

## Appendix B-1

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# R code to calculate the approximate group sizes for testing non-inferiority/
# superiority of a linear contrast for a designated power
#####

# 1. You need to specify the following values in the input part: significance level,
#    Type II error, the deviation from  $H_0$  to  $H_a$ , contrast coefficients,
#    and the corresponding unit cost, and variance for each group.
# 2. Based on different purposes, you can have different group size allocation ratios.
#    You can choose gamma=1 to minimize total cost (MC), gamma=2 to minimize total
#    sample size (MS) or gamma=3 for equal group size (ES).
# 3. We use the data from Clarke et al. (2005) and the sample size calculation is
#    demonstrated in the Section of illustrative example in this paper.
#####

# ----- procedure -----

                                # begin of the function of SizeInfe
SizeInfe = function(alpha, beta, Delta, lc, cost, variance, gamma)
{
# Step 1: Calculate the group size allocation ratio and the initial group sizes
                                # Equ.11 of allocation ratio for minimal total cost
gam1 = abs(lc/lc[1])*(variance/variance[1])^0.5*(cost[1]/cost)^0.5
gam2 = abs(lc/lc[1])*(variance/variance[1])^0.5          # for minimal sample size
gam3 = c(rep(1,length(lc)))                             # for equal group size
if (gamma == 1) {gam = gam1}
  else if (gamma == 2) {gam = gam2}
    else {gam = gam3}

zalpha = qnorm(1-alpha)          # quantile of the standard normal distribution
zbeta = qnorm(1-beta)

sum0 = lc^2 %*%(variance/gam)

zn1 = sum0*(zalpha+zbeta)^2/Delta^2 # initial first group size in Equ. 2
zn = zn1 * gam                    # initial sizes for all groups

# Step 2: Calculate the degrees of freedom and find the quantile of the t distribution
df = function(lc, variance, ns)  # find degrees of freedom in Equ. 14
{
  u1 = lc^2 * variance / ns
  sum1 = sum(u1)
  sum2 = sum(u1^2/(ns+1))
  df = sum1^2 / sum2 - 2
}

df1 = df(lc,variance,zn)
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talpha = qt(1-alpha, df1)          # quantile of the t distribution with df1
tbeta = qt(1-beta, df1)

# Step 3: Calculate the approximate group sizes

tn1 = sum0*(talpha+tbeta)^2/Delta^2 # approximate first group size in Equ. 15
tn = tn1 * gam                      # approximate group sizes for all groups
fn = ceiling(tn)                    # final group sizes after rounding up
Ton = sum(fn)                       # the resulting total size
Toc = fn %*% cost                    # the resulting total cost

list(gamma=gam, sum0=sum0, zalpha=zalpha, zbeta=zbeta, size.ini=zn, df=df1,
     talpha=talpha, tbeta=tbeta, size.t=tn, size.fixpower=fn, total.size=Ton,
     total.cost=Toc)

}                                     # end of the function of SizeInfe

# ----- input -----
alpha = 0.05                         # significance level
beta = 0.2                           # Type II error rate
Delta = 2.2                          # big Delta, the deviation from H0 to Ha

# In the following, you can change the number of elements in the parentheses based on
# the number of groups you are investigating.

lc = c(1/2,1/2,-1)                   # coefficients of contrast
cost = c(20,50,100)                  # unit cost for each group
variance = c(79.21,57.76,77.44)      # variance for each group
gamma = 1                             # gamma = 1 for minimal total cost (MC)
                                     # gamma = 2 for minimal total sample size (MS)
                                     # gamma = 3 for equal group size (ES)

