Supplemental Material

to accompany Pek. J., & Wu, H. (in press). Parameter uncertainty in structural equations models: Confidence sets and fungible estimates. *Psychological Methods*.

Results from four examples are presented below. The examples based on data from Bagozzi and Yi (1988), Cadell et al. (2014) and Dowling et al. (2004) have been fully presented in the paper. We only report parameter estimates in this document. The last example by Schmitt, Branscombe, Kobrynowicz, and Owen (2002) is presented in detail for two reasons: (a) to serve as an additional empirical example illustrating the computation and interpretation of confidence sets (CSs) and fungible parameter estimate (FPE) contours, and (b) the targeted illustration made use of this example to determine population generating models.

In all four examples, the covariance structure can be expressed as

$$\mathbf{\Sigma} = \mathbf{D}(\mathbf{\Lambda}\mathbf{\Phi}\mathbf{\Lambda}' + \mathbf{\Psi})\mathbf{D}$$

where $\mathbf{\Lambda}$ is the matrix of standardized factor loadings, $\mathbf{D}$ is a diagonal matrix of manifest variable standard deviations, and $\mathbf{\Phi}$ is the correlation matrix of the latent variables implied by the standard path coefficients among them. The diagonal matrix $\mathbf{\Psi}$ of the standardized unique variances are functions of the other parameters rather than independent parameters.

Example 1: Conversation with a Stranger

This example from Bagozzi and Warshaw (1988) as reported in Bagozzi and Yi (1988) investigates what motivates people to talk to an attractive stranger. The model includes four latent variables: Intention, Attitude, Social Norms and Trying. Intention was measured by two indicators, Attitude was measured by three indicators, and Social Norms and Trying are each measured by a single indicator. The path diagram of this model was included in the paper as Figure 1. The total sample size is $N = 250$.

Standardized parameter estimates are listed in Table 1. All estimates are significantly different from zero ($p < .05$) except for the path from Attitude to Trying.

Table 1: Standardized estimates in the Conversation with a Stranger example

Parameter	Estimate
Intention $\rightarrow y_1$	0.90
Intention $\rightarrow y_2$	0.94
Attitude $\rightarrow x_1$	0.78
Attitude $\rightarrow x_2$	0.89
Attitude $\rightarrow x_3$	0.82
Attitude $\longleftrightarrow$ SN	0.28
Attitude $\rightarrow$ Intention	0.64
SN $\rightarrow$ Intention	0.11
Attitude $\rightarrow$ Trying	0.05
Intention $\rightarrow$ Trying	0.49

Example 2: Posttraumatic Growth of Caregivers

In this example, we fit a latent variable multiple regression model to the data from Cadell et al. (2014). In this model, the latent variable Posttraumatic Growth (PTG) of caregivers is predicted by Meaning in Caregiving (MCG), Spirituality and Psychological Well-being (PWB). PTG is measured by five subscales; PWB is indicated by four indicators; MCG and spirituality are each measured by a single indicator and their standardized loading is set to the square root of the reliability reported in the paper. Spirituality also has a cross-loading to the Spiritual Change subscale of PTG. The path diagram of this model was included in the paper as Figure 3. The total sample size is $N = 273$.

Standardized parameter estimates are listed in Table 2. All estimates are significantly different from zero ($p < .05$) except for the path from spirituality to PTG.

Table 2: Standardized estimates in the PTG of Caregivers example

Parameter	Estimate
PTG $\rightarrow y_1$	0.72
PTG $\rightarrow y_2$	0.73
PTG $\rightarrow y_3$	0.86
PTG $\rightarrow y_4$	0.84
PTG $\rightarrow y_5$	0.42
PWB $\rightarrow x_1$	0.75
PWB $\rightarrow x_2$	0.68
PWB $\rightarrow x_3$	-0.84
PWB $\rightarrow x_4$	-0.47
Spirituality $\rightarrow y_5$	0.56
MCG $\longleftrightarrow$ Spirituality	0.57
MCG $\longleftrightarrow$ PWB	0.27
Spirituality $\longleftrightarrow$ PWB	0.23
MCG $\rightarrow$ PTG	0.87
Spirituality $\rightarrow$ PTG	-0.08
PWB $\rightarrow$ PTG	-0.15

Example 3: Adolescent Thriving

In Dowling et al. (2004), the direct and indirect effects of Spirituality on Thriving through Religiosity was examined on $N = 1,000$ youth who provided data on 47 variables in the Search Institute Young Adolescents and their Parents archival data set. A latent variable mediation model was fit to the covariance matrix of 14 selected items in their data. Spirituality was indicated by 2 MVs, Religiosity was indicated by 4 MVs, and Thriving was indicated by 8 MVs. The items chosen in our analysis are listed in Table 3.

