Online appendix to accompany:

Preacher, K. J., & Hancock, G. R. (submitted). *Meaningful aspects of change as novel random coefficients: A general method for reparameterizing longitudinal models*.

Contents

1. Mplus syntax for a reparameterized inverse quadratic function

The data may be obtained from Marie Davidian's website:
http://www4.stat.ncsu.edu/~davidian/data/theophylline.dat
The data file should be transformed to wide format prior to analysis.)

```
TITLE: davidian theophylline data, inverse quadratic;
DATA: FILE IS theophylline_wide.dat; !data file in ascii format
VARIABLE: NAMES ARE subject dose wt t1-t11 conc1-conc11; !all 25 variables
  !subject: subject ID
  !dose: oral dose of theophylline in mg/kg
  !wt: subject weight in kg
  !t1-t11: time since administration of dose in hours
  !conc1-conc11: theophylline concentration in mg/L
USEVARIABLES ARE conc1-conc11; !we plan to model 11 variables
CONSTRAINT ARE t1-t11; !individual-specific time scores for use in loadings
ANALYSIS: ESTIMATOR=ML; ITERATIONS=2000; CONVERGENCE=.001; !estimation options
!BOOTSTRAP=5000; !for bootstrap confidence intervals if desired
MODEL:

[conc1-conc11*](z1-z11); !mean trend (z's defined below)
conc1-conc11*.7(ve); !residual variances constrained equal
conc1-conc10 PWITH conc2-conc11*.2(ve2); !optional lag-1 residual covariances
b*.002(vb); c@0; m*.5(vm); !random coeff. variances (only for maximizer and theta1)
b WITH c@0; b WITH m*; c WITH m@0; !random coefficient covariances
[b@0 c@0 m@0]; !factor means constrained to zero
b BY conc1*(b1); b BY conc2-conc11*(b2-b11); !loadings for the theta1 parameter (b)
c BY conc1*(c1); c BY conc2-conc11*(c2-c11); !loadings for the theta2 parameter (c)
m BY conc1*(m1); m BY conc2-conc11*(m2-m11); !loadings for the maximizer parameter (m)
```

```
MODEL CONSTRAINT:

NEW(mb*.07 mc*.02 mm*1.77); !starts for theta1, theta2, and maximizer parameters

!intercepts constrained equal to target function
z1=t1/(mb*t1+mc*(mm^2+t1^2));
z2=t2/(mb*t2+mc*(mm^2+t2^2));
z3=t3/(mb*t3+mc*(mm^2+t3^2));
z4=t4/(mb*t4+mc*(mm^2+t4^2));
z5=t5/(mb*t5+mc*(mm^2+t5^2));
z6=t6/(mb*t6+mc*(mm^2+t6^2));
z7=t7/(mb*t7+mc*(mm^2+t7^2));
z8=t8/(mb*t8+mc*(mm^2+t8^2));
z9=t9/(mb*t9+mc*(mm^2+t9^2));
z10=t10/(mb*t10+mc*(mm^2+t10^2));
z11=t11/(mb*t11+mc*(mm^2+t11^2));

!loadings for "theta1" factor, constrained equal to d(y)/d(mb)
b1=-1*t1^2/(mb*t1+mc*(mm^2+t1^2))^2;
b2=-1*t2^2/(mb*t2+mc*(mm^2+t2^2))^2;
b3=-1*t3^2/(mb*t3+mc*(mm^2+t3^2))^2;
b4=-1*t4^2/(mb*t4+mc*(mm^2+t4^2))^2;
b5=-1*t5^2/(mb*t5+mc*(mm^2+t5^2))^2;
b6=-1*t6^2/(mb*t6+mc*(mm^2+t6^2))^2;
b7=-1*t7^2/(mb*t7+mc*(mm^2+t7^2))^2;
b8=-1*t8^2/(mb*t8+mc*(mm^2+t8^2))^2;
b9=-1*t9^2/(mb*t9+mc*(mm^2+t9^2))^2;
b10=-1*t10^2/(mb*t10+mc*(mm^2+t10^2))^2;
b11=-1*t11^2/(mb*t11+mc*(mm^2+t11^2))^2;

!loadings for "theta2" factor, constrained equal to d(y)/d(mc)
c1=-1*t1*(mm^2+t1^2)/(mb*t1+mc*(mm^2+t1^2))^2; !not strictly necessary
c2=-1*t2*(mm^2+t2^2)/(mb*t2+mc*(mm^2+t2^2))^2; !here because this
c3=-1*t3*(mm^2+t3^2)/(mb*t3+mc*(mm^2+t3^2))^2; !coefficient is fixed,
c4=-1*t4*(mm^2+t4^2)/(mb*t4+mc*(mm^2+t4^2))^2; !but included for
c5=-1*t5*(mm^2+t5^2)/(mb*t5+mc*(mm^2+t5^2))^2; !completeness.
c6=-1*t6*(mm^2+t6^2)/(mb*t6+mc*(mm^2+t6^2))^2;
c7=-1*t7*(mm^2+t7^2)/(mb*t7+mc*(mm^2+t7^2))^2;
c8=-1*t8*(mm^2+t8^2)/(mb*t8+mc*(mm^2+t8^2))^2;
c9=-1*t9*(mm^2+t9^2)/(mb*t9+mc*(mm^2+t9^2))^2;
c10=-1*t10*(mm^2+t10^2)/(mb*t10+mc*(mm^2+t10^2))^2;
c11=-1*t11*(mm^2+t11^2)/(mb*t11+mc*(mm^2+t11^2))^2;

!loadings for "maximizer" factor, constrained equal to d(y)/d(mm)
m1=-2*mm*mc*t1/(mb*t1+mc*(mm^2+t1^2))^2;
m2=-2*mm*mc*t2/(mb*t2+mc*(mm^2+t2^2))^2;
m3=-2*mm*mc*t3/(mb*t3+mc*(mm^2+t3^2))^2;
m4=-2*mm*mc*t4/(mb*t4+mc*(mm^2+t4^2))^2;
m5=-2*mm*mc*t5/(mb*t5+mc*(mm^2+t5^2))^2;
m6=-2*mm*mc*t6/(mb*t6+mc*(mm^2+t6^2))^2;
m7=-2*mm*mc*t7/(mb*t7+mc*(mm^2+t7^2))^2;
m8=-2*mm*mc*t8/(mb*t8+mc*(mm^2+t8^2))^2;
m9=-2*mm*mc*t9/(mb*t9+mc*(mm^2+t9^2))^2;
m10=-2*mm*mc*t10/(mb*t10+mc*(mm^2+t10^2))^2;
m11=-2*mm*mc*t11/(mb*t11+mc*(mm^2+t11^2))^2;

OUTPUT: TECH1; !CINTERVAL(BCBOOTSTRAP);
```

2. Mplus syntax for an unconditional bilinear spline model

```
TITLE: zerbe glucose;
```

```
DATA: FILE IS glucose.dat; !data file in ascii format
VARIABLE: NAMES ARE obese ph1-ph8; !all 9 variables
  !obese: obesity status, 0=control, 1=obese
  !ph1-ph8: plasma phosphate concentration repeated measures
USEVARIABLES ARE ph1-ph8; !we plan to model 8 variables
!ANALYSIS: BOOTSTRAP IS 1000; !for bootstrap confidence intervals if desired
MODEL:

[ph1-ph8*](z1-z8); !mean trend (z's defined below)
ph1-ph8*(ve); !residual variances constrained equal
w1-w4*; !random coefficient variances
w1 WITH w2*; w1 WITH w3*; w1 WITH w4*; !random coefficient covariances
w2 WITH w3*; w2 WITH w4*; w3 WITH w4*; !random coefficient covariances
[w1@0 w2@0 w3@0 w4@0]; !factor means constrained to zero
w1 BY ph1-ph8@1; !loadings for 1st factor, constrained to 1
w2 BY ph1*(w21); w2 BY ph2-ph8(w22-w28); !loadings for 2nd factor, defined below
w3 BY ph1*(w31); w3 BY ph2-ph8(w32-w38); !loadings for 3rd factor, defined below
w4 BY ph1*(w41); w4 BY ph2-ph8(w42-w48); !loadings for knot factor, defined below

MODEL CONSTRAINT:

!starts for parameters of the target function
NEW(mw1*3.4 mw2*-.26 mw3*.54 mw4*1.6 b1-b4);

!intercepts constrained equal to target function
z1=mw1+mw2*0.0+mw3*sqrt((0.0-mw4)^2);
z2=mw1+mw2*0.5+mw3*sqrt((0.5-mw4)^2);
z3=mw1+mw2*1.0+mw3*sqrt((1.0-mw4)^2);
z4=mw1+mw2*1.5+mw3*sqrt((1.5-mw4)^2);
z5=mw1+mw2*2.0+mw3*sqrt((2.0-mw4)^2);
z6=mw1+mw2*3.0+mw3*sqrt((3.0-mw4)^2);
z7=mw1+mw2*4.0+mw3*sqrt((4.0-mw4)^2);
z8=mw1+mw2*5.0+mw3*sqrt((5.0-mw4)^2);

!loadings for 2nd and 3rd factors, constrained to d(y)/d(mw2) and d(y)/d(mw3)
w21=0.0; w31=sqrt((0.0-mw4)^2);
w22=0.5; w32=sqrt((0.5-mw4)^2);
w23=1.0; w33=sqrt((1.0-mw4)^2);
w24=1.5; w34=sqrt((1.5-mw4)^2);
w25=2.0; w35=sqrt((2.0-mw4)^2);
w26=3.0; w36=sqrt((3.0-mw4)^2);
w27=4.0; w37=sqrt((4.0-mw4)^2);
w28=5.0; w38=sqrt((5.0-mw4)^2);

!loadings for knot factor, constrained to d(y)/d(mw4)
w41=(mw3*(mw4-0.0))/sqrt((0.0-mw4)^2);
w42=(mw3*(mw4-0.5))/sqrt((0.5-mw4)^2);
w43=(mw3*(mw4-1.0))/sqrt((1.0-mw4)^2);
w44=(mw3*(mw4-1.5))/sqrt((1.5-mw4)^2);
w45=(mw3*(mw4-2.0))/sqrt((2.0-mw4)^2);
w46=(mw3*(mw4-3.0))/sqrt((3.0-mw4)^2);
w47=(mw3*(mw4-4.0))/sqrt((4.0-mw4)^2);
w48=(mw3*(mw4-5.0))/sqrt((5.0-mw4)^2);

!parameters of original parameterization
b1=mw1+mw3*mw4;
b2=mw2-mw3;
b3=mw1-mw3*mw4;
b4=mw2+mw3;

OUTPUT: TECH1; !CINTERVAL(BCBOOTSTRAP);
```

Zerbe's (1979) data (N = 33), obtained from Zerbe (1979, p. 219).

```
0 3.4 3.3 3.0 2.6 2.2 2.5 3.4 4.4
0 3.7 2.6 2.6 1.9 2.9 3.2 3.1 3.9
0 4.0 4.1 3.1 2.3 2.9 3.1 3.9 4.0
0 3.6 3.0 2.2 2.8 2.9 3.9 3.8 4.0
0 4.1 3.8 2.1 3.0 3.6 3.4 3.6 3.7
0 3.8 2.2 2.0 2.6 3.8 3.6 3.0 3.5
0 3.8 3.0 2.4 2.5 3.1 3.4 3.5 3.7
0 4.4 3.9 2.8 2.1 3.6 3.8 4.0 3.9
0 5.0 4.0 3.4 3.4 3.3 3.6 4.0 4.3
0 3.7 3.1 2.9 2.2 1.5 2.3 2.7 2.8
0 3.7 2.6 2.6 2.3 2.9 2.2 3.1 3.9
0 4.4 3.7 3.1 3.2 3.7 4.3 3.9 4.8
0 4.7 3.1 3.2 3.3 3.2 4.2 3.7 4.3
1 4.3 3.3 3.0 2.6 2.2 2.5 2.4 3.4
1 5.0 4.9 4.1 3.7 3.7 4.1 4.7 4.9
1 4.6 4.4 3.9 3.9 3.7 4.2 4.8 5.0
1 4.3 3.9 3.1 3.1 3.1 3.1 3.6 4.0
1 3.1 3.1 3.3 2.6 2.6 1.9 2.3 2.7
1 4.8 5.0 2.9 2.8 2.2 3.1 3.5 3.6
1 3.7 3.1 3.3 2.8 2.9 3.6 4.3 4.4
1 5.4 4.7 3.9 4.1 2.8 3.7 3.5 3.7
1 3.0 2.5 2.3 2.2 2.1 2.6 3.2 3.5
1 4.9 5.0 4.1 3.7 3.7 4.1 4.7 4.9
1 4.8 4.3 4.7 4.6 4.7 3.7 3.6 3.9
1 4.4 4.2 4.2 3.4 3.5 3.4 3.9 4.0
1 4.9 4.3 4.0 4.0 3.3 4.1 4.2 4.3
1 5.1 4.1 4.6 4.1 3.4 4.2 4.4 4.9
1 4.8 4.6 4.6 4.4 4.1 4.0 3.8 3.8
1 4.2 3.5 3.8 3.6 3.3 3.1 3.5 3.9
1 6.6 6.1 5.2 4.1 4.3 3.8 4.2 4.8
1 3.6 3.4 3.1 2.8 2.1 2.4 2.5 3.5
1 4.5 4.0 3.7 3.3 2.4 2.3 3.1 3.3
1 4.6 4.4 3.8 3.8 3.8 3.6 3.8 3.8
```

3. Mplus syntax for a conditional bilinear spline model

The data may be obtained from Zerbe (1979, p. 219).

```
TITLE: zerbe glucose;
DATA: FILE IS glucose.dat; !data file in ascii format
DEFINE: xobese=abs(obese-1); !1=control; 0=obese
VARIABLE: NAMES ARE obese ph1-ph8; !all 9 variables
  !obese: obesity status, 0=control, 1=obese
  !ph1-ph8: plasma phosphate concentration repeated measures
USEVARIABLES ARE ph1-ph8 xobese; !we plan to model 9 variables
!ANALYSIS: BOOTSTRAP IS 1000; !for bootstrap confidence intervals if desired
MODEL:

[ph1-ph8*](z1-z8); !mean trend (z's defined below)
ph1-ph8*(ve); !residual variances constrained equal
w1-w4*; !random coefficient residual variances
w1 WITH w2*; w1 WITH w3*; w1 WITH w4*; !random coefficient residual covariances
w2 WITH w3*; w2 WITH w4*; w3 WITH w4*; !random coefficient residual covariances
[w1@0 w2@0 w3@0 w4@0]; !factor intercepts constrained to zero
w1 BY ph1-ph8@1; !loadings for 1st factor, constrained to 1
w2 BY ph1*(w21); w2 BY ph2-ph8(w22-w28); !loadings for 2nd factor, defined below
w3 BY ph1*(w31); w3 BY ph2-ph8(w32-w38); !loadings for 3rd factor, defined below
w4 BY ph1*(w41); w4 BY ph2-ph8(w42-w48); !loadings for knot factor, defined below
```

```
w1-w4 ON xobese; !regress random coefficients on obese.

MODEL CONSTRAINT:

!starts for parameters of the target function
NEW(mw1*3.5 mw2*-.21 mw3*.49 mw4*1.95 b1-b4);

!intercepts constrained equal to target function
z1=mw1+mw2*0.0+mw3*sqrt((0.0-mw4)^2);
z2=mw1+mw2*0.5+mw3*sqrt((0.5-mw4)^2);
z3=mw1+mw2*1.0+mw3*sqrt((1.0-mw4)^2);
z4=mw1+mw2*1.5+mw3*sqrt((1.5-mw4)^2);
z5=mw1+mw2*2.0+mw3*sqrt((2.0-mw4)^2);
z6=mw1+mw2*3.0+mw3*sqrt((3.0-mw4)^2);
z7=mw1+mw2*4.0+mw3*sqrt((4.0-mw4)^2);
z8=mw1+mw2*5.0+mw3*sqrt((5.0-mw4)^2);

!loadings for 2nd and 3rd factors, constrained to d(y)/d(mw2) and d(y)/d(mw3)
w21=0.0; w31=sqrt((0.0-mw4)^2);
w22=0.5; w32=sqrt((0.5-mw4)^2);
w23=1.0; w33=sqrt((1.0-mw4)^2);
w24=1.5; w34=sqrt((1.5-mw4)^2);
w25=2.0; w35=sqrt((2.0-mw4)^2);
w26=3.0; w36=sqrt((3.0-mw4)^2);
w27=4.0; w37=sqrt((4.0-mw4)^2);
w28=5.0; w38=sqrt((5.0-mw4)^2);

!loadings for knot factor, constrained to d(y)/d(mw4)
w41=(mw3*(mw4-0.0))/sqrt((0.0-mw4)^2);
w42=(mw3*(mw4-0.5))/sqrt((0.5-mw4)^2);
w43=(mw3*(mw4-1.0))/sqrt((1.0-mw4)^2);
w44=(mw3*(mw4-1.5))/sqrt((1.5-mw4)^2);
w45=(mw3*(mw4-2.0))/sqrt((2.0-mw4)^2);
w46=(mw3*(mw4-3.0))/sqrt((3.0-mw4)^2);
w47=(mw3*(mw4-4.0))/sqrt((4.0-mw4)^2);
w48=(mw3*(mw4-5.0))/sqrt((5.0-mw4)^2);

!parameters of original parameterization
b1=mw1+mw3*mw4;
b2=mw2-mw3;
b3=mw1-mw3*mw4;
b4=mw2+mw3;

OUTPUT: TECH1; !CINTERVAL(BCBOOTSTRAP);
```

