**Supplemental Materials for:**

Measurement Invariance of Big-Five Factors Over the Lifespan: ESEM Tests of Gender, Age, Plasticity, Maturity and La Dolce Vita Effects

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**1. Detailed Description of the Five-Factor Approach (FFA) Measures of Personality Used in the British Household Panel Survey (BHPS)**

In the documentation provided in the BHPS Technical Manual, (Taylor, et al., 2009, pp. a3-21 to a3-22) the FFA factors are described as follows:

*Extraversion refers to individual differences in sociability, gregariousness, level of activity, and the experience of positive affect. Agreeableness refers to individual differences in altruistic behavior, trust, warmth, and kindness. Conscientiousness refers to individual differences in self-control, task-orientation, and rule-abiding. Neuroticism refers to individual differences in the susceptibility to distress and the experience of negative emotions such as anxiety, anger, and depression. Finally, Openness to Experience refers to individual differences in the propensity for originality, creativity, and the acceptance of new ideas. The general agreement on the Big Five provides a standardized language for describing personality differences at the broadest levels and has facilitated the accumulation of knowledge concerning how personality traits are related to a broad range of life outcomes. Personality traits tend to be assessed using long questionnaires. However, recent scale-development studies have indicated that the Big Five traits can be reliably assessed with a small number of items (e.g., Gosling et al., 2003). […] Large scale nationally represented data are crucial for establishing that personality traits are in fact essential psychological constructs. Including personality in the BHPS will make it one of the best datasets in the world for study how personality traits are linked with real-world choices and reactions over time.*

**Wording of the 15 items used to measure the big-five personality items:**

I see myself as someone who . . .

*Agreeableness*

(A1R) 1 Is sometimes rude to others (reverse-scored).

(A2) 6 Has a forgiving nature.

(A3) 11 Is considerate and kind to almost everyone.

*Conscientiousness*

(C1) 2 Does a thorough job.

(C2R) 7 Tends to be lazy (reverse-scored).

(C3) 12 Does things efficiently.

*Extraversion*

(E1) 3 Is talkative.

(E2) 8 Is outgoing, sociable.

(E3R) 13 Is reserved (reverse-scored).

*Neuroticism*

(N1) 4 Worries a lot.

(N2) 9 Gets nervous easily.

(N3R) 14 Is relaxed, handles stress well (reverse-scored).

*Openness to Experience*

(O1) 5 Is original, comes up with new ideas.

(O2) 10 Values artistic, aesthetic experiences.

(O3) 15 Has an active imagination

For each item, there is a variable label (in parentheses) that identifies the big-five factor the item was designed to measure and indicates whether the item is reverse-scored (indicated by the suffix R). The number outside parentheses refers to the ordering of item in the actual British Household Panel Study.

**2. The Exploratory Structural Equation Modelling (ESEM) Approach**

 In the ESEM model (Asparouhov & Muthén, 2009; Marsh, Muthén, et al., 2009), there are p dependent variables Y = (Y1, ..., Yp) and q independent variables X = (X1, ...,Xq) and m latent variables η = (η1, ..., ηm) under the standard assumptions that the ε and ζ are normally distributed residuals with mean 0 and variance covariance matrix θ and ψ respectively. Λ is a factor loading matrix, whilst B and Γ are matrices of regression coefficients relating latent variables to each other.



 In order to estimate the parameters with maximum likelihood estimation (ML), additional constraints have to be imposed for identification. As in CFA analyses, the two typical approaches are to identify the metric of the latent variable by either fixing the variance of the latent variable to be 1.0 or by fixing one of the factor loadings for each factor typically to be 1.0.

