MFA simulation details

To simulate the standard signal detection theory (SDT; Swets & Green, 1963) model, we allowed two parameters to vary: d' and C. The noise and signal-plus-noise distributions were both assumed to be Gaussian with a standard deviation of 1. The mean of the noise distribution was fixed at 0, with the d' parameter being the mean of the signal-plusnoise distribution, and therefore, the difference in the means of the distributions. The Cparameter was specified as the offset from $\frac{d'}{2}$ (i.e. the mid-point between the means of the distributions), so that the absolute criterion was given by the sum of the C parameter and $\frac{d'}{2}$. Specifying the C parameter as an absolute value did not change the pattern of changes when using the MFA approach. To create the parameter space required to create the data space for MFA analysis, d' parameter values varied from 0 to 3.25 in 0.05 intervals, and C parameter values varied from -1.55 to 1.55 in 0.05 intervals, creating a total of 4,158 parameter combinations. For our manipulations of the parameter ranges, the bigger d'range varied from 0 to 20.25 in 0.05 intervals, the bigger C range varied from -1.55 to 10.55 in 0.05 intervals, the smaller d' range varied from 0.5 to 1.25 in 0.05 intervals, and the smaller C range varied from -0.25 to 0.25 in 0.05 intervals.

To simulate unequal variance signal detection theory (UV-SDT), all calculations remained identical, except that we allowed the standard deviation of the signal-plus-noise distribution to vary. The standard deviation of the signal-plus-noise distribution varied in separate intervals when greater than and smaller than the noise distribution's standard deviation, in order to maintain an equal ratio. When greater than the noise distribution's standard deviation, the standard deviation varied from 1 to 3 in intervals of .1, and when smaller than the noise distribution's standard deviation, the inverse of these values were used. With 41 different standard deviation parameter possibilities, there were a total of 170,478 parameter combinations.

The second set of simulations involved three models of perception. The Linear Integration Model (LIM; Anderson, 1981), the Fuzzy Logic Model of Perception (FLMP; Oden & Massaro, 1978), and a perceptual parameterization of the SDT model (Green & Swets, 1966). The functional forms of these models were identical to those used by Myung and Pitt (1997), which were:

$$\begin{split} LIM: & p_{i,j} &= \frac{\theta_i + \lambda_j}{2} \\ FLMP: & \\ p_{i,j} &= \frac{\theta_i \lambda_j}{\theta_i \lambda_j + (1 - \theta_i)(1 - \lambda_j)} \\ SDT: & \\ p_{i,j} &= \Phi[s_{i,j} \sqrt{|\Phi^{-1}(\theta)^2 + v_{i,j} \Phi^{-1}(\lambda)^2|}] \end{split}$$

Here, the task of interest was a categorization task with two response options, and two factors, the first with levels i = 1,...,n, and the second with levels j = 1,...,n. The θ and λ parameters represent the level of support for response option A, given the stimulus information provided by the first and second factors, respectively, both being bounded between 0 and 1. For the SDT model, $\Phi()$ is the standard cumulative normal density function, $s_{i,j}$ is the sign (+/-1) of $\Phi^{-1}(\theta_i) + \Phi^{-1}(\lambda)$, and $v_{i,j}$ is the sign (+/-1) of $\Phi^{-1}(\theta_i)\Phi^{-1}(\lambda)$.

For the simulations in our 2x2 design there were 4 dependent variables, being the probability of choosing response option A for each combination of the levels of the two factors. Both θ and λ for each factor level varied between 0 and 0.99, in 11 equally spaced values. Therefore, the total number of parameter combinations was 14,641 (11⁴).

Transformations

In order to test the stability of the MFA metric, we subjected the simulated data space to a series of common transformations. Simulated data were transformed after generation; there was no transformation applied to the parameter space. After performing each transformation the data were normalized relative to their maximum, to ensure data remained within the 0-1 range, as required for MFA. Failing to perform such a normalization procedure only resulted in more extreme and variable results. Any data that resulted in an undefined value due to the transformations were removed.

For the logarithmic transformation we took the natural logarithm of the data: $x \mapsto ln(x)$. For the exponential transform we used $x \mapsto e^x$. For the logit transformation: $x \mapsto log \frac{x}{1-x}$. For the probit transformation: $x \mapsto \Phi^{-1}(x)$, where Φ^{-1} is the inverse cumulative normal density function.

MFA calculation

All MFA complexity computations were based on the "R" (R Core Team, 2015) code provided by Veksler et al. (2015). We also wrote and tested an equivalent MATLAB version of the code, to ensure the reliability of our findings. All MFA grid sizes were, again, directly based on the provided code, and thus adhered to the suggested estimation procedure proposed by the authors.

NML simulation details

To assess the complexity metric provided by normalized maximum likelihood, we treated hit-rate and false-alarm-rate as continuous variables, and used the Monte Carlo integration method of Klauer and Kellen (2015), with 500 samples, to estimate the complexity metric, which involves integrating over the maximum likelihood values of all possible data values (i.e., the data space).

To estimate the maximum likelihood that the model could achieve for each sample of the data space, we used the differential evolution algorithm with 300 iterations, 50 particles, and a mutation factor of 0.0001. To improve the search efficiency of the algorithm, the parameters were bounded between reasonable values, being 0 and 5 for d', -2 and 2 for C, and 0 and 5 for the standard deviation of the signal-and-noise distribution, for UV-SDT.

The probability density function was solved using closed-form analytic solutions, with the predicted hit-rate and false-alarm rate being found through the same method as the MFA simulations. From there, the likelihood of the hit-rate and false-alarm-rate given the predicted rates was that of a joint binomial distribution, with no correlation between parameters.