4. Mplus syntax for a negative exponential function with intercept, asymptote, and rate

Chaiken's data are included in installations of LISREL:
http://www.ssicentral.com/

```
TITLE: chaiken, negative exponential, original parameterization;
DATA: FILE IS schaiken2.dat; !data file in ascii format
VARIABLE: NAMES ARE v1-v12 q1-q12; !all 24 variables
  !v1-v12: verbal skill acquisition repeated measures
  !q1-q12: quantitative skill acquisition repeated measures
ANALYSIS: ESTIMATOR IS ML; !use maximum likelihood estimation
MODEL:

!verbal
[v1-v12*](tv1-tv12); !mean trend for verbal
v1-v12*1.97(vv1); !residual variances constrained equal
```

```
v1-v11 PWITH v2-v12*(dv1); !lag-1 error covariance
v1-v10 PWITH v3-v12*(dv2); !lag-2, etc...
v1-v9 PWITH v4-v12*(dv3);
v1-v8 PWITH v5-v12*(dv4);
v1-v7 PWITH v6-v12*(dv5);
v1-v6 PWITH v7-v12*(dv6);
v1-v5 PWITH v8-v12*(dv7);
v1-v4 PWITH v9-v12*(dv8);
v1-v3 PWITH v10-v12*(dv9);
v1-v2 PWITH v11-v12*(dv10);
v1 WITH v12*(dv11);
[av@0 bv@0 cv@0]; !factor means constrained to zero
av*; bv*; cv*; !random coefficient variances
av WITH bv* cv*; bv WITH cv*; !random coefficient covariances
av BY v1*(av1); av BY v2-v12(av2-av12); !loadings for 1st factor
bv BY v1*(bv1); bv BY v2-v12(bv2-bv12); !loadings for 2nd factor
cv BY v1*(cv1); cv BY v2-v12(cv2-cv12); !loadings for 3rd factor

!quantitative
[q1-q12*](tq1-tq12); !mean trend for quantitative
q1-q12*1.21(vq1); !residual variances constrained equal
q1-q11 PWITH q2-q12*(dq1); !lag-1 error covariance
q1-q10 PWITH q3-q12*(dq2); !lat-2, etc...
q1-q9 PWITH q4-q12*(dq3);
q1-q8 PWITH q5-q12*(dq4);
q1-q7 PWITH q6-q12*(dq5);
q1-q6 PWITH q7-q12*(dq6);
q1-q5 PWITH q8-q12*(dq7);
q1-q4 PWITH q9-q12*(dq8);
q1-q3 PWITH q10-q12*(dq9);
q1-q2 PWITH q11-q12*(dq10);
q1 WITH q12*(dq11);
[aq@0 bq@0 cq@0]; !factor means constrained to zero
aq*; bq*; cq*; !random coefficient variances
aq WITH bq* cq*; bq WITH cq; !random coefficient covariances
aq BY q1*(aq1); aq BY q2-q12(aq2-aq12); !loadings for 1st factor
bq BY q1*(bq1); bq BY q2-q12(bq2-bq12); !loadings for 2nd factor
cq BY q1*(cq1); cq BY q2-q12(cq2-cq12); !loadings for 3rd factor

!cross-domain covariances
av WITH aq* bq*; bv WITH aq* bq*;

MODEL CONSTRAINT:

!starts for parameters of the target function and for lagged error covariances
NEW(mav*6.8 mbv*21 mcv*.7 maq*8.6 mbq*16.5 mcq*.7 rhov*.27 rhoq*.31);

!error covariances
dv1= vv1*rhov;    dq1= vq1*rhoq;
dv2= vv1*rhov^2;  dq2= vq1*rhoq^2;
dv3= vv1*rhov^3;  dq3= vq1*rhoq^3;
dv4= vv1*rhov^4;  dq4= vq1*rhoq^4;
dv5= vv1*rhov^5;  dq5= vq1*rhoq^5;
dv6= vv1*rhov^6;  dq6= vq1*rhoq^6;
dv7= vv1*rhov^7;  dq7= vq1*rhoq^7;
dv8= vv1*rhov^8;  dq8= vq1*rhoq^8;
dv9= vv1*rhov^9;  dq9= vq1*rhoq^9;
dv10=vv1*rhov^10; dq10=vq1*rhoq^10;
dv11=vv1*rhov^11; dq11=vq1*rhoq^11;

!verbal and quantitative intercepts constrained equal to target function
tv1 =mav-(mav-mbv)*exp(-1*mcv*( 1-1)); tq1 =maq-(maq-mbq)*exp(-1*mcq*( 1-1));
tv2 =mav-(mav-mbv)*exp(-1*mcv*( 2-1)); tq2 =maq-(maq-mbq)*exp(-1*mcq*( 2-1));
```

```
tv3 =mav-(mav-mbv)*exp(-1*mcv*( 3-1)); tq3 =maq-(maq-mbq)*exp(-1*mcq*( 3-1));
tv4 =mav-(mav-mbv)*exp(-1*mcv*( 4-1)); tq4 =maq-(maq-mbq)*exp(-1*mcq*( 4-1));
tv5 =mav-(mav-mbv)*exp(-1*mcv*( 5-1)); tq5 =maq-(maq-mbq)*exp(-1*mcq*( 5-1));
tv6 =mav-(mav-mbv)*exp(-1*mcv*( 6-1)); tq6 =maq-(maq-mbq)*exp(-1*mcq*( 6-1));
tv7 =mav-(mav-mbv)*exp(-1*mcv*( 7-1)); tq7 =maq-(maq-mbq)*exp(-1*mcq*( 7-1));
tv8 =mav-(mav-mbv)*exp(-1*mcv*( 8-1)); tq8 =maq-(maq-mbq)*exp(-1*mcq*( 8-1));
tv9 =mav-(mav-mbv)*exp(-1*mcv*( 9-1)); tq9 =maq-(maq-mbq)*exp(-1*mcq*( 9-1));
tv10=mav-(mav-mbv)*exp(-1*mcv*(10-1)); tq10=maq-(maq-mbq)*exp(-1*mcq*(10-1));
tv11=mav-(mav-mbv)*exp(-1*mcv*(11-1)); tq11=maq-(maq-mbq)*exp(-1*mcq*(11-1));
tv12=mav-(mav-mbv)*exp(-1*mcv*(12-1)); tq12=maq-(maq-mbq)*exp(-1*mcq*(12-1));

!loadings on 1st factor, constrained to d(y)/d(mav) and d(y)/d(maq)
av1 =1-exp(-1*mcv*( 1-1)); aq1 =1-exp(-1*mcq*( 1-1));
av2 =1-exp(-1*mcv*( 2-1)); aq2 =1-exp(-1*mcq*( 2-1));
av3 =1-exp(-1*mcv*( 3-1)); aq3 =1-exp(-1*mcq*( 3-1));
av4 =1-exp(-1*mcv*( 4-1)); aq4 =1-exp(-1*mcq*( 4-1));
av5 =1-exp(-1*mcv*( 5-1)); aq5 =1-exp(-1*mcq*( 5-1));
av6 =1-exp(-1*mcv*( 6-1)); aq6 =1-exp(-1*mcq*( 6-1));
av7 =1-exp(-1*mcv*( 7-1)); aq7 =1-exp(-1*mcq*( 7-1));
av8 =1-exp(-1*mcv*( 8-1)); aq8 =1-exp(-1*mcq*( 8-1));
av9 =1-exp(-1*mcv*( 9-1)); aq9 =1-exp(-1*mcq*( 9-1));
av10=1-exp(-1*mcv*(10-1)); aq10=1-exp(-1*mcq*(10-1));
av11=1-exp(-1*mcv*(11-1)); aq11=1-exp(-1*mcq*(11-1));
av12=1-exp(-1*mcv*(12-1)); aq12=1-exp(-1*mcq*(12-1));

!loadings on 2nd and 3rd factors for verbal, constrained to d(y)/d(mbv) and d(y)/d(mcv)
bv1 =exp(-1*mcv*( 1-1)); cv1 =(mav-mbv)*( 1-1)*exp(-1*mcv*( 1-1));
bv2 =exp(-1*mcv*( 2-1)); cv2 =(mav-mbv)*( 2-1)*exp(-1*mcv*( 2-1));
bv3 =exp(-1*mcv*( 3-1)); cv3 =(mav-mbv)*( 3-1)*exp(-1*mcv*( 3-1));
bv4 =exp(-1*mcv*( 4-1)); cv4 =(mav-mbv)*( 4-1)*exp(-1*mcv*( 4-1));
bv5 =exp(-1*mcv*( 5-1)); cv5 =(mav-mbv)*( 5-1)*exp(-1*mcv*( 5-1));
bv6 =exp(-1*mcv*( 6-1)); cv6 =(mav-mbv)*( 6-1)*exp(-1*mcv*( 6-1));
bv7 =exp(-1*mcv*( 7-1)); cv7 =(mav-mbv)*( 7-1)*exp(-1*mcv*( 7-1));
bv8 =exp(-1*mcv*( 8-1)); cv8 =(mav-mbv)*( 8-1)*exp(-1*mcv*( 8-1));
bv9 =exp(-1*mcv*( 9-1)); cv9 =(mav-mbv)*( 9-1)*exp(-1*mcv*( 9-1));
bv10=exp(-1*mcv*(10-1)); cv10=(mav-mbv)*(10-1)*exp(-1*mcv*(10-1));
bv11=exp(-1*mcv*(11-1)); cv11=(mav-mbv)*(11-1)*exp(-1*mcv*(11-1));
bv12=exp(-1*mcv*(12-1)); cv12=(mav-mbv)*(12-1)*exp(-1*mcv*(12-1));

!loadings on 2nd and 3rd factors for quant., constrained to d(y)/d(mbv) and d(y)/d(mcv)
bq1 =exp(-1*mcq*( 1-1)); cq1 =(maq-mbq)*( 1-1)*exp(-1*mcq*( 1-1));
bq2 =exp(-1*mcq*( 2-1)); cq2 =(maq-mbq)*( 2-1)*exp(-1*mcq*( 2-1));
bq3 =exp(-1*mcq*( 3-1)); cq3 =(maq-mbq)*( 3-1)*exp(-1*mcq*( 3-1));
bq4 =exp(-1*mcq*( 4-1)); cq4 =(maq-mbq)*( 4-1)*exp(-1*mcq*( 4-1));
bq5 =exp(-1*mcq*( 5-1)); cq5 =(maq-mbq)*( 5-1)*exp(-1*mcq*( 5-1));
bq6 =exp(-1*mcq*( 6-1)); cq6 =(maq-mbq)*( 6-1)*exp(-1*mcq*( 6-1));
bq7 =exp(-1*mcq*( 7-1)); cq7 =(maq-mbq)*( 7-1)*exp(-1*mcq*( 7-1));
bq8 =exp(-1*mcq*( 8-1)); cq8 =(maq-mbq)*( 8-1)*exp(-1*mcq*( 8-1));
bq9 =exp(-1*mcq*( 9-1)); cq9 =(maq-mbq)*( 9-1)*exp(-1*mcq*( 9-1));
bq10=exp(-1*mcq*(10-1)); cq10=(maq-mbq)*(10-1)*exp(-1*mcq*(10-1));
bq11=exp(-1*mcq*(11-1)); cq11=(maq-mbq)*(11-1)*exp(-1*mcq*(11-1));
bq12=exp(-1*mcq*(12-1)); cq12=(maq-mbq)*(12-1)*exp(-1*mcq*(12-1));

OUTPUT: TECH1 TECH4;
```

5. Mplus syntax for a negative exponential function with intercept, half-life, and ARC

Chaiken's data are included in installations of LISREL:
http://www.ssicentral.com/

```
TITLE: chaiken, negative exponential with halflife and arc;
```

```
DATA: FILE IS schaiken2.dat; !data file in ascii format
VARIABLE: NAMES ARE v1-v12 q1-q12; !all 24 variables
  !v1-v12: verbal skill acquisition repeated measures
  !q1-q12: quantitative skill acquisition repeated measures
ANALYSIS: ESTIMATOR IS ML; !use maximum likelihood estimation
MODEL:

!verbal
[v1-v12*](tv1-tv12); !mean trend for verbal
v1-v12*1.97(vv1); !residual variances constrained equal
v1-v11 PWITH v2-v12*(dv1); !lag-1 error covariance
v1-v10 PWITH v3-v12*(dv2); !lag-2, etc...
v1-v9 PWITH v4-v12*(dv3);
v1-v8 PWITH v5-v12*(dv4);
v1-v7 PWITH v6-v12*(dv5);
v1-v6 PWITH v7-v12*(dv6);
v1-v5 PWITH v8-v12*(dv7);
v1-v4 PWITH v9-v12*(dv8);
v1-v3 PWITH v10-v12*(dv9);
v1-v2 PWITH v11-v12*(dv10);
v1 WITH v12*(dv11);
[av@0 bv@0 cv@0]; !factor means constrained to zero
av*1.6; bv*92; cv*10; !random coefficient variances
av WITH bv*3.7 cv*; bv WITH cv*; !random coefficient covariances
av BY v1*(av1); av BY v2-v12(av2-av12); !loadings for 1st factor
bv BY v1*(bv1); bv BY v2-v12(bv2-bv12); !loadings for 2nd factor
cv BY v1*(cv1); cv BY v2-v12(cv2-cv12); !loadings for 3rd factor

!quantitative
[q1-q12*](tq1-tq12); !mean trend for quantitative
q1-q12*1.21(vq1); !residual variances constrained equal
q1-q11 PWITH q2-q12*(dq1); !lag-1 error covariance
q1-q10 PWITH q3-q12*(dq2); !lag-2, etc...
q1-q9 PWITH q4-q12*(dq3);
q1-q8 PWITH q5-q12*(dq4);
q1-q7 PWITH q6-q12*(dq5);
q1-q6 PWITH q7-q12*(dq6);
q1-q5 PWITH q8-q12*(dq7);
q1-q4 PWITH q9-q12*(dq8);
q1-q3 PWITH q10-q12*(dq9);
q1-q2 PWITH q11-q12*(dq10);
q1 WITH q12*(dq11);
[aq@0 bq@0 cq@0]; !factor means constrained to zero
aq*4; bq*35; cq*10; !random coefficient variances
aq WITH bq*7.4 cq*; bq WITH cq; !random coefficient covariances
aq BY q1*(aq1); aq BY q2-q12(aq2-aq12); !loadings for 1st factor
bq BY q1*(bq1); bq BY q2-q12(bq2-bq12); !loadings for 2nd factor
cq BY q1*(cq1); cq BY q2-q12(cq2-cq12); !loadings for 3rd factor

!cross-domain covariances
av WITH aq* bq* cq*; bv WITH aq* bq* cq*; cv WITH aq* bq* cq*;

MODEL CONSTRAINT:

!starts for parameters of the target function and for lagged error covariances
NEW(mav*1.7 mbv*-1.5 mcv*1.96 maq*4.2 mbq*-.9 mcq*1.99 rhov*.27 rhoq*.3 diff*0);

!error covariances
dv1= vv1*rhov;    dq1= vq1*rhoq;
dv2= vv1*rhov^2;  dq2= vq1*rhoq^2;
dv3= vv1*rhov^3;  dq3= vq1*rhoq^3;
dv4= vv1*rhov^4;  dq4= vq1*rhoq^4;
dv5= vv1*rhov^5;  dq5= vq1*rhoq^5;
```

```
dv6= vv1*rhov^6;  dq6= vq1*rhoq^6;
dv7= vv1*rhov^7;  dq7= vq1*rhoq^7;
dv8= vv1*rhov^8;  dq8= vq1*rhoq^8;
dv9= vv1*rhov^9;  dq9= vq1*rhoq^9;
dv10=vv1*rhov^10; dq10=vq1*rhoq^10;
dv11=vv1*rhov^11; dq11=vq1*rhoq^11;

!verbal intercepts constrained equal to target function
tv1 =mav+(mbv*11*(.5^(( 1-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv2 =mav+(mbv*11*(.5^(( 2-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv3 =mav+(mbv*11*(.5^(( 3-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv4 =mav+(mbv*11*(.5^(( 4-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv5 =mav+(mbv*11*(.5^(( 5-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv6 =mav+(mbv*11*(.5^(( 6-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv7 =mav+(mbv*11*(.5^(( 7-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv8 =mav+(mbv*11*(.5^(( 8-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv9 =mav+(mbv*11*(.5^(( 9-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv10=mav+(mbv*11*(.5^((10-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv11=mav+(mbv*11*(.5^((11-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv12=mav+(mbv*11*(.5^((12-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);

!loadings on 1st factor, constrained to d(y)/d(mav) and d(y)/d(maq)
0=1-av1 ; 0=1-aq1 ;
0=1-av2 ; 0=1-aq2 ;
0=1-av3 ; 0=1-aq3 ;
0=1-av4 ; 0=1-aq4 ;
0=1-av5 ; 0=1-aq5 ;
0=1-av6 ; 0=1-aq6 ;
0=1-av7 ; 0=1-aq7 ;
0=1-av8 ; 0=1-aq8 ;
0=1-av9 ; 0=1-aq9 ;
0=1-av10; 0=1-aq10;
0=1-av11; 0=1-aq11;
0=1-av12; 0=1-aq12;

!loadings on 2nd factor for verbal, constrained to d(y)/d(mbv)
bv1 =(11*(.5^(( 1-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv2 =(11*(.5^(( 2-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv3 =(11*(.5^(( 3-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv4 =(11*(.5^(( 4-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv5 =(11*(.5^(( 5-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv6 =(11*(.5^(( 6-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv7 =(11*(.5^(( 7-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv8 =(11*(.5^(( 8-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv9 =(11*(.5^(( 9-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv10=(11*(.5^((10-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv11=(11*(.5^((11-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv12=(11*(.5^((12-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);

!loadings on 3rd factor for verbal, constrained to d(y)/d(mcv)
cv1 =(.5^( 1/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 1-1)+(.5^(12/(mcv-1)))*(12- 1))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv2 =(.5^( 2/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 2-1)+(.5^(12/(mcv-1)))*(12- 2))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv3 =(.5^( 3/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 3-1)+(.5^(12/(mcv-1)))*(12- 3))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv4 =(.5^( 4/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 4-1)+(.5^(12/(mcv-1)))*(12- 4))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv5 =(.5^( 5/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 5-1)+(.5^(12/(mcv-1)))*(12- 5))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv6 =(.5^( 6/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 6-1)+(.5^(12/(mcv-1)))*(12- 6))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv7 =(.5^( 7/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 7-1)+(.5^(12/(mcv-1)))*(12- 7))*11*ln(.5))/
```

```
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv8 =(.5^( 8/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 8-1)+(.5^(12/(mcv-1)))*(12- 8))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv9 =(.5^( 9/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 9-1)+(.5^(12/(mcv-1)))*(12- 9))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv10=(.5^(10/(mcv-1))*mcv*((.5^(1/(mcv-1)))*(10-1)+(.5^(12/(mcv-1)))*(12-10))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv11=(.5^(11/(mcv-1))*mcv*((.5^(1/(mcv-1)))*(11-1)+(.5^(12/(mcv-1)))*(12-11))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv12=(.5^(12/(mcv-1))*mcv*((.5^(1/(mcv-1)))*(12-1)+(.5^(12/(mcv-1)))*(12-12))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));

!quantitative intercepts constrained equal to target function
tq1 =maq+(mbq*11*(.5^(( 1-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq2 =maq+(mbq*11*(.5^(( 2-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq3 =maq+(mbq*11*(.5^(( 3-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq4 =maq+(mbq*11*(.5^(( 4-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq5 =maq+(mbq*11*(.5^(( 5-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq6 =maq+(mbq*11*(.5^(( 6-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq7 =maq+(mbq*11*(.5^(( 7-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq8 =maq+(mbq*11*(.5^(( 8-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq9 =maq+(mbq*11*(.5^(( 9-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq10=maq+(mbq*11*(.5^((10-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq11=maq+(mbq*11*(.5^((11-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq12=maq+(mbq*11*(.5^((12-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);

!loadings on 2nd factor for quantitative, constrained to d(y)/d(mbv)
bq1 =(11*(.5^(( 1-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq2 =(11*(.5^(( 2-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq3 =(11*(.5^(( 3-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq4 =(11*(.5^(( 4-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq5 =(11*(.5^(( 5-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq6 =(11*(.5^(( 6-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq7 =(11*(.5^(( 7-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq8 =(11*(.5^(( 8-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq9 =(11*(.5^(( 9-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq10=(11*(.5^((10-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq11=(11*(.5^((11-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq12=(11*(.5^((12-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);

!loadings on 3rd factor for quantitative, constrained to d(y)/d(mcv)
cq1 =(.5^( 1/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 1-1)+(.5^(12/(mcq-1)))*(12- 1))*11*ln(.5))/
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
cq2 =(.5^( 2/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 2-1)+(.5^(12/(mcq-1)))*(12- 2))*11*ln(.5))/
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
cq3 =(.5^( 3/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 3-1)+(.5^(12/(mcq-1)))*(12- 3))*11*ln(.5))/
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
cq4 =(.5^( 4/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 4-1)+(.5^(12/(mcq-1)))*(12- 4))*11*ln(.5))/
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
cq5 =(.5^( 5/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 5-1)+(.5^(12/(mcq-1)))*(12- 5))*11*ln(.5))/
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
cq6 =(.5^( 6/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 6-1)+(.5^(12/(mcq-1)))*(12- 6))*11*ln(.5))/
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
cq7 =(.5^( 7/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 7-1)+(.5^(12/(mcq-1)))*(12- 7))*11*ln(.5))/
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
cq8 =(.5^( 8/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 8-1)+(.5^(12/(mcq-1)))*(12- 8))*11*ln(.5))/
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
cq9 =(.5^( 9/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 9-1)+(.5^(12/(mcq-1)))*(12- 9))*11*ln(.5))/
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
cq10=(.5^(10/(mcq-1))*mcq*((.5^(1/(mcq-1)))*(10-1)+(.5^(12/(mcq-1)))*(12-10))*11*ln(.5))/
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
cq11=(.5^(11/(mcq-1))*mcq*((.5^(1/(mcq-1)))*(11-1)+(.5^(12/(mcq-1)))*(12-11))*11*ln(.5))/
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
```

```
cq12=(.5^(12/(mcq-1))*mcq*((.5^(1/(mcq-1)))*(12-1)+(.5^(12/(mcq-1)))*(12-12))*11*ln(.5))/
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));

diff=mcv-mcq; !difference in the ARC parameters for verbal and quantitative

OUTPUT: TECH1 TECH4;
```