# ----- output -----
SizeInfe(alpha, beta, Delta, lc, cost, variance, gamma)

```

## Appendix B-2

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# R code to calculate the approximate group sizes for testing equivalence of a linear
# contrast for a designated power
#####
# 1. You need to specify the following values in the input part: significance level,
#    Type II error, equivalence margins (upper & lower), linear contrast for the
#    alternative hypothesis, contrast coefficients, and the corresponding unit cost,
#    and variance for each group.
# 2. Based on different purposes, you can have different group size allocation ratios.
#    You can choose gamma=1 to minimize total cost (MC), gamma=2 to minimize total
#    sample size (MS) or gamma=3 for equal group size (ES).
# 3. We use the data from Van Wier et al.(2012) and the sample size calculation is
#    demonstrated in the Section of illustrative example in this paper.
#####
#----- procedure -----
# begin of the function of SizeEqui
SizeEqui = function(alpha, beta, delta1, delta2, delta, lc, cost, variance, gamma)
{
# Step 1: Calculate the group size allocation ratios and the initial group sizes
# Equ. 11 of allocation ratio for minimal total cost
gam1 = abs(lc/lc[1])*(variance/variance[1])^0.5*(cost[1]/cost)^0.5
gam2 = abs(lc/lc[1])*(variance/variance[1])^0.5 # for minimal sample size
gam3 = c(rep(1,length(lc))) # for equal group size
if (gamma == 1) {gam = gam1}
else if (gamma == 2) {gam = gam2}
else {gam = gam3}
zalpha = qnorm(1-alpha) # quantile of the standard normal distribution
if (delta == (delta1+delta2)/2) {power = 1-beta/2 # given power in Equ. 8
} else {power = 1-beta } # given power in Equ. 9
zbeta = qnorm(power)
sum0 = lc^2 %*%(variance/gam)
eta = min(delta - delta1, delta2 - delta)
zn1 = sum0*(zalpha+zbeta)^2/eta^2 # initial first group size in Equ. 8 or 9
zn = zn1 * gam # initial sizes for all groups
# Step 2: Calculate the degrees of freedom and find the quantile of the t distribution
df = function(lc, variance, ns) # find degrees of freedom in Equ. 14
{
u1 = lc^2 * variance / ns
sum1 = sum(u1)
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sum2 = sum(u1^2/(ns+1))

df = sum1^2 / sum2 - 2

}

df1 = df(lc, variance, zn)

talpha = qt(1-alpha, df1)      # quantile of the t distribution with df1

tbeta = qt(power, df1)

# Step 3: Calculate the approximate group sizes

tn1 = sum0*(talpha+tbeta)^2/eta^2    # approximate first group size in Equ.15

tn = tn1 * gam; tn0 = tn           # approximate groups sizes for all groups

# Step 4: Check the power

tpower = function(ns)              # power function of t test

{

  v = sum0/ns[1]

  t1 = (delta-delta1)/v^0.5

  t2 = (delta2-delta)/v^0.5

  df = df(lc, variance, ns)

  talpha = qt(1-alpha, df)

  tpw = pt(-talpha, df, ncp=-t2)- pt(talpha, df, ncp=t1) # Equ.16 for noncentral t

}

tpw = tpower(tn); tpw0 = tpw        # tentative power

# Run iteration if the tentative power is not achieved the designated power

while(tpw < 1-beta)

{

  tn[1] = tn[1] + 0.2              # increment value 0.2

  tn = tn[1] * gam

  tpw = tpower(tn)

}

fn = ceiling(tn)                   # final group sizes after rounding up

Ton = sum(fn)                      # the resulting total size

Toc = fn %*% cost                  # the resulting total cost

list(gamma=gam, sum0=sum0, zalpha=zalpha, zbeta=zbeta, size.ini=zn, df=df1,

      talpha=talpha, tbeta=tbeta, size.t=tn0, power.ini=tpw0, power.final = tpw,

      size.fixpower=fn, total.size=Ton, total.cost=Toc)

}                                  # end of the function of SizeEqui

# ----- input -----

alpha = 0.05                      # significance level

beta = 0.2                        # Type II error rate

delta1 = -2                       # lower margin

delta2 = 2                        # upper margin

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delta = 0.2                                # linear contrast for the alternative hypothesis
# In the following, you can change the number of elements in the parentheses based on
# the number of groups you are investigating.
lc = c(-1,1/2,1/2)                        # coefficients of contrast
cost = c(20,69,65)                        # unit cost for each group
variance = c(48.79,38.10,32.81)           # variance for each group
gamma = 1                                # gamma = 1 for minimal total cost (MC)
                                           # gamma = 2 for minimal total sample size (MS)
                                           # gamma = 3 for equal group size (ES)
# ----- output -----
SizeEqui(alpha, beta, delta1, delta2, delta, lc, cost, variance, gamma)

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## Appendix B-3

```
# R code to calculate other non-fixed group sizes when some group sizes are fixed,  
# for testing equivalence of a linear contrast for a designated power  
#####  
# 1. You need to specify the following values in the input part: significance level,  
# Type II error, equivalence margins (upper & lower) and linear contrast for the  
# alternative hypothesis. You also need to specify the vectors of contrast  
# coefficients of fixed group(s) and non-fixed group(s), the corresponding  
# variances and unit costs, respectively. (see Input statement)  
# 2. Based on different purposes, you can have different group size allocation ratios.  
# You can choose gamma=1 to minimize total cost (MC), gamma=2 to minimize total  
# sample size (MS) or gamma=3 for equal non-fixed group size (ES).  
# 3. We use the data from Van Wier et al.(2012) and the sample size calculation is  
# demonstrated in the Section of illustrative example in this paper.  
#####  
#----- procedure -----  
  