The standardized effects of Spirituality on Religiosity, of Religiosity on Thriving, and of Spirituality on Thriving are 0.62, 0.25 and 0.71, respectively. The standardized factor loadings of the 14 MVs are presented below in Table 3. All parameter estimates are significant ($p < .05$) except for the effect of Religiosity on Thriving.

Table 3: Standardized loading of 14 items in Adolescent Thriving example

Item	Loading	Item	Loading
Spirituality		Thriving	
YAP315	0.55	YAP187	0.31
YAP284	0.48	YAP135	-0.55
Religiosity		YAP158	0.42
YAP262	0.62	YAP285	0.44
YAP216	0.61	YAP162	-0.27
YAP61	-0.19	YAP261	0.43
YAP253	0.22	YAP46	-0.34
		YAP140	-0.51

Items are from 319 Yong Adolescents and their Parents (YAP) survey.

Additional Example: Schmitt et al. (2002)

Confidence sets and FPEs are computed for a published example in social psychology where a LV mediation model is fit to 13 indicators collected from $N = 220$ women. Here, the effect of Perceived Discrimination against one's in-group on Psychological Well-being is hypothesized to be mediated by In-group Identification. Perceived Discrimination was indicated by four measured variables (MVs): in-group disadvantage (y_1), out-group privilege (y_2), prejudice across contexts (y_3), and past experience with discrimination (y_4). In-group Identification was indicated by four MVs: liking one's group (y_5), valuing one's group (y_6), having pride in one's group (y_7), and having positive experiences due to being a member of one's group (y_8). Finally, Psychological Well-being was indicated by five MVs: life satisfaction (y_9), self-esteem (y_{10}), positive affect (y_{11}), anxiety (y_{12}), and depression (y_{13}). To identify the model, all LVs were scaled according to their first indicators by fixing respective factor loadings to 1.0. Figure A is a path diagram of the model with maximum likelihood (ML) estimates. The estimated discrepancy value $\hat{F} = 0.576$, RMSEA = 0.069 with 90% CI = [0.052, 0.086], and CFI = 0.943.

Direct effect (k_f=1). The ML estimate for the direct effect of Perceived Discrimination on Psychological Well-being, controlling for In-group Identification, is $\hat{\theta}_f = -0.19$. The 95% profile likelihood CI = [-0.39, -0.01], and is defined by $\delta_{CS} = 0.0175$ in the scale of F. Consistent with Schmitt et al.'s (2002) findings, the direct effect of Perceived Discrimination on Well-being is significant, $p < .05$. In contrast, the profile likelihood FPEs were defined by $\tilde{\epsilon} = .005$ in the scale of RMSEA such that $\delta_{\epsilon} = 0.044$ in the scale of F. The two FPEs are (-0.53, 0.97) and have opposite signs, implying that the model's description of the direct effect of Perceived Discrimination to Well-being is deemed sensitive to a slight perturbation to model fit.

The 95% CI translates to a pair of FPEs defined by a perturbation of $\tilde{\epsilon} = .002$ in the scale of RMSEA. Alternatively, the pair of FPEs are numerically equivalent to a 99.81% CI. Thus, CIs can be translated to FPEs and vice versa. However, the conversion of one type of parameter uncertainty to the other in this example is not meaningful. The large CI obtained from the FPEs is not useful for inference according to conventional standards, and the small perturbation associated with CIs are not associated with an informative set of FPEs (i.e., the perturbation to model fit is relatively small to be an effective gauge of sensitivity of the model description of the data afforded by the FPEs).

***Indirect effect* ($k_f = 2$).** The indirect effect of Perceived Discrimination on Well-being through In-group Identification is examined as a set of focal parameters. The ML estimates for the effect of perceived discrimination on in-group identification and the effect of in-group identification on well-being is $\hat{\boldsymbol{\theta}}_f = (0.18, 0.20)'$. The 95% profile likelihood confidence region (CR) is defined by a perturbation of $\delta_{CS} = 0.0273$. Additionally, the FPE contour is defined by the same perturbation of $\tilde{\epsilon} = .005$, in the scale of RMSEA, which is $\delta_{\epsilon} = 0.044$ in the scale of F. Figure B below depicts the ML estimate as a bold dot, the 95% profile likelihood CR as crosses, and the FPE contour as open circles.