6. Mathematica code for obtaining partial derivatives of an exponential function

```
y = f[t_] = Exp[a + b * x]
```

$$e^{a+b\,x}$$

```
yp1 = D[f[t], a]
```

$$e^{a+b\,x}$$

```
yp2 = D[f[t], b]
```

$$e^{a+b\,x}\,x$$

```
Lambda = {{yp1, yp2}}
```

$$\left\{\left\{e^{a+b\,x},\ e^{a+b\,x}\,x\right\}\right\}$$

7. Mplus syntax for a quadratic function fit to Kanfer & Ackerman's data

Traditional structured latent curve:

```
TITLE: Cudeck and du Toit (2002) quadratic curve;
DATA: FILE IS kanfer.dat; !data file in ascii format
TYPE IS COVARIANCE MEANS; !input data are covariances and means
NOBSERVATIONS IS 140; !sample size is 140
VARIABLE: NAMES ARE y1-y9; !all 9 variables
  !y1-y9:
ANALYSIS: ESTIMATOR=ML; !use maximum likelihood estimation
ITERATIONS=5000; !use 5000 iterations
MODEL: y1-y9*9.9(v); [y1-y9@0]; !residual covariances constrained equal, intercepts = 0
[a0*16 ay*39](ma0 may); [ax@0]; !intercept and maximum means estimated, maximizer = 0
a0*123; ax*18; ay*53; !random coefficient variances
a0 WITH ax*-.8 ay*35; ax WITH ay*8.9; !random coefficient covariances
a0 BY y1-y9*(a01-a09); ax BY y1-y9*(ax1-ax9); ay BY y1-y9*(ay1-ay9); !loadings

MODEL CONSTRAINT: NEW(max*8.8); !define maximizer mean

!loadings on maximum, intercept, and maximizer factors, constrained to
!d(y)/d(may), d(y)/d(ma0), and d(y)/d(max)
ay1=1-((0/max)-1)^2; a01=((0/max)-1)^2; ax1=2*(may-ma0)*(0-max)*0/max^3;
ay2=1-((1/max)-1)^2; a02=((1/max)-1)^2; ax2=2*(may-ma0)*(1-max)*1/max^3;
ay3=1-((2/max)-1)^2; a03=((2/max)-1)^2; ax3=2*(may-ma0)*(2-max)*2/max^3;
ay4=1-((3/max)-1)^2; a04=((3/max)-1)^2; ax4=2*(may-ma0)*(3-max)*3/max^3;
ay5=1-((4/max)-1)^2; a05=((4/max)-1)^2; ax5=2*(may-ma0)*(4-max)*4/max^3;
ay6=1-((5/max)-1)^2; a06=((5/max)-1)^2; ax6=2*(may-ma0)*(5-max)*5/max^3;
ay7=1-((6/max)-1)^2; a07=((6/max)-1)^2; ax7=2*(may-ma0)*(6-max)*6/max^3;
ay8=1-((7/max)-1)^2; a08=((7/max)-1)^2; ax8=2*(may-ma0)*(7-max)*7/max^3;
```

```
ay9=1-((8/max)-1)^2; a09=((8/max)-1)^2; ax9=2*(may-ma0)*(8-max)*8/max^3;

OUTPUT: TECH1 TECH4;
```

Modified structured latent curve:

```
TITLE: Cudeck and du Toit (2002) quadratic curve;
DATA: FILE IS kanfer.dat; !data file in ascii format
TYPE IS COVARIANCE MEANS; !input data are covariances and means
NOBSERVATIONS IS 140; !sample size is 140
VARIABLE: NAMES ARE y1-y9; !all 9 variables
ANALYSIS: ESTIMATOR=ML; !use maximum likelihood estimation
ITERATIONS=5000; !use 5000 iterations
MODEL: [y1-y9*](my1-my9); !intercepts constrained equal to target function
y1-y9*9.9(v); !residual covariances constrained equal
[a0@0 ax@0 ay@0]; !factor means constrained to zero
a0*123; ax*18; ay*53; !random coefficient variances
a0 WITH ax*-.8 ay*35; ax WITH ay*8.9; !random coefficient covariances
a0 BY y1-y9*(a01-a09); ax BY y1-y9*(ax1-ax9); ay BY y1-y9*(ay1-ay9); !loadings

MODEL CONSTRAINT:
NEW(ma0*16 max*8.8 may*39); !define mean intercept, maximizer, and maximum

!intercepts constrained equal to target function
my1=may-(may-ma0)*((0/max)-1)^2;
my2=may-(may-ma0)*((1/max)-1)^2;
my3=may-(may-ma0)*((2/max)-1)^2;
my4=may-(may-ma0)*((3/max)-1)^2;
my5=may-(may-ma0)*((4/max)-1)^2;
my6=may-(may-ma0)*((5/max)-1)^2;
my7=may-(may-ma0)*((6/max)-1)^2;
my8=may-(may-ma0)*((7/max)-1)^2;
my9=may-(may-ma0)*((8/max)-1)^2;

!loadings on maximum, intercept, and maximizer factors, constrained to
!d(y)/d(may), d(y)/d(ma0), and d(y)/d(max)
ay1=1-((0/max)-1)^2; a01=((0/max)-1)^2; ax1=2*(may-ma0)*(0-max)*0/max^3;
ay2=1-((1/max)-1)^2; a02=((1/max)-1)^2; ax2=2*(may-ma0)*(1-max)*1/max^3;
ay3=1-((2/max)-1)^2; a03=((2/max)-1)^2; ax3=2*(may-ma0)*(2-max)*2/max^3;
ay4=1-((3/max)-1)^2; a04=((3/max)-1)^2; ax4=2*(may-ma0)*(3-max)*3/max^3;
ay5=1-((4/max)-1)^2; a05=((4/max)-1)^2; ax5=2*(may-ma0)*(4-max)*4/max^3;
ay6=1-((5/max)-1)^2; a06=((5/max)-1)^2; ax6=2*(may-ma0)*(5-max)*5/max^3;
ay7=1-((6/max)-1)^2; a07=((6/max)-1)^2; ax7=2*(may-ma0)*(6-max)*6/max^3;
ay8=1-((7/max)-1)^2; a08=((7/max)-1)^2; ax8=2*(may-ma0)*(7-max)*7/max^3;
ay9=1-((8/max)-1)^2; a09=((8/max)-1)^2; ax9=2*(may-ma0)*(8-max)*8/max^3;

OUTPUT: TECH1 TECH4;
```

Kanfer & Ackerman's (1989) data (N = 140), a mean vector and covariance matrix:

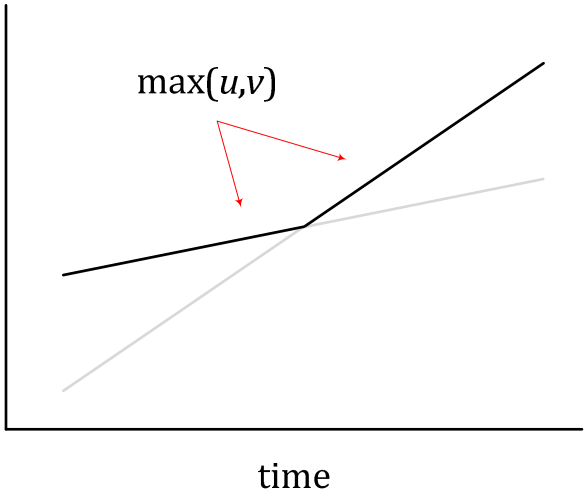
```
20.121 25.521 29.321 32.400 34.186 35.600 37.729 39.029 38.786
93.650
79.637 93.564
72.890 87.718 95.732
62.623 76.941 81.886 86.054
53.342 63.803 69.612 70.611 73.666
43.820 50.644 54.443 56.446 58.353 59.154
35.183 41.099 46.016 45.494 49.900 46.163 54.169
39.839 44.099 46.105 44.074 48.345 45.847 49.329 60.528
35.497 39.169 42.262 39.429 42.954 42.950 47.120 54.570 66.183
```

8. Derivation of Equation 11: Reparameterization of a bilinear spline target function

A common expression for the bilinear spline model is:

$$y = \begin{cases} \theta_1 + \theta_2 t & t \le \theta_\kappa \\ \theta_3 + \theta_4 t & t > \theta_\kappa \end{cases}$$

where θ_1 and θ_2 are the intercept and slope for the first segment, θ_3 and θ_4 are the intercept and slope for the second segment, and θ_κ is the knot. This expression is not convenient for linearization, so it needs to be changed. To perform the first step of the reparameterization, it is first necessary to decide whether the slope of the second segment is larger or smaller than the slope of the first segment. In most applications, this choice will be clear by inspection. In cases for which the slope of the second segment is greater than the slope of the first segment (as in our example), a bilinear spline model can be understood as the maximum of the two functions at any point *t*. Visually, at the knot point the linear equation for the second segment "takes over" where the equation for the first segment leaves off, so it is clear why the maximum would be chosen in this instance (if the slope of the second segment is expected to be less than that of the first, we would use the minimum rather than the minimum):



Harring, Cudeck, and du Toit (2006, Appx. C) and Kohli and Harring (2013, Appx. A) provide a convenient expression for the minimum of two functions:

$$\min(u,v) = \tfrac{1}{2}\left(u + v - \sqrt{(u-v)^2}\right),$$

from which we may derive a convenient expression for the maximum:

$$\max(u,v) = \tfrac{1}{2}\left(u + v + \sqrt{(u-v)^2} \right).$$

In words, the maximum of two functions *u* and *v* is the sum of the two functions plus the absolute value of their difference, all divided by 2. The first step of the reparameterization consists of substituting the earlier expressions for each segment into this max(.) function:

$$\begin{aligned}
\min(u,v) &= \tfrac{1}{2}\left(u + v + \sqrt{(u-v)^2} \right) \\
&= \tfrac{1}{2}\left(\theta_1 + \theta_2 t + \theta_3 + \theta_4 t + \sqrt{\left(\theta_1 + \theta_2 t - (\theta_3 + \theta_4 t)\right)^2} \right) \\
&= \tfrac{1}{2}\left((\theta_1 + \theta_3) + (\theta_2 + \theta_4) t + \sqrt{\left[(\theta_2 - \theta_4) t + (\theta_1 - \theta_3)\right]^2} \right)
\end{aligned}$$

Typically, the two functions join at the knot, in which case there are effectively four parameters, not five. The second step in this reparameterization involves eliminating one of the parameters by replacing it with an equivalent expression involving some of the other parameters. Returning to the original expressions for each segment, we observe that where $t = \theta_\kappa$,

$$\begin{aligned}
\theta_1 + \theta_2 \theta_\kappa &= \theta_3 + \theta_4 \theta_\kappa \\
\theta_1 - \theta_3 &= (\theta_4 - \theta_2)\theta_\kappa
\end{aligned}$$

Therefore, we can substitute $(\theta_4 - \theta_2)\theta_\kappa$ for $(\theta_1 - \theta_3)$ in the max(.) function, yielding:

$$\begin{aligned}
\max(u,v) &= \tfrac{1}{2}\left((\theta_1 + \theta_3) + (\theta_2 + \theta_4) t + \sqrt{\left[(\theta_2 - \theta_4) t + (\theta_1 - \theta_3)\right]^2} \right) \\
&= \tfrac{1}{2}\left((\theta_1 + \theta_3) + (\theta_2 + \theta_4) t + \sqrt{\left[(\theta_2 - \theta_4) t + (\theta_4 - \theta_2)\theta_\kappa\right]^2} \right) \\
&= \tfrac{1}{2}\left((\theta_1 + \theta_3) + (\theta_2 + \theta_4) t + \sqrt{\left[(\theta_4 - \theta_2)\theta_\kappa - (\theta_4 - \theta_2) t\right]^2} \right) \\
&= \tfrac{1}{2}\left((\theta_1 + \theta_3) + (\theta_2 + \theta_4) t + \sqrt{\left[(\theta_4 - \theta_2)(\theta_\kappa - t)\right]^2} \right) \\
&= \tfrac{1}{2}\left((\theta_1 + \theta_3) + (\theta_2 + \theta_4) t + \sqrt{(\theta_4 - \theta_2)^2 (\theta_\kappa - t)^2} \right) \\
&= \tfrac{1}{2}\left((\theta_1 + \theta_3) + (\theta_2 + \theta_4) t + (\theta_4 - \theta_2)\sqrt{(t - \theta_\kappa)^2} \right)
\end{aligned}$$

In the final step, bringing $(\theta_4 - \theta_2)$ outside the radical is permissible because we started with the assumption that $\theta_4 > \theta_2$, implying that $(\theta_4 - \theta_2)$ is a positive square root. Finally, given that we are

emphasizing the importance of θ_κ and do not care much about the other parts of the equation, we can make the following substitutions with no loss:

$$\omega_1 = \frac{\theta_1 + \theta_3}{2} \qquad \omega_2 = \frac{\theta_2 + \theta_4}{2} \qquad \omega_3 = \frac{\theta_4 - \theta_2}{2}$$

so that

$$y = \omega_1 + \omega_2 t + \omega_3 \sqrt{(t - \theta_\kappa)^2}\ .$$

9. Annotated output: inverse quadratic function

Only the most relevant output has been included below.

```
Mplus VERSION 7.2
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INPUT INSTRUCTIONS

  TITLE: davidian theophylline data, inverse quadratic;
  DATA: FILE IS theophylline_wide.dat; !data file in ascii format
  VARIABLE: NAMES ARE subject dose wt t1-t11 conc1-conc11; !all 25 variables
    !subject: subject ID
    !dose: oral dose of theophylline in mg/kg
    !wt: subject weight in kg
    !t1-t11: time since administration of dose in hours
    !conc1-conc11: theophylline concentration in mg/L
  USEVARIABLES ARE conc1-conc11; !we plan to model 11 variables
  CONSTRAINT ARE t1-t11; !individual-specific time scores for use in loadings
  ANALYSIS: ESTIMATOR=ML; ITERATIONS=2000; CONVERGENCE=.001; !estimation options
  !BOOTSTRAP=5000; !for bootstrap confidence intervals if desired
  MODEL:

  [conc1-conc11*](z1-z11); !mean trend (z's defined below)
  conc1-conc11*.7(ve); !residual variances constrained equal
  conc1-conc10 PWITH conc2-conc11*.2(ve2); !optional lag-1 residual covariances
  b*.002(vb); c@0; m*.5(vm); !random coeff. variances (only for maximizer and theta1)
  b WITH c@0; b WITH m*; c WITH m@0; !random coefficient covariances
  [b@0 c@0 m@0]; !factor means constrained to zero
  b BY conc1*(b1); b BY conc2-conc11*(b2-b11); !loadings for the theta1 parameter (b)
  c BY conc1*(c1); c BY conc2-conc11*(c2-c11); !loadings for the theta2 parameter (c)
  m BY conc1*(m1); m BY conc2-conc11*(m2-m11); !loadings for the maximizer parameter (m)

  MODEL CONSTRAINT:

  NEW(mb*.07 mc*.02 mm*1.77); !starts for theta1, theta2, and maximizer parameters

  !intercepts constrained equal to target function
  z1=t1/(mb*t1+mc*(mm^2+t1^2));
  z2=t2/(mb*t2+mc*(mm^2+t2^2));
  z3=t3/(mb*t3+mc*(mm^2+t3^2));
  z4=t4/(mb*t4+mc*(mm^2+t4^2));
  z5=t5/(mb*t5+mc*(mm^2+t5^2));
  z6=t6/(mb*t6+mc*(mm^2+t6^2));
  z7=t7/(mb*t7+mc*(mm^2+t7^2));
```

```
  z8=t8/(mb*t8+mc*(mm^2+t8^2));
  z9=t9/(mb*t9+mc*(mm^2+t9^2));
  z10=t10/(mb*t10+mc*(mm^2+t10^2));
  z11=t11/(mb*t11+mc*(mm^2+t11^2));

  !loadings for "theta1" factor, constrained equal to d(y)/d(mb)
  b1=-1*t1^2/(mb*t1+mc*(mm^2+t1^2))^2;
  b2=-1*t2^2/(mb*t2+mc*(mm^2+t2^2))^2;
  b3=-1*t3^2/(mb*t3+mc*(mm^2+t3^2))^2;
  b4=-1*t4^2/(mb*t4+mc*(mm^2+t4^2))^2;
  b5=-1*t5^2/(mb*t5+mc*(mm^2+t5^2))^2;
  b6=-1*t6^2/(mb*t6+mc*(mm^2+t6^2))^2;
  b7=-1*t7^2/(mb*t7+mc*(mm^2+t7^2))^2;
  b8=-1*t8^2/(mb*t8+mc*(mm^2+t8^2))^2;
  b9=-1*t9^2/(mb*t9+mc*(mm^2+t9^2))^2;
  b10=-1*t10^2/(mb*t10+mc*(mm^2+t10^2))^2;
  b11=-1*t11^2/(mb*t11+mc*(mm^2+t11^2))^2;

  !loadings for "theta2" factor, constrained equal to d(y)/d(mc)
  c1=-1*t1*(mm^2+t1^2)/(mb*t1+mc*(mm^2+t1^2))^2; !not strictly necessary
  c2=-1*t2*(mm^2+t2^2)/(mb*t2+mc*(mm^2+t2^2))^2; !here because this
  c3=-1*t3*(mm^2+t3^2)/(mb*t3+mc*(mm^2+t3^2))^2; !coefficient is fixed,
  c4=-1*t4*(mm^2+t4^2)/(mb*t4+mc*(mm^2+t4^2))^2; !but included for
  c5=-1*t5*(mm^2+t5^2)/(mb*t5+mc*(mm^2+t5^2))^2; !completeness.
  c6=-1*t6*(mm^2+t6^2)/(mb*t6+mc*(mm^2+t6^2))^2;
  c7=-1*t7*(mm^2+t7^2)/(mb*t7+mc*(mm^2+t7^2))^2;
  c8=-1*t8*(mm^2+t8^2)/(mb*t8+mc*(mm^2+t8^2))^2;
  c9=-1*t9*(mm^2+t9^2)/(mb*t9+mc*(mm^2+t9^2))^2;
  c10=-1*t10*(mm^2+t10^2)/(mb*t10+mc*(mm^2+t10^2))^2;
  c11=-1*t11*(mm^2+t11^2)/(mb*t11+mc*(mm^2+t11^2))^2;

  !loadings for "maximizer" factor, constrained equal to d(y)/d(mm)
  m1=-2*mm*mc*t1/(mb*t1+mc*(mm^2+t1^2))^2;
  m2=-2*mm*mc*t2/(mb*t2+mc*(mm^2+t2^2))^2;
  m3=-2*mm*mc*t3/(mb*t3+mc*(mm^2+t3^2))^2;
  m4=-2*mm*mc*t4/(mb*t4+mc*(mm^2+t4^2))^2;
  m5=-2*mm*mc*t5/(mb*t5+mc*(mm^2+t5^2))^2;
  m6=-2*mm*mc*t6/(mb*t6+mc*(mm^2+t6^2))^2;
  m7=-2*mm*mc*t7/(mb*t7+mc*(mm^2+t7^2))^2;
  m8=-2*mm*mc*t8/(mb*t8+mc*(mm^2+t8^2))^2;
  m9=-2*mm*mc*t9/(mb*t9+mc*(mm^2+t9^2))^2;
  m10=-2*mm*mc*t10/(mb*t10+mc*(mm^2+t10^2))^2;
  m11=-2*mm*mc*t11/(mb*t11+mc*(mm^2+t11^2))^2;

  OUTPUT: TECH1; !CINTERVAL(BCBOOTSTRAP);

INPUT READING TERMINATED NORMALLY



davidian theophylline data, inverse quadratic;

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                          12

Number of dependent variables                                   11
Number of independent variables                                  0
Number of continuous latent variables                            3

Observed dependent variables
```

```
  Continuous
   CONC1       CONC2       CONC3       CONC4       CONC5       CONC6
   CONC7       CONC8       CONC9       CONC10      CONC11

Continuous latent variables
   B           C           M

Estimator                                                       ML
Information matrix                                        OBSERVED
Maximum number of iterations                                  2000
Convergence criterion                                    0.100D-02
Maximum number of steepest descent iterations                   20

Input data file(s)
  theophylline_wide.dat

Input data format  FREE

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters                        8

Loglikelihood

          H0 Value                       -184.424

Information Criteria

          Akaike (AIC)                    384.848
          Bayesian (BIC)                  388.727
          Sample-Size Adjusted BIC        364.536
            (n* = (n + 2) / 24)

MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 B        BY
    CONC1            999.000      0.000    999.000    999.000
    CONC2            999.000      0.000    999.000    999.000
    CONC3            999.000      0.000    999.000    999.000
    CONC4            999.000      0.000    999.000    999.000
    CONC5            999.000      0.000    999.000    999.000
    CONC6            999.000      0.000    999.000    999.000
    CONC7            999.000      0.000    999.000    999.000
    CONC8            999.000      0.000    999.000    999.000
    CONC9            999.000      0.000    999.000    999.000
    CONC10           999.000      0.000    999.000    999.000
    CONC11           999.000      0.000    999.000    999.000

 C        BY
    CONC1            999.000      0.000    999.000    999.000
    CONC2            999.000      0.000    999.000    999.000
    CONC3            999.000      0.000    999.000    999.000
    CONC4            999.000      0.000    999.000    999.000
    CONC5            999.000      0.000    999.000    999.000
    CONC6            999.000      0.000    999.000    999.000
    CONC7            999.000      0.000    999.000    999.000
    CONC8            999.000      0.000    999.000    999.000
    CONC9            999.000      0.000    999.000    999.000
```

```
   CONC10             999.000      0.000    999.000    999.000
   CONC11             999.000      0.000    999.000    999.000

M        BY
   CONC1              999.000      0.000    999.000    999.000
   CONC2              999.000      0.000    999.000    999.000
   CONC3              999.000      0.000    999.000    999.000
   CONC4              999.000      0.000    999.000    999.000
   CONC5              999.000      0.000    999.000    999.000
   CONC6              999.000      0.000    999.000    999.000
   CONC7              999.000      0.000    999.000    999.000
   CONC8              999.000      0.000    999.000    999.000
   CONC9              999.000      0.000    999.000    999.000
   CONC10             999.000      0.000    999.000    999.000
   CONC11             999.000      0.000    999.000    999.000

B        WITH
   C                    0.000      0.000    999.000    999.000
   M                   -0.026      0.016     -1.572      0.116 !COVARIANCE OF MAXIMIZER
                                                                !AND THETA1 RANDOM
C        WITH                                                   !COEFFICIENTS
   M                    0.000      0.000    999.000    999.000

CONC1    WITH
   CONC2                0.226      0.077      2.943      0.003 !LAG-1 ERROR COVARIANCE
                                                                !
CONC2    WITH                                                   !
   CONC3                0.226      0.077      2.943      0.003 !
                                                                !
CONC3    WITH                                                   !
   CONC4                0.226      0.077      2.943      0.003 !
                                                                !
CONC4    WITH                                                   !
   CONC5                0.226      0.077      2.943      0.003 !
                                                                !
CONC5    WITH                                                   !
   CONC6                0.226      0.077      2.943      0.003 !
                                                                !
CONC6    WITH                                                   !
   CONC7                0.226      0.077      2.943      0.003 !
                                                                !
CONC7    WITH                                                   !
   CONC8                0.226      0.077      2.943      0.003 !
                                                                !
CONC8    WITH                                                   !
   CONC9                0.226      0.077      2.943      0.003 !
                                                                !
CONC9    WITH                                                   !
   CONC10               0.226      0.077      2.943      0.003 !
                                                                !
CONC10   WITH                                                   !
   CONC11               0.226      0.077      2.943      0.003 !

Means
   B                    0.000      0.000    999.000    999.000 !FACTOR MEANS FIXED = 0
   C                    0.000      0.000    999.000    999.000 !
   M                    0.000      0.000    999.000    999.000 !

Intercepts
   CONC1              999.000      0.000    999.000    999.000
   CONC2              999.000      0.000    999.000    999.000
   CONC3              999.000      0.000    999.000    999.000
   CONC4              999.000      0.000    999.000    999.000
```

```
    CONC5            999.000      0.000    999.000    999.000
    CONC6            999.000      0.000    999.000    999.000
    CONC7            999.000      0.000    999.000    999.000
    CONC8            999.000      0.000    999.000    999.000
    CONC9            999.000      0.000    999.000    999.000
    CONC10           999.000      0.000    999.000    999.000
    CONC11           999.000      0.000    999.000    999.000

 Variances
    B                  0.002      0.001      1.559      0.119 !VARIANCES OF THETA1,
    C                  0.000      0.000    999.000    999.000 !THETA2, AND MAXIMIZER
    M                  0.522      0.291      1.796      0.073 !COEFFICIENTS

 Residual Variances
    CONC1              0.731      0.119      6.119      0.000 !RESIDUAL VARIANCE, HELD
    CONC2              0.731      0.119      6.119      0.000 !EQUAL OVER TIME
    CONC3              0.731      0.119      6.119      0.000 !
    CONC4              0.731      0.119      6.119      0.000 !
    CONC5              0.731      0.119      6.119      0.000 !
    CONC6              0.731      0.119      6.119      0.000 !
    CONC7              0.731      0.119      6.119      0.000 !
    CONC8              0.731      0.119      6.119      0.000 !
    CONC9              0.731      0.119      6.119      0.000 !
    CONC10             0.731      0.119      6.119      0.000 !
    CONC11             0.731      0.119      6.119      0.000 !

New/Additional Parameters
    MB                 0.069      0.013      5.258      0.000 !MEAN THETA1 ESTIMATE
    MC                 0.017      0.001     14.672      0.000 !MEAN THETA2 ESTIMATE
    MM                 1.768      0.189      9.342      0.000 !MEAN MAXIMIZER ESTIMATE

QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.496E-07
       (ratio of smallest to largest eigenvalue)
```

