Supplemental Material to

On the origins of logarithmic number to position mapping

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1. The small-number advantage is not a motor effect

The small-number advantage effect – the faster deviation of the finger towards small numbers than towards large numbers – was taken in this study as a numeric effect. An alternative interpretation, however, could attribute this effect to a motor rather than a numeric process. In the present section, we describe two control experiments that refute this motor interpretation. These experiments were mentioned briefly in the main text (in the discussion of Experiment 1).

1.1. Experiment 6: Number-to-position task with left-handed participants

A motor interpretation of the small-number advantage may attribute the effect to the asymmetry resulting from the fact that all participants in Experiments 1-5 were right handed. For example, the types of muscle activity required to push the finger left or right, may make leftward movements faster than rightward movements. Such a view predicts a reversed effect (large-number advantage) if the experiment is performed by left-handed participants.

1.1.1. Method

Seventeen left-handed adults, aged 26;7 ± 3;9, participated in this experiment. Their mother tongue was Hebrew and they had no reported cognitive disorders. They performed the silent condition in Experiment 1 with 4 trials per target.

The horizontal movement onset time was calculated per trial using the method described in the main text. This succeeded for 79% of the trials by the automatic onset-detection algorithm, and for 98% after manual encoding. The factors affecting onset times were analyzed with a 2-way repeated measures ANOVA. The dependent measure was the horizontal movement onset time, the subject was a random factor, and there were 2 within-subject factors: the target side (< 20, left; or > 20, right), and a numeric factor given by the absolute distance between the target number and 20.

1.1.2. Results

The failed trial rate was 6.9% ± 6.8%. The endpoint error was 1.71 ± 0.49 numerical units, the endpoint bias was -0.68 ± 0.47 numerical units, and movement time was 1197 ± 166 ms. There was no significant difference between the left-handed and right-handed group in any of these measures ($t(33) < 1.75$, two-tailed $p > 0.09$) except the failed trial rate – the left-handed group had more errors ($t(33) = 2.3$, two-tailed $p = .03$). Fig. S1a shows the mean trajectories per target number.

Contrary to the interpretation of the small-number advantage as a motor effect, the left-handed participants showed a small-number advantage just like the right-handed participants in the previous experiments: the horizontal movement onset times (Fig. S1b) were smaller for target numbers < 20 (mean = 415 ms) than for target numbers > 20 (mean = 478 ms). The Side x Distance repeated measures ANOVA showed that this small-number advantage was significant (a main effect of Side, $F(1,16) = 12.87$, $p = .002$, $\eta_p^2 = .45$, $\eta^2 = .16$). The ANOVA also showed, similarly to Experiment 1, that movement onset times were affected by the target distance from the middle of the number line (main effect of Distance, $F(1,16) = 27.97$, $p < .001$, $\eta_p^2 = .64$, $\eta^2 = .10$), and there was no Side x Distance interaction ($F(1,16) = 1.57$, $p = .23$).

We compared the small-number advantage between the left-handed group (Experiment 6) and the right-handed group (silent condition in Experiment 1). The data of both experiments was submitted to a mixed-design ANOVA – we repeated the Side x Distance ANOVA, while adding the Experiment as a between-subject factor. The small-number advantage was not significantly different between the two experiments (Side x Condition interaction: F < 1).