In Figure B, the 95% CR does not contain the null value of $\boldsymbol{\theta}_{0f} = (0, 0)'$, implying that the estimated effects are jointly significant from the zero point at $p < .05$. The indirect effect, requiring both estimates to be jointly different from zero, is not significant at $p < .05$ because the CR includes values of zero for the effect of Perceived Discrimination on In-group Identification. Stated differently, the effect of Perceived Discrimination on In-group Identification can be zero or negative at certain positive levels of the effect of in-group identification on well-being. Conversely, the FPE contour takes on all combinations of positive and negative values for each of the effects, reflecting sensitivity of the model's description of the data via the indirect effect estimates due to a slight perturbation to model fit. The FPEs for the effect of Perceived Discrimination on In-group Identification are negative only when the effect of In-group Identification on Well-being is positive. Similarly, the FPEs for the effect of Perceived Discrimination on In-group Identification are positive only for negative effects of In-group Identification on Well-being. The unique information afforded by the CR and FPEs indicate that the indirect effect of Perceived Discrimination on Well-being through In-group Identification cannot be interpreted with rigor.

The CSs and FPEs in this two parameter illustration can also be translated to each other. The CR is associated with a perturbation of $\tilde{\epsilon} = .005$ in the scale of RMSEA and the FPE contour is equivalent to a 99.2% CR. Similar to the $k_f = 1$ example, the numerical equivalence of CSs and FPEs can be established, but the conversion of one type of parameter uncertainty to the other is not necessarily substantively meaningful in this illustration.

OpenMx Code

OpenMx (Neale et al., 2015) is a freely available program for extended structural equation modeling which runs as a package within the R environment (R Core Team, 2015). Here, we provide OpenMx code for the second and third examples as illustrated in the paper. Results for all three examples in this paper were produced by OpenMx version 2.6.9 on R version 3.3.2.

OpenMx by default calculates normal theory maximum likelihood, not maximum Wishart likelihood. The Wishart likelihood function was minimized to produce estimates in this paper. This was achieved by specifying $(N-1)\mathbf{S}/(N-2)$ as the sample covariance matrix and specifying $N-1$ as the sample size in OpenMx.

The present version of OpenMx has implemented the algorithm of Wu and Neale (2012) for the calculation of profile-likelihood CIs. One only needs to specify the confidence level. To calculate FPEs, one could either convert the disturbance to confidence level or change the `mxModel` construct as in the code below. To avoid failure of convergence when calculating the CIs, it is advisable to restrict both parameters of interest and other parameters in the model to a smaller range when setting the bounds of parameters. A CI is valid as long as the final solution of the search of this CI (available through `summary(fit)$CIdetail`) does not touch these artificial bounds. To avoid an error code, one can also modify the feasibility tolerance and the line search tolerance through `mxOption`.

To calculate a FPE contour or a CR for two parameters, one of the two parameters is fixed at selected locations within its pair of FPEs or its CI while the pair of FPEs or CI of the other parameter is calculated. These roles of the two parameters are then switched to produce additional points to make the FPE or CR smoother.