10. Annotated output: unconditional bilinear spline model

```
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INPUT INSTRUCTIONS

  TITLE: zerbe glucose;
  DATA: FILE IS glucose.dat; !data file in ascii format
  VARIABLE: NAMES ARE obese ph1-ph8; !all 9 variables
  USEVARIABLES ARE ph1-ph8; !we plan to model 8 variables
    !obese: obesity status, 0=control, 1=obese
    !ph1-ph8: plasma phosphate concentration repeated measures
  !ANALYSIS: BOOTSTRAP IS 1000; !for bootstrap confidence intervals if desired
  MODEL:

  [ph1-ph8*](z1-z8); !mean trend (z's defined below)
  ph1-ph8*(ve); !residual variances constrained equal
  w1-w4*; !random coefficient variances
  w1 WITH w2*; w1 WITH w3*; w1 WITH w4*; !random coefficient covariances
  w2 WITH w3*; w2 WITH w4*; w3 WITH w4*; !random coefficient covariances
  [w1@0 w2@0 w3@0 w4@0]; !factor means constrained to zero
  w1 BY ph1-ph8@1; !loadings for 1st factor, constrained to 1
  w2 BY ph1*(w21); w2 BY ph2-ph8(w22-w28); !loadings for 2nd factor, defined below
  w3 BY ph1*(w31); w3 BY ph2-ph8(w32-w38); !loadings for 3rd factor, defined below
```

```
  w4 BY ph1*(w41); w4 BY ph2-ph8(w42-w48); !loadings for knot factor, defined below

  MODEL CONSTRAINT:

  !starts for parameters of the target function
  NEW(mw1*3.4 mw2*-.26 mw3*.54 mw4*1.6 b1-b4);

  !intercepts constrained equal to target function
  z1=mw1+mw2*0.0+mw3*sqrt((0.0-mw4)^2);
  z2=mw1+mw2*0.5+mw3*sqrt((0.5-mw4)^2);
  z3=mw1+mw2*1.0+mw3*sqrt((1.0-mw4)^2);
  z4=mw1+mw2*1.5+mw3*sqrt((1.5-mw4)^2);
  z5=mw1+mw2*2.0+mw3*sqrt((2.0-mw4)^2);
  z6=mw1+mw2*3.0+mw3*sqrt((3.0-mw4)^2);
  z7=mw1+mw2*4.0+mw3*sqrt((4.0-mw4)^2);
  z8=mw1+mw2*5.0+mw3*sqrt((5.0-mw4)^2);

  !loadings for 2nd and 3rd factors, constrained to d(y)/d(mw2) and d(y)/d(mw3)
  w21=0.0; w31=sqrt((0.0-mw4)^2);
  w22=0.5; w32=sqrt((0.5-mw4)^2);
  w23=1.0; w33=sqrt((1.0-mw4)^2);
  w24=1.5; w34=sqrt((1.5-mw4)^2);
  w25=2.0; w35=sqrt((2.0-mw4)^2);
  w26=3.0; w36=sqrt((3.0-mw4)^2);
  w27=4.0; w37=sqrt((4.0-mw4)^2);
  w28=5.0; w38=sqrt((5.0-mw4)^2);

  !loadings for knot factor, constrained to d(y)/d(mw4)
  w41=(mw3*(mw4-0.0))/sqrt((0.0-mw4)^2);
  w42=(mw3*(mw4-0.5))/sqrt((0.5-mw4)^2);
  w43=(mw3*(mw4-1.0))/sqrt((1.0-mw4)^2);
  w44=(mw3*(mw4-1.5))/sqrt((1.5-mw4)^2);
  w45=(mw3*(mw4-2.0))/sqrt((2.0-mw4)^2);
  w46=(mw3*(mw4-3.0))/sqrt((3.0-mw4)^2);
  w47=(mw3*(mw4-4.0))/sqrt((4.0-mw4)^2);
  w48=(mw3*(mw4-5.0))/sqrt((5.0-mw4)^2);

  !parameters of original parameterization
  b1=mw1+mw3*mw4;
  b2=mw2-mw3;
  b3=mw1-mw3*mw4;
  b4=mw2+mw3;

  OUTPUT: TECH1; !CINTERVAL(BCBOOTSTRAP);

INPUT READING TERMINATED NORMALLY

zerbe glucose;

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                          33

Number of dependent variables                                    8
Number of independent variables                                  0
Number of continuous latent variables                            4

Observed dependent variables

  Continuous
   PH1         PH2         PH3         PH4         PH5         PH6
   PH7         PH8
```

Continuous latent variables
   W1   W2   W3   W4

Estimator	ML
Information matrix	OBSERVED
Maximum number of iterations	1000
Convergence criterion	0.500D-04
Maximum number of steepest descent iterations	20

Input data file(s)
  glucose.dat

Input data format FREE

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters	15

Loglikelihood

H0 Value	-176.244
H1 Value	-142.162

Information Criteria

Akaike (AIC)	382.487
Bayesian (BIC)	404.935
Sample-Size Adjusted BIC (n* = (n + 2) / 24)	358.147

Chi-Square Test of Model Fit

Value	68.163
Degrees of Freedom	29
P-Value	0.0001

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.202	
90 Percent C.I.	0.140	0.265
Probability RMSEA <= .05	0.000	

CFI/TLI

CFI	0.843
TLI	0.849

Chi-Square Test of Model Fit for the Baseline Model

Value	277.636
Degrees of Freedom	28
P-Value	0.0000

SRMR (Standardized Root Mean Square Residual)

Value	0.212

MODEL RESULTS

				Two-Tailed

		Estimate	S.E.	Est./S.E.	P-Value	
W1	BY					
	PH1	1.000	0.000	999.000	999.000	
	PH2	1.000	0.000	999.000	999.000	
	PH3	1.000	0.000	999.000	999.000	
	PH4	1.000	0.000	999.000	999.000	
	PH5	1.000	0.000	999.000	999.000	
	PH6	1.000	0.000	999.000	999.000	
	PH7	1.000	0.000	999.000	999.000	
	PH8	1.000	0.000	999.000	999.000	
W2	BY					
	PH1	0.000	0.000	999.000	999.000	
	PH2	0.500	0.000	999.000	999.000	
	PH3	1.000	0.000	999.000	999.000	
	PH4	1.500	0.000	999.000	999.000	
	PH5	2.000	0.000	999.000	999.000	
	PH6	3.000	0.000	999.000	999.000	
	PH7	4.000	0.000	999.000	999.000	
	PH8	5.000	0.000	999.000	999.000	
W3	BY					
	PH1	1.588	0.112	14.181	0.000	
	PH2	1.088	0.112	9.715	0.000	
	PH3	0.588	0.112	5.248	0.000	
	PH4	0.088	0.112	0.782	0.434	
	PH5	0.412	0.112	3.684	0.000	
	PH6	1.412	0.112	12.616	0.000	
	PH7	2.412	0.112	21.549	0.000	
	PH8	3.412	0.112	30.481	0.000	
W4	BY					
	PH1	0.539	0.040	13.436	0.000	
	PH2	0.539	0.040	13.436	0.000	
	PH3	0.539	0.040	13.436	0.000	
	PH4	0.539	0.040	13.436	0.000	
	PH5	-0.539	0.040	-13.436	0.000	
	PH6	-0.539	0.040	-13.436	0.000	
	PH7	-0.539	0.040	-13.436	0.000	
	PH8	-0.539	0.040	-13.436	0.000	
W1	WITH					!COVARIANCES OF RANDOM
	W2	-0.030	0.028	-1.064	0.287	!COEFFICIENTS
	W3	-0.024	0.030	-0.810	0.418	!
	W4	-0.008	0.085	-0.089	0.929	!
						!
W2	WITH					!
	W3	-0.009	0.009	-1.028	0.304	!
	W4	0.018	0.026	0.701	0.484	!
						!
W3	WITH					!
	W4	0.001	0.027	0.036	0.971	!
Means						
	W1	0.000	0.000	999.000	999.000	!FACTOR MEANS FIXED = 0
	W2	0.000	0.000	999.000	999.000	!
	W3	0.000	0.000	999.000	999.000	!
	W4	0.000	0.000	999.000	999.000	!
Intercepts						
	PH1	4.256	0.134	31.879	0.000	
	PH2	3.858	0.124	31.115	0.000	

```
    PH3                3.459      0.123     28.182      0.000
    PH4                3.061      0.130     23.537      0.000
    PH5                3.107      0.120     25.928      0.000
    PH6                3.388      0.104     32.705      0.000
    PH7                3.668      0.098     37.354      0.000
    PH8                3.949      0.105     37.477      0.000

 Variances
    W1                 0.457      0.135      3.397      0.001 !VARIANCES OF RANDOM
    W2                 0.013      0.010      1.291      0.197 !COEFFICIENTS
    W3                 0.027      0.013      2.029      0.042 !
    W4                 0.254      0.110      2.304      0.021 !

 Residual Variances
    PH1                0.103      0.013      8.124      0.000 !RESIDUAL VARIANCE, HELD EQUAL
    PH2                0.103      0.013      8.124      0.000 !OVER TIME
    PH3                0.103      0.013      8.124      0.000 !
    PH4                0.103      0.013      8.124      0.000 !
    PH5                0.103      0.013      8.124      0.000 !
    PH6                0.103      0.013      8.124      0.000 !
    PH7                0.103      0.013      8.124      0.000 !
    PH8                0.103      0.013      8.124      0.000 !

New/Additional Parameters
    MW1                3.400      0.128     26.470      0.000 !MEAN OF 1ST GROWTH PARAMETER
    MW2               -0.258      0.034     -7.526      0.000 !MEAN OF 2ND GROWTH PARAMETER
    MW3                0.539      0.040     13.436      0.000 !MEAN OF 3RD GROWTH PARAMETER
    MW4                1.588      0.112     14.181      0.000 !MEAN OF KNOT PARAMETER
    B1                 4.256      0.134     31.879      0.000 !INTERCEPT FOR FIRST SEGMENT
    B2                -0.797      0.066    -12.158      0.000 !SLOPE FOR FIRST SEGMENT
    B3                 2.545      0.171     14.881      0.000 !INTERCEPT FOR SECOND SEGMENT
    B4                 0.281      0.036      7.876      0.000 !SLOPE FOR SECOND SEGMENT

QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.260E-05
       (ratio of smallest to largest eigenvalue)
```

11. Annotated output: conditional bilinear spline model

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INPUT INSTRUCTIONS

  TITLE: zerbe glucose;
  DATA: FILE IS glucose.dat; !data file in ascii format
  DEFINE: xobese=abs(obese-1); !1=control; 0=obese
  VARIABLE: NAMES ARE obese ph1-ph8; !all 9 variables
    !obese: obesity status, 0=control, 1=obese
    !ph1-ph8: plasma phosphate concentration repeated measures
  USEVARIABLES ARE ph1-ph8 xobese; !we plan to model 9 variables
  !ANALYSIS: BOOTSTRAP IS 1000; !for bootstrap confidence intervals if desired
  MODEL:

  [ph1-ph8*](z1-z8); !mean trend (z's defined below)
  ph1-ph8*(ve); !residual variances constrained equal
  w1-w4*; !random coefficient residual variances
  w1 WITH w2*; w1 WITH w3*; w1 WITH w4*; !random coefficient residual covariances
  w2 WITH w3*; w2 WITH w4*; w3 WITH w4*; !random coefficient residual covariances
  [w1@0 w2@0 w3@0 w4@0]; !factor intercepts constrained to zero
```

```
  w1 BY ph1-ph8@1; !loadings for 1st factor, constrained to 1
  w2 BY ph1*(w21); w2 BY ph2-ph8(w22-w28); !loadings for 2nd factor, defined below
  w3 BY ph1*(w31); w3 BY ph2-ph8(w32-w38); !loadings for 3rd factor, defined below
  w4 BY ph1*(w41); w4 BY ph2-ph8(w42-w48); !loadings for knot factor, defined below
  w1-w4 ON xobese; !regress random coefficients on obese

  MODEL CONSTRAINT:

  !starts for parameters of the target function
  NEW(mw1*3.5 mw2*-.21 mw3*.49 mw4*1.95 b1-b4);

  !intercepts constrained equal to target function
  z1=mw1+mw2*0.0+mw3*sqrt((0.0-mw4)^2);
  z2=mw1+mw2*0.5+mw3*sqrt((0.5-mw4)^2);
  z3=mw1+mw2*1.0+mw3*sqrt((1.0-mw4)^2);
  z4=mw1+mw2*1.5+mw3*sqrt((1.5-mw4)^2);
  z5=mw1+mw2*2.0+mw3*sqrt((2.0-mw4)^2);
  z6=mw1+mw2*3.0+mw3*sqrt((3.0-mw4)^2);
  z7=mw1+mw2*4.0+mw3*sqrt((4.0-mw4)^2);
  z8=mw1+mw2*5.0+mw3*sqrt((5.0-mw4)^2);

  !loadings for 2nd and 3rd factors, constrained to d(y)/d(mw2) and d(y)/d(mw3)
  w21=0.0; w31=sqrt((0.0-mw4)^2);
  w22=0.5; w32=sqrt((0.5-mw4)^2);
  w23=1.0; w33=sqrt((1.0-mw4)^2);
  w24=1.5; w34=sqrt((1.5-mw4)^2);
  w25=2.0; w35=sqrt((2.0-mw4)^2);
  w26=3.0; w36=sqrt((3.0-mw4)^2);
  w27=4.0; w37=sqrt((4.0-mw4)^2);
  w28=5.0; w38=sqrt((5.0-mw4)^2);

  !loadings for knot factor, constrained to d(y)/d(mw4)
  w41=(mw3*(mw4-0.0))/sqrt((0.0-mw4)^2);
  w42=(mw3*(mw4-0.5))/sqrt((0.5-mw4)^2);
  w43=(mw3*(mw4-1.0))/sqrt((1.0-mw4)^2);
  w44=(mw3*(mw4-1.5))/sqrt((1.5-mw4)^2);
  w45=(mw3*(mw4-2.0))/sqrt((2.0-mw4)^2);
  w46=(mw3*(mw4-3.0))/sqrt((3.0-mw4)^2);
  w47=(mw3*(mw4-4.0))/sqrt((4.0-mw4)^2);
  w48=(mw3*(mw4-5.0))/sqrt((5.0-mw4)^2);

  !parameters of original parameterization
  b1=mw1+mw3*mw4;
  b2=mw2-mw3;
  b3=mw1-mw3*mw4;
  b4=mw2+mw3;

  OUTPUT: TECH1; !CINTERVAL(BCBOOTSTRAP);

INPUT READING TERMINATED NORMALLY

zerbe glucose;

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                          33

Number of dependent variables                                    8
Number of independent variables                                  1
Number of continuous latent variables                            4

Observed dependent variables
```

```
  Continuous
   PH1         PH2         PH3         PH4         PH5         PH6
   PH7         PH8

Observed independent variables
   XOBESE

Continuous latent variables
   W1          W2          W3          W4

Estimator                                                      ML
Information matrix                                       OBSERVED
Maximum number of iterations                                 1000
Convergence criterion                                  0.500D-04
Maximum number of steepest descent iterations                  20

Input data file(s)
  glucose.dat

Input data format  FREE

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters                       19

Loglikelihood

          H0 Value                       -161.967
          H1 Value                       -126.612

Information Criteria

          Akaike (AIC)                    361.934
          Bayesian (BIC)                  390.368
          Sample-Size Adjusted BIC        331.103
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                           70.711
          Degrees of Freedom                  33
          P-Value                         0.0001

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                         0.186
          90 Percent C.I.                  0.126  0.246
          Probability RMSEA <= .05         0.001

CFI/TLI

          CFI                              0.862
          TLI                              0.849

Chi-Square Test of Model Fit for the Baseline Model

          Value                          308.737
          Degrees of Freedom                  36
          P-Value                         0.0000
```

SRMR (Standardized Root Mean Square Residual)

	Value	0.192

MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	
W1	BY					
	PH1	1.000	0.000	999.000	999.000	
	PH2	1.000	0.000	999.000	999.000	
	PH3	1.000	0.000	999.000	999.000	
	PH4	1.000	0.000	999.000	999.000	
	PH5	1.000	0.000	999.000	999.000	
	PH6	1.000	0.000	999.000	999.000	
	PH7	1.000	0.000	999.000	999.000	
	PH8	1.000	0.000	999.000	999.000	
W2	BY					
	PH1	0.000	0.000	999.000	999.000	
	PH2	0.500	0.000	999.000	999.000	
	PH3	1.000	0.000	999.000	999.000	
	PH4	1.500	0.000	999.000	999.000	
	PH5	2.000	0.000	999.000	999.000	
	PH6	3.000	0.000	999.000	999.000	
	PH7	4.000	0.000	999.000	999.000	
	PH8	5.000	0.000	999.000	999.000	
W3	BY					
	PH1	1.951	0.123	15.799	0.000	
	PH2	1.451	0.123	11.749	0.000	
	PH3	0.951	0.123	7.699	0.000	
	PH4	0.451	0.123	3.649	0.000	
	PH5	0.049	0.123	0.401	0.689	
	PH6	1.049	0.123	8.501	0.000	
	PH7	2.049	0.123	16.600	0.000	
	PH8	3.049	0.123	24.700	0.000	
W4	BY					
	PH1	0.490	0.050	9.860	0.000	
	PH2	0.490	0.050	9.860	0.000	
	PH3	0.490	0.050	9.860	0.000	
	PH4	0.490	0.050	9.860	0.000	
	PH5	-0.490	0.050	-9.860	0.000	
	PH6	-0.490	0.050	-9.860	0.000	
	PH7	-0.490	0.050	-9.860	0.000	
	PH8	-0.490	0.050	-9.860	0.000	
W1	ON					!REGRESSION OF 1ST RANDOM
	XOBESE	-0.387	0.254	-1.521	0.128	!COEFFICIENT ON OBESITY
W2	ON					!REGRESSION OF 2ND RANDOM
	XOBESE	-0.117	0.067	-1.750	0.080	!COEFFICIENT ON OBESITY
W3	ON					!REGRESSION OF 3RD RANDOM
	XOBESE	0.124	0.079	1.560	0.119	!COEFFICIENT ON OBESITY
W4	ON					!REGRESSION OF RANDOM KNOT
	XOBESE	-1.013	0.253	-4.004	0.000	!COEFFICIENT ON OBESITY
W1	WITH					!COVARIANCES OF RANDOM
	W2	-0.041	0.027	-1.499	0.134	!COEFFICIENT RESIDUALS

```
    W3                -0.013      0.028     -0.464      0.643 !
    W4                -0.084      0.077     -1.093      0.274 !
                                                               !
 W2       WITH                                                 !
    W3                -0.006      0.008     -0.714      0.475 !
    W4                -0.001      0.023     -0.061      0.951 !
                                                               !
 W3       WITH                                                 !
    W4                 0.011      0.023      0.468      0.639 !

