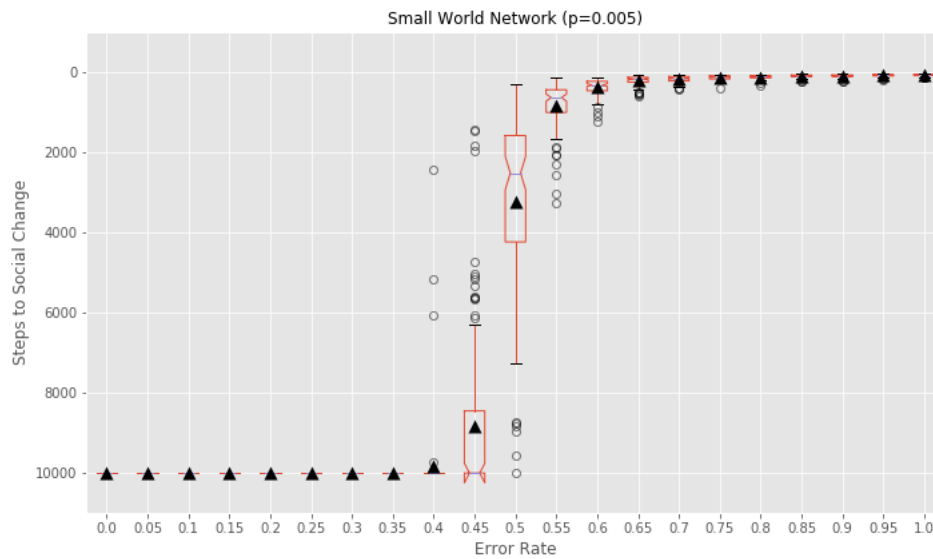


Fig S6A.2. The effect of Error Rates on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of initial minority on a small world network ($p = 0.005$). The tipping point is 0.45.

(a) The likelihood of social change



(b) Steps to social change



(c) The spread of initial minority

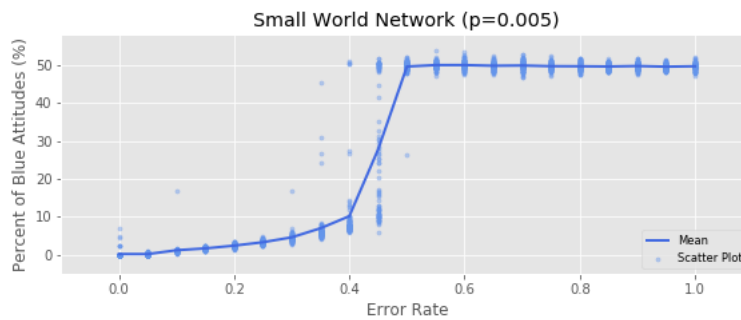


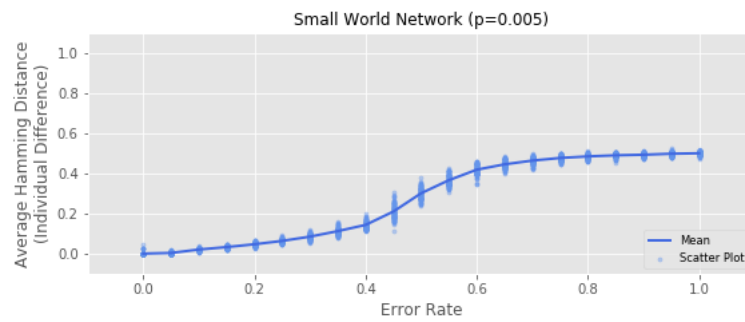
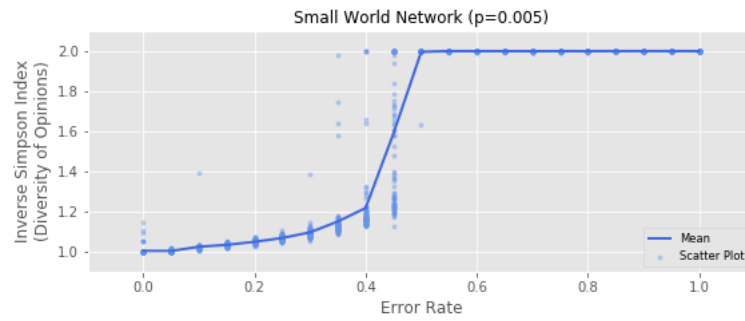
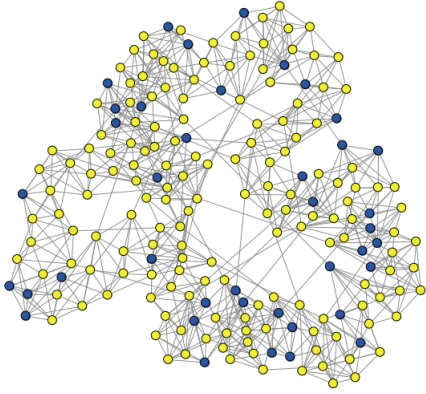
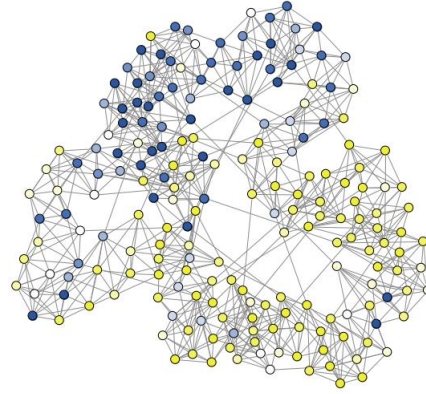
Fig S6A.3. Cultural diversity with varying Error Rates on small world networks ($p = 0.005$).

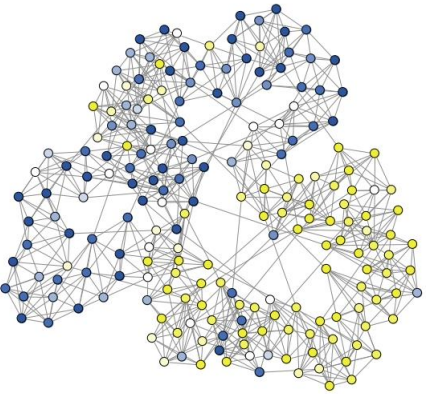
Fig S6A.4. The pattern of social change on a small world network ($p=0.05$). Error Rate = 0.50.



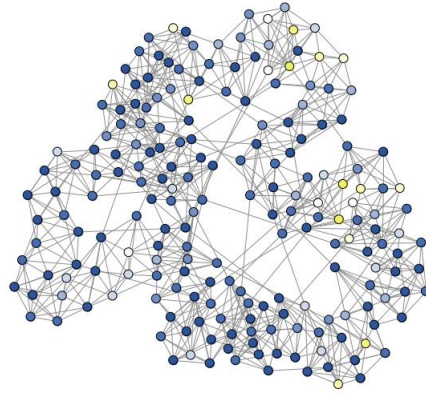
1. 20% initial minority (blue) agents are randomly distributed across the network.



2. Minority forms cliques of varying sizes. Once a clique forms, it tends to persist for a long time. These cliques gradually contract and expand.



3. In small world networks, minority opinions not just spread via clustered ties but via long ties formed by rewiring.



4. As a social change occurs, there is one minority clique connected as one component.

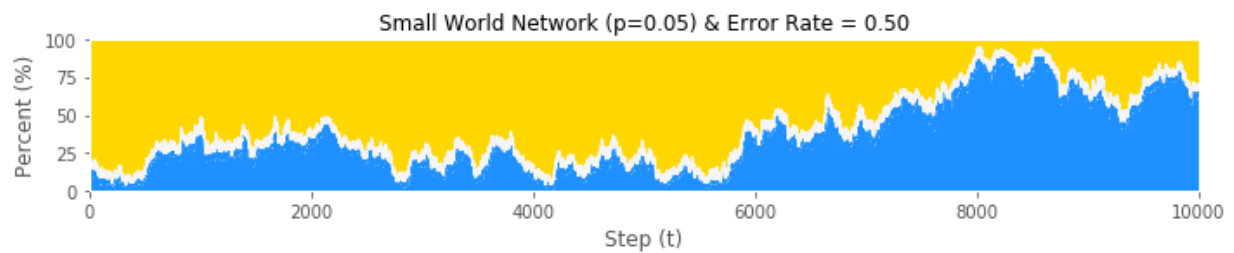
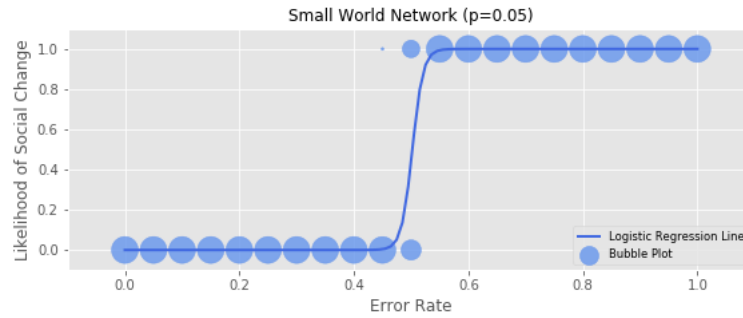
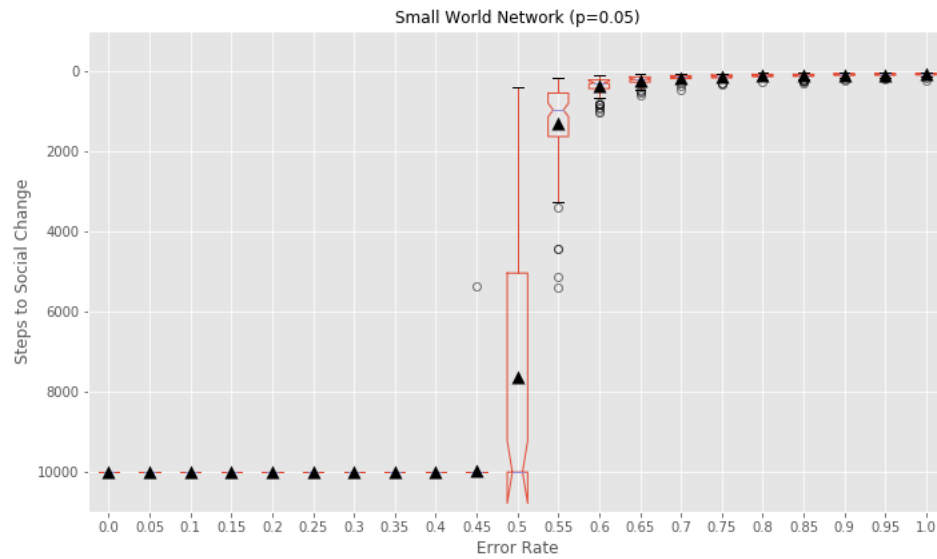


Fig S6A.5. The effect of Error Rates on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a small world network ($p=0.05$). The tipping point is 0.50.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