OpenMx code for Example 2: Posttraumatic Growth

```
library(OpenMx)


varnames<-c("MCG","RSES","LOTR","Spirit","Depression","Burden","PTG1","PTG2",
                  "PTG3","PTG4","PTG5")


L<-array(0,dim=c(11,4))
L[1,1]<-sqrt(.82) # MCG
L[c(4,10),2]<-c(sqrt(.93),.5) # Spirituality
L[c(2,3,5,6),3]<-c(.8,.7,-.8,-.1) # PWB
L[7:11,4]<-c(.7,.7,.8,.7,.7) #PTG
L.free<-array(FALSE,dim=c(11,4))
L.free[7:11,4]<-TRUE
L.free[c(2,3,5,6),3]<-TRUE
L.free[10,2]<-TRUE


## Data ##
R<-array(0,dim=c(11,11))
R[upper.tri(R,TRUE)]<-c(1,
                         .31, 1,
                         .16, .52, 1,
                         .51, .22, .24, 1,
                        -.16,-.63,-.56,-.18, 1,
                         .02,-.32,-.31,-.01, .44,  1,
```

```
                         .47, .17, .10, .35, .00,-.01,  1,
                         .51, .12, .02, .26, .02, .08, .60, 1,
                         .63, .16, .05, .31,-.04, .05, .62, .60, 1,
                         .53, .06, .00, .70, .07, .12, .54, .44, .54, 1,
                         .62, .18, .01, .27,-.03,-.01, .57, .61, .72, .52, 1)
R <- R + t(R) -diag(1,11)
Sd<-c(0.55, 0.67, 0.89, 0.74, 0.68, 0.98, 1.02, 1.09, 1.03, 1.32, 1.12)
S<-diag(Sd)%*%R%*%diag(Sd)
S<-(S+t(S))/2
dimnames(S)<-dimnames(R)<-list(varnames,varnames)
n<-273
S<-(n-1)*S/(n-2)

Lambda<-mxMatrix(type="Full",nrow=11,ncol=4,free=L.free,values=L,
                 ubound=.999,lbound=-.999,name="Lambda")
Phi<-mxMatrix(type="Symm",nrow=3,ncol=3,free=!diag(3),values=diag(3),
              lbound=-1,ubound=1,name="Phi")
eigenvalphi<-mxAlgebra(eigenval(Phi),name="eigenvalphi")
pdPhi<-mxConstraint(eigenvalphi>rbind(0,0,0),name="pdPhi")

Gamma<-mxMatrix(type="Full",nrow=3,ncol=1,free=TRUE,values=c(.7,0,0),
                lbound=-2,ubound=2,name="Gamma")
CovXiEta<-mxAlgebra(Phi%*%Gamma,name="CovXiEta")
Rho0<-mxAlgebra(Lambda%&%rbind(cbind(Phi,CovXiEta),cbind(t(CovXiEta),1)),
                name="Rho0")
R2<-mxAlgebra(t(Gamma)%&%Phi,name="R2")
pd<-mxConstraint(R2<1,name="pd")
```

```
D<-mxMatrix(type="Diag",nrow=11,ncol=11,free=T,value=1,lbound=0,name="D")
Sigma<-mxAlgebra(D%&%Rho0+ D*D-D%&%vec2diag(diag2vec(Rho0)),name="Sigma")
condVar<-mxAlgebra(t(omxSelectRows(Gamma,rbind(0,1,1)))
%&%omxSelectRowsAndCols(Phi,rbind(0,1,1))-
   (t(omxSelectRows(Gamma,rbind(0,1,1)))
    %*%omxSelectCols(omxSelectRows(Phi,rbind(0,1,1)),rbind(1,0,0)))^2,
    name="condVar");

Model<-mxModel("Model",Lambda,Phi,eigenvalphi,pdPhi,
               D,Rho0,Sigma,CovXiEta,Gamma,R2,pd,indirect,#VIF,
               mxCI("Gamma",.95),mxCI("condVar",.95),
               mxExpectationNormal(covariance="Sigma",dimnames=varnames),
               mxFitFunctionML(),mxData(observed=S,type="cov",numObs=n-1))
Model<-mxOption(Model,"Feasibility tolerance",1e-3)

fit0<-mxRun(Model,intervals=TRUE)
est0<-fit0@output$estimate
CI0<-fit0$output$confidenceIntervals
CIdetails0<-summary(fit0)$CIdetail

RMSEA0<-summary(fit0)$RMSEA
Tstat0<-summary(fit0)$Chi
df0<-summary(fit0)$degreesOfFreedom

#######################
```

```
FPE<-function(dT)
{
  Model<-omxSetParameters(Model,names(est0),values=est0)
  Model@intervals$Gamma$lowerdelta<-Model@intervals$Gamma$upperdelta<-dT
  Model@intervals$condVar$lowerdelta<-Model@intervals$condVar$upperdelta<-dT
  Model<-mxOption(Model,"Line search tolerance",1e-4)
  fit<-mxRun(Model,intervals=TRUE)
  CI<-fit$output$confidenceIntervals
  CI.code<-fit$output$confidenceIntervalCodes
  return(list(CI=CI,code=CI.code,detail=summary(fit)$CIdetail))
}

summary(fit0)

FPE(df0*(1+(RMSEA0+.001)^2*(n-1))-Tstat0)
FPE(df0*(1+(RMSEA0+.005)^2*(n-1))-Tstat0)
FPE(0.02*Tstat0)
FPE(0.05*Tstat0)
```