 The ESEM approach differs from the typical CFA approach in that all factor loadings are estimated, subject to constraints so that the model can be identified. In particular, when more than one factor is posited (m > 1.0), further constrains are required to achieve an identified solution. To resolve this problem, consider any m x m square matrix (m = number of factors), a square matrix that we refer to as H. In this (mxm) square matrix H one can replace the η vector by H η in the ESEM model (1-2) which will also alter the parameters in the model as well; Λ to Λ H−1, the α vector H α, the Γ matrix to H Γ, the B matrix to HBH−1 and the Ψ matrix to HΨH**T**. Since H has m2 elements, the ESEM model has a total of m2 indeterminacies that must be resolved. Two variations of this model are considered; one where factors are orthogonal so that the factor variance-covariance matrix (Ψ) is an identity matrix, and an oblique model where Ψ is an unrestricted correlation matrix (i.e., all correlations and residual correlations between the latent variables are estimated as free parameters). This model can also be extended to include a structured variance-covariance matrix (Ψ).

 For an orthogonal matrix H (i.e., a square m x m matrix H such that HHT = I), one can replace the η vector by H η and obtain an equivalent model in which the parameters are changed. EFA can resolve this non-identification problem by minimizing f(Λ\*) = f(Λ H−1), where f is a function called the rotation criteria or simplicity function (Asparouhov & Muthén, 2009; Jennrich & Sampson, 1966), typically such that among all equivalent Λ parameters the simplest solution is obtained. There are a total of m(m−1)/2 constraints in addition to m(m + 1)/2 constraints that are directly imposed on the Ψ matrix for a total of m2 constraints needed to identify the model. The identification for the oblique model is developed similarly such that a total of m2 constraints needed to identify the model are imposed. Although the requirement for m2 constraints is only a necessary condition and in some cases it may be insufficient, in most cases the model is identified if and only if the Fisher information matrix is not singular (Silvey, 1970). This method can be used in the ESEM framework as well (Asparouhov & Muthén, 2009; also see Hayashi & Marcoulides, 2006).

 The estimation of the ESEM model consists of several steps (Asparouhov & Muthén, 2009). Initially a SEM model is estimated using the ML estimator. The factor variance covariance matrix is specified as an identity matrix (ψ = I), giving m(m + 1)/2 restrictions. The EFA loading matrix (Λ), has all entries above the main diagonal (i.e., for the first m rows and column in the upper right hand corner of factor loading matrix, Λ), fixed to 0, providing remaining m(m − 1)/2 identifying restrictions. This initial, unrotated model provides starting values that can be subsequently rotated into an EFA model with m factors. The asymptotic distribution of all parameter estimates in this starting value model is also obtained. Then the ESEM variance covariance matrix is computed (based only on Λ ΛT + θ and ignoring the remaining part of the model).

 The correlation matrix is also computed and, using the delta method (Asparouhov & Muthén, 2009), the asymptotic distribution of the correlation matrix and the standardization factors are obtained. In addition, again using the delta method, the joint asymptotic distribution of the correlation matrix, standardization factors and all remaining parameters in the model are computed and used to obtain the standardized rotated solution based on the correlation matrix and its asymptotic distribution (Asparouhov & Muthén, 2009). This method is also extended to provide the asymptotic covariance of the standardized rotated solution, standardized unrotated solution, standardization factors, and all other parameters in the model. This asymptotic covariance is then used to compute the asymptotic distribution of the optimal rotation matrix H and all unrotated parameters which is then used to compute the rotated solution for the model and its asymptotic variance covariance.

 In Mplus multiple random starting values are used in the estimation process to protect against non-convergence and local minimums in the rotation algorithms. Although a wide variety of orthogonal and oblique rotation procedures are available, leading authorities on this topic (e.g., Asparouhov & Muthén, 2009; Browne, 2001; Jennrich, 2006) have recommended Geomin rotation, but made it clear that the researchers should explore alternative solutions with different rotation strategies. In the context of the present investigation, geomin ration had desirable theoretical and statistical rationales in that it was developed specifically to represent simple structure as originally conceived of by Thurstone (1947) . Geomin rotations also incorporate a complexity parameter consistent with Thrustone’s original proposal. As operationalized in Mplus, this complexity parameter (ε) takes on small positive value that increases with the number of factors (Browne, 2001; Asparouhov & Muthén, 2009). Increasing the ε alters the balance between the sizes of cross-loadings and factor correlations. As we were especially concerned with the sizes of factor correlations, we set the epsilon at a rather high value (.5) that resulted in somewhat lower factor correlations and somewhat higher cross-loadings. Nevertheless, consistent with recommendations, we explored a number of different rotations in preliminary analyses. There did not seem to be substantial differences results based on the various rotations, consistent with suggestions by Asparouhov & Muthén (2009) who concluded that “*In most ESEM applications the choice of the rotation criterion will have little or no effect on the rotated parameter estimates”* (p. 428)***.*** Although we had a clear basis for using the geomin rotation, we are not suggesting that this will always – or even generally – be the best rotation in other studies. Quite the contrary, following recommendations based on Asparouhov and Muthén (2009), Browne (2001) and others – as well as our own personal experience, we suggest that applied researchers should evaluate the theoretical and mathematical rationales for different rotations, experiment with a number of different rotations and complexity parameters, and chose the one that is most appropriate for their specific application. We also note that this is clearly an area where more research – using both simulation and real data – is needed.