 Intercepts
    PH1                4.509      0.156     28.814      0.000
    PH2                4.158      0.136     30.638      0.000
    PH3                3.807      0.125     30.533      0.000
    PH4                3.456      0.126     27.431      0.000
    PH5                3.154      0.153     20.563      0.000
    PH6                3.432      0.132     25.906      0.000
    PH7                3.711      0.126     29.539      0.000
    PH8                3.989      0.135     29.566      0.000
    W1                 0.000      0.000    999.000    999.000 !FACTOR INTERCEPTS FIXED = 0
    W2                 0.000      0.000    999.000    999.000 !
    W3                 0.000      0.000    999.000    999.000 !
    W4                 0.000      0.000    999.000    999.000 !

 Residual Variances
    PH1                0.103      0.013      8.124      0.000 !RESIDUAL VARIANCE, HELD EQUAL
    PH2                0.103      0.013      8.124      0.000 !OVER TIME
    PH3                0.103      0.013      8.124      0.000 !
    PH4                0.103      0.013      8.124      0.000 !
    PH5                0.103      0.013      8.124      0.000 !
    PH6                0.103      0.013      8.124      0.000 !
    PH7                0.103      0.013      8.124      0.000 !
    PH8                0.103      0.013      8.124      0.000 !
    W1                 0.421      0.126      3.349      0.001 !RESIDUAL VARIANCES OF RANDOM
    W2                 0.010      0.009      1.043      0.297 !COEFFICIENTS
    W3                 0.024      0.013      1.879      0.060 !
    W4                 0.076      0.082      0.929      0.353 !

New/Additional Parameters
    MW1                3.553      0.160     22.272      0.000 !CONDITIONAL MEANS OF ALL 4
    MW2               -0.212      0.042     -5.026      0.000 !GROWTH PARAMETERS
    MW3                0.490      0.050      9.860      0.000 !
    MW4                1.951      0.123     15.799      0.000 !
    B1                 4.509      0.156     28.814      0.000 !CONDITIONAL MEANS OF ORIGINAL
    B2                -0.702      0.080     -8.778      0.000 !INTERCEPTS AND SLOPES FOR
    B3                 2.596      0.219     11.845      0.000 !FIRST AND SECOND SEGMENTS
    B4                 0.278      0.046      6.080      0.000 !

QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.308E-05
       (ratio of smallest to largest eigenvalue)
```

12. Annotated output: negative exponential function with intercept, asymptote, and rate

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INPUT INSTRUCTIONS

  TITLE: chaiken, negative exponential, original parameterization;
```

```
DATA: FILE IS schaiken2.dat; !data file in ascii format
VARIABLE: NAMES ARE v1-v12 q1-q12; !all 24 variables
  !v1-v12: verbal skill acquisition repeated measures
  !q1-q12: quantitative skill acquisition repeated measures
ANALYSIS: ESTIMATOR IS ML; !use maximum likelihood estimation
MODEL:

!verbal
[v1-v12*](tv1-tv12); !mean trend for verbal
v1-v12*1.97(vv1); !residual variances constrained equal
v1-v11 PWITH v2-v12*(dv1); !lag-1 error covariance
v1-v10 PWITH v3-v12*(dv2); !lag-2, etc...
v1-v9 PWITH v4-v12*(dv3);
v1-v8 PWITH v5-v12*(dv4);
v1-v7 PWITH v6-v12*(dv5);
v1-v6 PWITH v7-v12*(dv6);
v1-v5 PWITH v8-v12*(dv7);
v1-v4 PWITH v9-v12*(dv8);
v1-v3 PWITH v10-v12*(dv9);
v1-v2 PWITH v11-v12*(dv10);
v1 WITH v12*(dv11);
[av@0 bv@0 cv@0]; !factor means constrained to zero
av*; bv*; cv*; !random coefficient variances
av WITH bv* cv*; bv WITH cv*; !random coefficient covariances
av BY v1*(av1); av BY v2-v12(av2-av12); !loadings for 1st factor
bv BY v1*(bv1); bv BY v2-v12(bv2-bv12); !loadings for 2nd factor
cv BY v1*(cv1); cv BY v2-v12(cv2-cv12); !loadings for 3rd factor

!quantitative
[q1-q12*](tq1-tq12); !mean trend for quantitative
q1-q12*1.21(vq1); !residual variances constrained equal
q1-q11 PWITH q2-q12*(dq1); !lag-1 error covariance
q1-q10 PWITH q3-q12*(dq2); !lag-2, etc...
q1-q9 PWITH q4-q12*(dq3);
q1-q8 PWITH q5-q12*(dq4);
q1-q7 PWITH q6-q12*(dq5);
q1-q6 PWITH q7-q12*(dq6);
q1-q5 PWITH q8-q12*(dq7);
q1-q4 PWITH q9-q12*(dq8);
q1-q3 PWITH q10-q12*(dq9);
q1-q2 PWITH q11-q12*(dq10);
q1 WITH q12*(dq11);
[aq@0 bq@0 cq@0]; !factor means constrained to zero
aq*; bq*; cq*; !random coefficient variances
aq WITH bq* cq*; bq WITH cq; !random coefficient covariances
aq BY q1*(aq1); aq BY q2-q12(aq2-aq12); !loadings for 1st factor
bq BY q1*(bq1); bq BY q2-q12(bq2-bq12); !loadings for 2nd factor
cq BY q1*(cq1); cq BY q2-q12(cq2-cq12); !loadings for 3rd factor

!cross-domain covariances
av WITH aq* bq*; bv WITH aq* bq*;

MODEL CONSTRAINT:

!starts for parameters of the target function and for lagged error covariances
NEW(mav*6.8 mbv*21 mcv*.7 maq*8.6 mbq*16.5 mcq*.7 rhov*.27 rhoq*.31);

!error covariances
dv1= vv1*rhov;    dq1= vq1*rhoq;
dv2= vv1*rhov^2;  dq2= vq1*rhoq^2;
dv3= vv1*rhov^3;  dq3= vq1*rhoq^3;
dv4= vv1*rhov^4;  dq4= vq1*rhoq^4;
dv5= vv1*rhov^5;  dq5= vq1*rhoq^5;
```

```
dv6= vv1*rhov^6;  dq6= vq1*rhoq^6;
dv7= vv1*rhov^7;  dq7= vq1*rhoq^7;
dv8= vv1*rhov^8;  dq8= vq1*rhoq^8;
dv9= vv1*rhov^9;  dq9= vq1*rhoq^9;
dv10=vv1*rhov^10; dq10=vq1*rhoq^10;
dv11=vv1*rhov^11; dq11=vq1*rhoq^11;

!verbal and quantitative intercepts constrained equal to target function
tv1 =mav-(mav-mbv)*exp(-1*mcv*( 1-1)); tq1 =maq-(maq-mbq)*exp(-1*mcq*( 1-1));
tv2 =mav-(mav-mbv)*exp(-1*mcv*( 2-1)); tq2 =maq-(maq-mbq)*exp(-1*mcq*( 2-1));
tv3 =mav-(mav-mbv)*exp(-1*mcv*( 3-1)); tq3 =maq-(maq-mbq)*exp(-1*mcq*( 3-1));
tv4 =mav-(mav-mbv)*exp(-1*mcv*( 4-1)); tq4 =maq-(maq-mbq)*exp(-1*mcq*( 4-1));
tv5 =mav-(mav-mbv)*exp(-1*mcv*( 5-1)); tq5 =maq-(maq-mbq)*exp(-1*mcq*( 5-1));
tv6 =mav-(mav-mbv)*exp(-1*mcv*( 6-1)); tq6 =maq-(maq-mbq)*exp(-1*mcq*( 6-1));
tv7 =mav-(mav-mbv)*exp(-1*mcv*( 7-1)); tq7 =maq-(maq-mbq)*exp(-1*mcq*( 7-1));
tv8 =mav-(mav-mbv)*exp(-1*mcv*( 8-1)); tq8 =maq-(maq-mbq)*exp(-1*mcq*( 8-1));
tv9 =mav-(mav-mbv)*exp(-1*mcv*( 9-1)); tq9 =maq-(maq-mbq)*exp(-1*mcq*( 9-1));
tv10=mav-(mav-mbv)*exp(-1*mcv*(10-1)); tq10=maq-(maq-mbq)*exp(-1*mcq*(10-1));
tv11=mav-(mav-mbv)*exp(-1*mcv*(11-1)); tq11=maq-(maq-mbq)*exp(-1*mcq*(11-1));
tv12=mav-(mav-mbv)*exp(-1*mcv*(12-1)); tq12=maq-(maq-mbq)*exp(-1*mcq*(12-1));

!loadings on 1st factor, constrained to d(y)/d(mav) and d(y)/d(maq)
av1 =1-exp(-1*mcv*( 1-1)); aq1 =1-exp(-1*mcq*( 1-1));
av2 =1-exp(-1*mcv*( 2-1)); aq2 =1-exp(-1*mcq*( 2-1));
av3 =1-exp(-1*mcv*( 3-1)); aq3 =1-exp(-1*mcq*( 3-1));
av4 =1-exp(-1*mcv*( 4-1)); aq4 =1-exp(-1*mcq*( 4-1));
av5 =1-exp(-1*mcv*( 5-1)); aq5 =1-exp(-1*mcq*( 5-1));
av6 =1-exp(-1*mcv*( 6-1)); aq6 =1-exp(-1*mcq*( 6-1));
av7 =1-exp(-1*mcv*( 7-1)); aq7 =1-exp(-1*mcq*( 7-1));
av8 =1-exp(-1*mcv*( 8-1)); aq8 =1-exp(-1*mcq*( 8-1));
av9 =1-exp(-1*mcv*( 9-1)); aq9 =1-exp(-1*mcq*( 9-1));
av10=1-exp(-1*mcv*(10-1)); aq10=1-exp(-1*mcq*(10-1));
av11=1-exp(-1*mcv*(11-1)); aq11=1-exp(-1*mcq*(11-1));
av12=1-exp(-1*mcv*(12-1)); aq12=1-exp(-1*mcq*(12-1));

!loadings on 2nd and 3rd factors for verbal, constrained to d(y)/d(mbv) and d(y)/d(mcv)
bv1 =exp(-1*mcv*( 1-1)); cv1 =(mav-mbv)*( 1-1)*exp(-1*mcv*( 1-1));
bv2 =exp(-1*mcv*( 2-1)); cv2 =(mav-mbv)*( 2-1)*exp(-1*mcv*( 2-1));
bv3 =exp(-1*mcv*( 3-1)); cv3 =(mav-mbv)*( 3-1)*exp(-1*mcv*( 3-1));
bv4 =exp(-1*mcv*( 4-1)); cv4 =(mav-mbv)*( 4-1)*exp(-1*mcv*( 4-1));
bv5 =exp(-1*mcv*( 5-1)); cv5 =(mav-mbv)*( 5-1)*exp(-1*mcv*( 5-1));
bv6 =exp(-1*mcv*( 6-1)); cv6 =(mav-mbv)*( 6-1)*exp(-1*mcv*( 6-1));
bv7 =exp(-1*mcv*( 7-1)); cv7 =(mav-mbv)*( 7-1)*exp(-1*mcv*( 7-1));
bv8 =exp(-1*mcv*( 8-1)); cv8 =(mav-mbv)*( 8-1)*exp(-1*mcv*( 8-1));
bv9 =exp(-1*mcv*( 9-1)); cv9 =(mav-mbv)*( 9-1)*exp(-1*mcv*( 9-1));
bv10=exp(-1*mcv*(10-1)); cv10=(mav-mbv)*(10-1)*exp(-1*mcv*(10-1));
bv11=exp(-1*mcv*(11-1)); cv11=(mav-mbv)*(11-1)*exp(-1*mcv*(11-1));
bv12=exp(-1*mcv*(12-1)); cv12=(mav-mbv)*(12-1)*exp(-1*mcv*(12-1));

!loadings on 2nd and 3rd factors for quant., constrained to d(y)/d(mbv) and d(y)/d(mcv)
bq1 =exp(-1*mcq*( 1-1)); cq1 =(maq-mbq)*( 1-1)*exp(-1*mcq*( 1-1));
bq2 =exp(-1*mcq*( 2-1)); cq2 =(maq-mbq)*( 2-1)*exp(-1*mcq*( 2-1));
bq3 =exp(-1*mcq*( 3-1)); cq3 =(maq-mbq)*( 3-1)*exp(-1*mcq*( 3-1));
bq4 =exp(-1*mcq*( 4-1)); cq4 =(maq-mbq)*( 4-1)*exp(-1*mcq*( 4-1));
bq5 =exp(-1*mcq*( 5-1)); cq5 =(maq-mbq)*( 5-1)*exp(-1*mcq*( 5-1));
bq6 =exp(-1*mcq*( 6-1)); cq6 =(maq-mbq)*( 6-1)*exp(-1*mcq*( 6-1));
bq7 =exp(-1*mcq*( 7-1)); cq7 =(maq-mbq)*( 7-1)*exp(-1*mcq*( 7-1));
bq8 =exp(-1*mcq*( 8-1)); cq8 =(maq-mbq)*( 8-1)*exp(-1*mcq*( 8-1));
bq9 =exp(-1*mcq*( 9-1)); cq9 =(maq-mbq)*( 9-1)*exp(-1*mcq*( 9-1));
bq10=exp(-1*mcq*(10-1)); cq10=(maq-mbq)*(10-1)*exp(-1*mcq*(10-1));
bq11=exp(-1*mcq*(11-1)); cq11=(maq-mbq)*(11-1)*exp(-1*mcq*(11-1));
bq12=exp(-1*mcq*(12-1)); cq12=(maq-mbq)*(12-1)*exp(-1*mcq*(12-1));
```

```
  OUTPUT: TECH1 TECH4;

INPUT READING TERMINATED NORMALLY

chaiken, negative exponential, original parameterization;

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                         228

Number of dependent variables                                   24
Number of independent variables                                  0
Number of continuous latent variables                            6

Observed dependent variables

  Continuous
   V1          V2          V3          V4          V5          V6
   V7          V8          V9          V10         V11         V12
   Q1          Q2          Q3          Q4          Q5          Q6
   Q7          Q8          Q9          Q10         Q11         Q12

Continuous latent variables
   AV          BV          CV          AQ          BQ          CQ

Estimator                                                       ML
Information matrix                                        OBSERVED
Maximum number of iterations                                  1000
Convergence criterion                                    0.500D-04
Maximum number of steepest descent iterations                   20

Input data file(s)
  schaiken2.dat

Input data format  FREE

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters                       31

Loglikelihood

          H0 Value                      -10314.946
          H1 Value                       -9097.350

Information Criteria

          Akaike (AIC)                   20691.891
          Bayesian (BIC)                 20798.201
          Sample-Size Adjusted BIC       20699.952
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                           2435.191
          Degrees of Freedom                   293
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)
```

Estimate	0.179	
90 Percent C.I.	0.173	0.186
Probability RMSEA <= .05	0.000	

CFI/TLI

CFI	0.730
TLI	0.745

Chi-Square Test of Model Fit for the Baseline Model

Value	8204.815
Degrees of Freedom	276
P-Value	0.0000

SRMR (Standardized Root Mean Square Residual)

Value	0.218

MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
AV	BY				
	V1	0.000	0.000	999.000	999.000
	V2	0.514	0.010	53.981	0.000
	V3	0.763	0.009	82.480	0.000
	V4	0.885	0.007	131.042	0.000
	V5	0.944	0.004	215.555	0.000
	V6	0.973	0.003	365.328	0.000
	V7	0.987	0.002	634.904	0.000
	V8	0.994	0.001	1126.553	0.000
	V9	0.997	0.000	2033.343	0.000
	V10	0.998	0.000	3721.911	0.000
	V11	0.999	0.000	6892.175	0.000
	V12	1.000	0.000	12886.546	0.000
BV	BY				
	V1	1.000	0.000	999.000	999.000
	V2	0.486	0.010	51.121	0.000
	V3	0.237	0.009	25.561	0.000
	V4	0.115	0.007	17.040	0.000
	V5	0.056	0.004	12.780	0.000
	V6	0.027	0.003	10.224	0.000
	V7	0.013	0.002	8.520	0.000
	V8	0.006	0.001	7.303	0.000
	V9	0.003	0.000	6.390	0.000
	V10	0.002	0.000	5.680	0.000
	V11	0.001	0.000	5.112	0.000
	V12	0.000	0.000	4.647	0.000
CV	BY				
	V1	0.000	0.000	999.000	999.000
	V2	-7.020	0.352	-19.949	0.000
	V3	-6.829	0.422	-16.191	0.000
	V4	-4.982	0.382	-13.039	0.000
	V5	-3.231	0.302	-10.712	0.000
	V6	-1.965	0.218	-9.012	0.000
	V7	-1.147	0.148	-7.744	0.000
	V8	-0.651	0.096	-6.772	0.000
	V9	-0.362	0.060	-6.009	0.000
	V10	-0.198	0.037	-5.395	0.000

```
    V11                -0.107      0.022     -4.892      0.000
    V12                -0.057      0.013     -4.474      0.000

 AQ       BY
    Q1                  0.000      0.000    999.000    999.000
    Q2                  0.502      0.012     43.058      0.000
    Q3                  0.752      0.012     64.796      0.000
    Q4                  0.877      0.009    101.169      0.000
    Q5                  0.939      0.006    163.257      0.000
    Q6                  0.970      0.004    271.095      0.000
    Q7                  0.985      0.002    461.203      0.000
    Q8                  0.992      0.001    800.636      0.000
    Q9                  0.996      0.001   1413.322      0.000
    Q10                 0.998      0.000   2529.599      0.000
    Q11                 0.999      0.000   4579.770      0.000
    Q12                 1.000      0.000   8371.340      0.000

 BQ       BY
    Q1                  1.000      0.000    999.000    999.000
    Q2                  0.498      0.012     42.642      0.000
    Q3                  0.248      0.012     21.321      0.000
    Q4                  0.123      0.009     14.214      0.000
    Q5                  0.061      0.006     10.661      0.000
    Q6                  0.030      0.004      8.528      0.000
    Q7                  0.015      0.002      7.107      0.000
    Q8                  0.008      0.001      6.092      0.000
    Q9                  0.004      0.001      5.330      0.000
    Q10                 0.002      0.000      4.738      0.000
    Q11                 0.001      0.000      4.264      0.000
    Q12                 0.000      0.000      3.877      0.000

 CQ       BY
    Q1                  0.000      0.000    999.000    999.000
    Q2                 -3.913      0.187    -20.931      0.000
    Q3                 -3.894      0.240    -16.220      0.000
    Q4                 -2.906      0.233    -12.481      0.000
    Q5                 -1.928      0.194     -9.932      0.000
    Q6                 -1.199      0.147     -8.177      0.000
    Q7                 -0.716      0.103     -6.922      0.000
    Q8                 -0.416      0.069     -5.988      0.000
    Q9                 -0.236      0.045     -5.270      0.000
    Q10                -0.132      0.028     -4.702      0.000
    Q11                -0.073      0.017     -4.243      0.000
    Q12                -0.040      0.010     -3.864      0.000

 AV       WITH                                                 !COVARIANCES OF VERBAL (V) AND
    BV                  3.783      0.976      3.875      0.000 !QUANTITATIVE (Q) RANDOM
    CV                 -0.147      0.059     -2.479      0.013 !COEFFICIENTS
    AQ                  1.664      0.235      7.097      0.000 !
    BQ                  2.051      0.606      3.384      0.001 !
                                                               !
 BV       WITH                                                 !
    CV                 -0.468      0.391     -1.196      0.232 !
    AQ                  8.242      1.468      5.616      0.000 !
    BQ                 23.425      4.200      5.578      0.000 !
                                                               !
 AQ       WITH                                                 !
    BQ                  7.414      0.985      7.530      0.000 !
    CQ                 -0.126      0.073     -1.733      0.083 !
    CV                 -0.205      0.082     -2.484      0.013 !
                                                               !
 BQ       WITH                                                 !
    CQ                  0.540      0.204      2.647      0.008 !
```