OpenMx code for Example 3: Adolescence Thriving

```
library(OpenMx)

L_rel<-c(.8,.5,-.1,.3)
L_sp<-c(.7,.5)
L_th<-c(.4,-.4,.3,.4,-.3,.5,-.3,-.4)
p_rel<-length(L_rel)
p_sp<-length(L_sp)
```

```
p_th<-length(L_th)
p_total<-p_rel+p_sp+p_th

lambda<-L<-cbind(c(rep(1,p_rel),rep(0,p_sp+p_th)),c(rep(0,p_rel),rep(1,p_sp),
                 rep(0,p_th)),c(rep(0,p_rel+p_sp),rep(1,p_th)))
label.L<-array(NA,dim=c(p_total,3))
label.L[L==1]<-c(paste("l_rel",1:p_rel,sep=""),paste("l_sp",1:p_sp,sep=""),
               paste("l_th",1:p_th,sep=""))
lambda[L==1]<-c(L_rel,L_sp,L_th)

n<-1000

S<-as.matrix(read.csv('DowlingCov.csv', header = FALSE))
S[is.na(S)] <- 0
S <- S + t(S) - diag(diag(S))
Sd<-sqrt(diag(S))

selected<-c(3,5,7,11,14,   20,   25,28,32,33,38,41,44,47)
S<-S[selected,selected]
Sd<-Sd[selected]
varnames=paste("item",1:p_total,sep="")
dimnames(S)<-list(varnames,varnames)
S<-(n-1)*S/(n-2)

Lambda<-mxMatrix(type="Full",nrow=p_total,ncol=3,free=as.logical(L),
                 values=lambda,
               label=label.L,ubound=.999,lbound=-.999,name="Lambda")
```

```
SpRel<-mxMatrix(type="Full",nrow=1,ncol=1,free=T,value=.4,
               label="sp_rel",ubound=.98,lbound=0,name="SpRel")
RelTh<-mxMatrix(type="Full",nrow=1,ncol=1,free=T,value=.4,
                label="rel_th",ubound=0.8,lbound=-1.5,name="RelTh")
SpTh<-mxMatrix(type="Full",nrow=1,ncol=1,free=T,value=.6,
                label="sp_th",ubound=2.4,lbound=0.15,name="SpTh")
covSpTh<-mxAlgebra(SpRel*RelTh+SpTh,name="covSpTh")
resTh<-mxAlgebra(1-RelTh*RelTh-SpTh*SpTh-2*RelTh*SpTh*SpRel,name="resTh")
Phipd<-mxConstraint(resTh>0,name="Phipd")
covRelTh<-mxAlgebra(RelTh+SpTh*SpRel,name="covRelTh")
Phi<-mxAlgebra(cbind(rbind(1,SpRel,covRelTh),rbind(SpRel,1,covSpTh),
                rbind(covRelTh,covSpTh,1)),name="Phi")
D<-mxMatrix(type="Full",nrow=p_total,ncol=1,free=T,value=Sd,
            label=c(paste("d_rl",1:p_rel,sep=""),paste("d_sp",1:p_sp,sep=""),
            paste("d_th",1:p_th,sep="")),lbound=0,name="D")
Sigma0<-mxAlgebra((vec2diag(D)%*%Lambda)%&%Phi,name="Sigma0")
Sigma<-mxAlgebra(Sigma0-vec2diag(diag2vec(Sigma0))+vec2diag(D*D),name="Sigma")
CI.direct<-mxCI(c("sp_th"),0.95,type="both")
CI.indirect<-mxCI(c("sp_rel","rel_th"),pchisq(df=1,qchisq(df=2,0.95)),type="both")
Model<-mxModel("Model",Lambda,SpRel,RelTh,SpTh,Phi,D,
               Sigma0,Sigma,covSpTh,resTh,covRelTh,Phipd,CI.indirect,CI.direct,
               mxExpectationNormal(covariance="Sigma",dimnames=varnames),
               mxFitFunctionML(),mxData(observed=S,type="cov",numObs=n-1))
Model<-mxOption(Model,"Feasibility tolerance",1e-2)

fit0<-mxRun(Model,intervals=TRUE)
est0<-fit0@output$estimate;
```

```
RMSEA0<-summary(fit0)$RMSEA;
Tstat0<-summary(fit0)$Chi;
df0<-summary(fit0)$degreesOfFreedom;

fit0$output$confidenceIntervals;# This gives the CIs
fit0$output$confidenceIntervalCodes;
 # Code 0 suggests that the calculations of CIs are OK.
summary(fit)$CIdetail