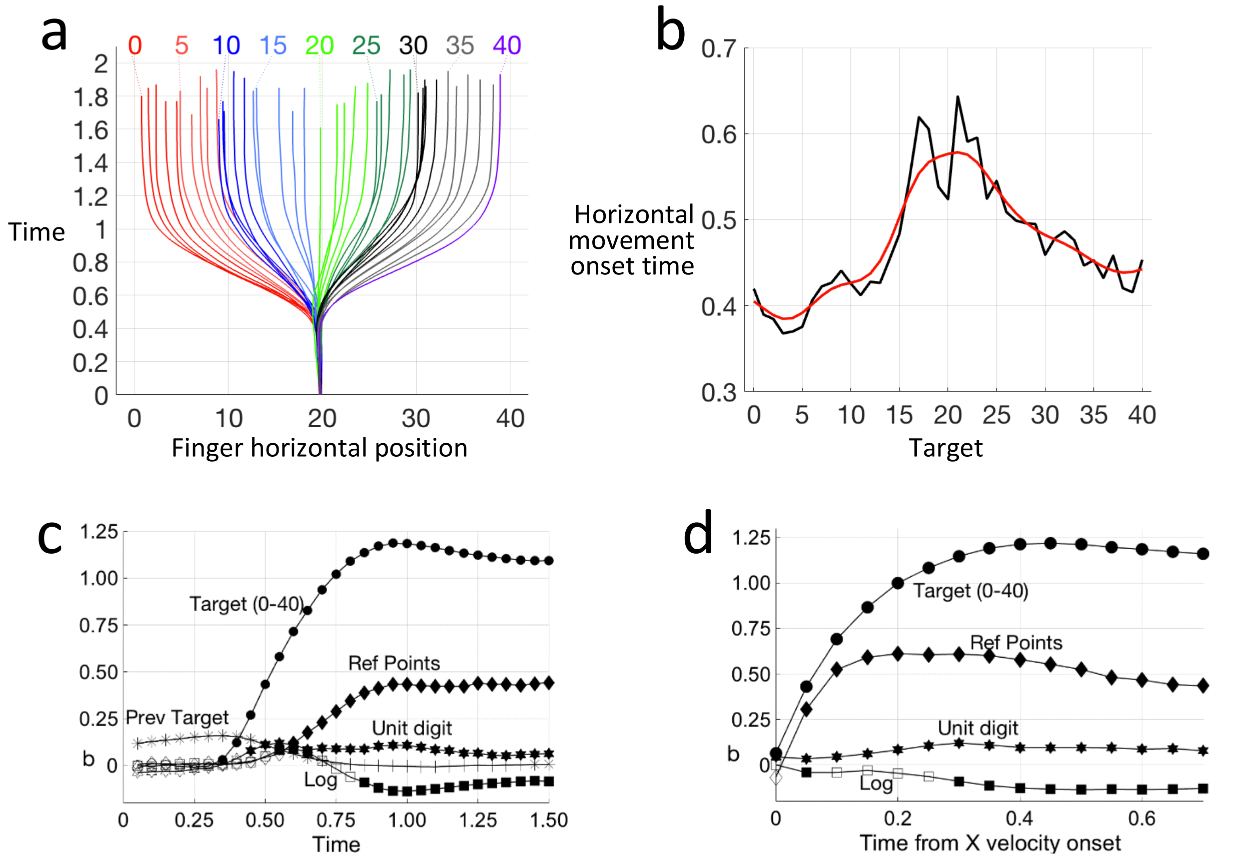

Fig. S1. Results of Experiment 6. (a) Median trajectories per target. (b) The mean horizontal movement onset time, averaged over all participants, as a function of target number. The red line shows the same data with Gaussian smoothing, σ = 2. (c) Regression b values, with the trajectories aligned by the target onset. (d) Regression b values, with the trajectories aligned by the horizontal movement onset time.

To further confirm that the left-handed group behaved similarly to the right-handed group, the trajectory data was submitted to the regression analyses presented in Experiment 1. The dependent variable was the implied endpoint and the predictors were the target number $N_{0\text{-}40}$, log'($N_{0\text{-}40}$), the unit digit, the spatial-reference-points-based bias function SRP, and the target number of the previous trial. One regression was run per participant and time point in 50 ms intervals. The per-subject regression b values of each time point and predictor were compared versus zero using t-test. The results (Fig. S1c) were very similar to the pattern observed in the silent condition in Experiment 1: dominant linear factor, transient logarithmic factor, SRP contribution in the late trajectory parts, and an effect of the previous trial in early trajectory parts. In a second regression, in which the trials were aligned by their horizontal movement onset time, the log effect was no longer significant (Fig. S1d), like in Experiment 1.