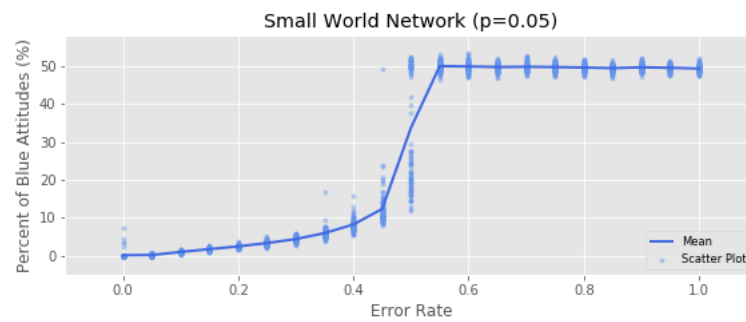


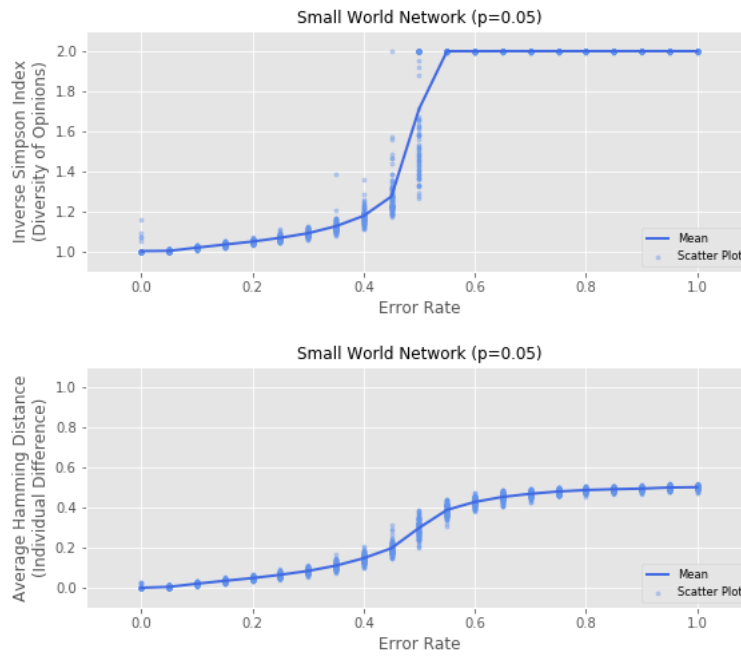
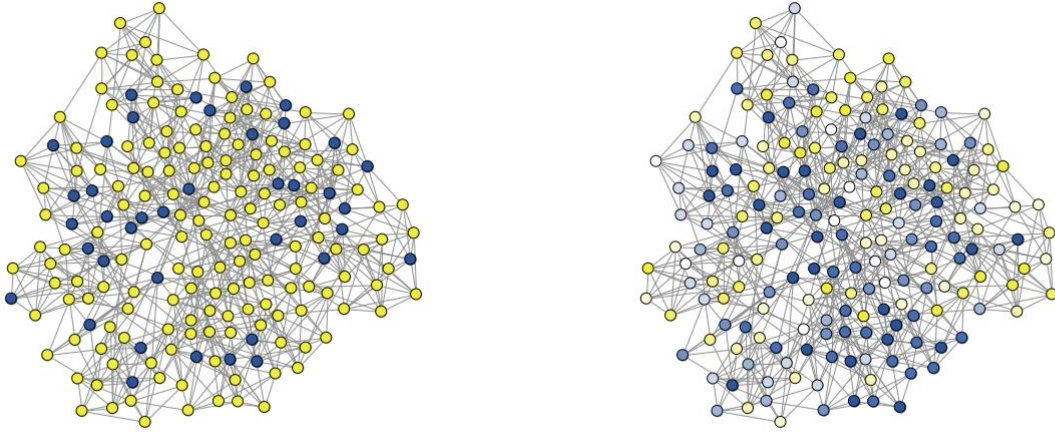
Fig S6A.6. Cultural diversity with varying Error Rates on small world networks ($p=0.05$).

Fig S6A.7. The pattern of social change on a random network ($p = 0.125$). Error Rate = 0.55.



1. 20% initial minority (blue) agents are randomly distributed across the network.
2. Social change occurs.

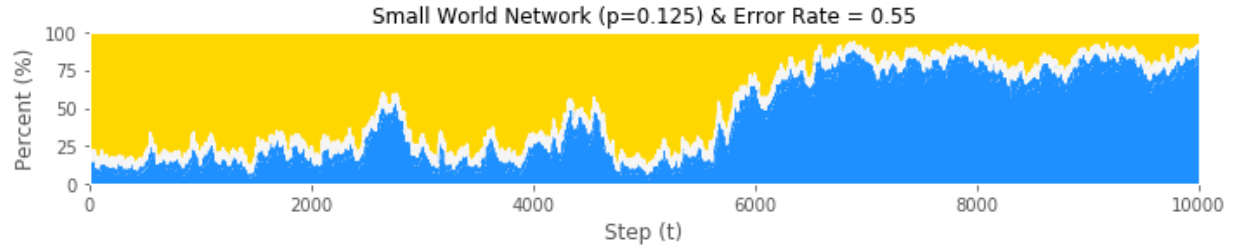
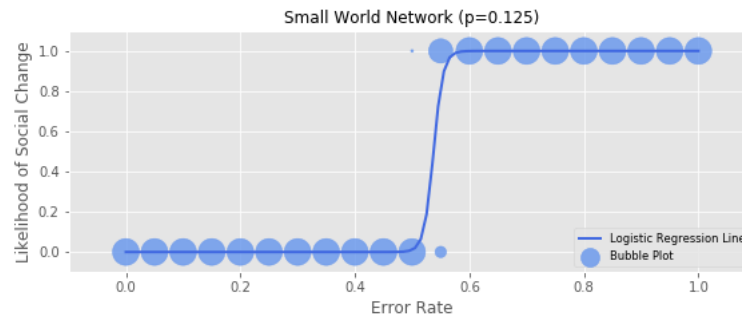
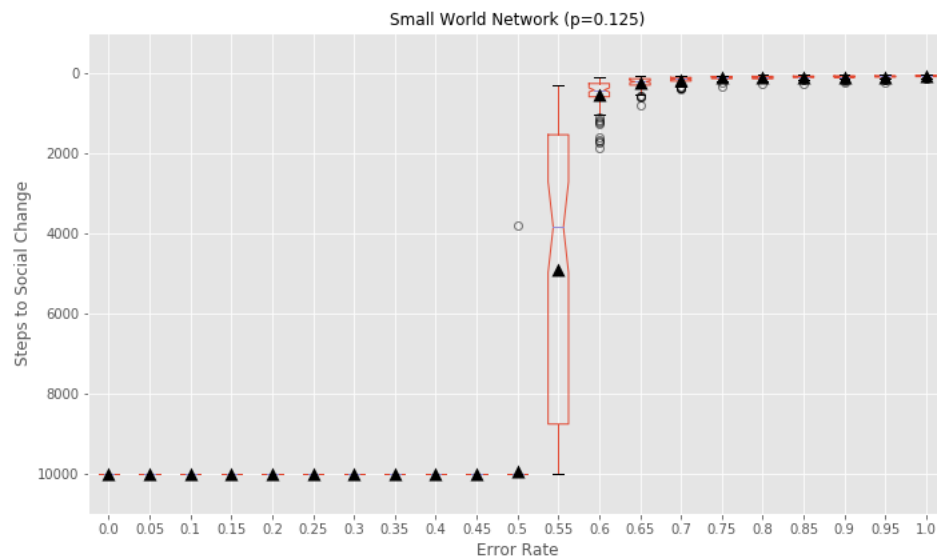


Fig S6A.8. . The effect of Error Rates on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a random network ($p = 0.125$). The tipping point is 0.55.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

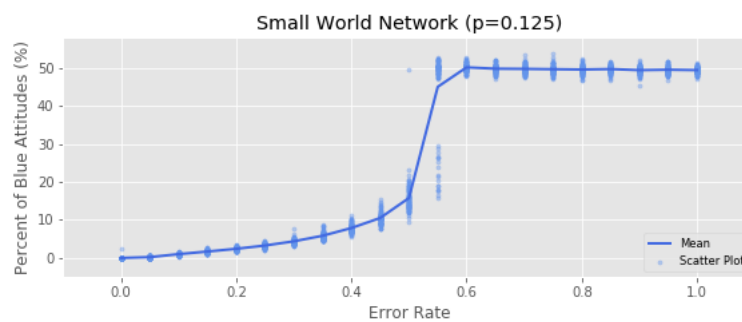


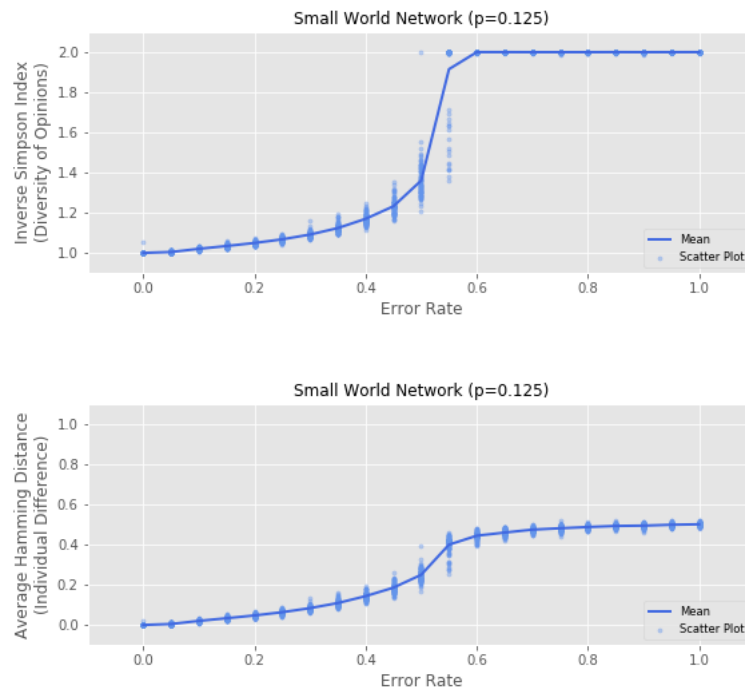
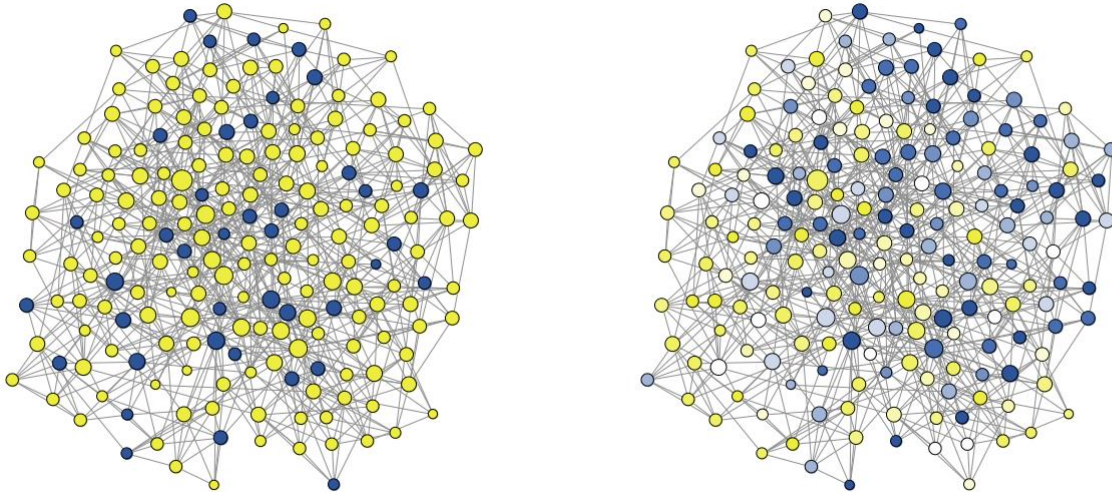
Fig S6A.9. Cultural diversity with varying Error Rates on random networks ($p = 0.125$).

Fig S6A.10. The pattern of social change on a random network ($p = 0.25$). Error Rate = 0.60.



1. 20% initial minority (blue) agents are randomly distributed across the network.
2. Social change occurs.

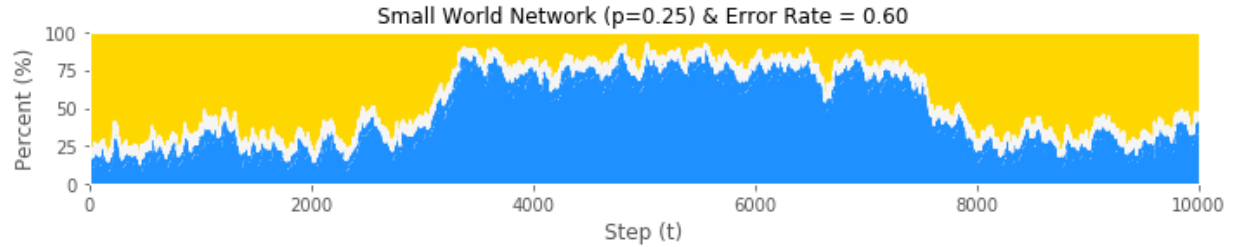
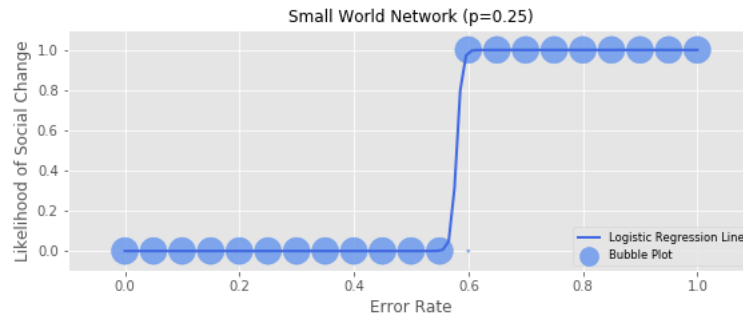
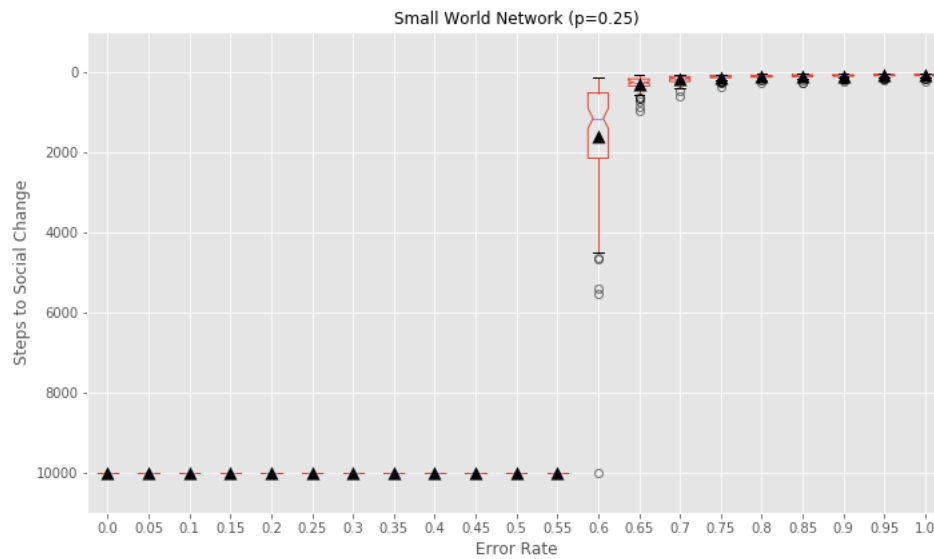


Fig S6A.11. The effect of Error Rates on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a random network ($p = 0.25$).