	CV	-0.392	0.232	-1.688	0.091	!
						!
CQ	WITH					!
	AV	-0.072	0.049	-1.484	0.138	!
	BV	0.371	0.318	1.166	0.243	!
	CV	0.036	0.018	1.956	0.050	!
V1	WITH					!AUTOREGRESSIVE RESIDUAL
	V2	0.544	0.092	5.905	0.000	!COVARIANCE STRUCTURE FOR
	V3	0.150	0.045	3.337	0.001	!VERBAL REPEATED MEASURES
	V4	0.041	0.018	2.321	0.020	!
	V5	0.011	0.006	1.779	0.075	!
	V6	0.003	0.002	1.442	0.149	!
	V7	0.001	0.001	1.212	0.226	!
	V8	0.000	0.000	1.045	0.296	!
	V9	0.000	0.000	0.919	0.358	!
	V10	0.000	0.000	0.820	0.412	!
	V11	0.000	0.000	0.740	0.459	!
	V12	0.000	0.000	0.674	0.500	!
						!
V2	WITH					!
	V3	0.544	0.092	5.905	0.000	!
	V4	0.150	0.045	3.337	0.001	!
	V5	0.041	0.018	2.321	0.020	!
	V6	0.011	0.006	1.779	0.075	!
	V7	0.003	0.002	1.442	0.149	!
	V8	0.001	0.001	1.212	0.226	!
	V9	0.000	0.000	1.045	0.296	!
	V10	0.000	0.000	0.919	0.358	!
	V11	0.000	0.000	0.820	0.412	!
	V12	0.000	0.000	0.740	0.459	!
						!
V3	WITH					!
	V4	0.544	0.092	5.905	0.000	!
	V5	0.150	0.045	3.337	0.001	!
	V6	0.041	0.018	2.321	0.020	!
	V7	0.011	0.006	1.779	0.075	!
	V8	0.003	0.002	1.442	0.149	!
	V9	0.001	0.001	1.212	0.226	!
	V10	0.000	0.000	1.045	0.296	!
	V11	0.000	0.000	0.919	0.358	!
	V12	0.000	0.000	0.820	0.412	!
						!
V4	WITH					!
	V5	0.544	0.092	5.905	0.000	!
	V6	0.150	0.045	3.337	0.001	!
	V7	0.041	0.018	2.321	0.020	!
	V8	0.011	0.006	1.779	0.075	!
	V9	0.003	0.002	1.442	0.149	!
	V10	0.001	0.001	1.212	0.226	!
	V11	0.000	0.000	1.045	0.296	!
	V12	0.000	0.000	0.919	0.358	!
						!
V5	WITH					!
	V6	0.544	0.092	5.905	0.000	!
	V7	0.150	0.045	3.337	0.001	!
	V8	0.041	0.018	2.321	0.020	!
	V9	0.011	0.006	1.779	0.075	!
	V10	0.003	0.002	1.442	0.149	!
	V11	0.001	0.001	1.212	0.226	!
	V12	0.000	0.000	1.045	0.296	!
						!
V6	WITH					!

```
   V7                 0.544      0.092      5.905      0.000 !
   V8                 0.150      0.045      3.337      0.001 !
   V9                 0.041      0.018      2.321      0.020 !
   V10                0.011      0.006      1.779      0.075 !
   V11                0.003      0.002      1.442      0.149 !
   V12                0.001      0.001      1.212      0.226 !
                                                             !
V7       WITH                                                !
   V8                 0.544      0.092      5.905      0.000 !
   V9                 0.150      0.045      3.337      0.001 !
   V10                0.041      0.018      2.321      0.020 !
   V11                0.011      0.006      1.779      0.075 !
   V12                0.003      0.002      1.442      0.149 !
                                                             !
V8       WITH                                                !
   V9                 0.544      0.092      5.905      0.000 !
   V10                0.150      0.045      3.337      0.001 !
   V11                0.041      0.018      2.321      0.020 !
   V12                0.011      0.006      1.779      0.075 !
                                                             !
V9       WITH                                                !
   V10                0.544      0.092      5.905      0.000 !
   V11                0.150      0.045      3.337      0.001 !
   V12                0.041      0.018      2.321      0.020 !
                                                             !
V10      WITH                                                !
   V11                0.544      0.092      5.905      0.000 !
   V12                0.150      0.045      3.337      0.001 !
                                                             !
V11      WITH                                                !
   V12                0.544      0.092      5.905      0.000 !

Q1       WITH                                                !AUTOREGRESSIVE RESIDUAL
   Q2                 0.345      0.053      6.528      0.000 !COVARIANCE STRUCTURE FOR
   Q3                 0.107      0.028      3.787      0.000 !QUANTITATIVE REPEATED MEASURES
   Q4                 0.033      0.012      2.662      0.008 !
   Q5                 0.010      0.005      2.051      0.040 !
   Q6                 0.003      0.002      1.668      0.095 !
   Q7                 0.001      0.001      1.405      0.160 !
   Q8                 0.000      0.000      1.214      0.225 !
   Q9                 0.000      0.000      1.068      0.285 !
   Q10                0.000      0.000      0.954      0.340 !
   Q11                0.000      0.000      0.862      0.389 !
   Q12                0.000      0.000      0.786      0.432 !
                                                             !
Q2       WITH                                                !
   Q3                 0.345      0.053      6.528      0.000 !
   Q4                 0.107      0.028      3.787      0.000 !
   Q5                 0.033      0.012      2.662      0.008 !
   Q6                 0.010      0.005      2.051      0.040 !
   Q7                 0.003      0.002      1.668      0.095 !
   Q8                 0.001      0.001      1.405      0.160 !
   Q9                 0.000      0.000      1.214      0.225 !
   Q10                0.000      0.000      1.068      0.285 !
   Q11                0.000      0.000      0.954      0.340 !
   Q12                0.000      0.000      0.862      0.389 !
                                                             !
Q3       WITH                                                !
   Q4                 0.345      0.053      6.528      0.000 !
   Q5                 0.107      0.028      3.787      0.000 !
   Q6                 0.033      0.012      2.662      0.008 !
   Q7                 0.010      0.005      2.051      0.040 !
   Q8                 0.003      0.002      1.668      0.095 !
```

```
   Q9                 0.001      0.001      1.405      0.160 !
   Q10                0.000      0.000      1.214      0.225 !
   Q11                0.000      0.000      1.068      0.285 !
   Q12                0.000      0.000      0.954      0.340 !
                                                              !
Q4       WITH                                                 !
   Q5                 0.345      0.053      6.528      0.000 !
   Q6                 0.107      0.028      3.787      0.000 !
   Q7                 0.033      0.012      2.662      0.008 !
   Q8                 0.010      0.005      2.051      0.040 !
   Q9                 0.003      0.002      1.668      0.095 !
   Q10                0.001      0.001      1.405      0.160 !
   Q11                0.000      0.000      1.214      0.225 !
   Q12                0.000      0.000      1.068      0.285 !
                                                              !
Q5       WITH                                                 !
   Q6                 0.345      0.053      6.528      0.000 !
   Q7                 0.107      0.028      3.787      0.000 !
   Q8                 0.033      0.012      2.662      0.008 !
   Q9                 0.010      0.005      2.051      0.040 !
   Q10                0.003      0.002      1.668      0.095 !
   Q11                0.001      0.001      1.405      0.160 !
   Q12                0.000      0.000      1.214      0.225 !
                                                              !
Q6       WITH                                                 !
   Q7                 0.345      0.053      6.528      0.000 !
   Q8                 0.107      0.028      3.787      0.000 !
   Q9                 0.033      0.012      2.662      0.008 !
   Q10                0.010      0.005      2.051      0.040 !
   Q11                0.003      0.002      1.668      0.095 !
   Q12                0.001      0.001      1.405      0.160 !
                                                              !
Q7       WITH                                                 !
   Q8                 0.345      0.053      6.528      0.000 !
   Q9                 0.107      0.028      3.787      0.000 !
   Q10                0.033      0.012      2.662      0.008 !
   Q11                0.010      0.005      2.051      0.040 !
   Q12                0.003      0.002      1.668      0.095 !
                                                              !
Q8       WITH                                                 !
   Q9                 0.345      0.053      6.528      0.000 !
   Q10                0.107      0.028      3.787      0.000 !
   Q11                0.033      0.012      2.662      0.008 !
   Q12                0.010      0.005      2.051      0.040 !
                                                              !
Q9       WITH                                                 !
   Q10                0.345      0.053      6.528      0.000 !
   Q11                0.107      0.028      3.787      0.000 !
   Q12                0.033      0.012      2.662      0.008 !
                                                              !
Q10      WITH                                                 !
   Q11                0.345      0.053      6.528      0.000 !
   Q12                0.107      0.028      3.787      0.000 !
                                                              !
Q11      WITH                                                 !
   Q12                0.345      0.053      6.528      0.000 !

Means
   AV                 0.000      0.000    999.000    999.000 !FACTOR MEANS FIXED = 0
   BV                 0.000      0.000    999.000    999.000 !
   CV                 0.000      0.000    999.000    999.000 !
   AQ                 0.000      0.000    999.000    999.000 !
   BQ                 0.000      0.000    999.000    999.000 !
```

CQ	0.000	0.000	999.000	999.000	!
Intercepts					
V1	21.278	0.661	32.192	0.000	
V2	13.865	0.374	37.074	0.000	
V3	10.260	0.238	43.076	0.000	
V4	8.506	0.164	51.793	0.000	
V5	7.653	0.126	60.760	0.000	
V6	7.238	0.109	66.405	0.000	
V7	7.036	0.103	68.566	0.000	
V8	6.938	0.101	69.035	0.000	
V9	6.890	0.100	68.996	0.000	
V10	6.867	0.100	68.879	0.000	
V11	6.856	0.100	68.789	0.000	
V12	6.850	0.100	68.733	0.000	
Q1	16.492	0.404	40.778	0.000	
Q2	12.541	0.261	47.962	0.000	
Q3	10.575	0.202	52.297	0.000	
Q4	9.597	0.171	56.083	0.000	
Q5	9.111	0.155	58.658	0.000	
Q6	8.868	0.148	59.868	0.000	
Q7	8.748	0.145	60.270	0.000	
Q8	8.688	0.144	60.347	0.000	
Q9	8.658	0.144	60.330	0.000	
Q10	8.643	0.143	60.299	0.000	
Q11	8.636	0.143	60.275	0.000	
Q12	8.632	0.143	60.261	0.000	
Variances					
AV	1.652	0.207	7.981	0.000	!VARIANCES OF RANDOM
BV	92.144	8.816	10.452	0.000	!COEFFICIENTS
CV	0.272	0.044	6.232	0.000	!
AQ	4.187	0.423	9.908	0.000	!
BQ	35.772	3.456	10.351	0.000	!
CQ	0.179	0.030	6.058	0.000	!
Residual Variances					
V1	1.974	0.092	21.355	0.000	!VERBAL RESIDUAL VARIANCE
V2	1.974	0.092	21.355	0.000	!HELD EQUAL OVER TIME
V3	1.974	0.092	21.355	0.000	!
V4	1.974	0.092	21.355	0.000	!
V5	1.974	0.092	21.355	0.000	!
V6	1.974	0.092	21.355	0.000	!
V7	1.974	0.092	21.355	0.000	!
V8	1.974	0.092	21.355	0.000	!
V9	1.974	0.092	21.355	0.000	!
V10	1.974	0.092	21.355	0.000	!
V11	1.974	0.092	21.355	0.000	!
V12	1.974	0.092	21.355	0.000	!
Q1	1.112	0.055	20.403	0.000	!QUANTITATIVE RESIDUAL VARIANCE
Q2	1.112	0.055	20.403	0.000	!HELD EQUAL OVER TIME
Q3	1.112	0.055	20.403	0.000	!
Q4	1.112	0.055	20.403	0.000	!
Q5	1.112	0.055	20.403	0.000	!
Q6	1.112	0.055	20.403	0.000	!
Q7	1.112	0.055	20.403	0.000	!
Q8	1.112	0.055	20.403	0.000	!
Q9	1.112	0.055	20.403	0.000	!
Q10	1.112	0.055	20.403	0.000	!
Q11	1.112	0.055	20.403	0.000	!
Q12	1.112	0.055	20.403	0.000	!

New/Additional Parameters

```
    MAV                6.845      0.100     68.667      0.000 !VERBAL MEAN ASYMPTOTE
    MBV               21.278      0.661     32.192      0.000 !VERBAL MEAN INITIAL VALUE
    MCV                0.721      0.020     36.845      0.000 !VERBAL MEAN RATE PARAMETER
    MAQ                8.629      0.143     60.242      0.000 !QUANT. MEAN ASYMPTOTE
    MBQ               16.492      0.404     40.778      0.000 !QUANT. MEAN INITIAL VALUE
    MCQ                0.698      0.023     29.765      0.000 !QUANT. MEAN RATE PARAMETER
    RHOV               0.276      0.036      7.597      0.000 !AUTOREGRESSIVE PARAMETER (V)
    RHOQ               0.310      0.035      8.910      0.000 !AUTOREGRESSIVE PARAMETER (Q)


QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.168E-03
       (ratio of smallest to largest eigenvalue)
```