 # This gives the parameter values in the final iteration
 # of the search for the CIs.

res<-cov2cor(S)-mxEval(Lambda%&%Phi,fit0)
diag(res)<-0
SRMR0<-sqrt(sum(res^2)/p_total/(p_total+1)) # SRMR

#######################

### FPE ###

FPE<-function(dT)
{
  Model<-omxSetParameters(Model,names(est0),values=est0)
  Model@intervals[["sp_rel"]]$lowerdelta<-Model@intervals[["sp_rel"]]$upperdelta<-dT
  Model@intervals[["rel_th"]]$lowerdelta<-Model@intervals[["rel_th"]]$upperdelta<-dT
  Model@intervals[["sp_th"]]$lowerdelta<-Model@intervals[["sp_th"]]$upperdelta<-dT

  fit<-mxRun(Model,intervals=TRUE)
```

```
  CI<-fit$output$confidenceIntervals
  CI.code<-fit$output$confidenceIntervalCodes
  return(list(CI=CI,code=CI.code,detail=summary(fit)$CIdetail))
}

###########################


CR<-function(par1,par2,par1.values,dT)
{
  ## Plot a CR with parameter par1 taking values within its CI
  ##        for each of which a CI of parameter par2 is plotted.
  ## par1.values lie between 0 and 1 specifying the locations of
  ##    par1 taken within its CI.
  ## dT is the desired increase in test statistic

  Model<-omxSetParameters(Model,names(est0),values=est0)
  Model@intervals<-Model@intervals[c(par1,par2)];
  Model@intervals[par2][[1]]$lowerdelta<-Model@intervals[par2][[1]]$upperdelta<-dT
  Model@intervals[par1][[1]]$lowerdelta<-Model@intervals[par1][[1]]$upperdelta<-dT

  fit0<-mxRun(Model,intervals=TRUE);
  est0<-fit0$output$estimate
  CI0<-fit0$output$confidenceIntervals
  CIdetails0<-summary(fit0)$CIdetail
  Tstat0<-summary(fit0)$Chi
  Tstat<-Tstat0+dT
```

```
L<-length(par1.values)
m<-CI0[par1,"lbound"]
M<-CI0[par1,"ubound"]
P1<-c(m,m*(1-par1.values)+M*par1.values,M);
P2.upper<-P2.lower<-rep(NA, L+2);
P2.upper[1]<-P2.lower[1]<-CIdetails0[CIdetails0$parameter==par1&CIdetails0$side==
            "lower",par2]
P2.upper[L+2]<-P2.lower[L+2]<-CIdetails0[CIdetails0$parameter==par1&CIdetails0$side==
            "upper",par2]

### These are the nuisance parameter values corresponding to
### the limits of CI of par1.

initial.lower<-as.matrix(CIdetails0[CIdetails0$parameter==par1&CIdetails0$side==
                "lower",names(est0)])
initial.upper<-as.matrix(CIdetails0[CIdetails0$parameter==par1&CIdetails0$side==
                "upper",names(est0)])

interpol<-function(i){
  initial.upper*par1.values[i]+initial.lower*(1-par1.values[i])
  }
initial<-sapply(1:L,FUN=interpol)

Model@intervals<-Model@intervals[par2]

cCI<-function(i)
{
```

```
    Model<-omxSetParameters(Model,par1,free=FALSE,values=P1[i+1])
    Model<-omxSetParameters(Model,names(est0),values=initial[,i])
    fit<-mxRun(Model,intervals=FALSE)
    est<-fit$output$estimate
    Model@intervals[par2][[1]]$lowerdelta<-Model@intervals[par2][[1]]$upperdelta
            <-Tstat-summary(fit)$Chi
    Model<-omxSetParameters(Model,names(est),values=est)