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

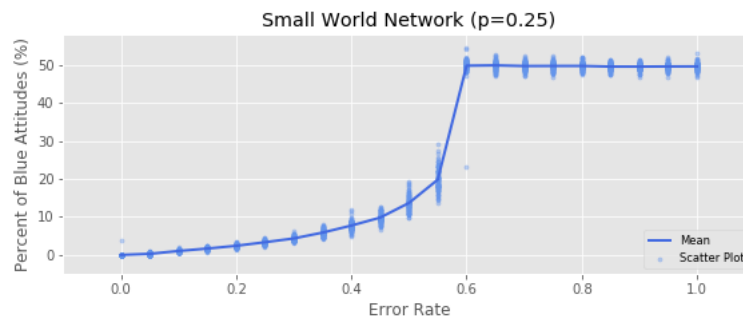


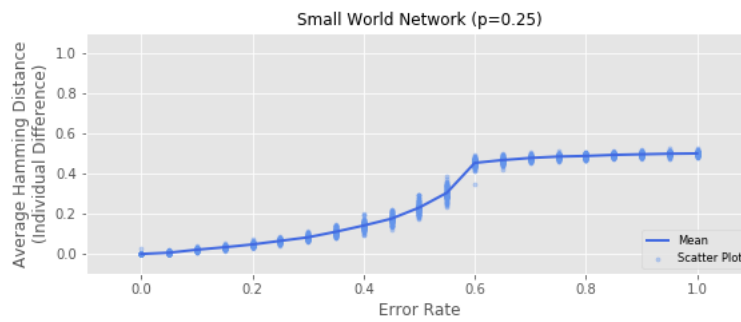
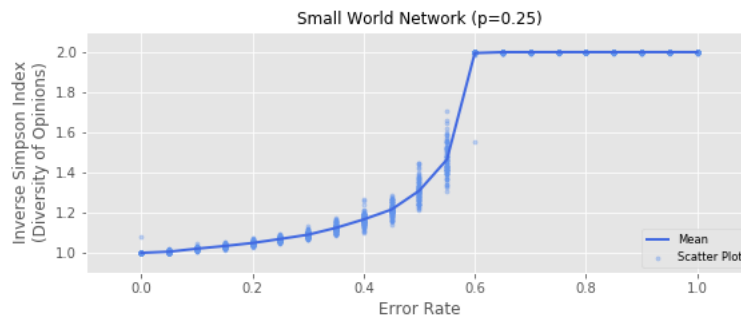
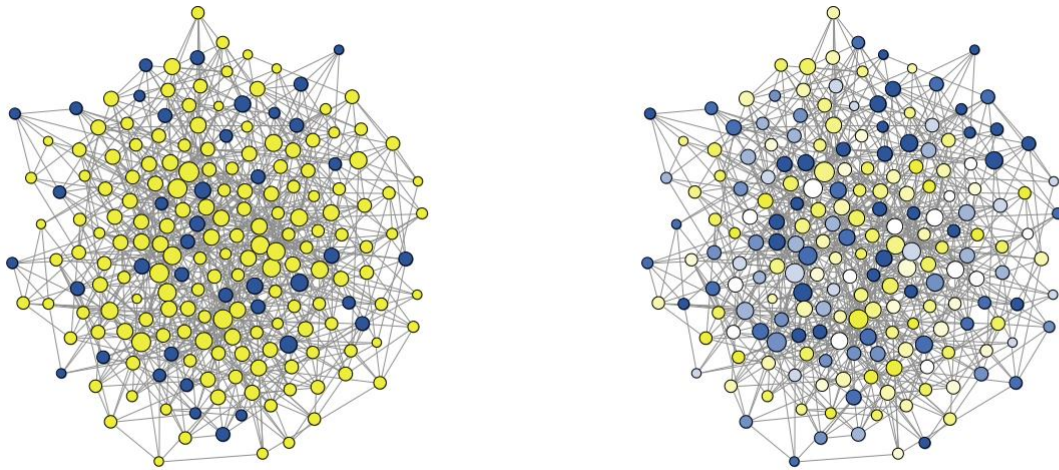
Fig S6A.12. Cultural diversity with varying Error Rates on a random network ($p = 0.25$)

Fig S6A.13. The pattern of social change on a random network ($p = 0.50$). Error Rate = 0.60.



1. 20% initial minority (blue) agents are randomly distributed across the network.
2. Social change occurs.

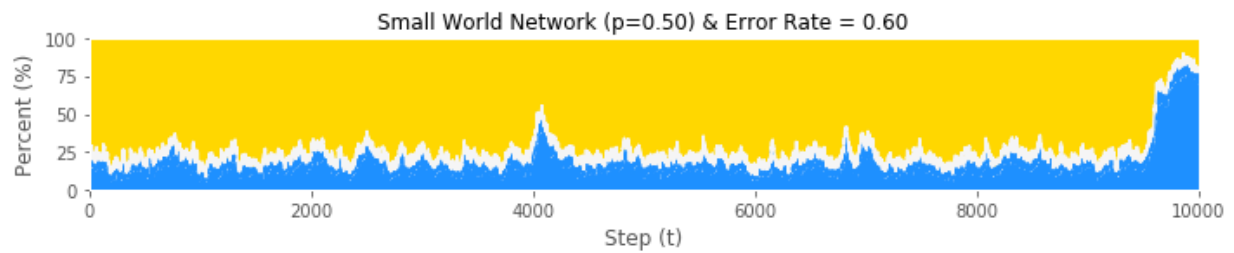
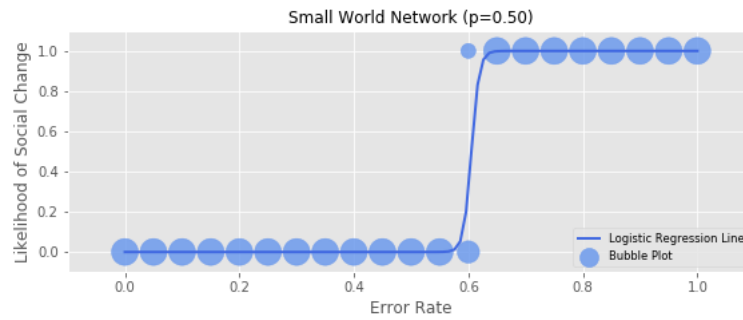
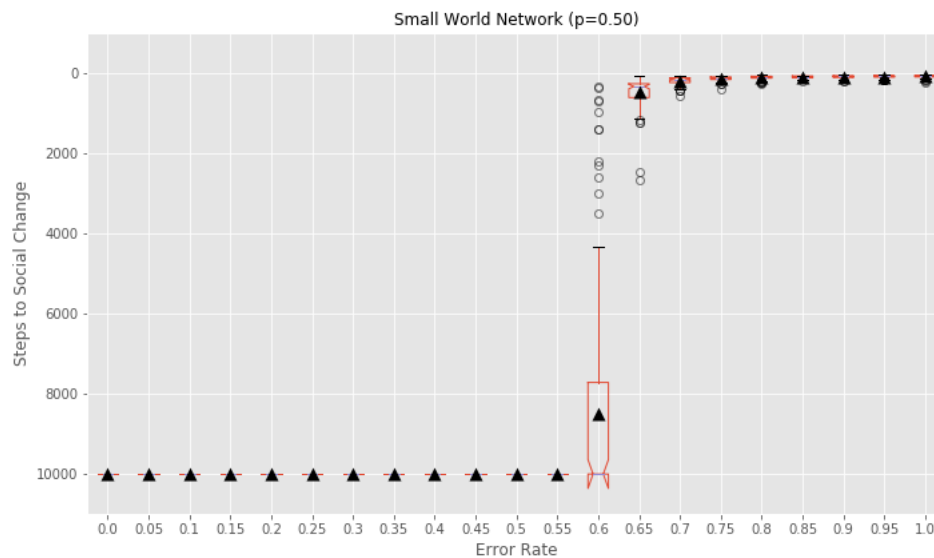


Fig S6A.14. The effect of Error Rates on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a small world network ($p = 0.50$). The tipping point is 0.60.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

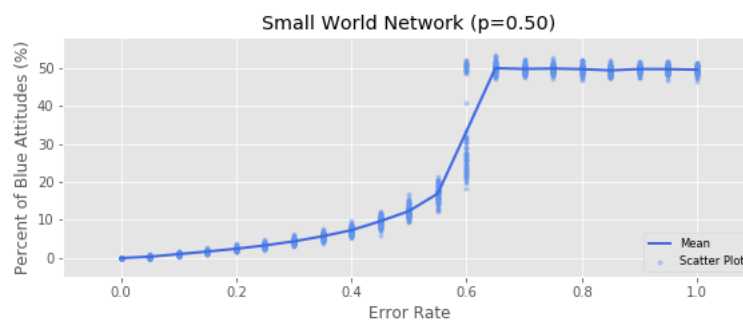


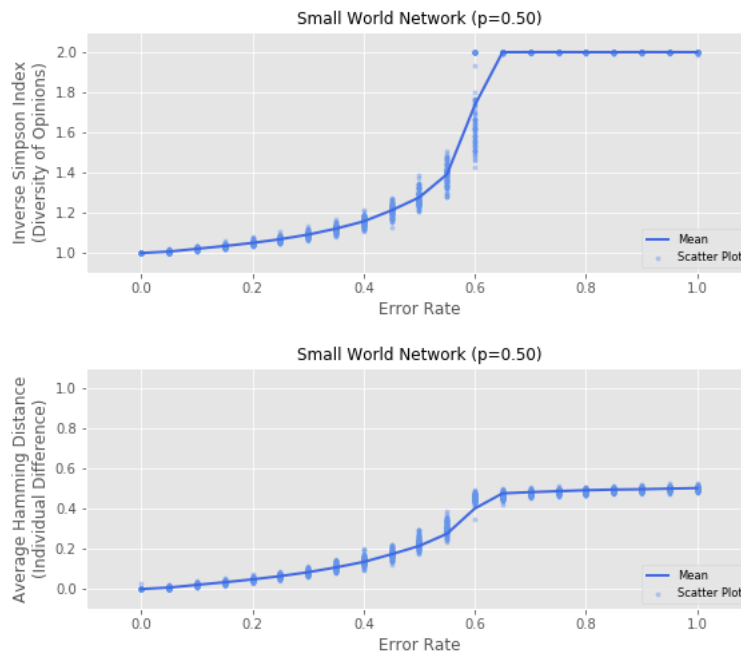
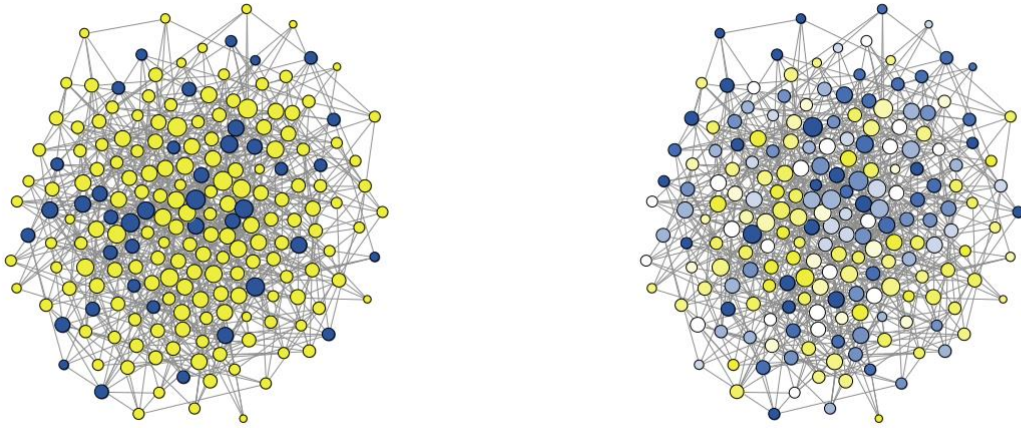
Fig S6A.15. Cultural diversity with varying Error Rates on a random network ($p = 0.50$)

Fig S6A.16. The pattern of social change on a random network ($p = 1.00$). Error Rate = 0.65.



