

Supplemental Material: Derivation of the Area of Historical Differences

The Area of Historical Differences (AHD) is defined as the area between the aging trajectories of the earliest born cohort (ebc) and the latest born cohort (lbc) in the sample.

Generally an aging trajectory is given by:

$$y'_{yob,age} = i + (a_1 * age) + (a_2 * age^2) + (a_3 * age^3) + (b_1 * yob) + (b_2 * yob * age) + (b_3 * yob * age^2) + (b_4 * yob * age^3) \quad (A1)$$

Since yob (i.e., year of birth) is zero for the ebc the function of the aging trajectory of the ebc is:

$$f_{ebc}(age) = i + a_1 * age + a_2 * age^2 + a_3 * age^3 \quad (A2)$$

The function of the aging trajectory of the lbc is:

$$f_{lbc}(age) = i + a_1 * age + a_2 * age^2 + a_3 * age^3 + (b_1 * yob_{lbc}) + (b_2 * yob_{lbc} * age) + (b_3 * yob_{lbc} * age^2) + (b_4 * yob_{lbc} * age^3) \quad (A3)$$

Assuming that $f_{ebc}(age)$ lies under $f_{lbc}(age)$ the area between in the interval $[ul, ll]$ is calculated:

$$AHD = \int_{ul}^{ll} f_{lbc}(age)d(age) - \int_{ul}^{ll} f_{ebc}(age)d(age) =$$

$$AHD = \int_{ul}^{ll} (f_{lbc}(age) - f_{ebc}(age))d(age) =$$

$$AHD = \int_{ul}^{ll} g(age)d(age) \quad (A4)$$

$$g(age) = f_{lbc} - f_{ebc} \quad (A5)$$

Inserting Equations A2 and A3 we derive:

$$g(age) = i + a_1 * age + a_2 * age^2 + a_3 * age^3 + (b_1 * yob_{lbc}) + (b_2 * yob_{lbc} * age) + (b_3 * yob_{lbc} * age^2) + (b_4 * yob_{lbc} * age^3) - (i + a_1 * age + a_2 * age^2 + a_3 * age^3) \quad (A6)$$

Which then is reduced to:

$$g(age) = (b_1 * yob_{lbc}) + (b_2 * yob_{lbc} * age) + (b_3 * yob_{lbc} * age^2) + (b_4 * yob_{lbc} * age^3) \quad (A7)$$

Derivation of the AHD

With integration we obtain $G(\text{age})$ as the primitive of $g(\text{age})$

$$G(\text{age}) = (b_1 * yob_{lbc} * \text{age}) + \frac{1}{2} * (b_2 * yob_{lbc} * \text{age}^2) + \frac{1}{3} * (b_3 * yob_{lbc} * \text{age}^3) + \frac{1}{4} * (b_4 * yob_{lbc} * \text{age}^4) \quad (\text{A8})$$

Now we calculate the integral $[ul, ll]$

Given:

$$AHD = \int_{ll}^{ul} (b_1 * yob_{lbc}) + (b_2 * yob_{lbc} * \text{age}) + (b_3 * yob_{lbc} * \text{age}^2) + (b_4 * yob_{lbc} * \text{age}^3) = G(ul) - G(ll) \quad (\text{A9})$$

$$G(ul) = (b_1 * yob_{lbc} * ul) + \frac{1}{2} * (b_2 * yob_{lbc} * ul^2) + \frac{1}{3} * (b_3 * yob_{lbc} * ul^3) + \frac{1}{4} * (b_4 * yob_{lbc} * ul^4) \quad (\text{A10})$$

$$G(ll) = (b_1 * yob_{lbc} * ll) + \frac{1}{2} * (b_2 * yob_{lbc} * ll^2) + \frac{1}{3} * (b_3 * yob_{lbc} * ll^3) + \frac{1}{4} * (b_4 * yob_{lbc} * ll^4) \quad (\text{A11})$$

The integral is given by:

$$AHD = (b_1 * yob_{lbc} * ul) + \frac{1}{2} * (b_2 * yob_{lbc} * ul^2) + \frac{1}{3} * (b_3 * yob_{lbc} * ul^3) + \frac{1}{4} * (b_4 * yob_{lbc} * ul^4) - (b_1 * yob_{lbc} * ll) - \frac{1}{2} * (b_2 * yob_{lbc} * ll^2) - \frac{1}{3} * (b_3 * yob_{lbc} * ll^3) - \frac{1}{4} * (b_4 * yob_{lbc} * ll^4) \quad (\text{A12})$$

Which can be rewritten for convenience as

$$AHD = (b_1 * yob_{lbc}) * (ul - ll) + \frac{1}{2} * (b_2 * yob_{lbc}) * (ul^2 - ll^2) + \frac{1}{3} * (b_3 * yob_{lbc}) * (ul^3 - ll^3) + \frac{1}{4} * (b_4 * yob_{lbc}) * (ul^4 - ll^4) \quad (\text{A13})$$

The upper limit (ul) is given by the highest age in the sample. The lower limit (ll) is given by the lowest age in the sample. Please note that the estimation of the AHD in this form is only warranted if every point estimate at any age of the aging trajectories of the lbc is of a higher value as the respective point estimate of the aging trajectory of the ebc. If the aging trajectories of the lbc and the ebc intersect, new lower and upper limits of the integrals need to be chosen and different AHCs before and after the intersection need to be calculated.