    fit<-mxRun(Model,intervals=TRUE)
    CI<-fit$output$confidenceIntervals
    CI.code<-fit$output$confidenceIntervalCodes
    return(c(CI,CI.code))
  }
  CI<-vapply(1:L,FUN=cCI,FUN.VALUE=c(0,0,0,0,0));
  P2.upper[2:(L+1)]<-CI[3,]
  P2.lower[2:(L+1)]<-CI[1,];
  return(list(P1=P1,P2.l=P2.lower,P2.u=P2.upper,P2.code=CI[c(4,5),]))
}

### Compute and plot CRs and FPEs ############

  CR.indirect1<-CR("sp_rel","rel_th",(1:9)/10,qchisq(df=2,.95))
  CR.indirect2<-CR("rel_th","sp_rel",(1:9)/10,qchisq(df=2,.95))
  CR.indirect<-cbind(c(rep(CR.indirect1$P1,2),CR.indirect2$P2.l,CR.indirect2$P2.u),
                    c(CR.indirect1$P2.l,CR.indirect1$P2.u,rep(CR.indirect2$P1,2)))

  dT<-df0*(1+(RMSEA0+0.001)^2*(n-1))-Tstat0
  FPR1<-CR("sp_rel","rel_th",(1:9)/10,dT)
```

```
FPR2<-CR("rel_th","sp_rel",(1:9)/10,dT)
FPR01<-cbind(c(rep(FPR1$P1,2),FPR2$P2.l,FPR2$P2.u),
             c(FPR1$P2.l,FPR1$P2.u,rep(FPR2$P1,2)))

dT<-0.03*(n-1)
FPR1<-CR("sp_rel","rel_th",(1:9)/10,dT)
FPR2<-CR("rel_th","sp_rel",(1:9)/10,dT)
FPR02<-cbind(c(rep(FPR1$P1,2),FPR2$P2.l,FPR2$P2.u),
             c(FPR1$P2.l,FPR1$P2.u,rep(FPR2$P1,2)))

dT<-Tstat0*0.02
FPR1<-CR("sp_rel","rel_th",(1:9)/10,dT)
FPR2<-CR("rel_th","sp_rel",(1:9)/10,dT)
FPR.F01<-cbind(c(rep(FPR1$P1,2),FPR2$P2.l,FPR2$P2.u),
               c(FPR1$P2.l,FPR1$P2.u,rep(FPR2$P1,2)))

dT<-Tstat0*0.05
FPR1<-CR("sp_rel","rel_th",(1:9)/10,dT)
FPR2<-CR("rel_th","sp_rel",(1:9)/10,dT)
FPR.F02<-cbind(c(rep(FPR1$P1,2),FPR2$P2.l,FPR2$P2.u),
               c(FPR1$P2.l,FPR1$P2.u,rep(FPR2$P1,2)))

plot(CR.indirect[,1],CR.indirect[,2],pch=".",xlab="Spirituality->Religiosity",
      ylab="Religiosity->Thriving",xlim=c(0,1),ylim=c(-1.5,.8))
points(FPR01[,1],FPR01[,2],pch="+")
points(FPR02[,1],FPR02[,2],pch="x")
points(FPR.F01[,1],FPR.F01[,2],pch="*")
```

```
points(FPR.F02[,1],FPR.F02[,2])

####

dT<-df0*(1+(RMSEA0+0.001)^2*(n-1))-Tstat0
FPR1<-CR("sp_rel","sp_th",(1:9)/10,dT)
FPR2<-CR("sp_th","sp_rel",(1:9)/10,dT)
FPR01<-cbind(c(rep(FPR1$P1,2),FPR2$P2.l,FPR2$P2.u),
             c(FPR1$P2.l,FPR1$P2.u,rep(FPR2$P1,2)))

dT<-30
FPR1<-CR("sp_rel","sp_th",(1:9)/10,dT)
FPR2<-CR("sp_th","sp_rel",(1:9)/10,dT)
FPR02<-cbind(c(rep(FPR1$P1,2),FPR2$P2.l,FPR2$P2.u),
             c(FPR1$P2.l,FPR1$P2.u,rep(FPR2$P1,2)))

dT<-Tstat0*0.02
FPR1<-CR("sp_rel","sp_th",(1:9)/10,dT)
FPR2<-CR("sp_th","sp_rel",(1:9)/10,dT)
FPR.F01<-cbind(c(rep(FPR1$P1,2),FPR2$P2.l,FPR2$P2.u),
               c(FPR1$P2.l,FPR1$P2.u,rep(FPR2$P1,2)))

dT<-Tstat0*0.05
FPR1<-CR("sp_rel","sp_th",(1:9)/10,dT)
FPR2<-CR("sp_th","sp_rel",(1:9)/10,dT)
FPR.F02<-cbind(c(rep(FPR1$P1,2),FPR2$P2.l,FPR2$P2.u),
               c(FPR1$P2.l,FPR1$P2.u,rep(FPR2$P1,2)))
```

```
plot(FPR01[,1],FPR01[,2],pch="+",xlab="Spirituality->Religiosity",
        ylab="Spirituality->Thriving",xlim=c(0,1),ylim=c(0,2.4))
points(FPR02[,1],FPR02[,2],pch="x")
points(FPR.F01[,1],FPR.F01[,2],pch="*")
points(FPR.F02[,1],FPR.F02[,2])
```