1. 20% initial minority (blue) agents are randomly distributed across the network.
2. Social change occurs.

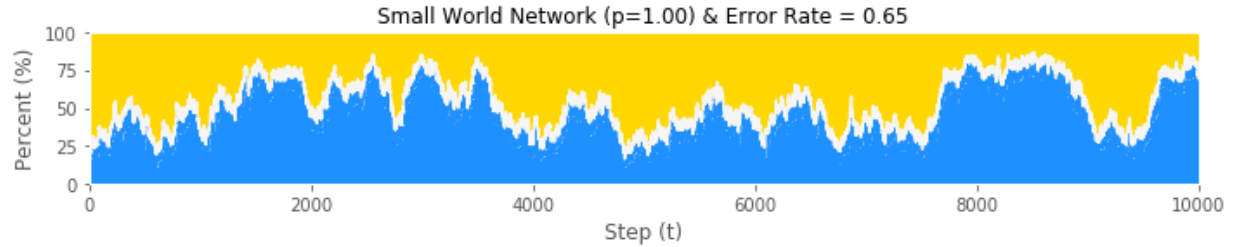
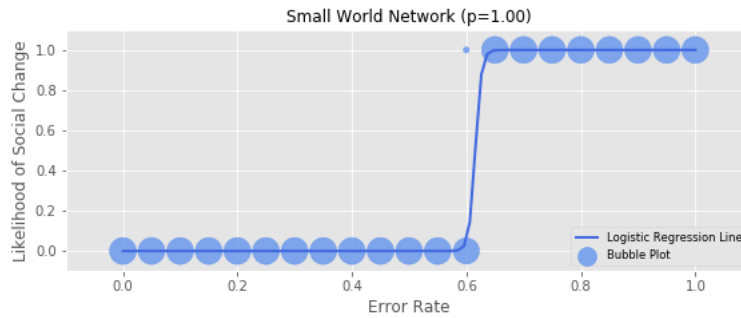
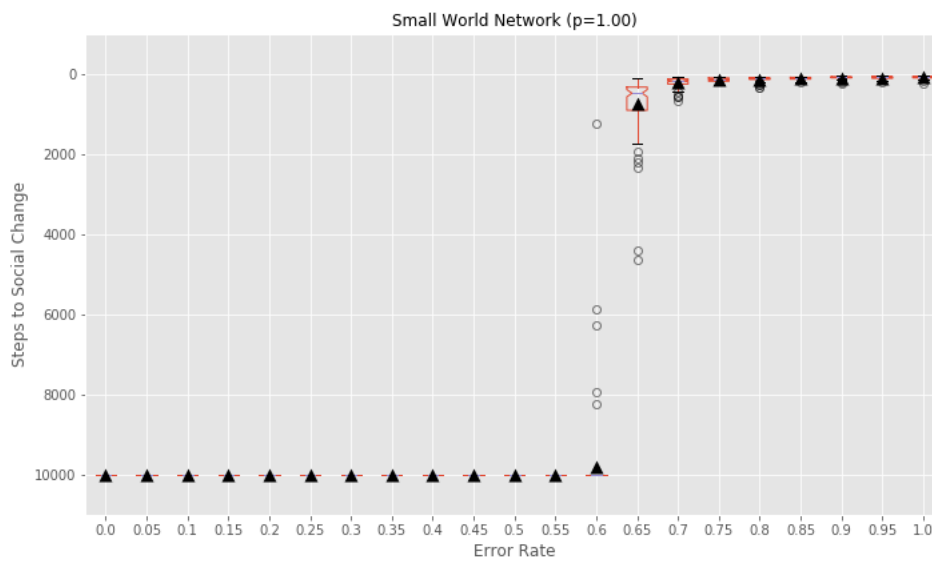


Fig S6A.17. The effect of Error Rates on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a random network ($p = 1.00$). The tipping point is 0.65.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

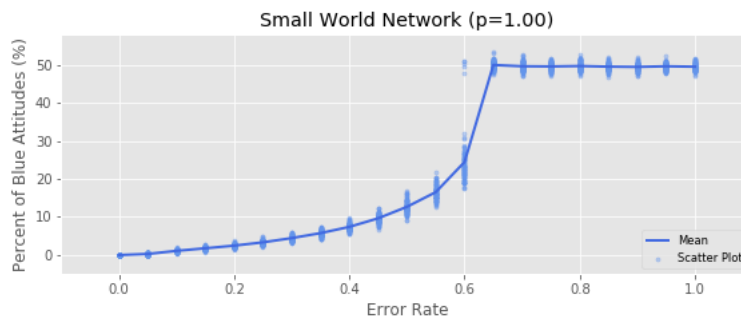
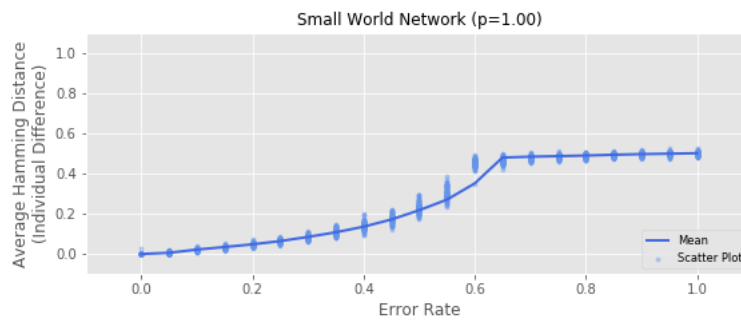
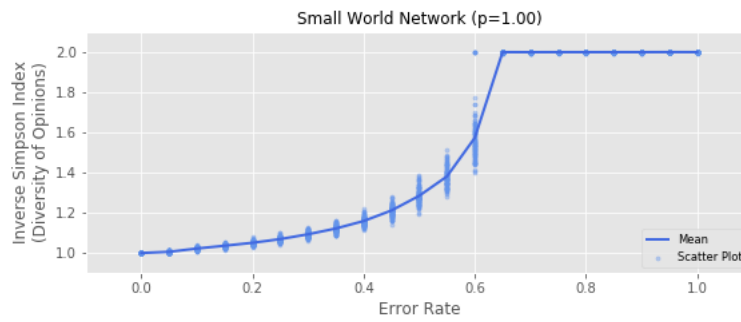
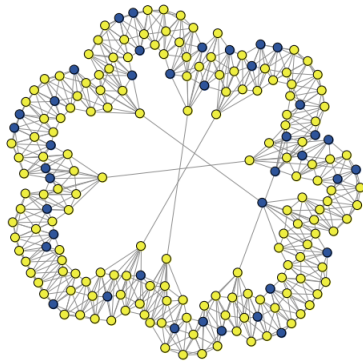


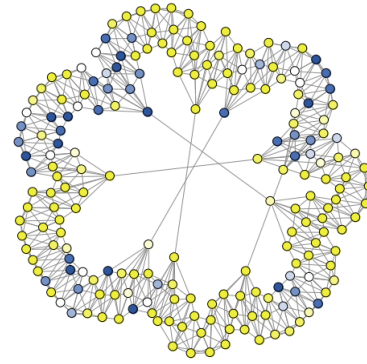
Fig S6A.18. Cultural diversity with varying Error Rates on a random network ($p = 1.00$)

Supplementary Online Material 6B
The Patterns of Social Change and Diversity with Varying Leniency Thresholds
on the Watts-Strogatz Small world networks (N=200, m=4)

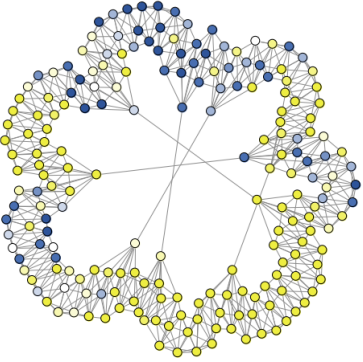
Fig S6B.1. The pattern of social change on a small world network ($p = 0.005$). Leniency threshold = 0.55.



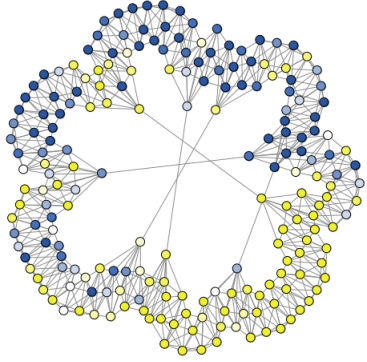
1. 20% initial minority (blue) agents are randomly distributed across the network.



2. Minority forms cliques of varying sizes.



3. In small world networks, minority opinions not just spread via clustered ties but via long ties formed by rewiring. When two minority cliques connected via a long tie, they reinforce each other and become a persistent large minority group.



4. Social change occurs. As the simulation goes, two groups (blue & yellow) emerge. High global diversity but low local diversity indicates societal polarization/segregation.

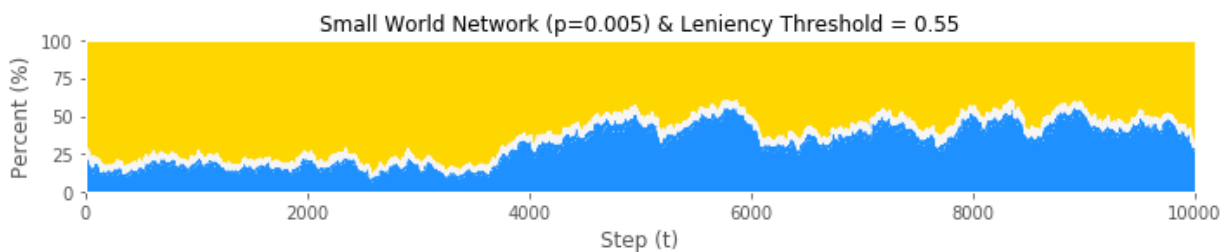
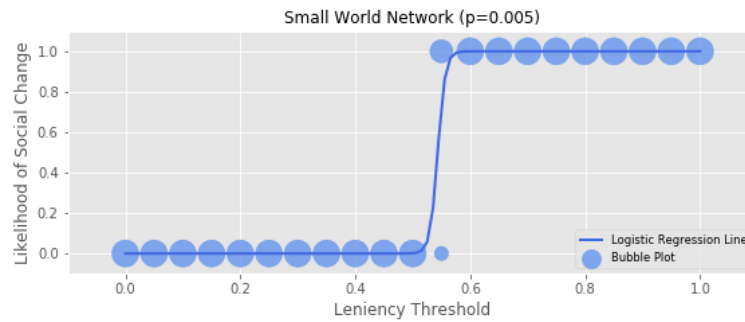
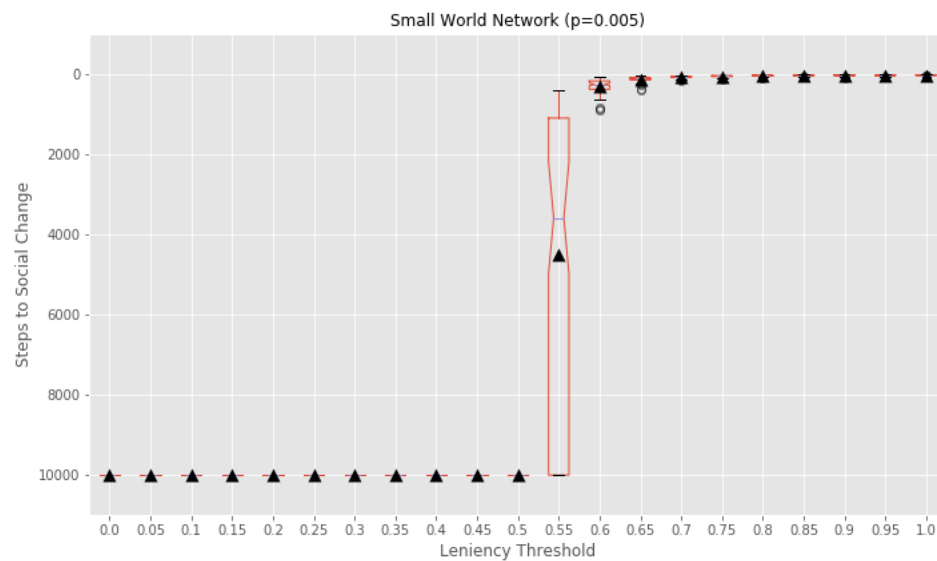


Fig S6B.2. The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of initial minority on a small world network ($p = 0.005$). The tipping point is 0.55.