13. Annotated output: negative exponential function with intercept, half-life, and ARC

```
Mplus VERSION 7.2
MUTHEN & MUTHEN
06/10/2014   5:55 PM

INPUT INSTRUCTIONS

  TITLE: chaiken, negative exponential with halflife and arc;
  DATA: FILE IS schaiken2.dat; !data file in ascii format
  VARIABLE: NAMES ARE v1-v12 q1-q12; !all 24 variables
    !v1-v12: verbal skill acquisition repeated measures
    !q1-q12: quantitative skill acquisition repeated measures
  ANALYSIS: ESTIMATOR IS ML; !use maximum likelihood estimation
  MODEL:

  !verbal
  [v1-v12*](tv1-tv12); !mean trend for verbal
  v1-v12*1.97(vv1); !residual variances constrained equal
  v1-v11 PWITH v2-v12*(dv1); !lag-1 error covariance
  v1-v10 PWITH v3-v12*(dv2); !lag-2, etc...
  v1-v9 PWITH v4-v12*(dv3);
  v1-v8 PWITH v5-v12*(dv4);
  v1-v7 PWITH v6-v12*(dv5);
  v1-v6 PWITH v7-v12*(dv6);
  v1-v5 PWITH v8-v12*(dv7);
  v1-v4 PWITH v9-v12*(dv8);
  v1-v3 PWITH v10-v12*(dv9);
  v1-v2 PWITH v11-v12*(dv10);
  v1 WITH v12*(dv11);
  [av@0 bv@0 cv@0]; !factor means constrained to zero
  av*1.6; bv*92; cv*10; !random coefficient variances
  av WITH bv*3.7 cv*; bv WITH cv*; !random coefficient covariances
  av BY v1*(av1); av BY v2-v12(av2-av12); !loadings for 1st factor
  bv BY v1*(bv1); bv BY v2-v12(bv2-bv12); !loadings for 2nd factor
  cv BY v1*(cv1); cv BY v2-v12(cv2-cv12); !loadings for 3rd factor

  !quantitative
  [q1-q12*](tq1-tq12); !mean trend for quantitative
  q1-q12*1.21(vq1); !residual variances constrained equal
  q1-q11 PWITH q2-q12*(dq1); !lag-1 error covariance
  q1-q10 PWITH q3-q12*(dq2); !lag-2, etc...
  q1-q9 PWITH q4-q12*(dq3);
  q1-q8 PWITH q5-q12*(dq4);
  q1-q7 PWITH q6-q12*(dq5);
  q1-q6 PWITH q7-q12*(dq6);
  q1-q5 PWITH q8-q12*(dq7);
```

```
q1-q4 PWITH q9-q12*(dq8);
q1-q3 PWITH q10-q12*(dq9);
q1-q2 PWITH q11-q12*(dq10);
q1 WITH q12*(dq11);
[aq@0 bq@0 cq@0]; !factor means constrained to zero
aq*4; bq*35; cq*10; !random coefficient variances
aq WITH bq*7.4 cq*; bq WITH cq; !random coefficient covariances
aq BY q1*(aq1); aq BY q2-q12(aq2-aq12); !loadings for 1st factor
bq BY q1*(bq1); bq BY q2-q12(bq2-bq12); !loadings for 2nd factor
cq BY q1*(cq1); cq BY q2-q12(cq2-cq12); !loadings for 3rd factor

!cross-domain covariances
av WITH aq* bq* cq*; bv WITH aq* bq* cq*; cv WITH aq* bq* cq*;

MODEL CONSTRAINT:

!starts for parameters of the target function and for lagged error covariances
NEW(mav*1.7 mbv*-1.5 mcv*1.96 maq*4.2 mbq*-.9 mcq*1.99 rhov*.27 rhoq*.3 diff*0);

!error covariances
dv1= vv1*rhov;    dq1= vq1*rhoq;
dv2= vv1*rhov^2;  dq2= vq1*rhoq^2;
dv3= vv1*rhov^3;  dq3= vq1*rhoq^3;
dv4= vv1*rhov^4;  dq4= vq1*rhoq^4;
dv5= vv1*rhov^5;  dq5= vq1*rhoq^5;
dv6= vv1*rhov^6;  dq6= vq1*rhoq^6;
dv7= vv1*rhov^7;  dq7= vq1*rhoq^7;
dv8= vv1*rhov^8;  dq8= vq1*rhoq^8;
dv9= vv1*rhov^9;  dq9= vq1*rhoq^9;
dv10=vv1*rhov^10; dq10=vq1*rhoq^10;
dv11=vv1*rhov^11; dq11=vq1*rhoq^11;

!verbal intercepts constrained equal to target function
tv1 =mav+(mbv*11*(.5^(( 1-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv2 =mav+(mbv*11*(.5^(( 2-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv3 =mav+(mbv*11*(.5^(( 3-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv4 =mav+(mbv*11*(.5^(( 4-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv5 =mav+(mbv*11*(.5^(( 5-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv6 =mav+(mbv*11*(.5^(( 6-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv7 =mav+(mbv*11*(.5^(( 7-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv8 =mav+(mbv*11*(.5^(( 8-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv9 =mav+(mbv*11*(.5^(( 9-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv10=mav+(mbv*11*(.5^((10-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv11=mav+(mbv*11*(.5^((11-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv12=mav+(mbv*11*(.5^((12-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);

!loadings on 1st factor, constrained to d(y)/d(mav) and d(y)/d(maq)
0=1-av1 ; 0=1-aq1 ;
0=1-av2 ; 0=1-aq2 ;
0=1-av3 ; 0=1-aq3 ;
0=1-av4 ; 0=1-aq4 ;
0=1-av5 ; 0=1-aq5 ;
0=1-av6 ; 0=1-aq6 ;
0=1-av7 ; 0=1-aq7 ;
0=1-av8 ; 0=1-aq8 ;
0=1-av9 ; 0=1-aq9 ;
0=1-av10; 0=1-aq10;
0=1-av11; 0=1-aq11;
0=1-av12; 0=1-aq12;

!loadings on 2nd factor for verbal, constrained to d(y)/d(mbv)
bv1 =(11*(.5^(( 1-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv2 =(11*(.5^(( 2-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
```

```
bv3 =(11*(.5^(( 3-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv4 =(11*(.5^(( 4-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv5 =(11*(.5^(( 5-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv6 =(11*(.5^(( 6-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv7 =(11*(.5^(( 7-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv8 =(11*(.5^(( 8-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv9 =(11*(.5^(( 9-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv10=(11*(.5^((10-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv11=(11*(.5^((11-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv12=(11*(.5^((12-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);

!loadings on 3rd factor for verbal, constrained to d(y)/d(mcv)
cv1 =(.5^( 1/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 1-1)+(.5^(12/(mcv-1)))*(12- 1))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv2 =(.5^( 2/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 2-1)+(.5^(12/(mcv-1)))*(12- 2))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv3 =(.5^( 3/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 3-1)+(.5^(12/(mcv-1)))*(12- 3))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv4 =(.5^( 4/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 4-1)+(.5^(12/(mcv-1)))*(12- 4))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv5 =(.5^( 5/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 5-1)+(.5^(12/(mcv-1)))*(12- 5))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv6 =(.5^( 6/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 6-1)+(.5^(12/(mcv-1)))*(12- 6))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv7 =(.5^( 7/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 7-1)+(.5^(12/(mcv-1)))*(12- 7))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv8 =(.5^( 8/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 8-1)+(.5^(12/(mcv-1)))*(12- 8))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv9 =(.5^( 9/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 9-1)+(.5^(12/(mcv-1)))*(12- 9))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv10=(.5^(10/(mcv-1))*mcv*((.5^(1/(mcv-1)))*(10-1)+(.5^(12/(mcv-1)))*(12-10))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv11=(.5^(11/(mcv-1))*mcv*((.5^(1/(mcv-1)))*(11-1)+(.5^(12/(mcv-1)))*(12-11))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv12=(.5^(12/(mcv-1))*mcv*((.5^(1/(mcv-1)))*(12-1)+(.5^(12/(mcv-1)))*(12-12))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));

!quantitative intercepts constrained equal to target function
tq1 =maq+(mbq*11*(.5^(( 1-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq2 =maq+(mbq*11*(.5^(( 2-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq3 =maq+(mbq*11*(.5^(( 3-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq4 =maq+(mbq*11*(.5^(( 4-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq5 =maq+(mbq*11*(.5^(( 5-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq6 =maq+(mbq*11*(.5^(( 6-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq7 =maq+(mbq*11*(.5^(( 7-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq8 =maq+(mbq*11*(.5^(( 8-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq9 =maq+(mbq*11*(.5^(( 9-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq10=maq+(mbq*11*(.5^((10-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq11=maq+(mbq*11*(.5^((11-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
tq12=maq+(mbq*11*(.5^((12-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);

!loadings on 2nd factor for quantitative, constrained to d(y)/d(mbv)
bq1 =(11*(.5^(( 1-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq2 =(11*(.5^(( 2-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq3 =(11*(.5^(( 3-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq4 =(11*(.5^(( 4-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq5 =(11*(.5^(( 5-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq6 =(11*(.5^(( 6-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq7 =(11*(.5^(( 7-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq8 =(11*(.5^(( 8-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq9 =(11*(.5^(( 9-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq10=(11*(.5^((10-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
bq11=(11*(.5^((11-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);
```

```
  bq12=(11*(.5^((12-1)/(mcq-1))))/((.5^(11/(mcq-1)))-1);

  !loadings on 3rd factor for quantitative, constrained to d(y)/d(mcv)
  cq1 =(.5^( 1/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 1-1)+(.5^(12/(mcq-1)))*(12- 1))*11*ln(.5))/
  (((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
  cq2 =(.5^( 2/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 2-1)+(.5^(12/(mcq-1)))*(12- 2))*11*ln(.5))/
  (((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
  cq3 =(.5^( 3/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 3-1)+(.5^(12/(mcq-1)))*(12- 3))*11*ln(.5))/
  (((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
  cq4 =(.5^( 4/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 4-1)+(.5^(12/(mcq-1)))*(12- 4))*11*ln(.5))/
  (((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
  cq5 =(.5^( 5/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 5-1)+(.5^(12/(mcq-1)))*(12- 5))*11*ln(.5))/
  (((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
  cq6 =(.5^( 6/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 6-1)+(.5^(12/(mcq-1)))*(12- 6))*11*ln(.5))/
  (((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
  cq7 =(.5^( 7/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 7-1)+(.5^(12/(mcq-1)))*(12- 7))*11*ln(.5))/
  (((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
  cq8 =(.5^( 8/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 8-1)+(.5^(12/(mcq-1)))*(12- 8))*11*ln(.5))/
  (((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
  cq9 =(.5^( 9/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 9-1)+(.5^(12/(mcq-1)))*(12- 9))*11*ln(.5))/
  (((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
  cq10=(.5^(10/(mcq-1))*mcq*((.5^(1/(mcq-1)))*(10-1)+(.5^(12/(mcq-1)))*(12-10))*11*ln(.5))/
  (((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
  cq11=(.5^(11/(mcq-1))*mcq*((.5^(1/(mcq-1)))*(11-1)+(.5^(12/(mcq-1)))*(12-11))*11*ln(.5))/
  (((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
  cq12=(.5^(12/(mcq-1))*mcq*((.5^(1/(mcq-1)))*(12-1)+(.5^(12/(mcq-1)))*(12-12))*11*ln(.5))/
  (((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));

  diff=mcv-mcq; !difference in the ARC parameters for verbal and quantitative

  OUTPUT: TECH1 TECH4;

INPUT READING TERMINATED NORMALLY

chaiken, negative exponential with halflife and arc;

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                         228

Number of dependent variables                                   24
Number of independent variables                                  0
Number of continuous latent variables                            6

Observed dependent variables

  Continuous
   V1          V2          V3          V4          V5          V6
   V7          V8          V9          V10         V11         V12
   Q1          Q2          Q3          Q4          Q5          Q6
   Q7          Q8          Q9          Q10         Q11         Q12

Continuous latent variables
   AV          BV          CV          AQ          BQ          CQ

Estimator                                                       ML
Information matrix                                        OBSERVED
Maximum number of iterations                                  1000
Convergence criterion                                    0.500D-04
Maximum number of steepest descent iterations                   20

Input data file(s)
```

schaiken2.dat

Input data format FREE

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters 31

Loglikelihood

H0 Value	-10314.946
H1 Value	-9097.350

Information Criteria

Akaike (AIC)	20691.891
Bayesian (BIC)	20798.201
Sample-Size Adjusted BIC (n* = (n + 2) / 24)	20699.952

Chi-Square Test of Model Fit

Value	2435.191
Degrees of Freedom	293
P-Value	0.0000

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.179	
90 Percent C.I.	0.173	0.186
Probability RMSEA <= .05	0.000	

CFI/TLI

CFI	0.730
TLI	0.745

Chi-Square Test of Model Fit for the Baseline Model

Value	8204.815
Degrees of Freedom	276
P-Value	0.0000

SRMR (Standardized Root Mean Square Residual)

Value	0.218

MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
AV	BY				
V1		1.000	0.000	999.000	999.000
V2		1.000	0.000	999.000	999.000
V3		1.000	0.000	999.000	999.000
V4		1.000	0.000	999.000	999.000
V5		1.000	0.000	999.000	999.000
V6		1.000	0.000	999.000	999.000
V7		1.000	0.000	999.000	999.000
V8		1.000	0.000	999.000	999.000

V9	1.000	0.000	999.000	999.000
V10	1.000	0.000	999.000	999.000
V11	1.000	0.000	999.000	999.000
V12	1.000	0.000	999.000	999.000
BV BY				
V1	-11.004	0.001	*********	0.000
V2	-5.352	0.105	-50.919	0.000
V3	-2.603	0.102	-25.510	0.000
V4	-1.266	0.074	-17.018	0.000
V5	-0.616	0.048	-12.768	0.000
V6	-0.300	0.029	-10.216	0.000
V7	-0.146	0.017	-8.515	0.000
V8	-0.071	0.010	-7.299	0.000
V9	-0.034	0.005	-6.387	0.000
V10	-0.017	0.003	-5.678	0.000
V11	-0.008	0.002	-5.110	0.000
V12	-0.004	0.001	-4.646	0.000
CV BY				
V1	-0.064	0.011	-5.731	0.000
V2	-7.906	0.161	-49.135	0.000
V3	-7.676	0.009	-822.989	0.000
V4	-5.596	0.102	-54.932	0.000
V5	-3.628	0.137	-26.529	0.000
V6	-2.206	0.126	-17.478	0.000
V7	-1.287	0.099	-13.030	0.000
V8	-0.730	0.070	-10.386	0.000
V9	-0.406	0.047	-8.633	0.000
V10	-0.222	0.030	-7.386	0.000
V11	-0.120	0.019	-6.454	0.000
V12	-0.064	0.011	-5.731	0.000
AQ BY				
Q1	1.000	0.000	999.000	999.000
Q2	1.000	0.000	999.000	999.000
Q3	1.000	0.000	999.000	999.000
Q4	1.000	0.000	999.000	999.000
Q5	1.000	0.000	999.000	999.000
Q6	1.000	0.000	999.000	999.000
Q7	1.000	0.000	999.000	999.000
Q8	1.000	0.000	999.000	999.000
Q9	1.000	0.000	999.000	999.000
Q10	1.000	0.000	999.000	999.000
Q11	1.000	0.000	999.000	999.000
Q12	1.000	0.000	999.000	999.000
BQ BY				
Q1	-11.005	0.001	-8371.135	0.000
Q2	-5.476	0.129	-42.427	0.000
Q3	-2.725	0.128	-21.267	0.000
Q4	-1.356	0.096	-14.190	0.000
Q5	-0.675	0.063	-10.647	0.000
Q6	-0.336	0.039	-8.520	0.000
Q7	-0.167	0.024	-7.101	0.000
Q8	-0.083	0.014	-6.087	0.000
Q9	-0.041	0.008	-5.327	0.000
Q10	-0.021	0.004	-4.735	0.000
Q11	-0.010	0.002	-4.262	0.000
Q12	-0.005	0.001	-3.875	0.000
CQ BY				
Q1	-0.079	0.016	-4.809	0.000

	Q2	-7.717	0.196	-39.405	0.000	
	Q3	-7.661	0.020	-387.668	0.000	
	Q4	-5.713	0.118	-48.419	0.000	
	Q5	-3.788	0.167	-22.730	0.000	
	Q6	-2.356	0.159	-14.841	0.000	
	Q7	-1.406	0.128	-11.015	0.000	
	Q8	-0.816	0.093	-8.756	0.000	
	Q9	-0.464	0.064	-7.266	0.000	
	Q10	-0.260	0.042	-6.209	0.000	
	Q11	-0.144	0.026	-5.420	0.000	
	Q12	-0.079	0.016	-4.809	0.000	
AV	WITH					!COVARIANCES OF VERBAL (V) AND
	BV	-0.193	0.084	-2.303	0.021	!QUANTITATIVE (Q) RANDOM
	CV	-0.131	0.051	-2.592	0.010	!COEFFICIENTS
	AQ	1.664	0.235	7.097	0.000	!
	BQ	-0.035	0.046	-0.765	0.444	!
	CQ	-0.037	0.025	-1.494	0.135	!
						!
BV	WITH					!
	CV	0.025	0.030	0.834	0.404	!
	AQ	-0.597	0.126	-4.718	0.000	!
	BQ	0.122	0.028	4.403	0.000	!
	CQ	-0.021	0.014	-1.434	0.152	!
						!
AQ	WITH					!
	BQ	-0.293	0.068	-4.288	0.000	!
	CQ	-0.064	0.036	-1.780	0.075	!
						!
BQ	WITH					!
	CQ	-0.031	0.008	-3.696	0.000	!
						!
CV	WITH					!
	AQ	-0.183	0.071	-2.558	0.011	!
	BQ	0.015	0.016	0.966	0.334	!
	CQ	0.016	0.008	2.008	0.045	!
V1	WITH					!AUTOREGRESSIVE RESIDUAL
	V2	0.544	0.092	5.905	0.000	!COVARIANCE STRUCTURE FOR
	V3	0.150	0.045	3.337	0.001	!VERBAL REPEATED MEASURES
	V4	0.041	0.018	2.321	0.020	!
	V5	0.011	0.006	1.779	0.075	!
	V6	0.003	0.002	1.442	0.149	!
	V7	0.001	0.001	1.212	0.226	!
	V8	0.000	0.000	1.045	0.296	!
	V9	0.000	0.000	0.919	0.358	!
	V10	0.000	0.000	0.820	0.412	!
	V11	0.000	0.000	0.740	0.459	!
	V12	0.000	0.000	0.674	0.500	!
						!
V2	WITH					!
	V3	0.544	0.092	5.905	0.000	!
	V4	0.150	0.045	3.337	0.001	!
	V5	0.041	0.018	2.321	0.020	!
	V6	0.011	0.006	1.779	0.075	!
	V7	0.003	0.002	1.442	0.149	!
	V8	0.001	0.001	1.212	0.226	!
	V9	0.000	0.000	1.045	0.296	!
	V10	0.000	0.000	0.919	0.358	!
	V11	0.000	0.000	0.820	0.412	!
	V12	0.000	0.000	0.740	0.459	!
						!
V3	WITH					!

	V4		0.544	0.092	5.905	0.000 !
	V5		0.150	0.045	3.337	0.001 !
	V6		0.041	0.018	2.321	0.020 !
	V7		0.011	0.006	1.779	0.075 !
	V8		0.003	0.002	1.442	0.149 !
	V9		0.001	0.001	1.212	0.226 !
	V10		0.000	0.000	1.045	0.296 !
	V11		0.000	0.000	0.919	0.358 !
	V12		0.000	0.000	0.820	0.412 !
						!
V4		WITH				!
	V5		0.544	0.092	5.905	0.000 !
	V6		0.150	0.045	3.337	0.001 !
	V7		0.041	0.018	2.321	0.020 !
	V8		0.011	0.006	1.779	0.075 !
	V9		0.003	0.002	1.442	0.149 !
	V10		0.001	0.001	1.212	0.226 !
	V11		0.000	0.000	1.045	0.296 !
	V12		0.000	0.000	0.919	0.358 !
						!
V5		WITH				!
	V6		0.544	0.092	5.905	0.000 !
	V7		0.150	0.045	3.337	0.001 !
	V8		0.041	0.018	2.321	0.020 !
	V9		0.011	0.006	1.779	0.075 !
	V10		0.003	0.002	1.442	0.149 !
	V11		0.001	0.001	1.212	0.226 !
	V12		0.000	0.000	1.045	0.296 !
						!
V6		WITH				!
	V7		0.544	0.092	5.905	0.000 !
	V8		0.150	0.045	3.337	0.001 !
	V9		0.041	0.018	2.321	0.020 !
	V10		0.011	0.006	1.779	0.075 !
	V11		0.003	0.002	1.442	0.149 !
	V12		0.001	0.001	1.212	0.226 !
						!
V7		WITH				!
	V8		0.544	0.092	5.905	0.000 !
	V9		0.150	0.045	3.337	0.001 !
	V10		0.041	0.018	2.321	0.020 !
	V11		0.011	0.006	1.779	0.075 !
	V12		0.003	0.002	1.442	0.149 !
						!
V8		WITH				!
	V9		0.544	0.092	5.905	0.000 !
	V10		0.150	0.045	3.337	0.001 !
	V11		0.041	0.018	2.321	0.020 !
	V12		0.011	0.006	1.779	0.075 !
						!
V9		WITH				!
	V10		0.544	0.092	5.905	0.000 !
	V11		0.150	0.045	3.337	0.001 !
	V12		0.041	0.018	2.321	0.020 !
						!
V10		WITH				!
	V11		0.544	0.092	5.905	0.000 !
	V12		0.150	0.045	3.337	0.001 !
						!
V11		WITH				!
	V12		0.544	0.092	5.905	0.000 !
Q1		WITH				!AUTOREGRESSIVE RESIDUAL

	Q2	0.345	0.053	6.528	0.000	!COVARIANCE STRUCTURE FOR
	Q3	0.107	0.028	3.787	0.000	!QUANTITATIVE REPEATED MEASURES
	Q4	0.033	0.012	2.662	0.008	!
	Q5	0.010	0.005	2.051	0.040	!
	Q6	0.003	0.002	1.668	0.095	!
	Q7	0.001	0.001	1.405	0.160	!
	Q8	0.000	0.000	1.214	0.225	!
	Q9	0.000	0.000	1.068	0.285	!
	Q10	0.000	0.000	0.954	0.340	!
	Q11	0.000	0.000	0.862	0.389	!
	Q12	0.000	0.000	0.786	0.432	!
						!
Q2	WITH					!
	Q3	0.345	0.053	6.528	0.000	!
	Q4	0.107	0.028	3.787	0.000	!
	Q5	0.033	0.012	2.662	0.008	!
	Q6	0.010	0.005	2.051	0.040	!
	Q7	0.003	0.002	1.668	0.095	!
	Q8	0.001	0.001	1.405	0.160	!
	Q9	0.000	0.000	1.214	0.225	!
	Q10	0.000	0.000	1.068	0.285	!
	Q11	0.000	0.000	0.954	0.340	!
	Q12	0.000	0.000	0.862	0.389	!
						!
Q3	WITH					!
	Q4	0.345	0.053	6.528	0.000	!
	Q5	0.107	0.028	3.787	0.000	!
	Q6	0.033	0.012	2.662	0.008	!
	Q7	0.010	0.005	2.051	0.040	!
	Q8	0.003	0.002	1.668	0.095	!
	Q9	0.001	0.001	1.405	0.160	!
	Q10	0.000	0.000	1.214	0.225	!
	Q11	0.000	0.000	1.068	0.285	!
	Q12	0.000	0.000	0.954	0.340	!
						!
Q4	WITH					!
	Q5	0.345	0.053	6.528	0.000	!
	Q6	0.107	0.028	3.787	0.000	!
	Q7	0.033	0.012	2.662	0.008	!
	Q8	0.010	0.005	2.051	0.040	!
	Q9	0.003	0.002	1.668	0.095	!
	Q10	0.001	0.001	1.405	0.160	!
	Q11	0.000	0.000	1.214	0.225	!
	Q12	0.000	0.000	1.068	0.285	!
						!
Q5	WITH					!
	Q6	0.345	0.053	6.528	0.000	!
	Q7	0.107	0.028	3.787	0.000	!
	Q8	0.033	0.012	2.662	0.008	!
	Q9	0.010	0.005	2.051	0.040	!
	Q10	0.003	0.002	1.668	0.095	!
	Q11	0.001	0.001	1.405	0.160	!
	Q12	0.000	0.000	1.214	0.225	!
						!
Q6	WITH					!
	Q7	0.345	0.053	6.528	0.000	!
	Q8	0.107	0.028	3.787	0.000	!
	Q9	0.033	0.012	2.662	0.008	!
	Q10	0.010	0.005	2.051	0.040	!
	Q11	0.003	0.002	1.668	0.095	!
	Q12	0.001	0.001	1.405	0.160	!
						!
Q7	WITH					!