With ESEM it is possible to constrain the loadings to be equal across two or more sets of EFA blocks in which the different blocks represent multiple discrete groups or multiple occasions for the same group. This is accomplished by first estimating an unrotated solution with all loadings constrained to be equal across groups or over time. If the starting solutions in the rotation algorithm are the same, and no loading standardizing is used, the optimal rotation matrix will be the same as well as the subsequent rotated solutions. Thus obtaining a model with invariant rotated Λ\* amounts to estimating a model with invariant unrotated Λ, a standard task in maximum likelihood estimation.

 For an oblique rotation it is also possible to test the invariance of the factor variance-covariance matrix (Ψ) matrix across the groups. To obtain non-invariant Ψs an unrotated solution with Ψ = I is specified in the first group and an unrestricted Ψ is specified in all other groups. Note that this unrestricted specification means that Ψ is not a correlation matrix as factor variances are freely estimated. It is not possible in the ESEM framework to estimate a model where in the subsequent groups the Ψ matrix is an unrestricted correlation matrix, because even if the factor variances are constrained to be 1 in the unrotated solution, they will not be 1 in the rotated solution. However, it is possible to estimate an unrestricted Ψ in all but the first group and after the rotation the rotated Ψ can be constrained to be invariant or varying across groups. Similarly, when the rotated and unrotated loadings are invariant across groups, it is possible to test the invariance of the factor intercept and the structural regression coefficients. These coefficients can also be invariant or varying across groups simply by estimating the invariant or group-varying unrotated model. However, in this framework only full invariance can be tested in relation to parameters in Ψ and Λ in that it is not possible to have measurement invariance for one EFA factor but not for the other EFA factors. Similar restrictions apply to the factor variance covariance, intercepts and regression coefficients, although it is possible to have partial invariance in the ε matrix of residuals. (It is however, possible to have different blocks of ESEM factors such that invariance constraints are imposed in one block, but not the other). Furthermore, if the ESEM model contains both EFA factors and CFA factors, then all of the typical strategies for the SEM factors can be pursued with the CFA factors.

**3. MIMIC/Multiple-group Hybrid Model of Age Effects.**

Studies of age differences in FFA factors typically suffer the same methodological shortcomings as those of gender differences. However, the tests of invariance become even more complex in that age is a continuous variable rather that a natural categorical variable with a few discrete groups (like gender). Valid interpretations of age differences assume that that there is measurement invariance across all possible intervals of the age continuum under consideration. Multiple-group tests of invariance for continuous variables require that the continuous variables be divided into a relatively small number of groups, but there are inherent dangers in transforming continuous variables into categorical variables (MacCallum, Zhang, Preacher & Rucker, 2002). Within the CFA literature, there are traditionally two approaches to this problem, each with counter-balancing strengths and limitations: The MIMIC and the multiple-group approaches (e.g., Marsh, Tracey & Craven, 2006). The MIMIC model regresses the latent variables (the FFA factors) onto other variables (continuous, like age, or categorical, like gender). However, there are important limitations in the capability of the MIMIC model to evaluate invariance assumptions: Only the invariance of item intercepts and factor means can be tested. In the multiple group approach, it is possible to pursue the more rigorous tests of invariance presented in Table 1. However, for continuous variables, these tests require researchers to transform continuous variables into a relatively small number of categories that constitute the multiple groups. Marsh, Tracey and Craven (2006) proposed a hybrid approach involving an integration of interpretations based on both MIMIC and multiple group approaches. The multiple group approach is used to test invariance assumptions that cannot easily be tested with the MIMIC approach and the MIMIC approach is used to infer differences in relation to a score continuum rather than discrete groups. So long as the two approaches converge to similar interpretations, there is support for the construct validity of these interpretations (e.g., Marsh, Martin & Hau, 2006).