(a) The likelihood of social change



(b) Steps to social change



(c) The spread of initial minority

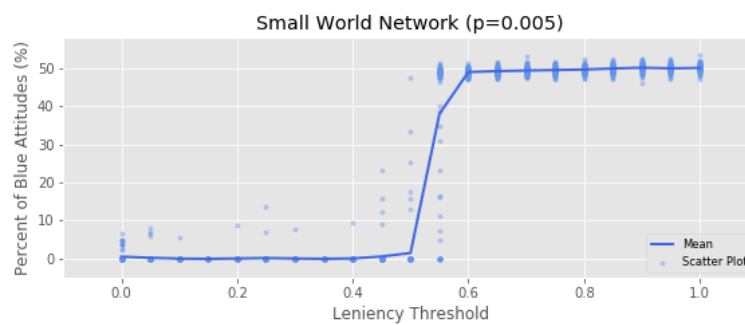


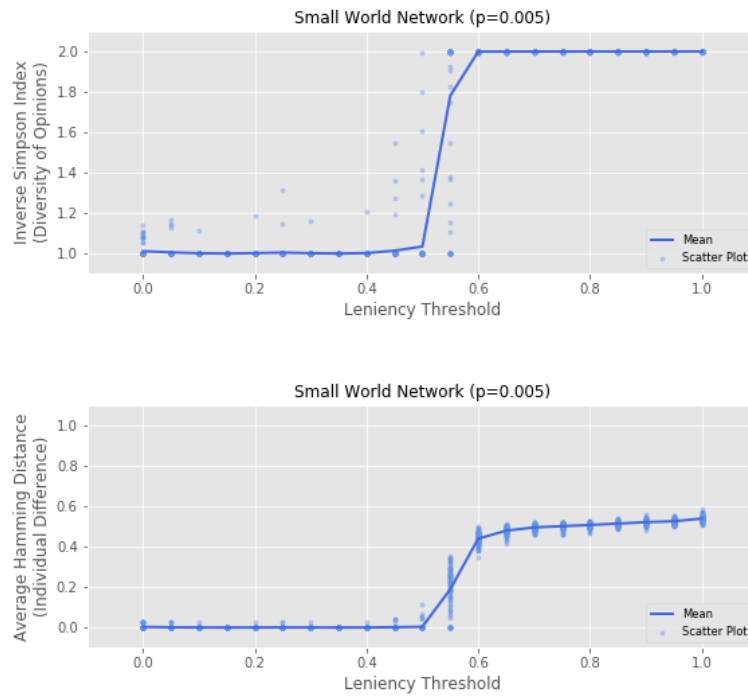
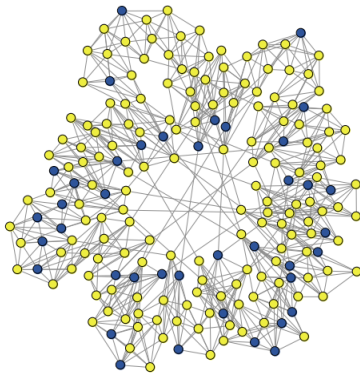
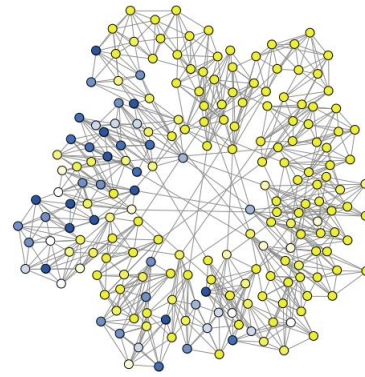
Fig S6B.3. Cultural diversity with varying leniency thresholds on small world networks ($p = 0.005$).

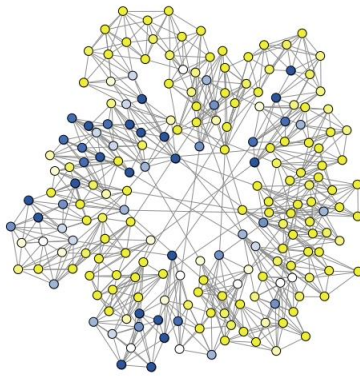
Fig S6B.4. The pattern of social change on a small world network ($p=0.05$). Leniency threshold = 0.55.



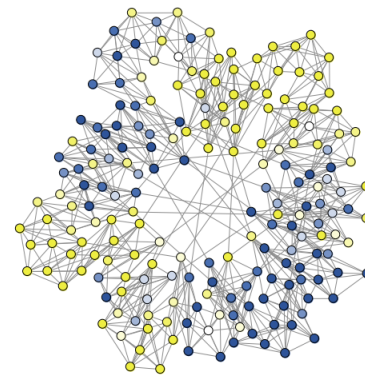
1. 20% initial minority (blue) agents are randomly distributed across the network.



2. Minority forms cliques of varying sizes.



3. In small world networks, minority opinions not just spread via clustered ties but via long ties formed by rewiring. When multiple minority cliques connected via a long tie, they reinforce one another and become a persistent large minority group.



4. When social change occurs, there is one minority clique connected via both clustered ties and long ties as one component. High global diversity but low local diversity indicates societal polarization.

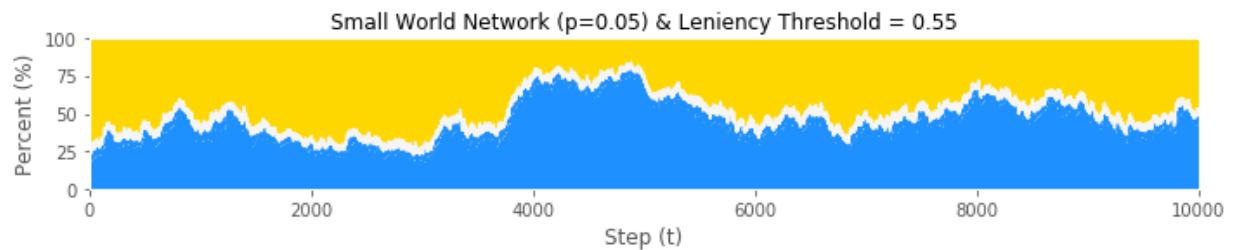
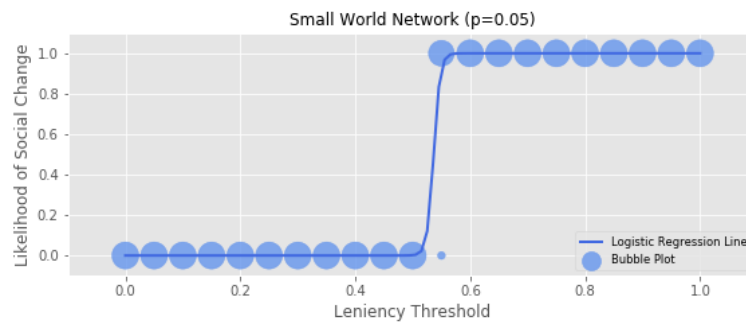
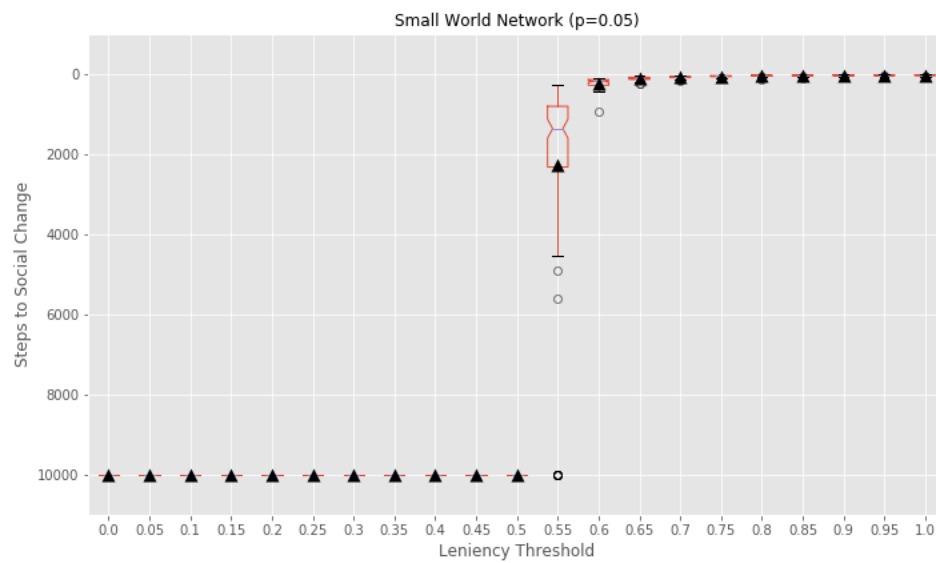


Fig S6B.5. The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a small world network ($p=0.05$). The tipping point is 0.55.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

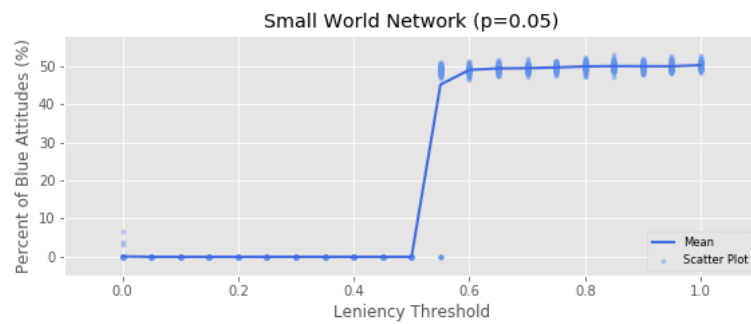


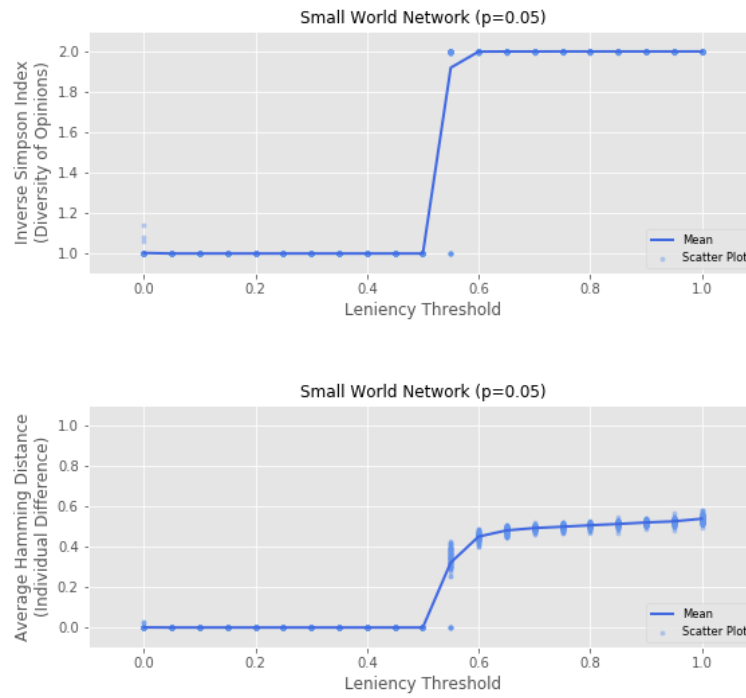
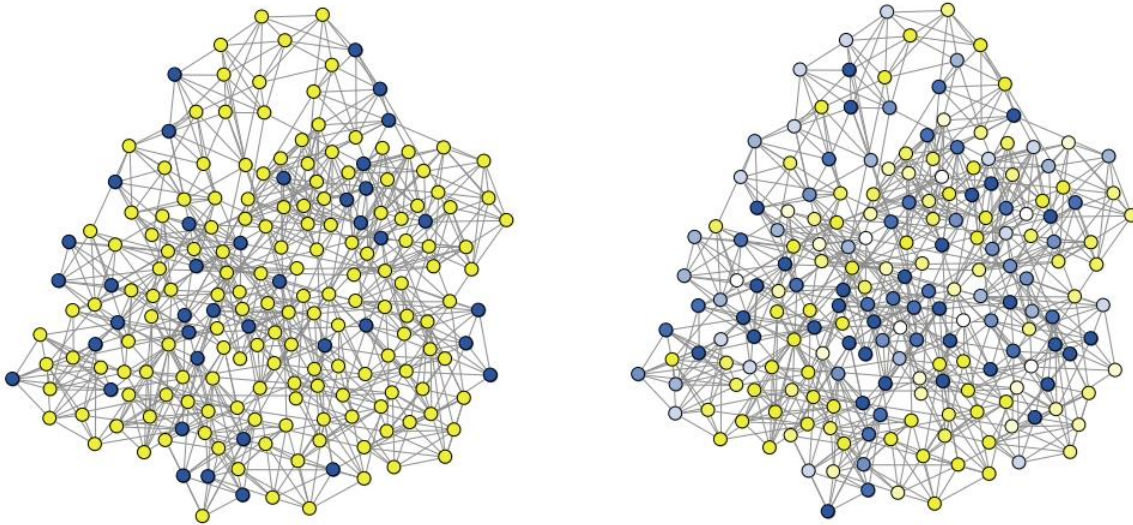
Fig S6B.6. Cultural diversity with varying leniency thresholds on small world networks ($p=0.05$).