Q8	0.345	0.053	6.528	0.000	!
Q9	0.107	0.028	3.787	0.000	!
Q10	0.033	0.012	2.662	0.008	!
Q11	0.010	0.005	2.051	0.040	!
Q12	0.003	0.002	1.668	0.095	!
					!
Q8 WITH					!
Q9	0.345	0.053	6.528	0.000	!
Q10	0.107	0.028	3.787	0.000	!
Q11	0.033	0.012	2.662	0.008	!
Q12	0.010	0.005	2.051	0.040	!
					!
Q9 WITH					!
Q10	0.345	0.053	6.528	0.000	!
Q11	0.107	0.028	3.787	0.000	!
Q12	0.033	0.012	2.662	0.008	!
					!
Q10 WITH					!
Q11	0.345	0.053	6.528	0.000	!
Q12	0.107	0.028	3.787	0.000	!
					!
Q11 WITH					!
Q12	0.345	0.053	6.528	0.000	!
Means					
AV	0.000	0.000	999.000	999.000	!FACTOR MEANS FIXED = 0
BV	0.000	0.000	999.000	999.000	!
CV	0.000	0.000	999.000	999.000	!
AQ	0.000	0.000	999.000	999.000	!
BQ	0.000	0.000	999.000	999.000	!
CQ	0.000	0.000	999.000	999.000	!
Intercepts					
V1	21.278	0.661	32.192	0.000	
V2	13.865	0.374	37.074	0.000	
V3	10.260	0.238	43.076	0.000	
V4	8.506	0.164	51.793	0.000	
V5	7.653	0.126	60.759	0.000	
V6	7.238	0.109	66.405	0.000	
V7	7.036	0.103	68.566	0.000	
V8	6.938	0.101	69.035	0.000	
V9	6.890	0.100	68.996	0.000	
V10	6.867	0.100	68.879	0.000	
V11	6.856	0.100	68.788	0.000	
V12	6.850	0.100	68.732	0.000	
Q1	16.492	0.404	40.779	0.000	
Q2	12.541	0.261	47.963	0.000	
Q3	10.576	0.202	52.297	0.000	
Q4	9.597	0.171	56.084	0.000	
Q5	9.111	0.155	58.658	0.000	
Q6	8.868	0.148	59.868	0.000	
Q7	8.748	0.145	60.270	0.000	
Q8	8.688	0.144	60.347	0.000	
Q9	8.658	0.144	60.330	0.000	
Q10	8.643	0.143	60.299	0.000	
Q11	8.636	0.143	60.275	0.000	
Q12	8.632	0.143	60.261	0.000	
Variances					
AV	1.652	0.207	7.981	0.000	!VARIANCES OF RANDOM
BV	0.712	0.069	10.383	0.000	!COEFFICIENTS
CV	0.216	0.023	9.563	0.000	!
AQ	4.187	0.423	9.908	0.000	!

```
    BQ                 0.208      0.021     10.104      0.000 !
    CQ                 0.046      0.006      7.922      0.000 !

 Residual Variances
    V1                 1.974      0.092     21.355      0.000 !VERBAL RESIDUAL VARIANCE
    V2                 1.974      0.092     21.355      0.000 !HELD EQUAL OVER TIME
    V3                 1.974      0.092     21.355      0.000 !
    V4                 1.974      0.092     21.355      0.000 !
    V5                 1.974      0.092     21.355      0.000 !
    V6                 1.974      0.092     21.355      0.000 !
    V7                 1.974      0.092     21.355      0.000 !
    V8                 1.974      0.092     21.355      0.000 !
    V9                 1.974      0.092     21.355      0.000 !
    V10                1.974      0.092     21.355      0.000 !
    V11                1.974      0.092     21.355      0.000 !
    V12                1.974      0.092     21.355      0.000 !
    Q1                 1.112      0.055     20.403      0.000 !QUANTITATIVE RESIDUAL VARIANCE
    Q2                 1.112      0.055     20.403      0.000 !HELD EQUAL OVER TIME
    Q3                 1.112      0.055     20.403      0.000 !
    Q4                 1.112      0.055     20.403      0.000 !
    Q5                 1.112      0.055     20.403      0.000 !
    Q6                 1.112      0.055     20.403      0.000 !
    Q7                 1.112      0.055     20.403      0.000 !
    Q8                 1.112      0.055     20.403      0.000 !
    Q9                 1.112      0.055     20.403      0.000 !
    Q10                1.112      0.055     20.403      0.000 !
    Q11                1.112      0.055     20.403      0.000 !
    Q12                1.112      0.055     20.403      0.000 !

New/Additional Parameters
    MAV                6.845      0.100     68.666      0.000 !VERBAL MEAN ASYMPTOTE
    MBV               -1.312      0.058    -22.498      0.000 !VERBAL MEAN ARC PARAMETER
    MCV                1.962      0.026     75.155      0.000 !VERBAL MEAN HALF-LIFE
    MAQ                8.629      0.143     60.242      0.000 !QUANT. MEAN ASYMPTOTE
    MBQ               -0.715      0.031    -23.159      0.000 !QUANT. MEAN ARC PARAMETER
    MCQ                1.993      0.033     59.738      0.000 !QUANT. MEAN HALF-LIFE
    RHOV               0.276      0.036      7.597      0.000 !AUTOREGRESSIVE PARAMETER (V)
    RHOQ               0.310      0.035      8.910      0.000 !AUTOREGRESSIVE PARAMETER (Q)
    DIFF              -0.031      0.042     -0.749      0.454 !DIFFERENCE IN HALF-LIFES

QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.214E-04
       (ratio of smallest to largest eigenvalue)
```

14. Annotated output: quadratic function fit to Kanfer & Ackerman's data (traditional)

```
Mplus VERSION 7.2
MUTHEN & MUTHEN
06/10/2014  11:00 PM

INPUT INSTRUCTIONS

  TITLE: Cudeck and du Toit (2002) quadratic curve;
  DATA: FILE IS kanfer.dat; !data file in ascii format
  TYPE IS COVARIANCE MEANS; !input data are covariances and means
  NOBSERVATIONS IS 140; !sample size is 140
  VARIABLE: NAMES ARE y1-y9; !all 9 variables
  ANALYSIS: ESTIMATOR=ML; !use maximum likelihood estimation
  ITERATIONS=5000; !use 5000 iterations
  MODEL: y1-y9*9.9(v); [y1-y9@0]; !residual covariances constrained equal, intercepts = 0
  [a0*16 ay*39](ma0 may); [ax@0]; !intercept and maximum means estimated, maximizer = 0
```

```
  a0*123; ax*18; ay*53; !random coefficient variances
  a0 WITH ax*-.8 ay*35; ax WITH ay*8.9; !random coefficient covariances
  a0 BY y1-y9*(a01-a09); ax BY y1-y9*(ax1-ax9); ay BY y1-y9*(ay1-ay9); !loadings

  MODEL CONSTRAINT: NEW(max*8.8); !define maximizer mean

  !loadings on maximum, intercept, and maximizer factors, constrained to
  !d(y)/d(may), d(y)/d(ma0), and d(y)/d(max)
  ay1=1-((0/max)-1)^2; a01=((0/max)-1)^2; ax1=2*(may-ma0)*(0-max)*0/max^3;
  ay2=1-((1/max)-1)^2; a02=((1/max)-1)^2; ax2=2*(may-ma0)*(1-max)*1/max^3;
  ay3=1-((2/max)-1)^2; a03=((2/max)-1)^2; ax3=2*(may-ma0)*(2-max)*2/max^3;
  ay4=1-((3/max)-1)^2; a04=((3/max)-1)^2; ax4=2*(may-ma0)*(3-max)*3/max^3;
  ay5=1-((4/max)-1)^2; a05=((4/max)-1)^2; ax5=2*(may-ma0)*(4-max)*4/max^3;
  ay6=1-((5/max)-1)^2; a06=((5/max)-1)^2; ax6=2*(may-ma0)*(5-max)*5/max^3;
  ay7=1-((6/max)-1)^2; a07=((6/max)-1)^2; ax7=2*(may-ma0)*(6-max)*6/max^3;
  ay8=1-((7/max)-1)^2; a08=((7/max)-1)^2; ax8=2*(may-ma0)*(7-max)*7/max^3;
  ay9=1-((8/max)-1)^2; a09=((8/max)-1)^2; ax9=2*(may-ma0)*(8-max)*8/max^3;

  OUTPUT: TECH1 TECH4;

INPUT READING TERMINATED NORMALLY

Cudeck and du Toit (2002) quadratic curve;

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                         140

Number of dependent variables                                    9
Number of independent variables                                  0
Number of continuous latent variables                            3

Observed dependent variables

  Continuous
   Y1          Y2          Y3          Y4          Y5          Y6
   Y7          Y8          Y9

Continuous latent variables
   A0          AX          AY

Estimator                                                       ML
Information matrix                                      EXPECTED
Maximum number of iterations                                5000
Convergence criterion                                  0.500D-04
Maximum number of steepest descent iterations                 20

Input data file(s)
  kanfer.dat

Input data format  FREE

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters                       10

Loglikelihood

          H0 Value                       -3743.236
          H1 Value                       -3645.210
```

Information Criteria

Akaike (AIC)	7506.473
Bayesian (BIC)	7535.889
Sample-Size Adjusted BIC (n* = (n + 2) / 24)	7504.251

Chi-Square Test of Model Fit

Value	196.054
Degrees of Freedom	44
P-Value	0.0000

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.157	
90 Percent C.I.	0.135	0.180
Probability RMSEA <= .05	0.000	

CFI/TLI

CFI	0.909
TLI	0.925

Chi-Square Test of Model Fit for the Baseline Model

Value	1702.962
Degrees of Freedom	36
P-Value	0.0000

SRMR (Standardized Root Mean Square Residual)

Value	0.076

MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
A0	BY				
Y1		1.000	0.000	999.000	999.000
Y2		0.760	0.012	64.221	0.000
Y3		0.552	0.020	27.380	0.000
Y4		0.378	0.025	15.099	0.000
Y5		0.237	0.026	8.959	0.000
Y6		0.128	0.024	5.275	0.000
Y7		0.053	0.019	2.819	0.005
Y8		0.010	0.010	1.064	0.287
Y9		0.001	0.003	0.252	0.801
AX	BY				
Y1		0.000	0.000	999.000	999.000
Y2		-0.519	0.046	11.153	0.000
Y3		-0.884	0.071	12.430	0.000
Y4		-1.097	0.075	14.701	0.000
Y5		-1.157	0.060	19.389	0.000
Y6		-1.065	0.044	24.352	0.000
Y7		-0.819	0.081	10.172	0.000
Y8		-0.421	0.162	2.603	0.009
Y9		0.130	0.270	-0.482	0.630
AY	BY				

```
    Y1                 0.000      0.000    999.000    999.000
    Y2                 0.240      0.012     20.318      0.000
    Y3                 0.448      0.020     22.194      0.000
    Y4                 0.622      0.025     24.853      0.000
    Y5                 0.763      0.026     28.917      0.000
    Y6                 0.872      0.024     35.897      0.000
    Y7                 0.947      0.019     50.687      0.000
    Y8                 0.990      0.010    103.048      0.000
    Y9                 0.999      0.003    336.999      0.000

 A0       WITH                                                 !COVARIANCES OF RANDOM
    AX                -8.248      4.168     -1.979      0.048 !COEFFICIENTS
    AY                34.623      7.489      4.623      0.000 !
                                                               !
 AX       WITH                                                 !
    AY                 8.864      6.611      1.341      0.180 !

 Means
    A0                20.732      0.846     24.510      0.000 !MEAN INTERCEPT (ESTIMATED)
    AX                 0.000      0.000    999.000    999.000 !MEAN MAXIMIZER (FIXED = 0)
    AY                38.772      0.655     59.185      0.000 !MEAN MAXIMUM (ESTIMATED)

 Intercepts
    Y1                 0.000      0.000    999.000    999.000 !REPEATED MEASURE INTERCEPTS
    Y2                 0.000      0.000    999.000    999.000 !ALL FIXED = 0
    Y3                 0.000      0.000    999.000    999.000 !
    Y4                 0.000      0.000    999.000    999.000 !
    Y5                 0.000      0.000    999.000    999.000 !
    Y6                 0.000      0.000    999.000    999.000 !
    Y7                 0.000      0.000    999.000    999.000 !
    Y8                 0.000      0.000    999.000    999.000 !
    Y9                 0.000      0.000    999.000    999.000 !

 Variances
    A0                93.610     11.976      7.816      0.000 !VARIANCE OF INTERCEPT FACTOR
    AX                18.019      6.969      2.585      0.010 !VARIANCE OF MAXIMIZER FACTOR
    AY                54.708      8.395      6.517      0.000 !VARIANCE OF MAXIMUM FACTOR

 Residual Variances
    Y1                 9.921      0.484     20.494      0.000 !RESIDUAL VARIANCE HELD EQUAL
    Y2                 9.921      0.484     20.494      0.000 !OVER TIME
    Y3                 9.921      0.484     20.494      0.000 !
    Y4                 9.921      0.484     20.494      0.000 !
    Y5                 9.921      0.484     20.494      0.000 !
    Y6                 9.921      0.484     20.494      0.000 !
    Y7                 9.921      0.484     20.494      0.000 !
    Y8                 9.921      0.484     20.494      0.000 !
    Y9                 9.921      0.484     20.494      0.000 !

New/Additional Parameters
    MAX                7.787      0.412     18.924      0.000 !MEAN MAXIMIZER


QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.200E-03
       (ratio of smallest to largest eigenvalue)
```

15. Annotated output: quadratic function fit to Kanfer & Ackerman's data (modified)

```
Mplus VERSION 7.2
MUTHEN & MUTHEN
```

06/10/2014 11:00 PM

INPUT INSTRUCTIONS

```
  TITLE: Cudeck and du Toit (2002) quadratic curve;
  DATA: FILE IS kanfer.dat; !data file in ascii format
  TYPE IS COVARIANCE MEANS; !input data are covariances and means
  NOBSERVATIONS IS 140; !sample size is 140
  VARIABLE: NAMES ARE y1-y9; !all 9 variables
  ANALYSIS: ESTIMATOR=ML; !use maximum likelihood estimation
  ITERATIONS=5000; !use 5000 iterations
  MODEL: [y1-y9*](my1-my9); !intercepts constrained equal to target function
  y1-y9*9.9(v); !residual covariances constrained equal
  [a0@0 ax@0 ay@0]; !factor means constrained to zero
  a0*123; ax*18; ay*53; !random coefficient variances
  a0 WITH ax*-.8 ay*35; ax WITH ay*8.9; !random coefficient covariances
  a0 BY y1-y9*(a01-a09); ax BY y1-y9*(ax1-ax9); ay BY y1-y9*(ay1-ay9); !loadings

  MODEL CONSTRAINT:
  NEW(ma0*16 max*8.8 may*39); !define mean intercept, maximizer, and maximum

  !intercepts constrained equal to target function
  my1=may-(may-ma0)*((0/max)-1)^2;
  my2=may-(may-ma0)*((1/max)-1)^2;
  my3=may-(may-ma0)*((2/max)-1)^2;
  my4=may-(may-ma0)*((3/max)-1)^2;
  my5=may-(may-ma0)*((4/max)-1)^2;
  my6=may-(may-ma0)*((5/max)-1)^2;
  my7=may-(may-ma0)*((6/max)-1)^2;
  my8=may-(may-ma0)*((7/max)-1)^2;
  my9=may-(may-ma0)*((8/max)-1)^2;

  !loadings on maximum, intercept, and maximizer factors, constrained to
  !d(y)/d(may), d(y)/d(ma0), and d(y)/d(max)
  ay1=1-((0/max)-1)^2; a01=((0/max)-1)^2; ax1=2*(may-ma0)*(0-max)*0/max^3;
  ay2=1-((1/max)-1)^2; a02=((1/max)-1)^2; ax2=2*(may-ma0)*(1-max)*1/max^3;
  ay3=1-((2/max)-1)^2; a03=((2/max)-1)^2; ax3=2*(may-ma0)*(2-max)*2/max^3;
  ay4=1-((3/max)-1)^2; a04=((3/max)-1)^2; ax4=2*(may-ma0)*(3-max)*3/max^3;
  ay5=1-((4/max)-1)^2; a05=((4/max)-1)^2; ax5=2*(may-ma0)*(4-max)*4/max^3;
  ay6=1-((5/max)-1)^2; a06=((5/max)-1)^2; ax6=2*(may-ma0)*(5-max)*5/max^3;
  ay7=1-((6/max)-1)^2; a07=((6/max)-1)^2; ax7=2*(may-ma0)*(6-max)*6/max^3;
  ay8=1-((7/max)-1)^2; a08=((7/max)-1)^2; ax8=2*(may-ma0)*(7-max)*7/max^3;
  ay9=1-((8/max)-1)^2; a09=((8/max)-1)^2; ax9=2*(may-ma0)*(8-max)*8/max^3;

  OUTPUT: TECH1 TECH4;
```

INPUT READING TERMINATED NORMALLY

Cudeck and du Toit (2002) quadratic curve;

SUMMARY OF ANALYSIS

Number of groups	1
Number of observations	140
Number of dependent variables	9
Number of independent variables	0
Number of continuous latent variables	3

Observed dependent variables

  Continuous

Y1	Y2	Y3	Y4	Y5	Y6

```
   Y7          Y8          Y9

Continuous latent variables
   A0          AX          AY

Estimator                                                      ML
Information matrix                                       EXPECTED
Maximum number of iterations                                 5000
Convergence criterion                                  0.500D-04
Maximum number of steepest descent iterations                  20

Input data file(s)
  kanfer.dat

Input data format  FREE

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters                       10

Loglikelihood

          H0 Value                       -3743.236
          H1 Value                       -3645.210

Information Criteria

          Akaike (AIC)                    7506.473
          Bayesian (BIC)                  7535.889
          Sample-Size Adjusted BIC        7504.251
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                            196.054
          Degrees of Freedom                    44
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.157
          90 Percent C.I.                    0.135  0.180
          Probability RMSEA <= .05           0.000

CFI/TLI

          CFI                                0.909
          TLI                                0.925

Chi-Square Test of Model Fit for the Baseline Model

          Value                           1702.962
          Degrees of Freedom                    36
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.076

MODEL RESULTS
```

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	
A0	BY					
Y1		1.000	0.000	999.000	999.000	
Y2		0.760	0.012	64.222	0.000	
Y3		0.552	0.020	27.380	0.000	
Y4		0.378	0.025	15.099	0.000	
Y5		0.237	0.026	8.959	0.000	
Y6		0.128	0.024	5.275	0.000	
Y7		0.053	0.019	2.819	0.005	
Y8		0.010	0.010	1.064	0.287	
Y9		0.001	0.003	0.252	0.801	
AX	BY					
Y1		0.000	0.000	999.000	999.000	
Y2		-0.519	0.046	-11.154	0.000	
Y3		-0.884	0.071	-12.430	0.000	
Y4		-1.097	0.075	-14.701	0.000	
Y5		-1.157	0.060	-19.389	0.000	
Y6		-1.065	0.044	-24.352	0.000	
Y7		-0.819	0.081	-10.172	0.000	
Y8		-0.421	0.162	-2.603	0.009	
Y9		0.130	0.270	0.482	0.630	
AY	BY					
Y1		0.000	0.000	999.000	999.000	
Y2		0.240	0.012	20.318	0.000	
Y3		0.448	0.020	22.194	0.000	
Y4		0.622	0.025	24.854	0.000	
Y5		0.763	0.026	28.918	0.000	
Y6		0.872	0.024	35.897	0.000	
Y7		0.947	0.019	50.688	0.000	
Y8		0.990	0.010	103.050	0.000	
Y9		0.999	0.003	336.998	0.000	
A0	WITH					!COVARIANCES OF RANDOM
AX		-8.248	4.168	-1.979	0.048	!COEFFICIENTS
AY		34.624	7.489	4.623	0.000	!
						!
AX	WITH					!
AY		8.864	6.610	1.341	0.180	!
Means						
A0		0.000	0.000	999.000	999.000	!FACTOR MEANS FIXED = 0
AX		0.000	0.000	999.000	999.000	!
AY		0.000	0.000	999.000	999.000	!
Intercepts						
Y1		20.732	0.846	24.510	0.000	!REPEATED MEASURE INTERCEPTS
Y2		25.067	0.764	32.809	0.000	!FIXED EQUAL TO THE TARGET
Y3		28.808	0.726	39.688	0.000	!FUNCTION
Y4		31.954	0.705	45.333	0.000	!
Y5		34.505	0.682	50.579	0.000	!
Y6		36.461	0.651	55.983	0.000	!
Y7		37.821	0.620	60.982	0.000	!
Y8		38.587	0.615	62.745	0.000	!
Y9		38.758	0.675	57.397	0.000	!
Variances						
A0		93.610	11.976	7.816	0.000	!VARIANCE OF INTERCEPT FACTOR
AX		18.019	6.969	2.585	0.010	!VARIANCE OF MAXIMIZER FACTOR
AY		54.708	8.395	6.517	0.000	!VARIANCE OF MAXIMUM FACTOR

Residual Variances					
Y1	9.921	0.484	20.494	0.000	!RESIDUAL VARIANCE HELD EQUAL
Y2	9.921	0.484	20.494	0.000	!OVER TIME
Y3	9.921	0.484	20.494	0.000	!
Y4	9.921	0.484	20.494	0.000	!
Y5	9.921	0.484	20.494	0.000	!
Y6	9.921	0.484	20.494	0.000	!
Y7	9.921	0.484	20.494	0.000	!
Y8	9.921	0.484	20.494	0.000	!
Y9	9.921	0.484	20.494	0.000	!
New/Additional Parameters					
MA0	20.732	0.846	24.510	0.000	!MEAN INTERCEPT
MAX	7.787	0.412	18.924	0.000	!MEAN MAXIMIZER
MAY	38.772	0.655	59.185	0.000	!MEAN MAXIMUM

QUALITY OF NUMERICAL RESULTS

Condition Number for the Information Matrix (ratio of smallest to largest eigenvalue)	0.647E-05