1.2. Experiment 7: Point towards an arrow

If the small-number advantage has a motor origin, it should also appear in a non-numeric mapping-to-position task. Experiment 7 did exactly that: it was almost identical with the silent condition in Experiment 1, but the stimuli were presented non-numerically, as an arrow pointing to the target location.

1.2.1. Method

Nineteen right-handed adults, aged 32;3 ± 12;9, participated in this experiment. Their mother tongue was Hebrew and they had no reported cognitive disorders.

The method was similar to the silent condition in Experiment 1, with a single difference: the target stimulus was not a number, but a downward-pointing arrow placed at the target location along the top line. The participants were instructed "to move their finger towards the arrow". Each target arrow could appear in one of 41 positions (corresponding with the positions of the numbers 0-40), and each position was presented four times.

The horizontal movement onset time was calculated per trial using the method described in the main text. This succeeded for 81% of the trials by the automatic onset-detection algorithm, and for 95% after manual encoding. The factors affecting onset times were analyzed with a 2-way repeated measures ANOVA: the dependent measure was the horizontal movement onset time, the subject was a random factor, and there were 2 within-subject factors: the target side (left or right), and a numeric factor given by the absolute distance between the target number and the middle location.

1.2.2. Results

The failed trial rate was 1.85% ± 1.92%. The endpoint error was 0.40 ± 0.11 numerical units, the endpoint bias was 0.11 ± 0.15 numerical units, and movement time was 730 ± 154 ms. Fig. S2a shows the mean trajectories per target location.

Contrary to the interpretation of the small-number advantage as a motor effect, the participants did not show a left-side advantage. In fact, they showed the opposite pattern – a right-side advantage: the horizontal movement onset times (Fig. S2b) were larger for left-side target locations (mean = 227 ms) than for right-side locations (mean = 215 ms). The Side x Distance repeated measures ANOVA showed that this large-number advantage was significant (a main effect of Side, $F(1,16) = 4.77$, $p = .04$, $\eta_p^2 = .21$, $\eta^2 = .06$). Movement onset times were also affected by the distance of the target location from the middle (main effect of Distance, $F(1,16) = 35.74$, $p < .001$, $\eta_p^2 = .67$, $\eta^2 = .10$), and there was no Side x Distance interaction ($F < 1$).

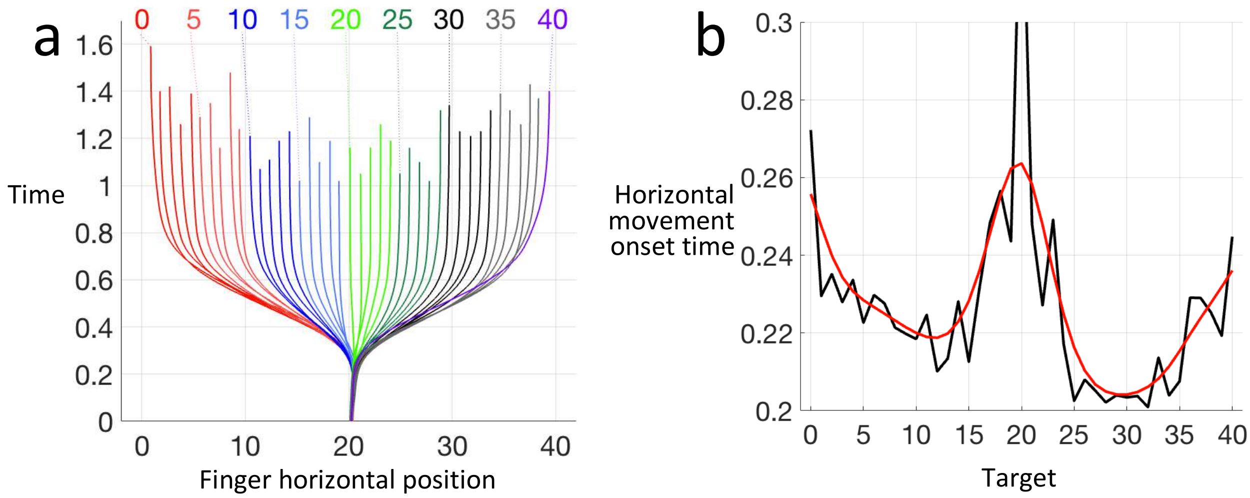

Fig. S2. Results of Experiment 7 – pointing to arrow. (a) Median trajectories per target. (b) The mean horizontal movement onset time, averaged over all participants, as a function of target number. The red line shows the same data with Gaussian smoothing, σ = 2.