Fig S6B.7. The pattern of social change on a random network ($p = 0.125$). Leniency threshold = 0.55.



1. 20% initial minority (blue) agents are randomly distributed across the network.
2. Social change occurs. Both global and local diversity indices are high, which indicates that there are the equal number of two positions coexists without segregation – Diversity regime.

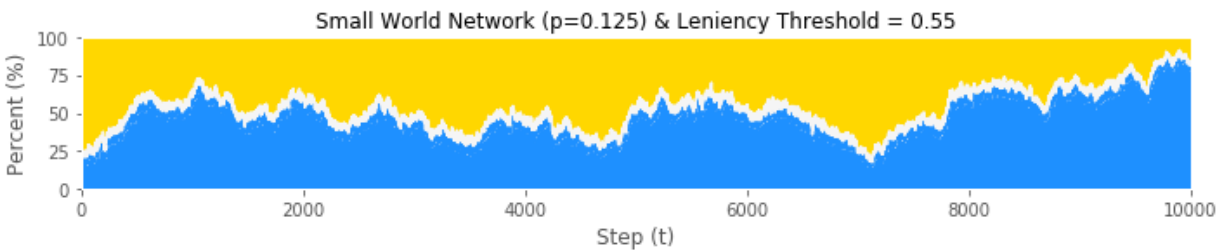
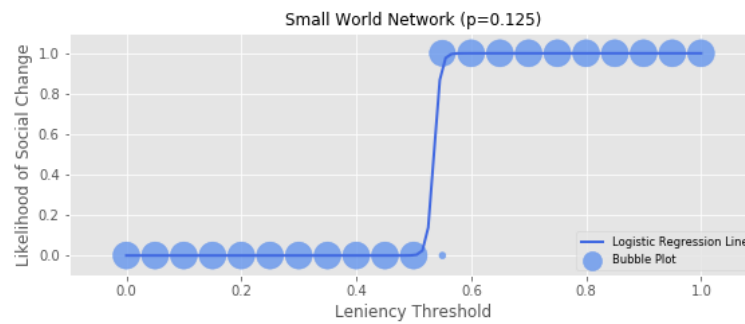
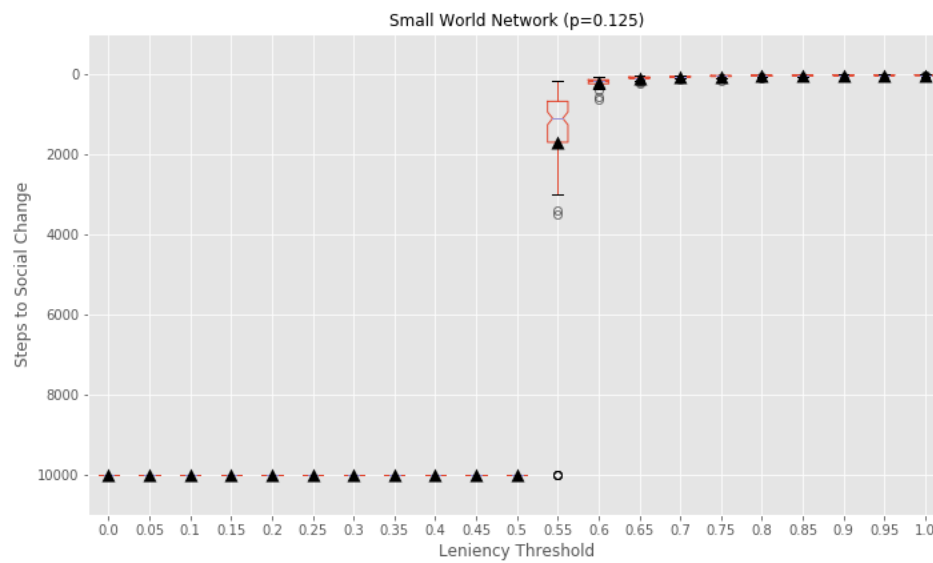


Fig S6B.8. . The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a random network ($p = 0.125$). The tipping point is 0.55.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

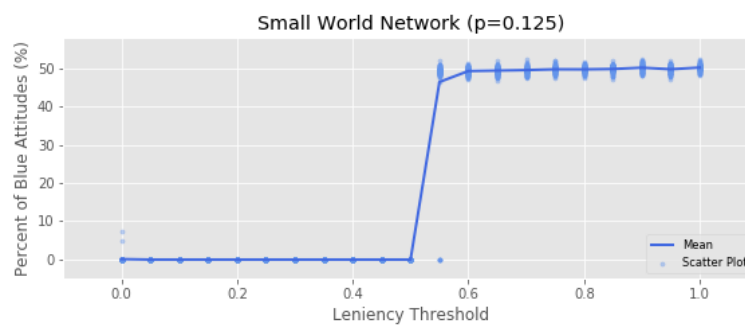


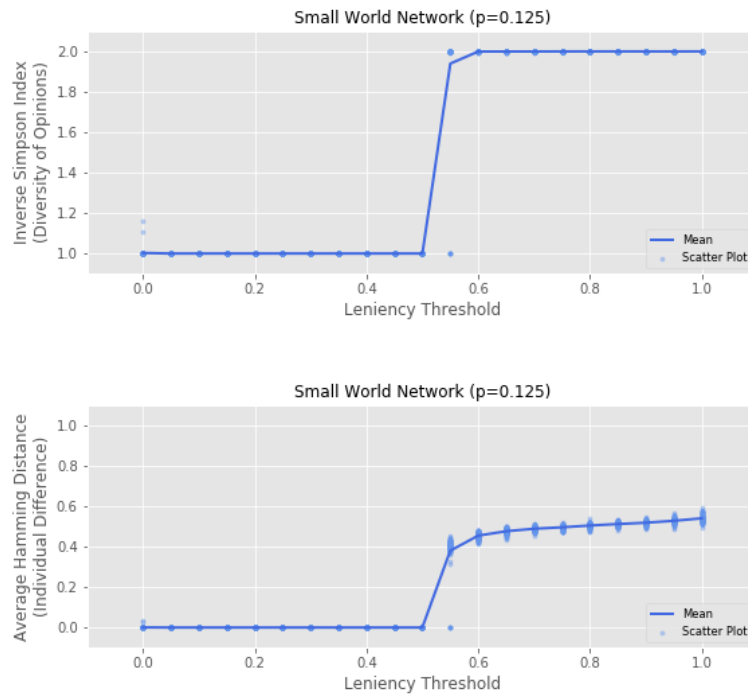
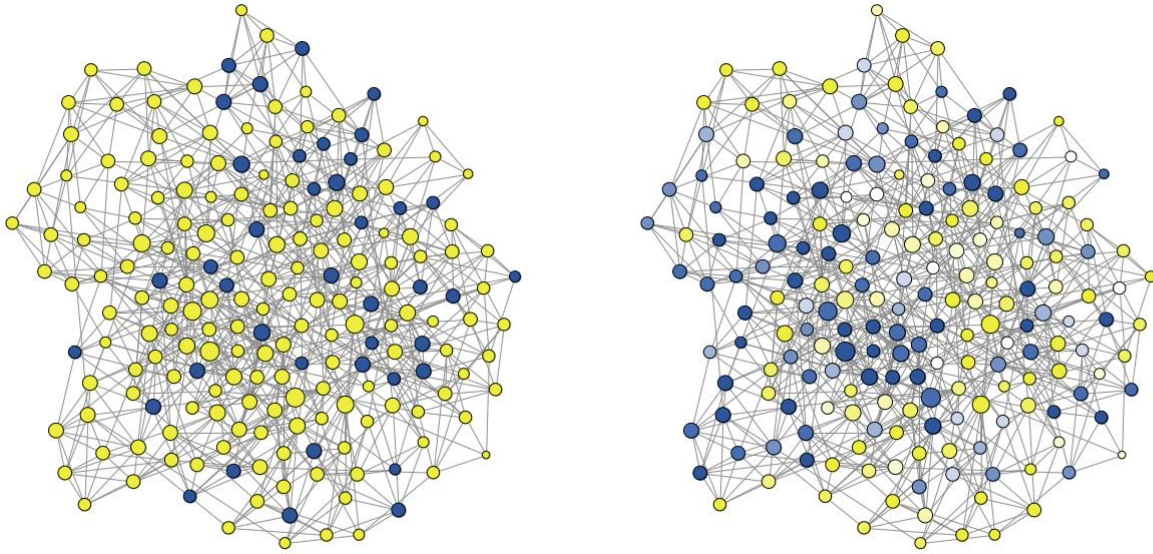
Fig S6B.9. Cultural diversity with varying leniency thresholds on random networks ($p = 0.125$).

Fig S6B.10. The pattern of social change on a random network ($p = 0.25$). Leniency threshold = 0.55.



1. 20% initial minority (blue) agents are randomly distributed across the network.
2. Social change occurs. Both global and local diversity indices are high, which indicates the equal number of two positions coexist without segregation – Diversity regime.

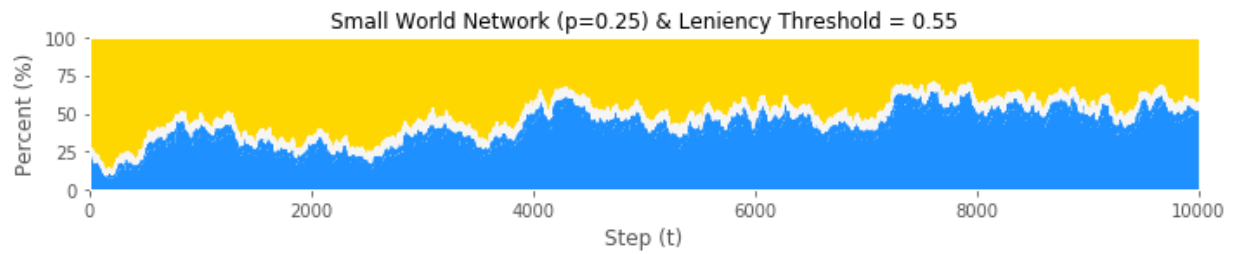
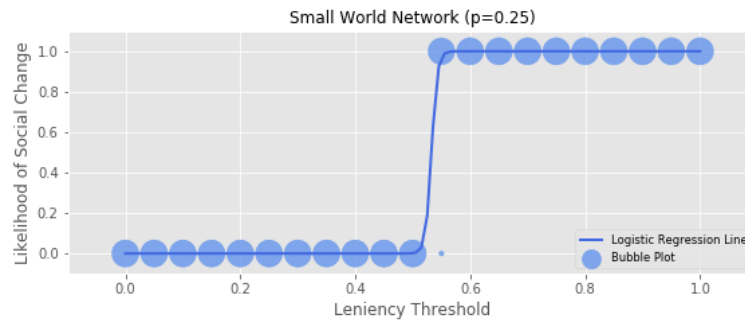
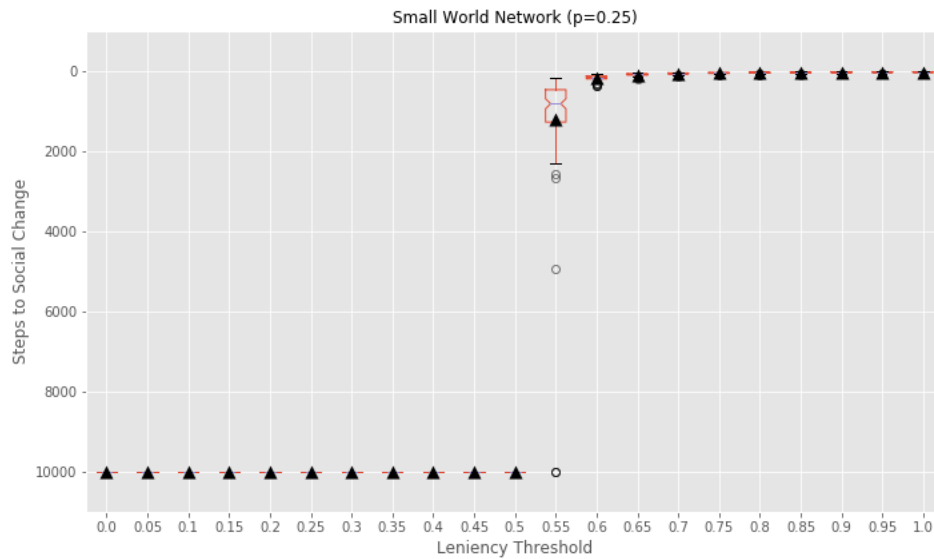