In the present investigation, we extend this hybrid approach in two important ways. First, following Marsh, Lüdtke et al. (2010), we demonstrate how this hybrid approach can be translated into the ESEM framework so that researchers do not have to rely on potentially problematic CFA models. Second, we demonstrate how the MIMIC and multiple group approaches can both be incorporated into a single ESEM model. This means that researchers do not have to rely on the subjective integration of results from two entirely different models and do not have to rely on the fortuitous combination of results from the two models to justify interpretations of their results. Here we extend the logic of the hybrid approach by actually adding the MIMIC age (linear and quadratic) variables to the multiple group model (based on sex-age groups). This allows us to evaluate more formally whether information in the continuous age effects are lost by forming categories and, if so, to estimate the combined age effects due to both representations of age.

The interaction between gender and age is substantively important to interpretations of both gender and age effects. However, the introduction of interactions into latent variable models presents a whole host of new complications. Particularly relevant to the present investigation, invariance assumptions apply not only to the distinct gender groups and to age groups (or regions along the age continuum), but also to groups (or regions) defined by all possible combinations of these two variables. Although tests of invariance have rarely been applied to the interaction of two variables, we illustrate how the use of ESEM to the hybrid integration of multiple-group and MIMIC models can be extended to include interactions between variables.

**4. ESEM-Within-CFA Models (ES-W-C): An Extension of ESEM.**

The ESEM approach is very flexible, but there are still some tests that are easily done with CFA/SEM models that cannot easily be accomplished with ESEM models as currently operationalized in Mplus (e.g., constraints on group specific correlations among factors, tests of higher-order factor models, fully latent latent curve models, factor mixture models, etc.). Of particular relevance to the present investigation, applied researchers cannot easily place constraints on latent means estimated in multiple groups models to test linear and non-linear effects based on a single grouping variable such as age or interaction effects between two grouping variables such as age-by-gender interactions. Here we propose an extension of the ESEM approach to address this limitation that makes the ESEM approach much more flexible – what we refer to as ESEM-Within-CFA (ES-W-C).

The ES-W-C approach introduced here is based in large part on an EFA-Within-CFA proposal by Jöreskog (1969; also see Muthén & Muthén, 2009, slides 133-146). EFA-Within-CFA provided SEs for EFA parameter estimates and greater flexibility in specifying factor structures; potentially important limitations in EFA at that time. However, SEs for EFA solutions, the inclusion CUs, and a variety of goodness of fit measures are now available through standard statistical packages such as the ESEM routine in Mplus (also see Dolan et al., 2009). Nevertheless, for the aforementioned cases and when preliminary analyses show that ESEM solutions are superior to CFA solutions, ES-W-C may represent an interesting complementary alternative to ESEM.

As the original EFA-within-CFA model must contains the same number of restrictions as the EFA– m2 (where m=number of factors) – it was proposed that researchers look for referent indicators that have near-zero loadings on all but the latent constructs that they are designed to measure (from initial runs of EFA solutions). The cross-loadings for these indicators were then constrained to be exactly zero in a corresponding CFA model where each of the remaining loadings on all factors are freely estimated and the factors variances are constrained to be 1. Under appropriate conditions, the resulting CFA parameter estimates will approximate the corresponding EFA model. In our ES-W-C approach, we offer an amendment to this approach:

* In preliminary analyses, compare ESEM and CFA models in relation to goodness of fit and parameter estimates. If the ESEM solution is not clearly superior, ESEM or ES-W-C models should not be pursued.
* If preliminary ESEM models are better than CFA models, a complete ESEM analysis should be done to identify the best model in relation to goodness of fit and substantive interpretations.
* If there is a need to conduct an additional analysis that cannot be easily implemented within the ESEM framework, then all parameter estimates from the final ESEM solution should be used as starting values to estimate the ES-W-C model.
* We now need to add a total of m2 constraints (where m=number of factors) to the ES-W-C model so that it is identified. This is usually done by fixing selected parameter estimates to the values obtained from the ESEM solution. One convenient way to do this is to:
	+ Fix the m factor variances (by default these parameters are fixed to be 1.0 in a single-group ESEM solution or for the first group of a multiple group solution).
	+ Select an “anchor” item (referent indicator) for each factor that has a large loading for the factor it is designed to measure and small cross-loadings on other factors. Then, fix these small cross-loadings to their values from the ESEM solution. This allows for a higher level of convergence with the ESEM solution than the classical method of fixing these cross-loadings to zero. (Because the ESEM parameter estimates have already been included, this merely means fixing these estimates so that they are not freely estimated).
	+ For all other parameter estimates, the pattern of fixed and free estimates should be the same as in the selected ESEM solution (i.e., if the parameter is free in the ESEM solution it is free in the ES-W-C solution; if the parameter is fixed or constrained in the ESEM solution it should be the same in the ES-W-C solution).
	+ It should be noted that the mean structure from the ES-W-C solution can be identified as in a standard CFA model (while using the ESEM start values when possible). This is usually done by freely estimating all items intercepts and constraining all factor means to be zero (or the factor means from the first group in a multiple group solution).

The ES-W-C solution will have the same df and, within rounding error, the same chi-square value, the same goodness of fit statistics, and – most importantly – the same parameter estimates as the ESEM solution. In this sense, it is equivalent to the ESEM solution. The ES-W-C approach is stronger then the EFA-Within-CFA approach in that the parameter estimates are the same as in ESEM rather than approximations, particularly when anchor items with zero factor loadings do not exist. Furthermore, the ESEM approach used to generate the starting parameter estimates for the ES-W-C approach is more flexible than the traditional EFA approach used in the EFA-Within-CFA strategy. Importantly, the applied researcher has more flexibility in terms of how to constrain or further modify the ES-W-C model (as it is a true CFA model) than with the ESEM model upon which it is based.

In the present investigation, for example, we added constraints on the estimated latent means to represent linear and quadratic effects of age, gender, and age-by-gender interactions – and tests of statistical significance of these effects (see appendix for Mplus codes). Because these constraints are merely a re-parameterization of the observed means, they did not alter the goodness of fit and other parameter estimates. However, it also possible to add further limitations or estimate additional parameters that do change the df and thus influence other parameter estimates (e.g., freeing or constraining parameters in specific groups). Particularly if the changes are substantial, it is important to evaluate critically the new parameter estimates in relation to substantive interpretations as well as to assess changes in goodness of fit, but this is also the case for any ESEM or CFA solution. Although the ES-W-C approach is very general, it is particularly useful in defining contrasts on latent means (like those used in traditional ANOVA approaches) and testing the invariance of parameters linked to these constraints that cannot normally be evaluated in the ESEM approach. Nevertheless, as this approach has not been widely used in applied research, caution is needed in its application when the ES-W-C model differs substantially from the ESEM solution upon which it was based.

**Model constraints in ES-W-C used to define contrasts to test main and interaction effects of sex and age are as following:**

**TITLE:** Mplus Syntax Converting ESEM solution Into ES-W-C solution (with unit Weights)

**DATA:** FILE IS bhpsindrespX4.DAT;

**VARIABLE:**

NAMES ARE b5a1r b5a2 b5a3 b5c1 b5c2r b5c3 b5e1 b5e2 b5e3r b5n1 b5n2 b5n3r

b5o1 b5o2 b5o3 fsb5n fsb5a fsb5e fsb5c fsb5o ohid a3s2 oXRWTUK2;