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Figure A. Path Diagram for Schmitt et al.'s (2002) LV Mediation Model

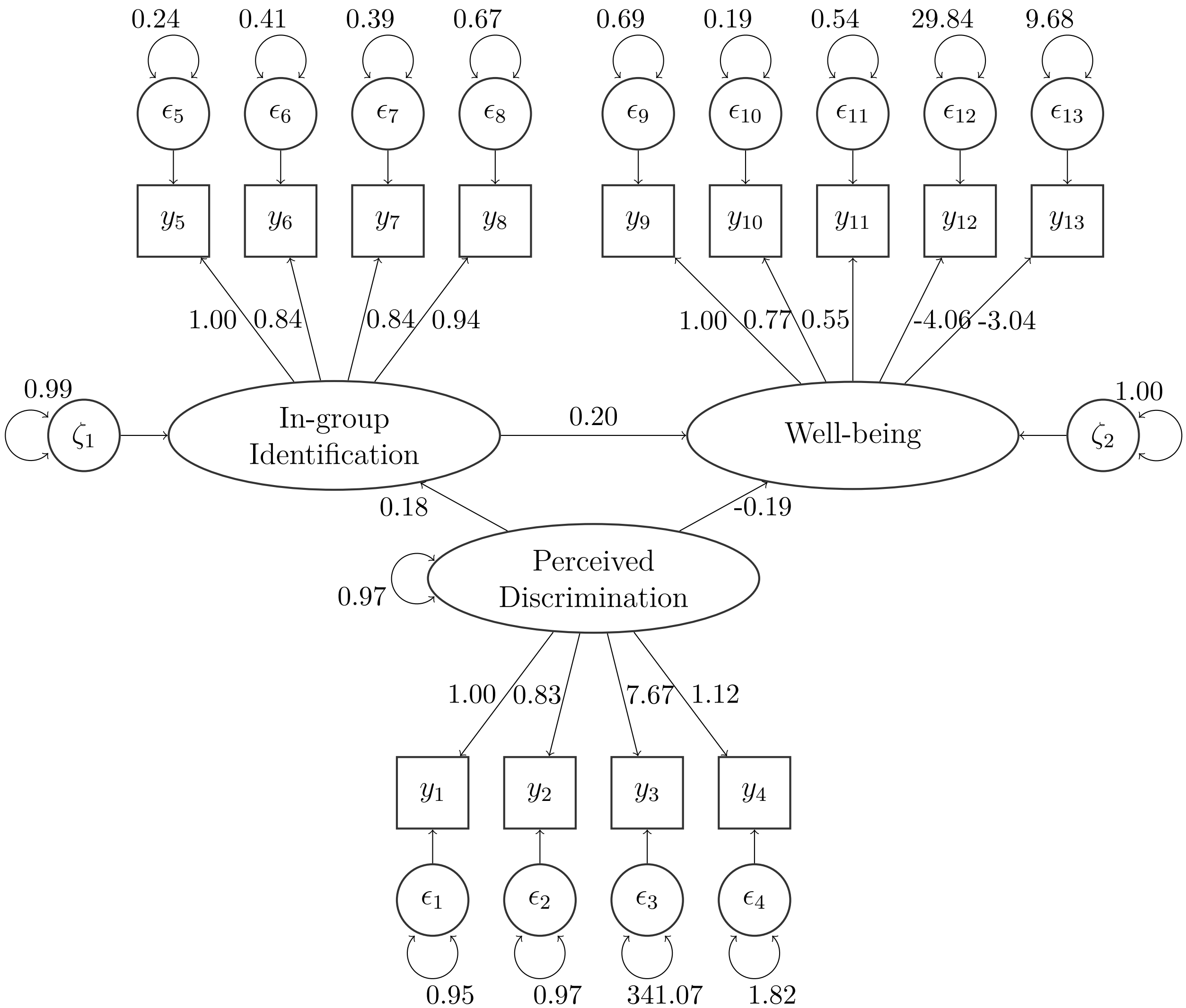

Figure B. Profile Likelihood Confidence Bounds and Fungible Contours for the Indirect Effect of Perceived Discrimination on Well-being.