Fig S6B.11. The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a random network ($p = 0.25$). The tipping point is 0.55.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

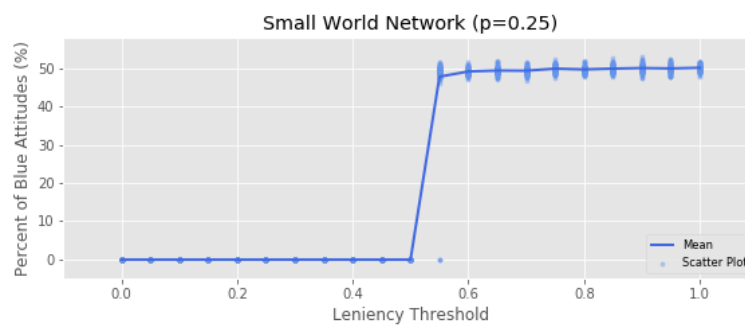


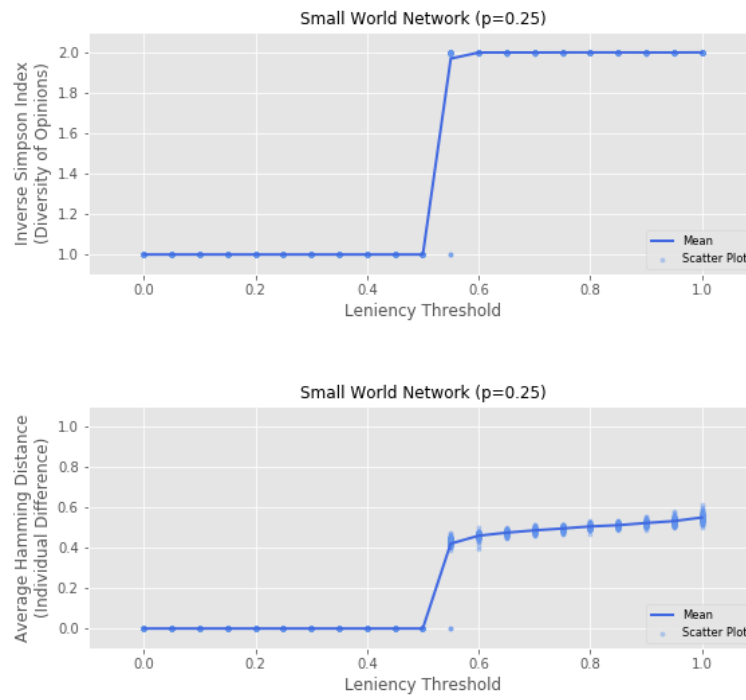
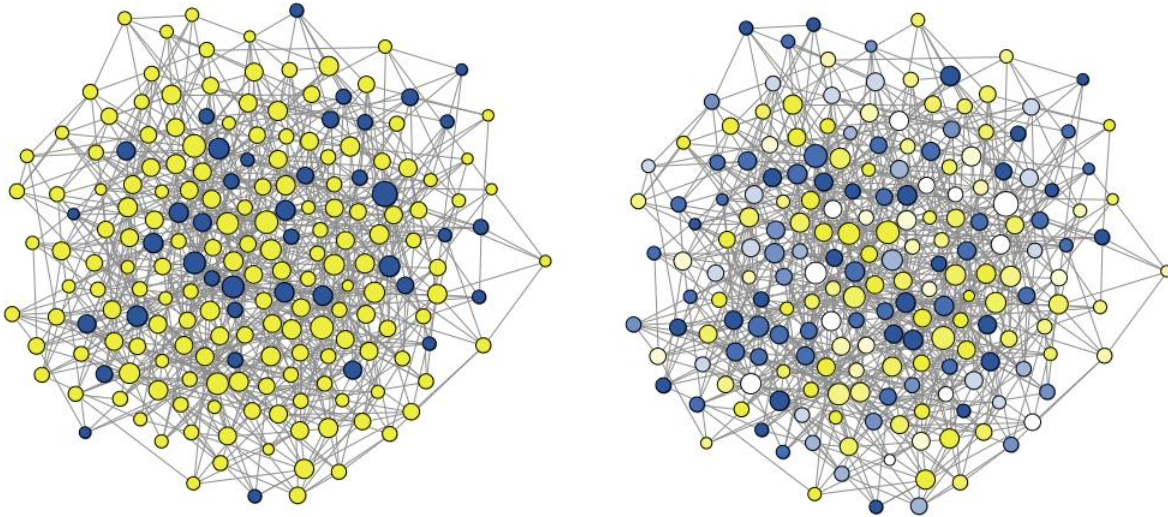
Fig S6B.12. Cultural diversity with varying leniency thresholds on a random network ($p = 0.25$)

Fig S6B.13. The pattern of social change on a random network ($p = 0.50$). Leniency threshold = 0.55.



1. 20% initial minority (blue) agents are randomly distributed across the network.
2. Social change occurs. Both global and local diversity indices reached maximum level, indicating the equal number of two positions coexist without segregation – Diversity regime.

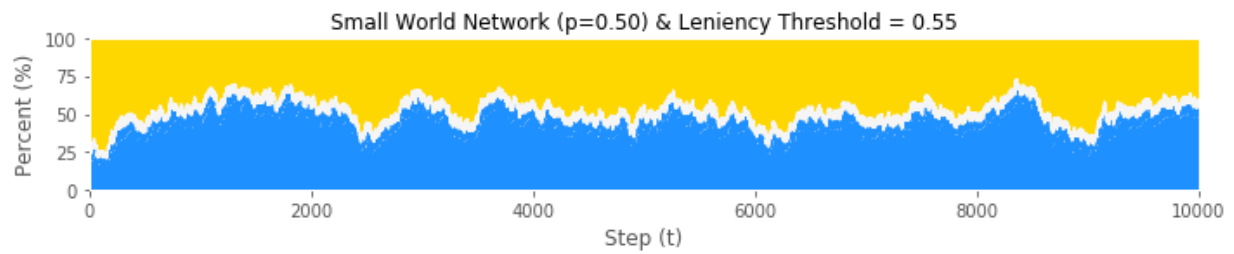
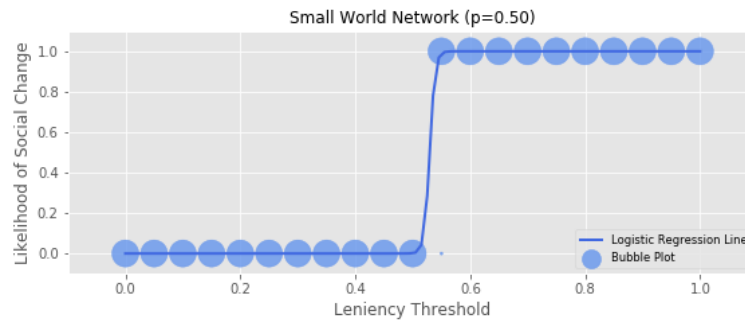
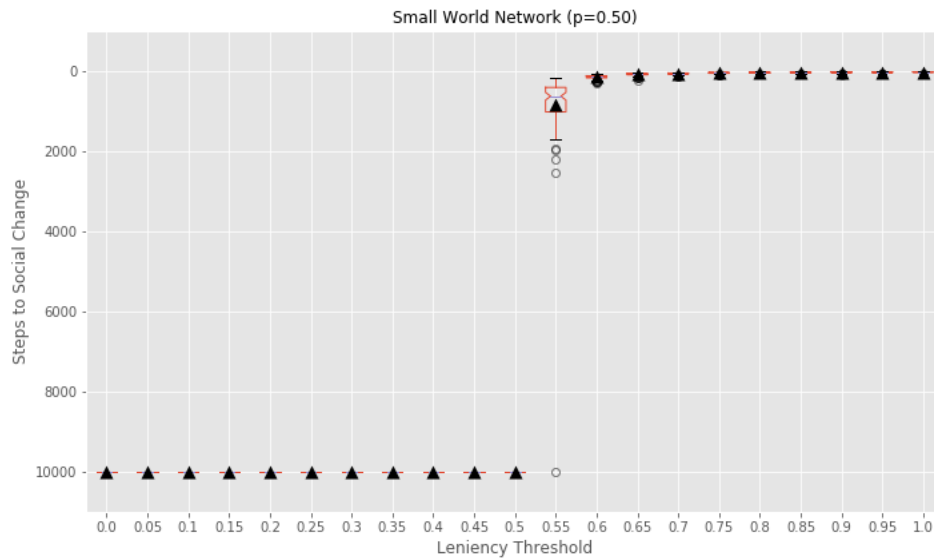


Fig S6B.14. The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a random network ($p = 0.50$). The tipping point is 0.55.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