*! b5a1r-b5o3 are the 15 items that define the FFA factors;*

*! ohid = household cluster id variable;*

*! a3s2= group identifiers for the 6 (3 age x 2 sex) groups;*

*! oXRWTUK2 = weighting variable supplied with the data;*

usevariables b5a1r-b5o3 ohid;

weight is oXRWTUK2; *! sampling weight*

missing are all (-9.99); *! little missing, default=FIML*

cluster is ohid; *! complex design cluster by household*

GROUPING IS a3s2 (11=A1S1, 12=A1S2,21=A2S1,22=A2S2,31=A3S1,32=A3S2);

*! grouping variable that defines the 6 groups (3 age x 2 gender)*

**ANALYSIS:**

type=complex; ESTIMATOR=MLR;

*! syntax for the first group (A1S1) in which starting values (\*) and fixed values (@)are copied from ESEM.*

*! Numbers in parentheses are used to make constrained parameter estimates invariant across two or more*

*! groups (i.e. parameters with the same number in parentheses are constrained to equality).*

*! Model in the first group.*

**Model A1S1:**

*! Factor loadings of all items on Factor 1. Thus item B5A1R loads on factor F1.*

*! The factor loading is freely estimated (as indicated the by \*), but has a starting value of 0.603*

*! The factor loadings of the anchor items are fixed as indicated by the “@”instead of the \**

*! (e.g. the factor loading of item B5C3 on F1 is fixed at 0.198)*

F1 BY B5A1R \* 0.603;

F1 BY B5A2 \* 0.729;

F1 BY B5A3 \* 0.869;

F1 BY B5C1 \* -0.073;

F1 BY B5C2R \* 0.010;

F1 BY B5C3 @ 0.198;

F1 BY B5E1 @ 0.018;

F1 BY B5E2 \* 0.279;

F1 BY B5E3R \* -0.239;

F1 BY B5N1 @ -0.043;

F1 BY B5N2 \* 0.144;

F1 BY B5N3R \* -0.310;

F1 BY B5O1 \* -0.093;

F1 BY B5O2 \* 0.119;

F1 BY B5O3 @ 0.130;

*! A similar section should be added for the remaining factors*

……..

*! Factor covariances;*

F2 with F1 \* 0.265;

F3 with F1 \* 0.134;

F3 with F2 \* 0.188;

F4 with F1 \* -0.067;

F4 with F2 \* -0.134;

F4 with F3 \* -0.206;

F5 with F1 \* 0.236;

F5 with F2 \* 0.214;

F5 with F3 \* 0.275;

F5 with F4 \* 0.072;

*! Correlated uniquenesses;*

B5A1R WITH B5C2R \* 0.345 (1);

B5A1R WITH B5E3R \* 0.035 (2);

B5A1R WITH B5N3R \* 0.108 (3);

B5C2R WITH B5E3R \* 0.162 (4);

B5C2R WITH B5N3R \* 0.161 (5);

B5E3R WITH B5N3R \* 0.243 (6);

*! the numbers in parentheses are used to label a particular parameter estimate so that it can be*

*! referenced in future constraints (e.g., constrained to be invariant over groups).*

*! Items Intercepts;*

 [ b5a1r \* 5.539 ] (31) ;

 [ B5A2 \* 5.014 ] (32) ;

 [ B5A3 \* 5.159 ] (33) ;

 [ B5C1 \* 5.161 ] (34) ;

 [ B5C2R \* 4.389 ] (135) ;

 [ B5C3 \* 5.104 ] (36) ;

 [ B5E1 \* 4.681 ] (37) ;

 [ B5E2 \* 5.183 ] (138) ;

 [ B5E3R \* 4.214 ] (39) ;

 [ B5N1 \* 3.387 ] (140) ;

 [ B5N2 \* 3.476 ] (141) ;

 [ B5N3R \* 3.477 ] (42) ;

 [ B5O1 \* 4.778 ] (43) ;

 [ B5O2 \* 4.296 ] (144) ;

 [ B5O3 \* 5.307 ] (45) ;

*! Factor Means (constrained to be zero in the first group);*

[F1@0.000]; [F2@0.000]; [F3@0.000]; [F4@0.000]; [F5@0.000];

*! Factor variances (constrained to be 1 in the first group);*

F1@1.0; F2@1.0; F3@1.0; F4@1.0; F5@1.0;