1.3. Conclusion from Experiments 6 and 7

Both experiments clearly refute the motor hypothesis as an interpretation of the small-number advantage: the effect, which was observed for right-handed participants in Experiments 1-5, was observed in the number-to-position task also for left-handed participants, whose motor movements are reversed. In contrast, the effect did not exist when right-handed participants pointed to arrows, a task that involved the same motor responses as the number-to-position task.

Taken together, the experiment clearly refute the notion of a motor-driven advantage of the left side (or of the non-dominant side). In fact, Experiment 7 even suggests the opposite: when the number-processing part was eliminated, we observed earlier movement towards locations on the right side than to locations on the left side.

2. The horizontal movement onset time detection algorithm: methodological notes

2.1. Separating intentional movements from jitter

The onset detection algorithm used the velocities during the time window 0-250 ms as a baseline for random movements. It is hard to know whether a movement is intentional or not, however, we can show that the horizontal velocities in this early time window are categorically different from the velocities in later time windows.

Fig. S3 shows the distribution of horizontal velocities in different 125-ms time windows (the data is from the silent condition in Experiment 1). It clearly shows that the velocity distribution hardly changed in the first three time windows, up to 375 ms post stimulus onset. Only in the next time window (375 – 500 ms) velocities start building up. By 500 ms, the velocities are already quite close to their peak value.

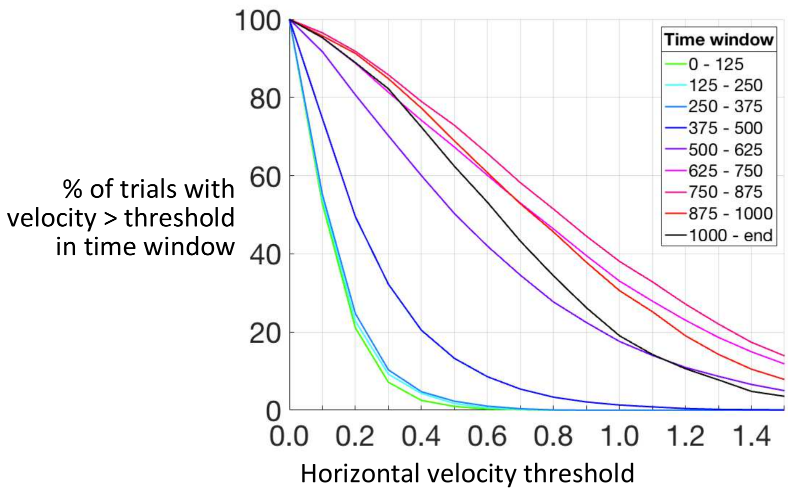

Fig. S3. Velocities in different time windows in Experiment 1 silent condition. Until 375 ms the velocities are low and remain quite unchanged. Then the velocities start increasing quickly, and after 500 ms the velocity distribution remains relatively stable.

Unsurprisingly, this figure is in almost perfect match with the regression analysis (Fig. 3a). The finger doesn't start deviating sideways before 350-400 ms, and this can be seen not only when inspecting the finger direction (implied endpoint) over all trials in a regression analysis, but even when inspecting the detailed, per-trial velocity information, as done here.

This figure strongly suggests that intentional movements are almost non-existent, or at least extremely small, in the first 375 ms of each trial. Thus our assumption, that the first 250 ms of each trial are random movements or jitter, is even conservative.