# begin of the function of SizeEquiFix  
SizeEquiFix = function(alpha, beta, delta1, delta2, delta, lc, cost, variance, gamma)  
{  
# Step 1: Calculate the allocation ratios and the initial non-fixed group sizes  
# Equ. 11 of allocation ratio for minimal total cost  
gam1 = abs(la/lc[1])*(vara / vara[1])^0.5 *(costa[1]/costa)^0.5  
gam2 = abs(la/lc[1])*(vara / vara[1])^0.5 # for minimal sample size  
gam3 = c(rep(1,length(la))) # for equal non-fixed group size  
if (gamma == 1) {gam = gam1}  
else if (gamma == 2) {gam = gam2}  
else {gam = gam3}  
zalpha = qnorm(1-alpha) # quantile of the standard normal distribution  
if (delta == (delta1+delta2)/2) {power = 1-beta/2 # given power in Equ. 8  
} else {power = 1-beta } # given power in Equ. 9  
zbeta = qnorm(power)  
suma = la^2 %*%(vara/gam)  
eta = min(delta - delta1, delta2 - delta)  
zdd = eta^2 - (zalpha+zbeta)^2*(lb^2 %*% (varb/nb)) # zdd needs to be positive  
if (zdd > 0.5) # The arbitrary value 0.5 is to set the  
# threshold to prevent from running too long.  
zn1 = suma*(zalpha+zbeta)^2/zdd # initial first non-fixed group size in Equ. 13  
zna = zn1 * gam # initial other non-fixed group sizes  
zn = c(zna, nb) # initial sizes for all groups
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# Step 2: Calculate the degrees of freedom and find the quantile of the t distribution
df = function(lc, variance, ns)      # find degrees of freedom in Equ. 14
{
  u1 = lc^2 * variance / ns
  sum1 = sum(u1)
  sum2 = sum(u1^2/(ns+1))
  df = sum1^2 / sum2 - 2
}

df1 = df(lc, variance, zn)

talpha = qt(1-alpha, df1)           # quantile of the t distribution with df1
tbeta = qt(power, df1)

# Step 3: Calculate the approximate group sizes
tdd = eta^2 - (talpha+tbeta)^2*(lb^2 %*% (varb/nb))

if (tdd > 0.5)                       # approximate first non-fixed group size in Equ. 15
  tn1 = suma*(talpha+tbeta)^2/tdd
  tna = tn1 * gam                    # approximate other non-fixed group sizes
  tn = c(tna, nb); tn0 = tn         # approximate sizes for all groups

# Step 4: Check the power
tpower = function(ns)               # power function of t test
{
  v = suma/ns[1] + lb^2 %*% varb/nb
  t1 = (delta-delta1)/v^0.5
  t2 = (delta2-delta)/v^0.5
  df = df(lc, variance, ns)
  talpha = qt(1-alpha, df)
  tpw = pt(-talpha ,df, ncp=-t2)- pt(talpha ,df, ncp=t1) # Equ. 16 for noncentral t
}

tpw = tpower(tn); tpw0 = tpw        # tentative power

# Run iteration if the tentative power is not achieved the designated power
while(tpw < 1-beta)
{
  tn[1] = tn[1] + 0.2                # increment value 0.2
  tn = c(tn[1]*gam, nb)
  tpw = tpower(tn)
}

fn = ceiling(tn)                    # final group sizes after rounding up
Ton = sum(fn)                       # the resulting total size
Toc = fn %*% cost                    # the resulting total cost

list(gamma=gam, suma=suma, zalpha=zalpha, zbeta=zbeta, size.ini=zn, df=df1,

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    talpha=talpha,tbeta=tbeta, size.t=tn0, power.ini=tpw0, power.final = tpw,
    size.fixpower=fn, total.size=Ton, total.cost=Toc)
}                                     # end of the function of SizeEquiFix

#----- input -----
alpha = 0.05          # significance level
beta = 0.2            # Type II error rate
delta1 = -2           # lower margin
delta2 = 2            # upper margin
delta = 0.2           # linear contrast for the alternative hypothesis

# In the following, we use b to represent the fixed group, a to represent other
# non-fixed groups. For example, if we fix the first group size, nb = c(200), then
# the corresponding coefficients of contrast for the fixed group lb = c(-1), and for
# other non-fixed groups la = c(1/2,1/2). You can also change the nummber of elements
# in the parentheses (vector) based on the number of groups you are investigating.
# Note that if the fixed group size is not sufficient, the program will print the
# error message "tn1 not found" and will be terminated.

nb = c(200)           # fixed group size n1 = 200
la = c(1/2,1/2)       # coefficients for other non-fixed groups
lb = c(-1)            # coefficient for the fixed group
lc = c(la,lb)         # coefficients for all groups
costa = c(69,65)      # unit costs for other non-fixed groups
costb = c(20)         # unit cost for the fixed group
cost = c(costa, costb) # unit costs for all groups
vara = c(38.10,32.81) # variances for other non-fixed groups
varb = c(48.79)       # variance for the fixed group
variance = c(vara, varb)

gamma = 1              # gamma = 1 for minimal total cost
                      # gamma = 2 for minimal sample size
                      # gamma = 3 for equal group size

#----- output -----
SizeEquiFix(alpha, beta, delta1, delta2, delta, lc, cost, variance, gamma)

```