*! Residual Variances;*

B5A1R\*1.596 (51);

B5A2\*1.515 (52);

B5A3\*0.480 (53);

B5C1\*1.415 (54);

B5C2R\*2.159 (55);

B5C3\*0.346 (56);

B5E1\*0.967 (57);

B5E2\*1.115 (58);

B5E3R\*1.715 (59);

B5N1\*1.345 (60);

B5N2\*1.492 (61);

B5N3R\*1.345 (62);

B5O1\*0.927 (63);

B5O2\*1.664 (64);

B5O3\*1.121 (65);

*! New variables are defined as the means of the latent factors.*

*! A1S1\_m1 = mean of F1 (first factor) in group A1S1 (Age = 1, s = 1; young males)*

[f1](A1S1\_m1); [f2](A1S1\_m2); [f3](A1S1\_m3); [f4](A1S1\_m4); [f5](A1S1\_m5);

*! The fixed and free values for the remaining 5 groups are specified according to*

*! the pattern of fixed and free values and parameter estimates in the selected ESEM solution;*

……..

*! Latent means for the same factor across the six age-gender groups are constrained*

*! to sum to 0 (for purposes of identification);*

**MODEL CONSTRAINT:** *!factor 1*

0 = A1S1\_m1 + A2S1\_m1 + A3S1\_m1 + A1S2\_m1 + A2S2\_m1 + A3S2\_m1;

0 = A1S1\_m2 + A2S1\_m2 + A3S1\_m2 + A1S2\_m2 + A2S2\_m2 + A3S2\_m2;

0 = A1S1\_m3 + A2S1\_m3 + A3S1\_m3 + A1S2\_m3 + A2S2\_m3 + A3S2\_m3;

0 = A1S1\_m4 + A2S1\_m4 + A3S1\_m4 + A1S2\_m4 + A2S2\_m4 + A3S2\_m4;

0 = A1S1\_m5 + A2S1\_m5 + A3S1\_m5 + A1S2\_m5 + A2S2\_m5 + A3S2\_m5;

*! Using traditional contrasts as in a typical linear model, groups means are used to define the main effects*

*! of gender (male vs. female), age (linear and quadratic) and their interaction. Here are the contrasts*

*! for factor 1 (F1) based on the six age-by-sex latent means: (A1S1 – A3S2) for the first factor (\_m1).*

NEW(MSEXF1);

MSEXF1 = (+1\* A1S1\_m1) + (+1\* A2S1\_m1) + (+1\* A3S1\_m1) +

(-1\* A1S2\_m1) + (-1\* A2S2\_m1) + (-1\* A3S2\_m1);

NEW(lAGEF1);

LAGEF1= (-1.0\* A1S1\_m1) + (0\* A1S2\_m1) + (+1.0\* A3S1\_m1) +

(-1.0\* A1S2\_m1) + (0\* A2S2\_m1) + (+1.0\* A3S2\_m1);

NEW(QAGEf1);

QAGEf1=(1.0\* A1S1\_m1) + (-2.0\* A1S2\_m1) + (1.0\* A3S1\_m1) +

(1.0\* A1S1\_m1) + (-2.0\* A2S2\_m1) + (1.0\* A3S2\_m1);

NEW(SXAGLF1);

SXAGLF1= (-1.0\* A1S1\_m1) + (0\* A1S2\_m1) + (+1.0\* A3S1\_m1) +

(+1.0\* A1S2\_m1) + (0\* A2S2\_m1) + (-1.0\* A3S2\_m1);

NEW(SXAGQF1);

SXAGQF1=(+1.0\* A1S1\_m1) + (-2.0\* A2S2\_m1) + (+1.0\* A3S1\_m1) +

(-1.0\* A1S1\_m1) + (2.0\* A2S2\_m1) + (-1.0\* A3S2\_m1);

……..

*! The same pattern of constraints is applied to each of the FFA factors (F2-F5) for*

*! latent means for the remaining five factors (\_m2, \_m3, \_m4, \_m5).*

5. **References in Supplemental Materials (not in main article)**

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