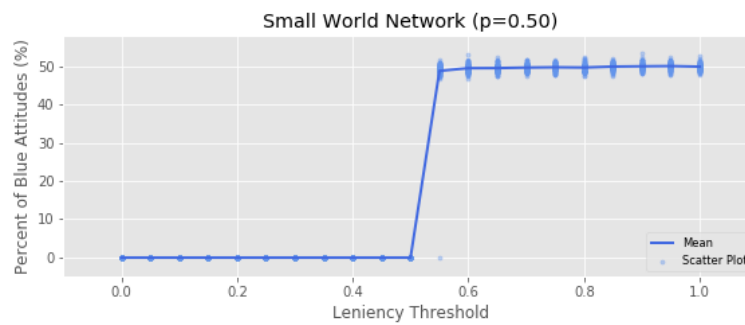


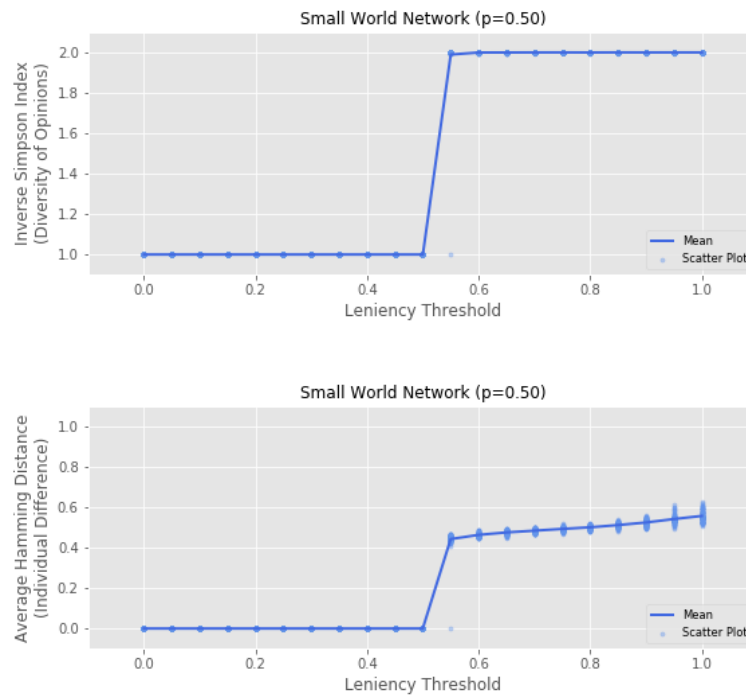
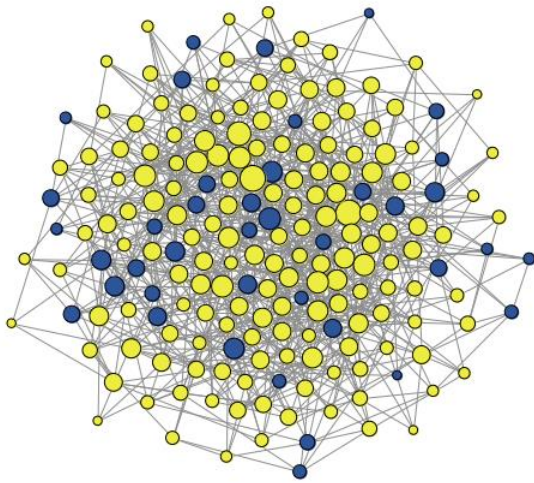
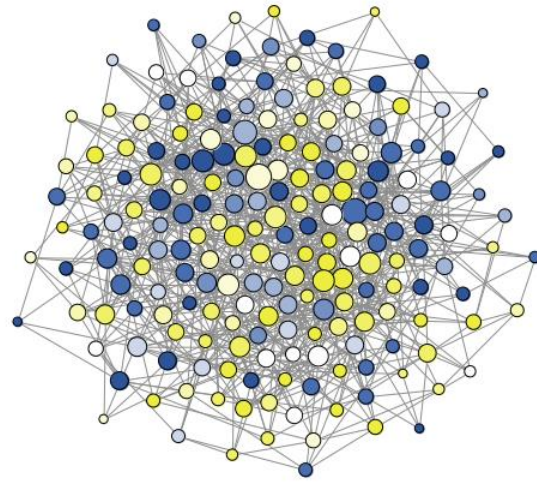
Fig S6B.15. Cultural diversity with varying leniency thresholds on a random network ($p = 0.50$)

Fig S6B.16. The pattern of social change on a random network ($p = 1.00$). Leniency threshold = 0.55.



1. 20% initial minority (blue) agents are randomly distributed across the network.



2. Social change occurs. Both global and local diversity indices reach maximum level, indicating the equal number of two positions coexist without segregation – Diversity regime.

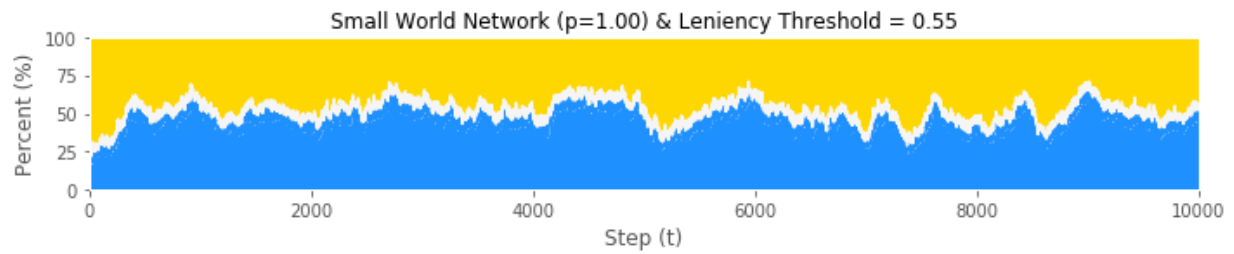
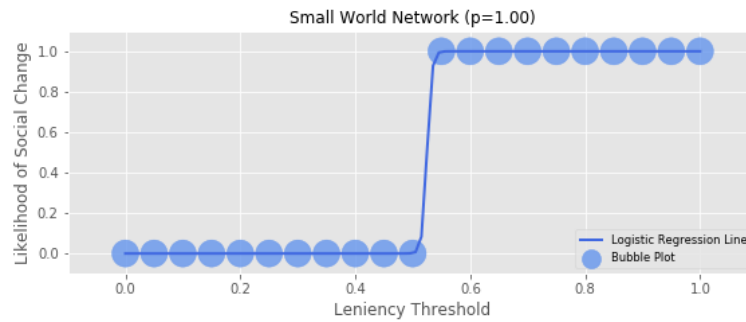
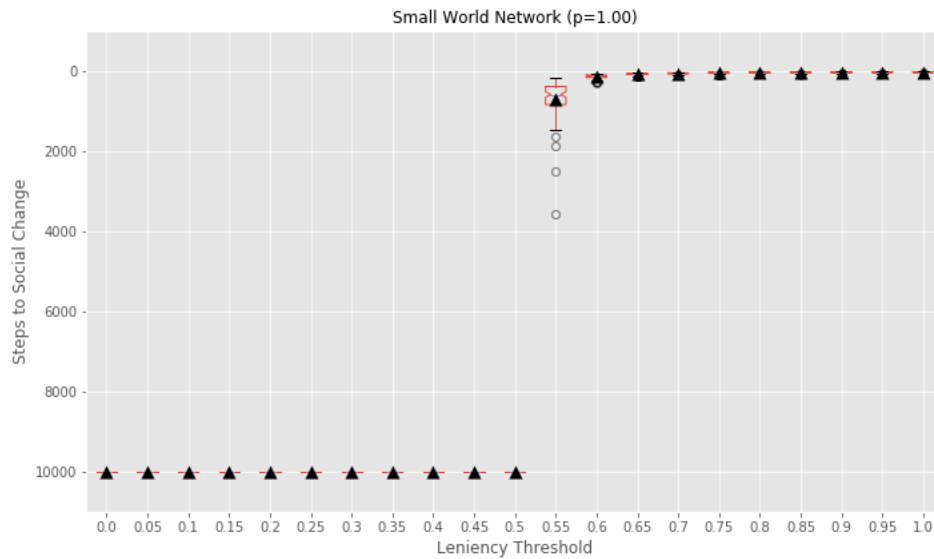


Fig S6B.17. The effect of leniency thresholds on (a) the likelihood of social change, (b) time/steps to social change, and (c) the spread of the initial minority on a random network ($p = 1.00$). The tipping point is 0.55.

(a) The likelihood of social change



(b) Time/steps to social change



(c) The spread of initial minority

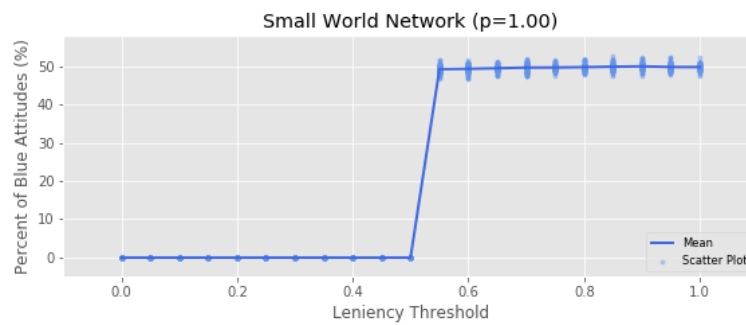
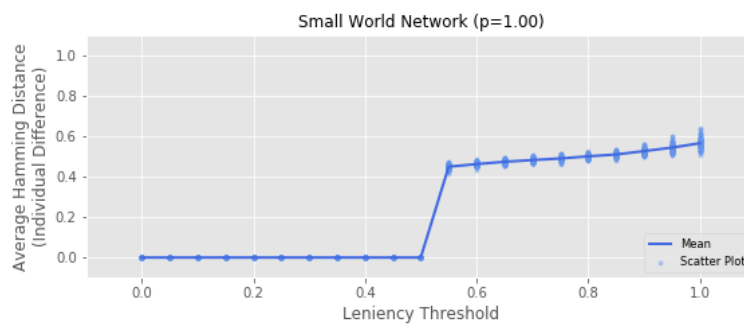
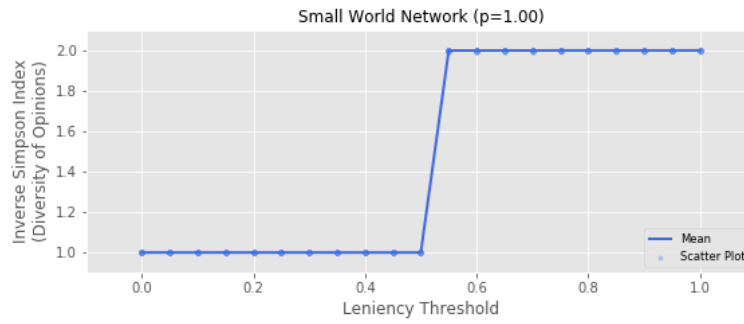


Fig S6B.18. Cultural diversity with varying leniency thresholds on a random network ($p = 1.00$)

Supplementary Online Material 7
The Patterns of Social Change and Diversity
on the Girvan-Newman Community Structure Network (N=200, m=4)

Fig S7.1. The patterns of social change in a modular network with 5% intergroup connections.

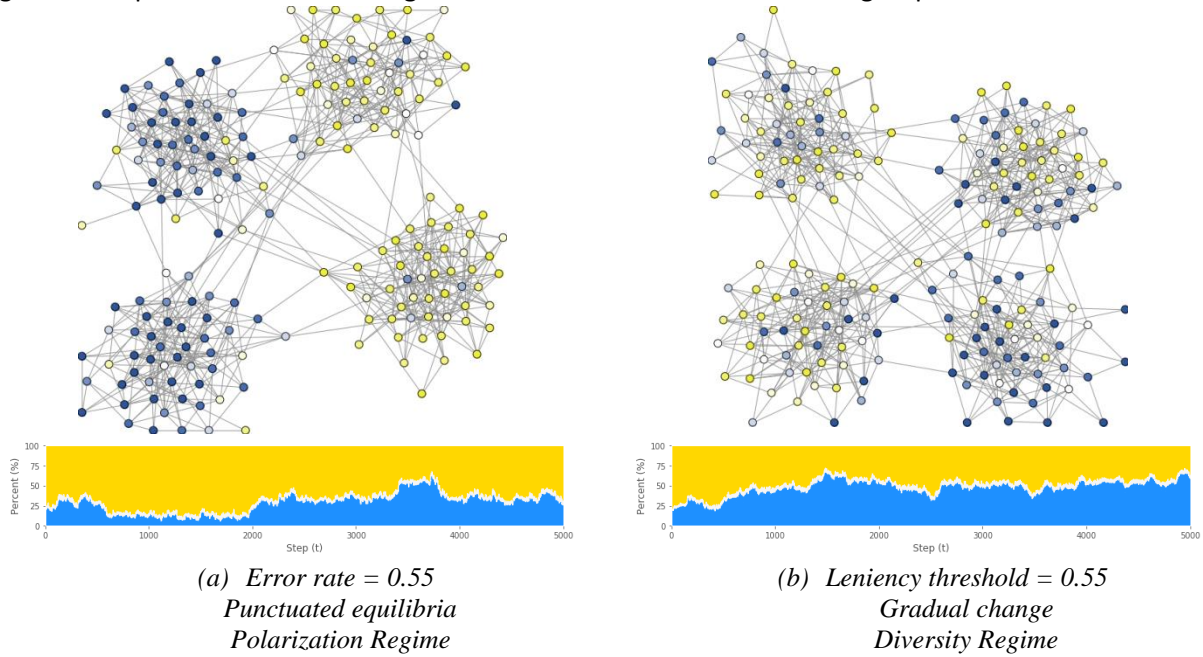


Table S7.1. The percentage of runs when a social change occurred (100 runs per cell)

Intergroup connections	Leniency Threshold			Error Rate		
	0.05	0.45	0.55	0.05	0.45	0.55
5%	0%	0%	98%	0%	2%	37%
10%	0%	0%	100%	0%	0%	2%
20%	0%	0%	99%	0%	0%	0%
30%	0%	0%	100%	0%	0%	0%

Table S7.2. The percentage of minority opinion holders (blue agents). Values in parentheses are non-dominant results.

Intergroup connections	Leniency Threshold			Error Rate		
	0.05	0.45	0.55	0.05	0.45	0.55
5%	24.96%	24.57%	47.71% (0%)	24.97%	4.25% (49.25%)	9.58% (47.89%)
10%	24.91%	14.25%	47.53%	24.88%	2.90%	8.59% (46.75%)
20%	19.87%	0%	47.36% (0%)	17.96%	2.57%	8.23%
30%	0.00%	0%	47.44%	0%	2.52%	8.34%

Fig S7.2. The pattern of minority spreads and social change with varying leniency thresholds and error rates on community structure networks of different intergroup connections.