2.2. Another measure for horizontal movement onset

To validate our conclusions about horizontal movement onsets, we examined an additional measure of horizontal movement onset time. This measure, *acceleration initiation time*, is the time when we first observe strong leftward or rightward acceleration. An acceleration peak was defined as a time window of at least 150 ms during which the acceleration was constantly above a certain threshold (0.175 numerical units / sec^2), and the acceleration initiation time was defined as the beginning of the first acceleration peak. To obtain accelerations, the horizontal velocities (calculated as described in the main text) were smoothed with Gaussian (σ = 20 ms) and derived.

The pattern of acceleration initiation times was very similar to the pattern of the horizontal movement onset times obtained with our onset-detection algorithm – both in visual inspection (Fig. S4) and when applying the Condition x Side x Distance repeated measures ANOVA as described in the main text. Specifically, acceleration initiation times were later in color naming than in the silent condition (main effect of Condition, $F(1,17) = 73.47$, $p < .001$, $\eta_p^2 = .81$, $\eta^2 = .35$). There was a small-number advantage (main effect of Side, $F(1,17) = 7.78$, $p = .12$, $\eta_p^2 = .31$, $\eta^2 = .01$). Unlike the onset-detection algorithm described in the main text, here we did not observe a difference in small-number advantage between the two conditions (no Condition x Side interaction, $F < 1$). Finally, acceleration initiation times were later for target numbers near the middle of the number line (main effect of Distance, $F(1,17) = 60.8$, $p < .001$, $\eta_p^2 = .78$, $\eta^2 = .04$), and this effect was marginally stronger in color naming than in the silent condition (Condition x Distance interaction, $F(1,17) = 3.50$, $p = .08$, $\eta_p^2 = .17$, $\eta^2 < .01$).

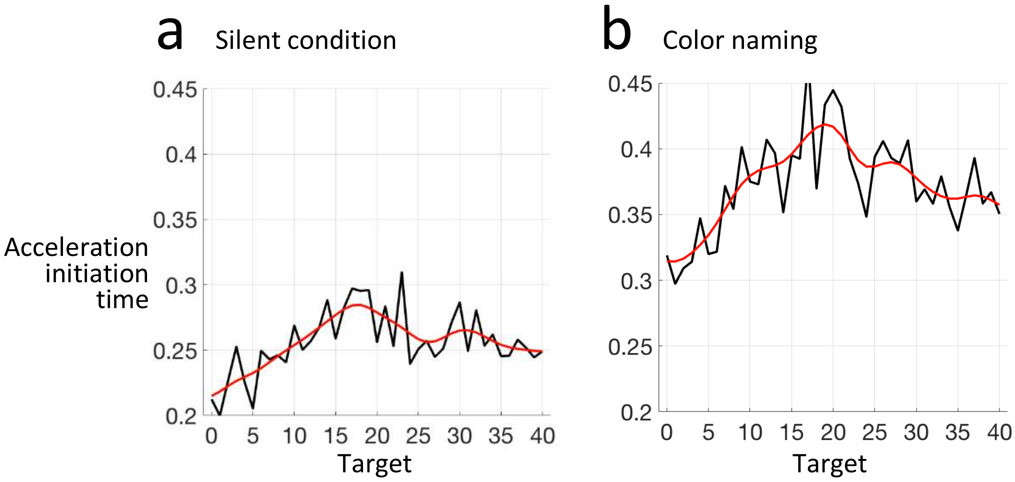

Fig. S4. Horizontal movement onset times in Experiment 1 as measured by the *acceleration initiation time* – the beginning of the earliest horizontal acceleration peak. This measure too shows a small-number advantage, i.e., earlier horizontal movement onset for smaller numbers than for large numbers. The red line shows the same data with Gaussian smoothing, σ = 2.

Thus, the analysis of acceleration initiation times replicated the main findings obtained with the onset detection algorithm in the main text. Acceleration initiation times may have the advantage of being a more intuitive measure, and that the way to calculate them is simpler. However, we believe that the onset-detection algorithm is a truer measure of the participants' intention to move. For example, on some trials the acceleration initiation time may reflect an initial bias, prior to processing the target number.